Decoherence of CBO

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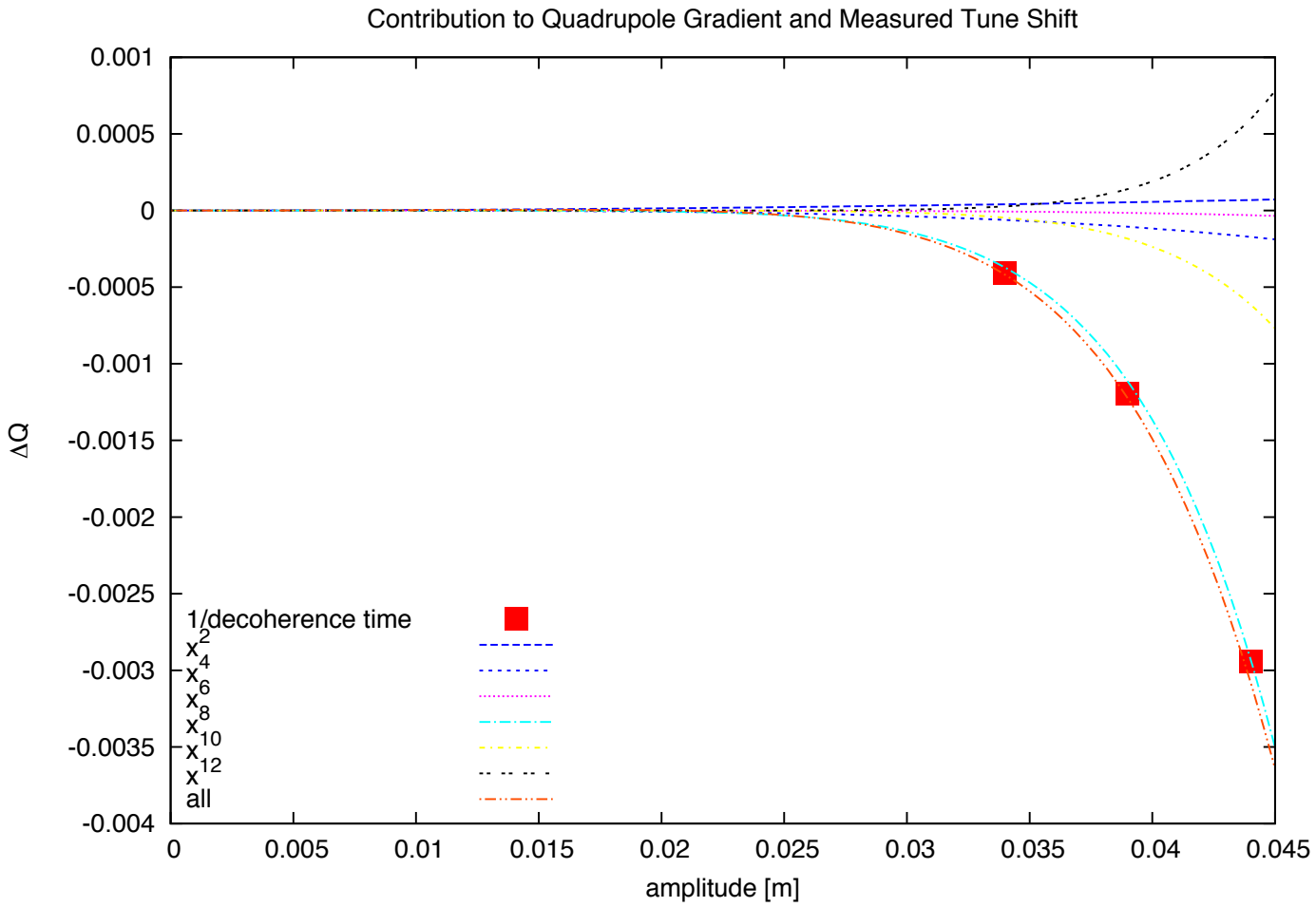
Decoherence

- Betatron tune (number of wavelengths/turn) is determined by quadrupole voltage
  - V=± 31kV => Q_x = 0.90386 Q_y = 0.43271
  - f_{rev} = 6.71 MHz
  - $f_h = Q_x f_{rev}$, $f_y = Q_y f_{rev}$
- An amplitude dependence results from quad nonlinearity
- Energy dependence due to chromaticity
  - $Q_x' = -0.14$ $Q_y' = 0.31$

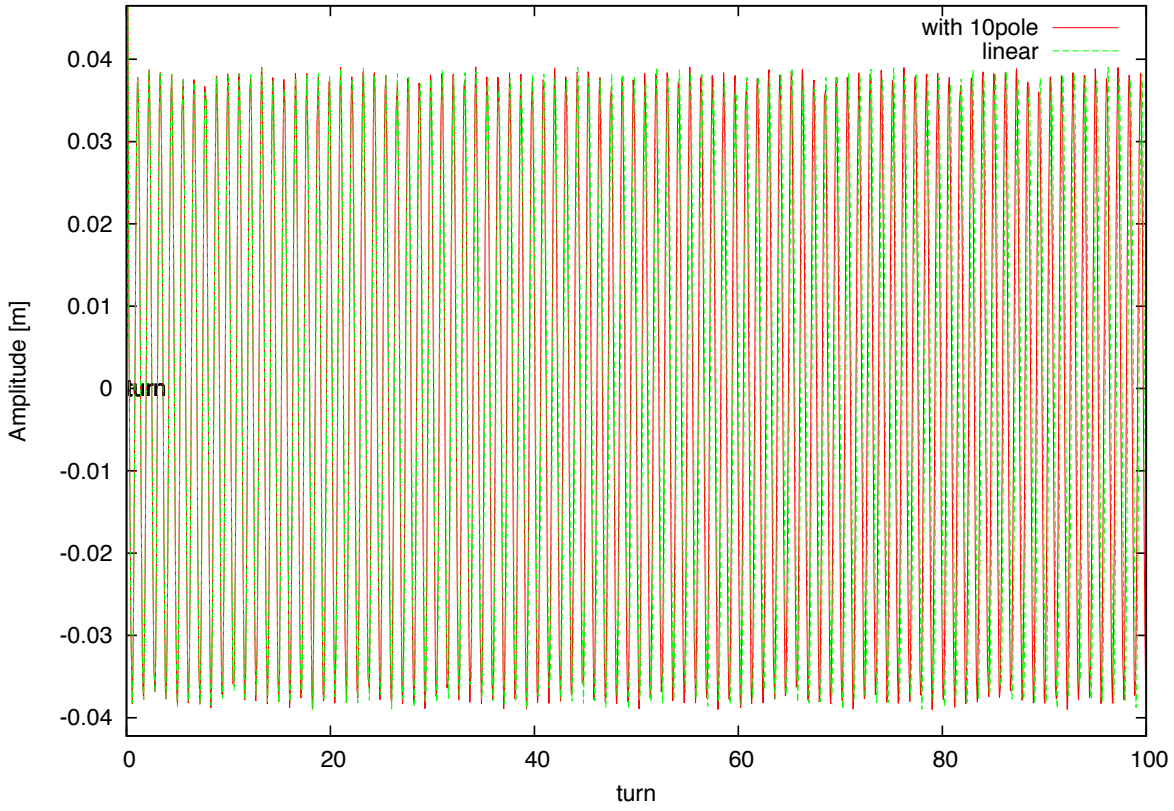
Damping

- No real damping.
  - Amplitude of oscillation of each muon is COM
- Muons with different amplitude and energy decohere
- Decoherence time [turns] ~ $1/\Delta Q$

Amplitude dependent tune

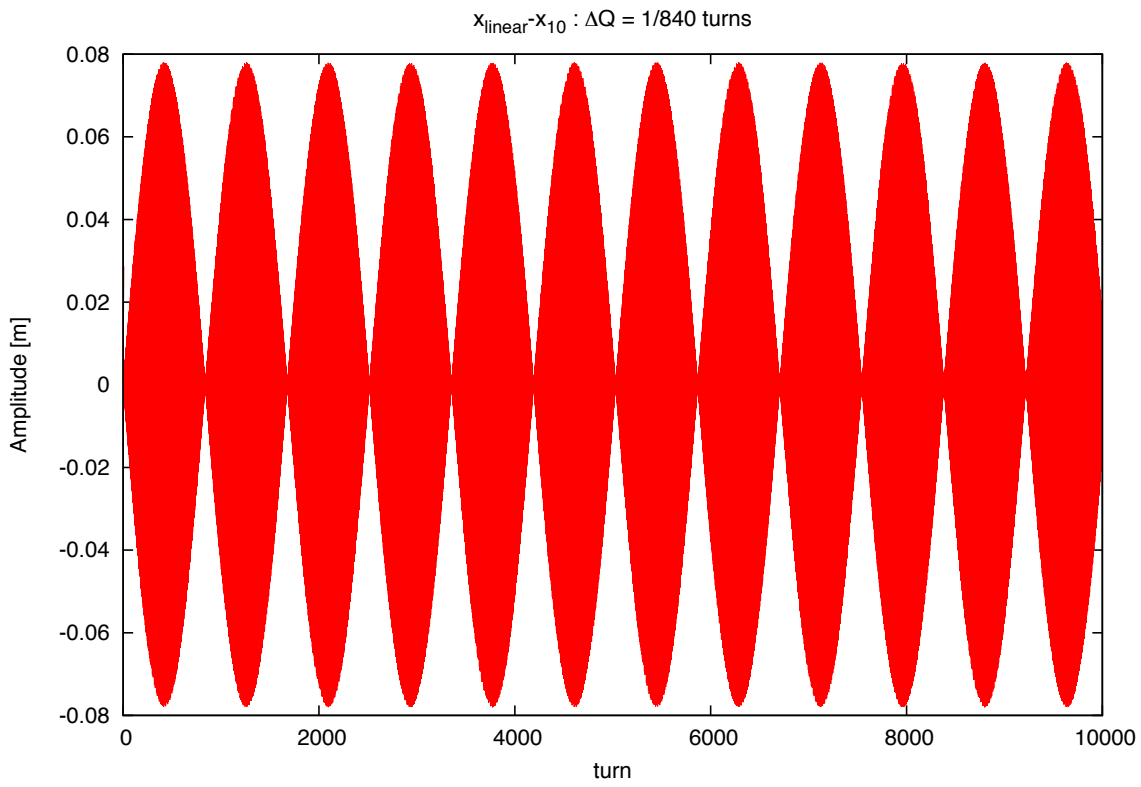


Amplitude dependent tune



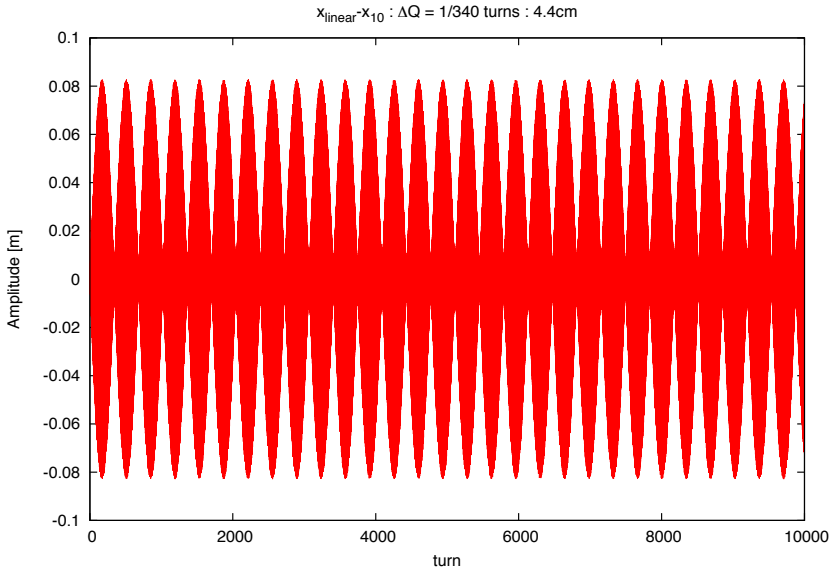
Oscillation of single muon with and without quad 10 pole term
4mm peak-to-peak

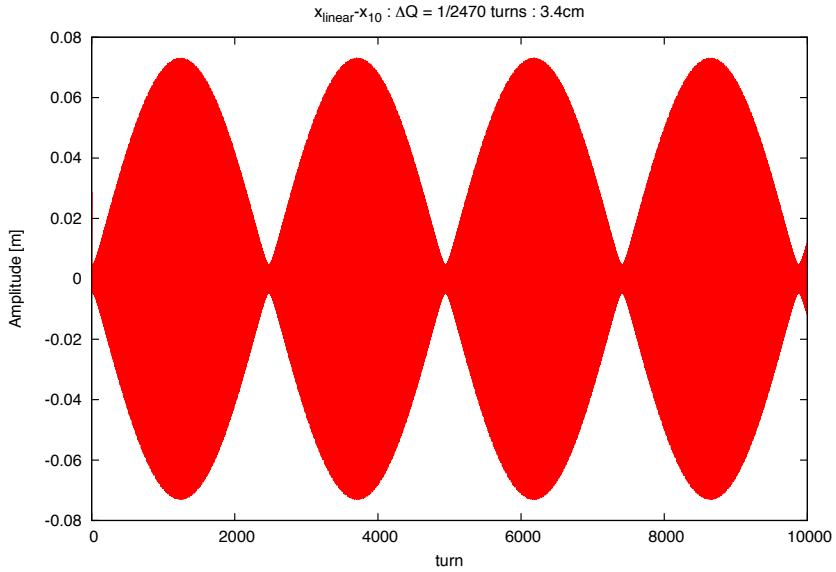
Envelope of difference => ΔQ

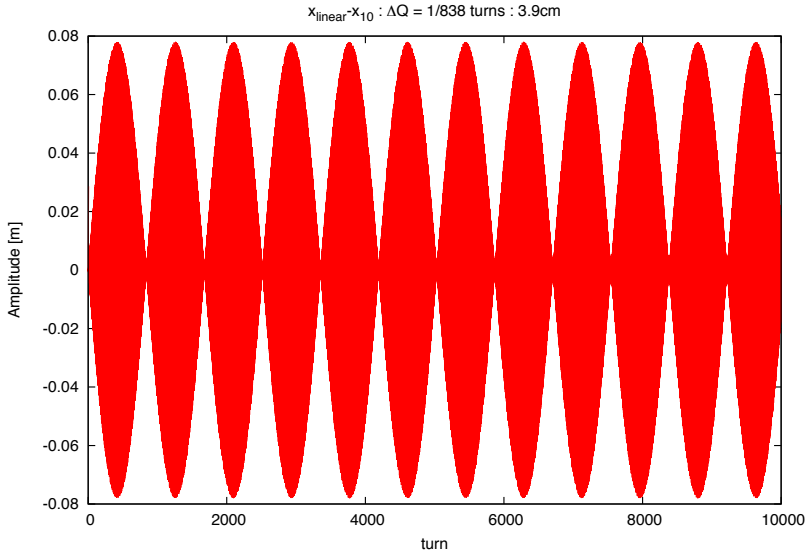


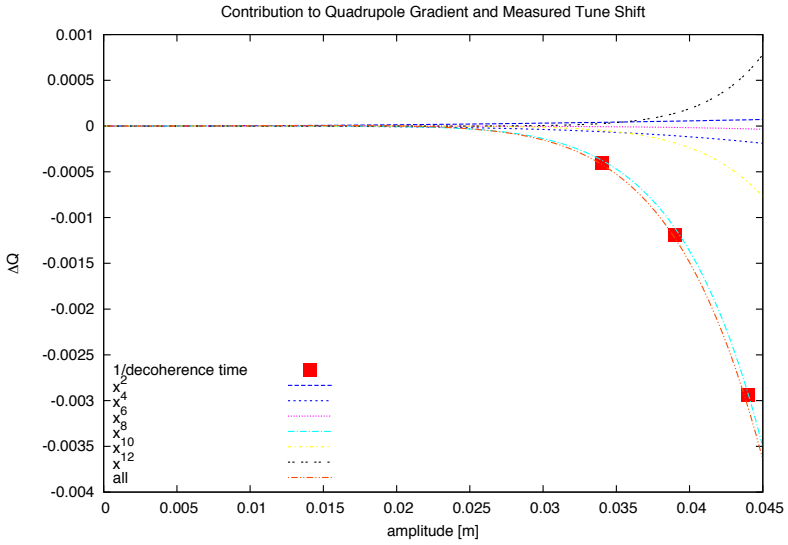
180 deg out of phase after 420 turns

Difference with and without quad 10 pole term (4mm peak-to-peak)

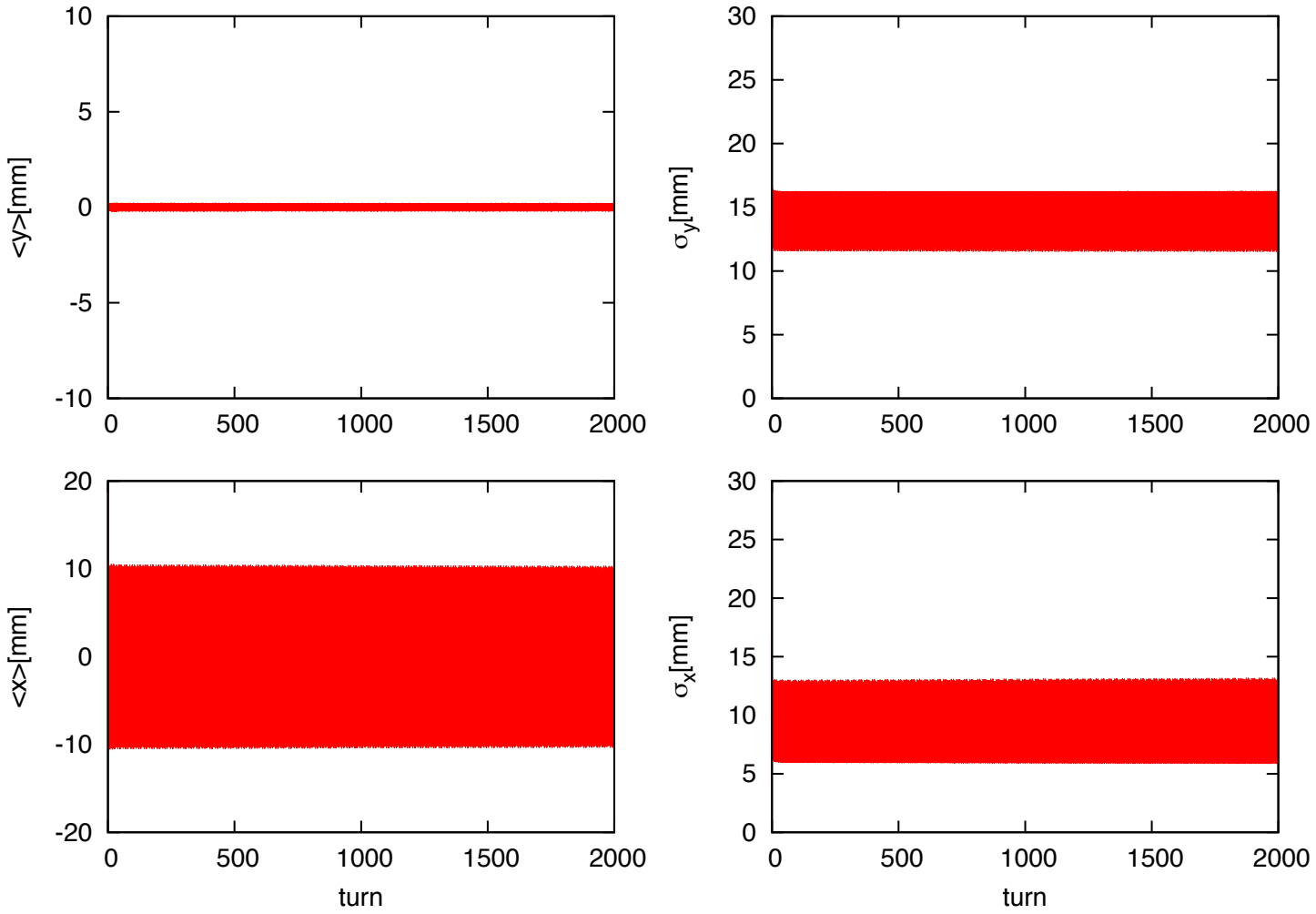

x_{linear}-x_{10} : ΔQ = 1/340 turns : 4.4cm
Amplitude [m]
turn
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
0
2000
4000
6000
8000
10000


x_{linear}-x_{10} : ΔQ = 1/2470 turns : 3.4cm
Amplitude [m]
turn
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
2000
4000
6000
8000
10000


x_{linear}-x_{10} : ΔQ = 1/838 turns : 3.9cm
Amplitude [m]
turn
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
2000
4000
6000
8000
10000


Contribution to Quadrupole Gradient and Measured Tune Shift
ΔQ
amplitude [m]
1/decoherence time
x^2
x^4
x^6
x^8
x^{10}
x^{12}
all
0.001
0.0005
0
-0.0005
-0.001
-0.0015
-0.002
-0.0025
-0.003
-0.0035
-0.004
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045

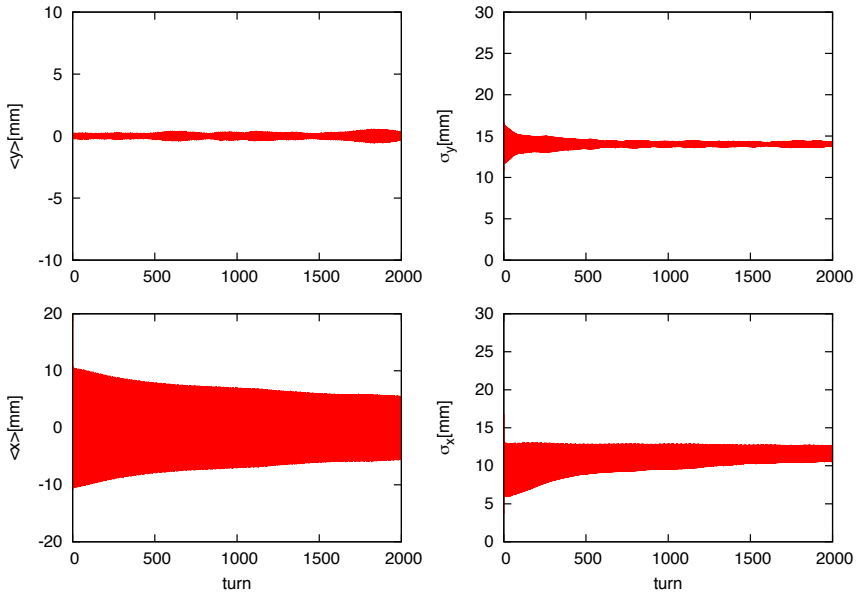
Time dependence of centroid and width



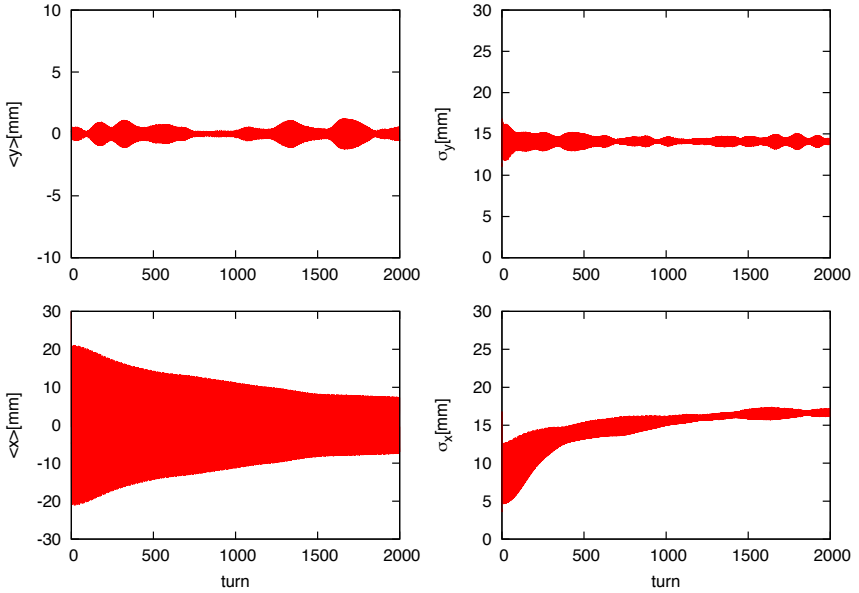
No quad multipoles and no energy spread

Time dependence of centroid and width

All quad multipoles included. Zero energy spread



10mm peak-to-peak



20mm peak-to-peak

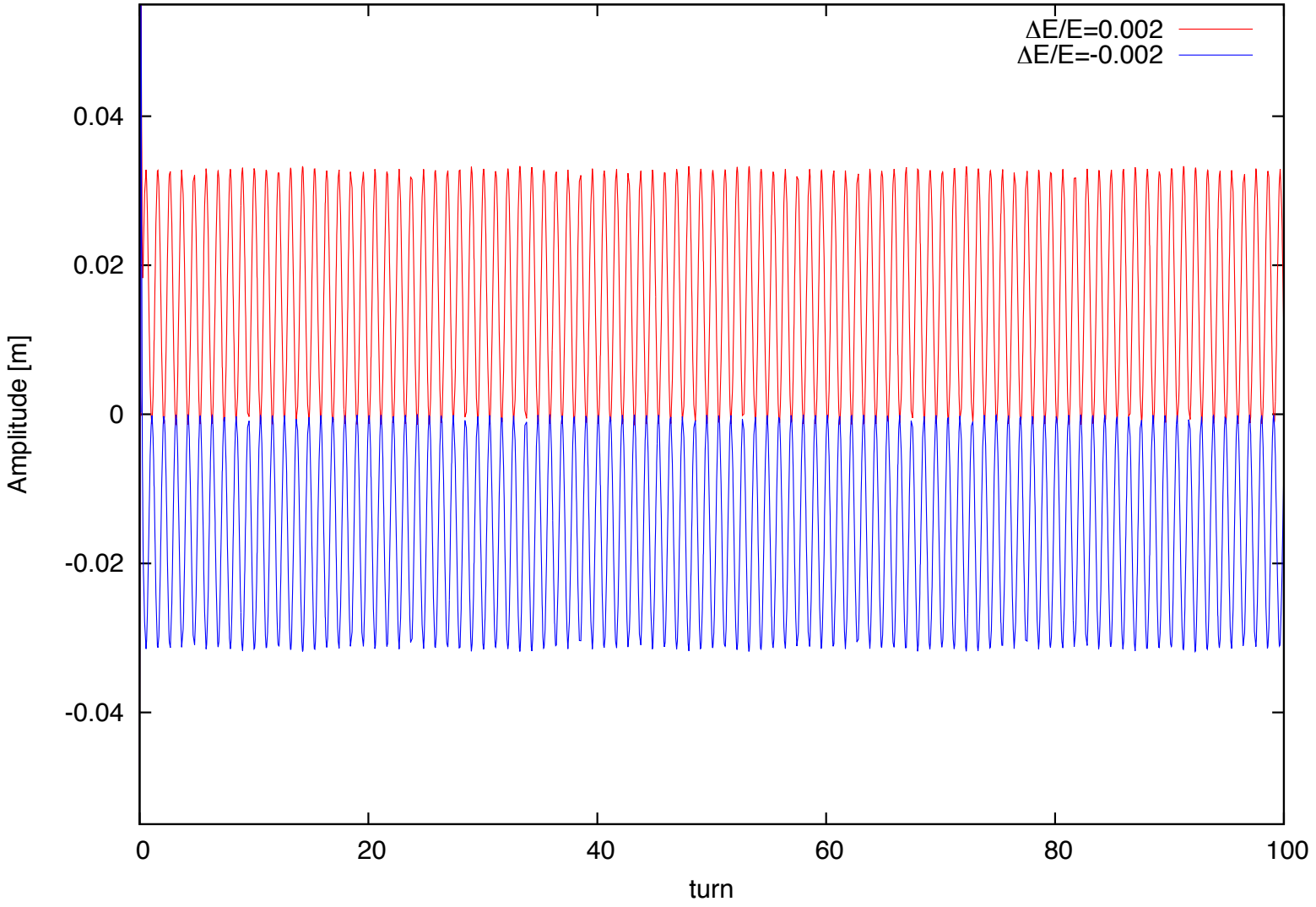
Energy dependent tune

$$Q_x' = -0.14 \qquad Q_y' = 0.31$$

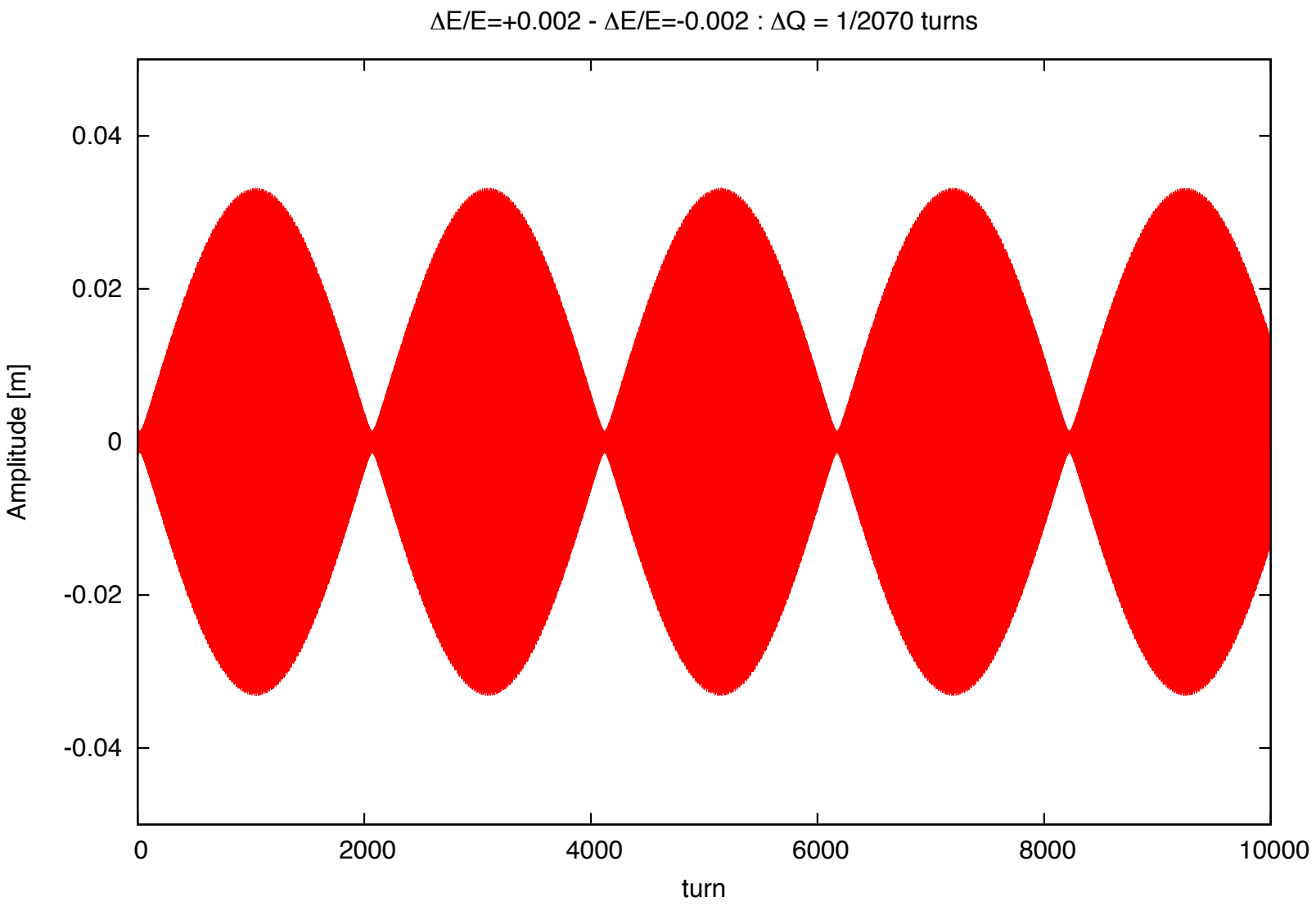
$\Delta E/E = \pm 0.002 \Rightarrow \Delta Q \approx (\Delta E/E)\, Q_x' \approx (2)2.8\text{ E-}4 = 1/1785$

=> Decoherence time due to energy spread ~ 1785 turns

Single muon energy ±0.002



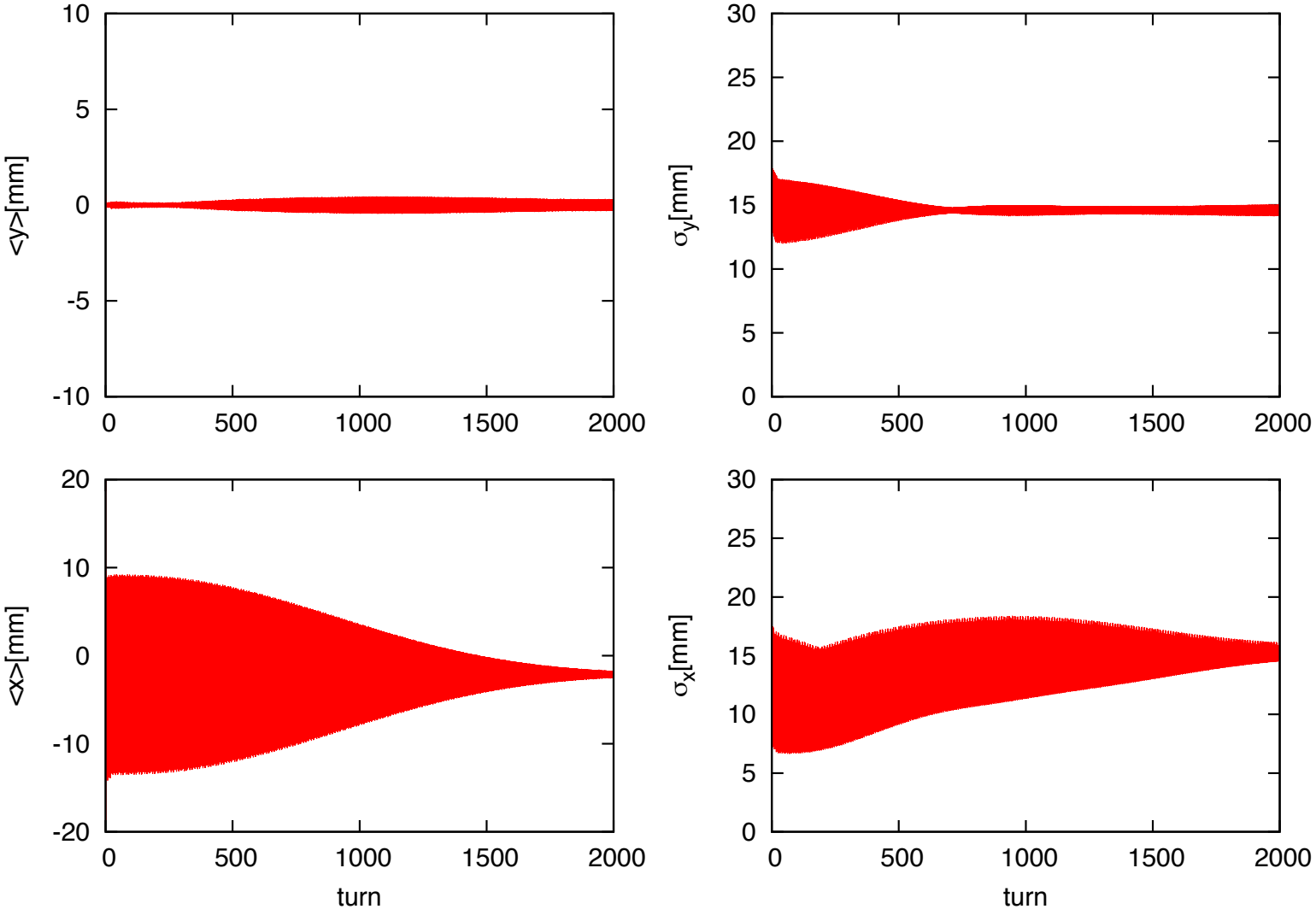
Difference of $\Delta E/E = \pm 0.002$



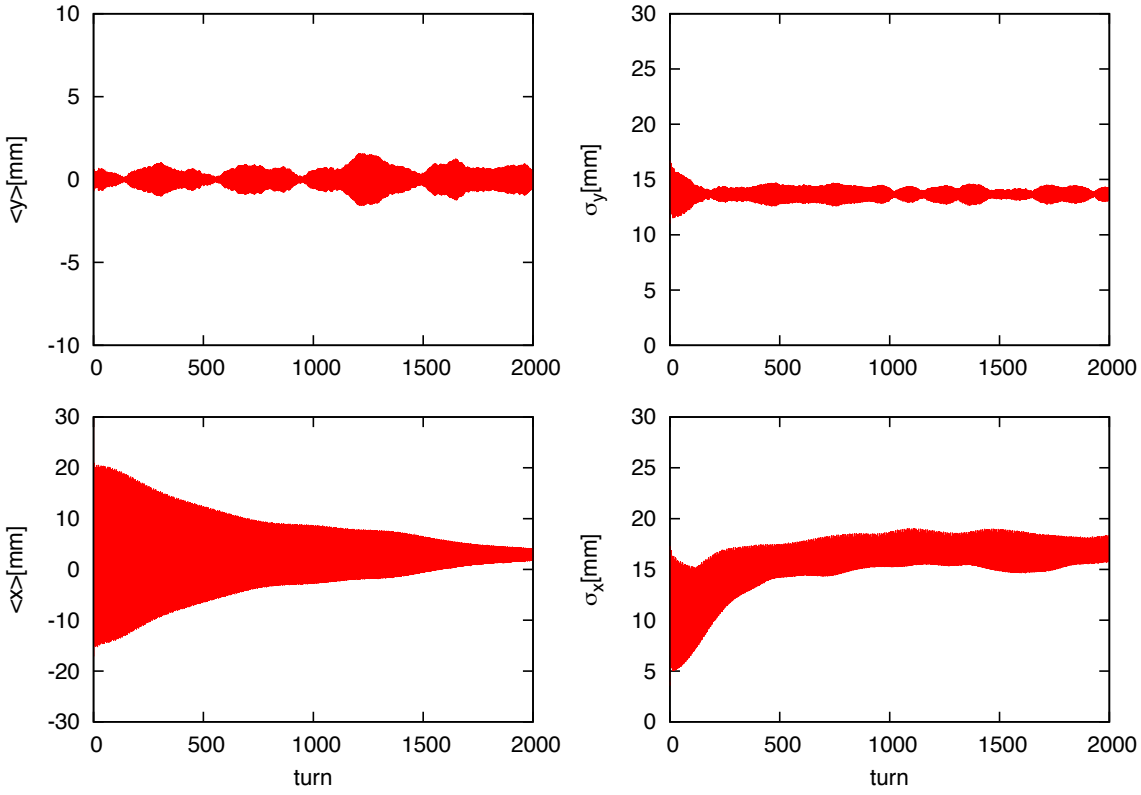
Decoherence due to energy dependent tune

$\sigma_e/E = 0.00112$

No quad multipoles



Energy and amplitude dependence



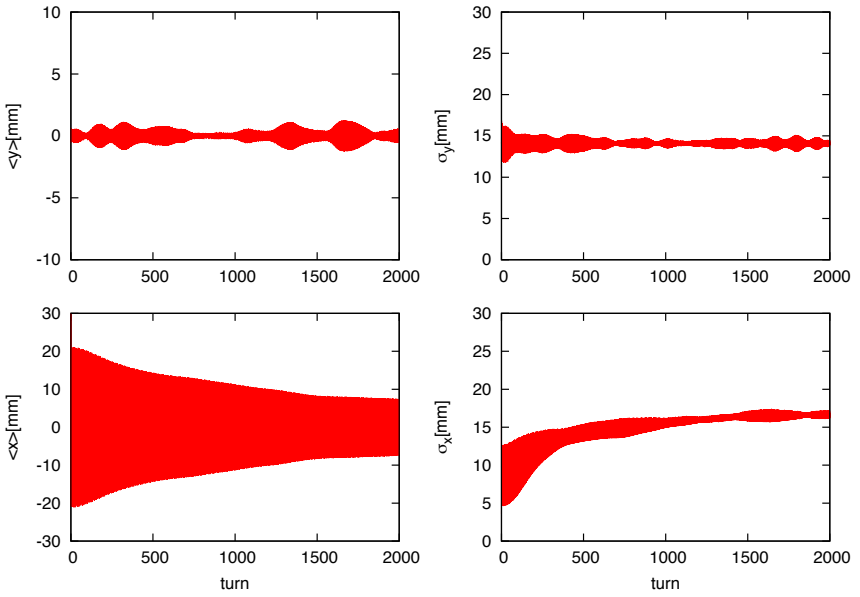
$\sigma_e/E = 0.00112$
All quad multipoles

Conclusion

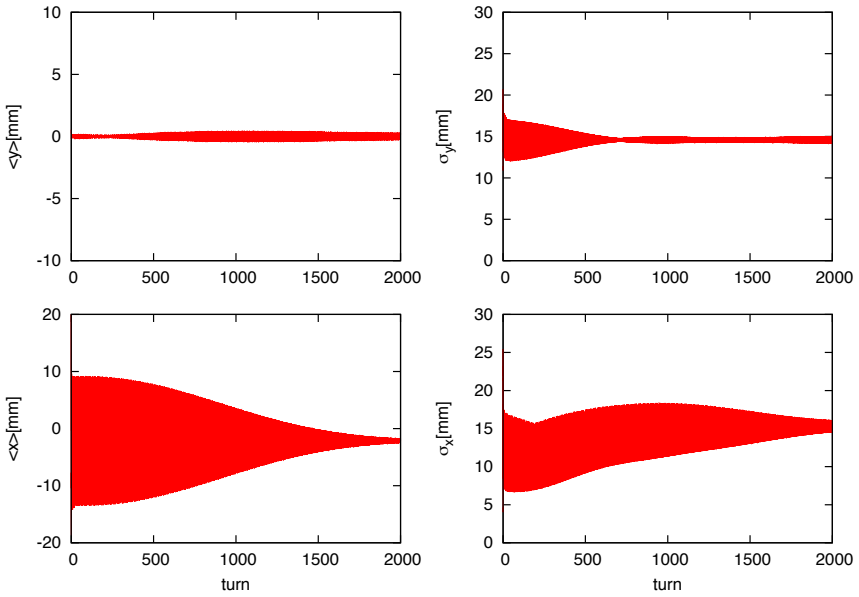
Chromatic decoherence time ≈ 1000 turns

Amplitude dependent decoherence time (due to quad nonlinearity) ≈ 2000 turns

(Betatron modulation more effectively damped by amplitude dependence)



Amplitude dependence
(No energy spread)



Energy dependence
(No multipoles)