Homework: Equilibrium Traffic Flow

4.1, 4.4, 4.8

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4.1

Given a speed-density relationship of $v = v_f(1 - k/k_j)$ and the relationship $q = k \times v$, we can derive flow-density and speed-flow relationships as follows:

$$
\begin{aligned}
q &= kv \\
q &= kv_f(1 - \frac{k}{k_j}) \\
q &= kv_f - \frac{k^2 v_f}{k_j} \qquad (1)
\end{aligned}
$$

and

$$
\begin{aligned}
k = \frac{q}{v}, \quad & v = v_f(1 - \frac{k}{k_j}) \\
\implies & v = v_f(1 - \frac{q/v}{k_j}) \\
\implies & v^2 = v_f v - \frac{q v_f}{k_j}. \qquad (2)
\end{aligned}
$$

From 1 we can find q_m (the capacity) and k_m (the density at capacity) by determining the maximum of the flow-density relationship (where $\frac{dq}{dk} = 0$):

$$
\begin{aligned}
q &= kv_f - \frac{k^2 v_f}{k_j} \\
\frac{dq}{dk} = 0 &= v_f - 2\frac{k_m v_f}{k_j} \\
0 &= v_f(1 - 2\frac{k_m}{k_j}) \\
0 &= 1 - 2\frac{k_m}{k_j} \quad \text{if} v_f \neq 0 \\
k_m &= \frac{k_j}{2} \qquad (3)
\end{aligned}
$$

$$(4)$$

$$
\begin{aligned}
q_m &= k_m v_f (1 - \frac{k_m}{k_j}) \\
q_m &= \frac{k_j v_f}{2}(1 - \frac{k_j/2}{k_j}) \\
q_m &= \frac{k_j v_f}{4} \qquad (5)
\end{aligned}
$$