HW3

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3.1

In this problem we are developing a gravity trip distribution model of the form

$$T_{ij} = \frac{P_i A_j^* (t_{ij})^{-b}}{\sum_{j' \in J} A_{j'}^* (t_{ij'})^{-b}}.$$

Much of the code required to do so was provided in the problem statement; it is provided below:

```r
#' Gravity Model
#' @param p vector of productions, length n
#' @param A vector of attractions, lenth n
#' @param C matrix of impedances, dim n x n
#' @param b impedance parameter
gravity <- function(p, a, C, b){
  # output matrix (all 0 here)
  trips <- matrix(0, nrow = length(p), ncol = length(a))
  # loop over all rows (production)
  for (i in 1:length(p)) {
    bottomA <- sum(a * C[i, ]^(-b)) # denominator

    # loop over all columns (attraction)
    for (j in 1:length(a)) {
      # calculate gravity model for trips from i to j
      topA <- a[j] * C[i,j]^(-b)
      trips[i, j] <-  p[i] * topA / bottomA
    }
  }

  return(trips)
}
```

```r
#' Function to balance gravity model
#' @param p vector of productions, length n
#' @param A vector of attractions, length n
#' @param C matrix of impedances, dim n x n
#' @param b impedance parameter
#' @param tolerance Acceptable change in trips matrix
balance_gravity <- function(p, a, C, b, tolerance) {

  # define starting values
  k <- 0 #iteration counter
  astar <- a # starting unadjusted attractions
  trips0 <- matrix(0, nrow = length(p), ncol = length(a)) #initial T is 0's
  error <- Inf # first time through, error is Infinite

  # loop through algorithm
  while(error > tolerance){
    # compute gravity model with adjusted attractions, using your function
    trips <- gravity(p, astar, C, b)

    # calculate the error as the change in trips in successive iterations
    error <- sum(abs(trips - trips0))

    # protect against infinite loops, increment values
    if (k > 100) break # maximum of 100 iterations
    k <- k + 1
    trips0 <- trips
    astar <- astar * a / colSums(trips) # next iteration astar
  }

  return(trips)
}
```

We also were given a 3-zone system with its respective production/attraction rates and costs:

```r
prod <- c(100, 200, 100)
attr <- c(200, 50, 150)
costs <- matrix(c(2, 5, 4,
                  5, 2, 3,
                  4, 3, 2),
                byrow = T,
                nrow = 3)
```

Using this balanced gravity model and the provided system, we can calculate the trip distribution:

```r
dist1 <- balance_gravity(a = attr, p = prod, C = costs, b = 0.5, tolerance = 0.01)
dist1
```

```
##          [,1]      [,2]     [,3]
## [1,] 62.50975  8.329009 29.16124
## [2,] 91.54031 30.492846 77.96684
## [3,] 45.94993 11.178146 42.87193
```

If we check the row and column sums, we find that they match our production and attraction vectors:

```r
rowSums(dist1)
```

```
## [1] 100 200 100
```

```r
colSums(dist1)
```

```
## [1] 200  50 150
```