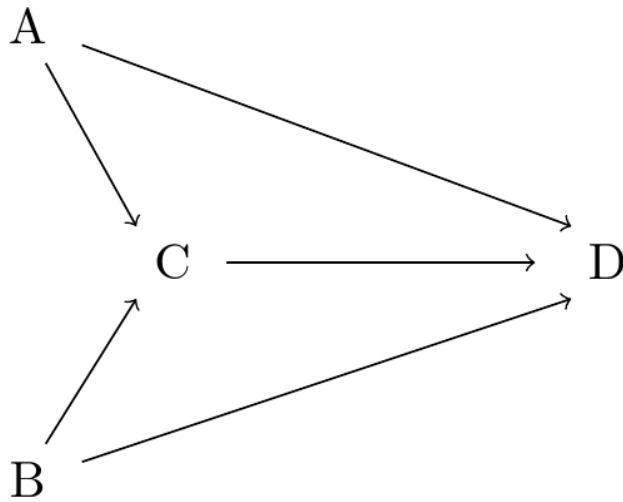# Homework 5: Network Assignmnet and Validation

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Given a network of



and link travel time equations of

$$t_{AD} = 20 + 0.01q_{AD}$$
$$t_{AC} = 10 + 0.005q_{AC}$$
$$t_{CD} = 12 + 0.005q_{CD}$$
$$t_{BC} = 7.25 + 0.005q_{BC}$$
$$t_{BD} = 20 + 0.01q_{BD},$$

```
timeConsts <- c(20, 10, 12, 7.25, 20)
timeCoeffs <- c(0.01, 0.005, 0.005, 0.005, 0.01)
totalVols <- c(7000, 5000)
```

with 7000 trips from A to D and 500 from B to D:

## 5.1

We set up a matrix to solve the system of equations:

$$\begin{array}{cccccccccc|c}
q_{AD} & q_{AC} & q_{CD} & q_{BC} & q_{BD} & t_{AD} & t_{AC} & t_{CD} & t_{BC} & t_{BD} & = \\
0.01 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -20 \\
0 & 0.005 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -10 \\
0 & 0 & 0.005 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -12 \\
0 & 0 & 0 & 0.005 & 0 & 0 & 0 & 0 & -1 & 0 & -7.25 \\
0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & -1 & -20 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7000 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 5000 \\
0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

```r
matrix(c(0.01,0,0,0,0,-1,0,0,0,0,-20,
0,0.005,0,0,0,0,-1,0,0,0,-10,
0,0,0.005,0,0,0,0,-1,0,0,-12,
0,0,0,0.005,0,0,0,0,-1,0,-7.25,
0,0,0,0,0.01,0,0,0,0,-1,-20,
0,0,0,0,0,-1,1,1,0,0,0,
0,0,0,0,0,0,0,0,1,1,-1,0,
1,1,0,0,0,0,0,0,0,0,0,totalVols[1],
0,0,0,1,1,0,0,0,0,0,0,totalVols[2],
0,1,-1,1,0,0,0,0,0,0,0
        ),
      byrow = T, ncol = 11) %>%
  rref()
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]    [,11]
## [1,]    1    0    0    0    0    0    0    0    0     0 4050.00
## [2,]    0    1    0    0    0    0    0    0    0     0 2950.00
## [3,]    0    0    1    0    0    0    0    0    0     0 4750.00
## [4,]    0    0    0    1    0    0    0    0    0     0 1800.00
## [5,]    0    0    0    0    1    0    0    0    0     0 3200.00
## [6,]    0    0    0    0    0    1    0    0    0     0   60.50
## [7,]    0    0    0    0    0    0    1    0    0     0   24.75
## [8,]    0    0    0    0    0    0    0    1    0     0   35.75
## [9,]    0    0    0    0    0    0    0    0    1     0   16.25
## [10,]   0    0    0    0    0    0    0    0    0     1   52.00
```

###TODO: write more about this

## 5.2

```r
iter0 <- tibble(
  "$n$" = character(),
  "$q_{AD}$" = numeric(),
  "$q_{AC}$" = numeric(),
  "$q_{CD}$" = numeric(),
  "$q_{BC}$" = numeric(),
  "$q_{BD}$" = numeric(),
```

```r
  "$t_{AD}$" = numeric(),
  "$t_{AC}$" = numeric(),
  "$t_{CD}$" = numeric(),
  "$t_{BC}$" = numeric(),
  "$t_{BD}$" = numeric(),
  "$T_{AD}$" = numeric(),
  "$T_{ACD}$" = numeric(),
  "$T_{BD}$" = numeric(),
  "$T_{BCD}$" = numeric()
  )

iter0[1,] <- c(list(as.character(0),0,0,0,0,0),
               timeConsts,
               list(timeConsts[1],
                    sum(timeConsts[2:3]),
                    timeConsts[5],
                    sum(timeConsts[4:3])
                    )
               )
```

## 5.3

## 5.4

```r
n = 5

fhwa <- iter0
f <- iter0[,2:6]

for(i in 1:(n+1)){
  phi <- 1/i

  AVol <- if(fhwa[i,]$`$T_{AD}$` < fhwa[i,]$`$T_{ACD}$`) 1 else c(2,3)
  BVol <- if(fhwa[i,]$`$T_{BD}$` < fhwa[i,]$`$T_{BCD}$`) 5 else c(4,3)

  f[i,] <- 0
  f[i,AVol] <- f[i,AVol] + totalVols[1]
  f[i,BVol] <- f[i,BVol] + totalVols[2]

  fhwa[i+1,1] = as.character(i)
  fhwa[i+1,2:6] =
    (1-phi)*fhwa[i,2:6] + phi*f[i,]

  incrVols <- fhwa[i+1,2:6] %>% as_vector() %>% unname

  fhwa[i+1,7:11] =
    (timeConsts + timeCoeffs * incrVols) %>%
    as.list()
  fhwa[i+1,12:15] =
    as.list(c(
```

```
      fhwa[i+1,7],
      sum(fhwa[i+1,8:9]),
      fhwa[i+1,11],
      sum(fhwa[i+1,10:9])
    ))

}

fhwa[n+2,] <- fhwa[n+1,]
fhwa[n+2,1] <- "Final"

fhwa %>% my_kbl() %>%
  column_spec(c(1,6,11,13), border_right = T) %>%
  row_spec(n+1, hline_after = T)
```

| $n$ | $q_{AD}$ | $q_{AC}$ | $q_{CD}$ | $q_{BC}$ | $q_{BD}$ | $t_{AD}$ | $t_{AC}$ | $t_{CD}$ | $t_{B...}$ |
|-------|----------|----------|------|----------|----------|----------|----------|------|----|
| 0     | 0.00     | 0.00     | 0    | 0.00     | 0.00     | 20.00    | 10.00    | 12   |    |
| 1     | 7000.00  | 0.00     | 5000 | 5000.00  | 0.00     | 90.00    | 10.00    | 37   | 3  |
| 2     | 3500.00  | 3500.00  | 6000 | 2500.00  | 2500.00  | 55.00    | 27.50    | 42   | 1  |
| 3     | 4666.67  | 2333.33  | 4000 | 1666.67  | 3333.33  | 66.67    | 21.67    | 32   | 1  |
| 4     | 3500.00  | 3500.00  | 6000 | 2500.00  | 2500.00  | 55.00    | 27.50    | 42   | 1  |
| 5     | 4200.00  | 2800.00  | 4800 | 2000.00  | 3000.00  | 62.00    | 24.00    | 36   | 1  |
| Final | 4200.00  | 2800.00  | 4800 | 2000.00  | 3000.00  | 62.00    | 24.00    | 36   | 1  |