

CHAPTER 24

Multiscale Traffic Flow Modeling

Thus far, this book has presented traffic flow theory progressively from macroscopic to microscopic to picoscopic and from the “obvious” field observations to simple equilibrium models to involved dynamic models to complicated driver-vehicle-environment closed-loop systems. It is natural to extend the line of thinking to multiscale modeling, where high-level models provide system-wide overview, while low-level models describe local operation details. In addition, it is critical to adopt a consistent modeling approach to ensure the coupling between different levels.¹

24.1 INTRODUCTION

Anyone who used maps has probably had the following experience. Fifteen years ago, a 1:10,000 paper map was needed to view a city (e.g., Amherst, MA, USA), while a 1:1,000,000 paper map was needed to view a state (e.g., Massachusetts). If the scale was changed, a new map was needed. Today, using digital maps (e.g., Google Maps), one is able to view the entire country, and then progressively zoom in to view Massachusetts, Amherst, and even the University of Massachusetts Amherst campus, all seamlessly and within a single system.

Similarly, it is desirable that traffic simulation allows an analyst to zoom in to examine low-level details and zoom out to view system-wide performance within the same simulation process. Figure 24.1 illustrates such a paradigm. The background represents a *macroscopic* view of traffic operation in an entire region. This is analogous to viewing traffic from 10,000 m above the ground and the traffic appears to be a compressible fluid whose states (speed, flow, density, etc.) propagate like waves. As one zooms in to a local area of the region, a *mesoscopic* view is obtained. This is like viewing traffic from 3000 m above the ground, where the sense of waves recedes and a scene of particles emerges. As one further zooms in to a segment of the roadway, a *microscopic* view results. Similarly to watching traffic from 1000 m above the ground, the scene is dominated by moving particles that

¹ This chapter is reproduced from [114].

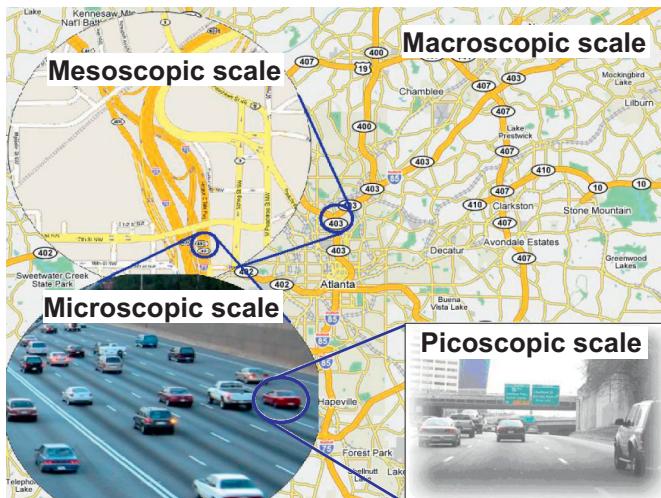


Figure 24.1 Multiscale traffic flow modeling.

interact with each other so as to maintain safe positions in a traffic stream. Finally, if one focuses on a few neighboring vehicles, a *picoscopic* view is achieved as if one were operating one of the vehicles. As such, one has to interact with the driving environment (e.g., roadway, signs, signals), make control decisions, and manage vehicle dynamic responses to travel safely. If such a “zoomable” simulation becomes available, one would be able to translate a traffic flow representation at multiple scales—for example, to trace a low-level event all the way to a high-level representation and, conversely, to decompose a global problem into one or more local deficiencies. As such, the “zoomable” simulation will transform the way that traffic flow is analyzed and transportation problems are addressed.

The objective of this chapter is to address multiscale traffic flow modeling with inherent consistency. The term “consistency” here concerns the coupling among models at different scales—that is, how less detailed models are derived from more detailed models and, conversely, how more detailed models are aggregated to less detailed models. Only consistent multiscale models are able to provide the theoretical foundation for the above “zoomable” traffic simulation. The chapter is organized as follows. [Section 24.2](#) takes a broad perspective on a spectrum of four modeling scales. Modeling objectives and model properties at each scale are discussed, and existing efforts are reviewed. [Section 24.3](#) presents the multiscale approach based on the field theory. The modeling strategy at each scale is discussed,

and some special cases are formulated at both the microscopic scale and the macroscopic scale. The emphasis of this multiscale approach is to ensure coupling among different modeling scales. Concluding remarks and future directions are presented in [Section 24.4](#).

24.2 THE SPECTRUM OF MODELING SCALES

The modeling of traffic flow can be performed at, but is not limited to, four scales—namely, picoscopic, microscopic, mesoscopic, and macroscopic, from the most to the least detailed. Considering that the definitions of these modeling scales are rather vague, implicit, or absent in the literature, this section attempts to provide an explicit definition so that existing and future models are easily classified and related. Such a definition is tabulated in [Figure 24.2](#) for each of the four modeling scales on the basis their properties (i.e., rows in the table), and literature related to each modeling scale is reviewed in subsequent subsections. The first three rows (“state variable,” “variable description,” and “state diagram”) are discussed in this section, and

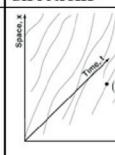
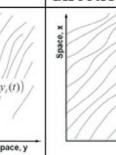
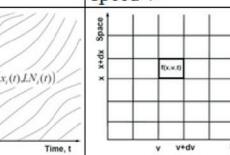
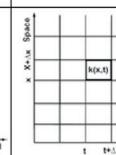
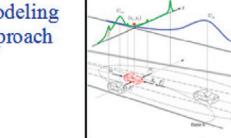
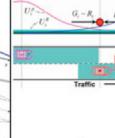
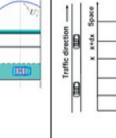
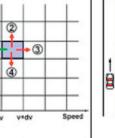
| Scale | Picoscopic | Microscopic | Mesoscopic | Macroscopic |
|----------------------|---|---|---|---|
| State variable | $(x_i(t), y_i(t))$ $i = 1, 2, 3, \dots$ $0 < t < \infty$ | $(x_i(t), LN_i(t))$ $LN \in \{1, 2, \dots, n\}$ | $f(x, v, t)$ | $k(x, t)$ |
| Variable description | Vehicle trajectory in longitudinal x and lateral y directions | Vehicle trajectory in x direction and lane # LN in y direction | Distribution of a vehicle at location x and time t with speed v | Concentration of vehicles at location x and time t |
| State diagram |  |  |  |  |
| Underlying principle | Control theory System dynamics Field theory | Field theory | Statistical mechanics | Fluid dynamics |
| Modeling approach |  |  |  |  |
| Model coupling | | Pico-Micro | Micro-Meso | Meso- Macro |
| | | | Micro - Macro | |

Figure 24.2 The spectrum of modeling scales.

the remaining three rows (“underlying principle,” “modeling approach,” and “model coupling”) will be described in the next section.

24.2.1 The Picoscopic Scale

Picoscopic modeling should be able to represent traffic flow so that the trajectory of each vehicle, $(x_i(t), y_i(t))$, where $i \in \{1, 2, 3, \dots, I\}$ denotes the vehicle ID, can be tracked in both the longitudinal x direction and the lateral y direction over time $t \geq 0$. Knowing these vehicle trajectories, one can completely determine the state and dynamics of the traffic system. Therefore, $(x_i(t), y_i(t))$ is the state variable (one or a set of variables that characterizes the state of a system). The corresponding state diagram (a graphical representation that illustrates the dynamics or evolution of system state) consists of these vehicle trajectories in a three-dimensional domain (x, y, t) .

Picoscopic models are mainly of interest in automotive engineering. Dynamic vehicle models with varying degrees of freedom have been proposed [132, 133]. A myriad of driver models have been reported to assist various aspects of automotive engineering, including vehicle handling and stability. Control theory was widely applied in modeling vehicle control [134, 135]. Models in this category typically incorporate one or more feedback loops. These loops are used by the controller to adjust its output to minimize control error. Human drivers can better perform reasoning using vague terms than can controllers. This observation allows the use of fuzzy logic [136, 137], which controls vehicles on the basis of some predefined rules. To allow implicit driving rules, artificial neural networks [138, 139] learn “driving experiences” from training processes and then apply the learned experiences in future driving. Several literature surveys of driver models are available [140–142].

24.2.2 The Microscopic Scale

Microscopic modeling should be able to represent traffic flow so that the trajectory of each vehicle can be tracked in the longitudinal direction $x_i(t)$, with the lateral direction being discretized by lanes $\text{LN}_i(t)$ where $\text{LN} \in \{1, 2, \dots, n\}$. Hence, $(x_i(t), \text{LN}_i(t))$ is a state variable that describes the state and dynamics of traffic flow at this scale, and the corresponding state diagram consists of vehicle trajectories in a two-dimensional domain (x, t) .

Within the traffic flow community, microscopic models treat driver-vehicle units as massless particles with personalities. The behavior of these particles is governed by car-following models in the longitudinal direction and discrete-choice (e.g., lane-changing and gap-acceptance) models in

the lateral direction. Car-following models describe how a vehicle (the follower) responds to the vehicle in front of it (the leader). For example, stimulus-response models [55, 56] assume that the follower's response (e.g., desired acceleration) is the result of stimuli (e.g., spacing and relative speed) from the leader, desired measure models [52, 57] assume that the follower always attempts to achieve his desired gains (e.g., speed and safety), psychophysical models [64, 111] introduce perception thresholds that trigger driver reactions, and rule-based models [67] apply “IF-THEN” rules to mimic driver decision making. Lane-changing and gap-acceptance models describe how a driver arrives at a lane change decision and how the driver executes such a decision, respectively. Approaches to lane changing include mandatory and discretionary lane changing [143, 144], adaptive acceleration mandatory and discretionary lane changing [145, 146], and autonomous vehicle control [147]. The following have been attempted to model gap acceptance: deterministic models [148–150], probabilistic models [151–153], and neuro-fuzzy hybrid models [154]. More surveys on microscopic models can be found in the literature [3, 155].

24.2.3 The Mesoscopic Scale

Mesoscopic modeling should be able to represent traffic flow so that the probability of the presence of a vehicle at a longitudinal location x with speed v at time t is tracked. The lateral direction is of interest only if it provides passing opportunities. The state diagram typically involves a two-dimensional domain (x, v) at an instant t , and the domain is partitioned into cells with space increment dx and speed increment dv . The state variable is a distribution function $f(x, v, t)$ such that $f(x, v, t)dx dv$ denotes the probability of having a vehicle within space range $(x, x + dx)$ and speed range $(v, v + dv)$ at time t . Knowing the distribution function $f(x, v, t)$, one can determine the dynamics of the system statistically.

Conventional mesoscopic traffic flow models come in three flavors. First, models such as the one in TRANSIMS [156] take a cellular automata approach, where the space domain (representing the longitudinal direction of a highway) is partitioned into short segments typically 7.5 m long. If it is occupied, a segment is able to store only one vehicle. Vehicles are then modeled as hopping from one segment to another, so their movement and speed are discretized and can take only some predetermined values. Second, models such as those implemented in DynaMIT [157] and DYNASMART [158] keep track of the motion of individual vehicles, but their speeds are determined with use of macroscopic models (such as an equilibrium

speed-density relationship) instead of microscopic car-following models. Third, truly mesoscopic models such as the one postulated by Prigogine and his coworkers [159] are based on nonequilibrium statistical mechanics or kinetic theory, which draws an analogy between classical particles and highway vehicles. However, Prigogine's model was criticized [160] for (1) lacking a theoretical basis, (2) lacking realism (e.g., car following, driver preferences, and vehicle lengths), and (3) lacking satisfactory agreement with empirical data. Many efforts have been made to improve Prigogine's model by addressing criticisms 2 and 3. For example, Paveri-Fontana [161] considered a driver's desired speeds, Helbing [162] adapted the desired speeds to speed limits and road conditions, Phillips [39, 163] incorporated vehicle lengths, Nelson [164] accounted for vehicle acceleration behavior, and Klar and Wegener [165, 166] included a stochastic microscopic model. Surveys of existing approaches are available in Ref. [167].

24.2.4 The Macroscopic Scale

Macroscopic modeling should be able to represent traffic flow so that only local aggregation of traffic flow (e.g., density k , speed u , and flow q) over space (longitudinal) x and time t is tracked. Traffic density $k(x, t)$ is a good candidate of state variable because, unlike flow and speed, density is an unambiguous indicator of the traffic condition. The state diagram typically involves a two-dimensional domain (x, t) . Knowing $k(x, t)$, one can determine the dynamics of the system macroscopically.

Conventional macroscopic traffic flow models describe the propagation of traffic disturbances as waves. A fundamental basis for formulating wave propagation is the law of conservation. The first-order form of the law is mass/vehicle conservation, which is used to create first-order models [24, 25]. In addition, momentum and energy are other forms of conservation. A model is of a higher order if it incorporates the latter forms of conservation [37, 38]. Since the limited benefit offered by higher-order models often does not justify their added complexity [47], numerical approximation and macroscopic simulation have been centered on first-order models—for example, KRONOS [27], the kinematic waves model [31], the cell transmission model [28, 29], FREQ [26], and CORQ [168]. More surveys of macroscopic models can be found in the literature [3].

24.2.5 Issues of Multiscale Modeling

Remarkably, existing models at the same scale typically follow different modeling approaches, and hence it is difficult to relate these models to each

other. In addition, models at different modeling scales are rarely coupled. For example, a macroscopic model typically lacks a microscopic basis, and a microscopic model does not have its macroscopic counterpart.

Therefore, an ideal multiscale modeling approach should emphasize not only model quality at each individual scale but also the coupling between different scales. Only models formulated with such an approach are able to support the “zoomable” traffic simulation discussed in [Section 24.1](#). As such, the resulting state diagram at a more detailed scale contains the necessary information to reproduce a less detailed diagram, as illustrated in [Figure 24.2](#). For example, the microscopic diagram is simply a projection of the picoscopic diagram onto the x - t plane, and the macroscopic state diagram can be completely reconstructed from the microscopic diagram with use of Edie’s definition of traffic flow characteristics [4, 6].

24.3 THE MULTISCALE APPROACH

The objective of this section is to pursue the above multiscale modeling approach and develop strategies to formulate a spectrum of models with inherent consistency. The approach starts at the picoscopic scale by formulating a model that is mathematically amenable to representing the natural way of human thinking while complying with physical principles; the microscopic model can be simplified from the picoscopic model yet still captures the essential mechanisms of vehicle motion and interaction; the mesoscopic model can be derived from the microscopic model on the basis of principles of nonequilibrium statistical mechanics; the macroscopic model can be derived from the mesoscopic model by application of the principles of fluid dynamics. See [Figure 24.2](#) for a summary of the underlying principles, the modeling approaches, and modeling coupling.

24.3.1 Picoscopic Modeling

This section consolidates and highlights the presentation in Chapters 18 and 21 as follows. To conform to real-world driving experiences, the picoscopic model should mimic the way that a driver operates his/her vehicle and responds to the driving environment. On the basis of the principles of control theory, a driver-vehicle-environment closed-loop control system has been developed. [Figure 24.3](#) illustrates the components of the system and its control flow, including feedback loops.

This system consists of a driver model and a vehicle model which interact with each other as well as with the driving environment. The

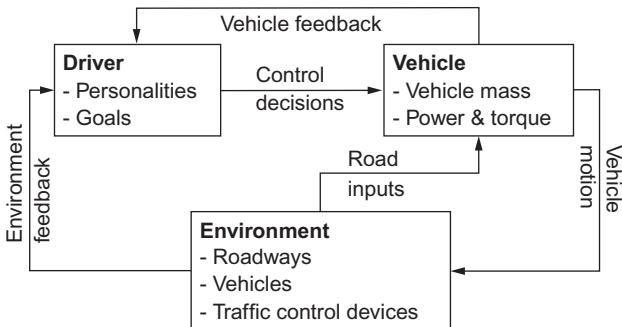


Figure 24.3 The closed-loop system.

driver receives information from the environment, such as roadways, traffic control devices, and the presence of other vehicles. The driver also receives information from his/her own vehicle, such as speed, acceleration, and yaw rate. These sources of information, together with driver properties and goals, are used to determine driving strategies (such as steering and accelerating/braking). The driving strategies are fed forward to the vehicle, which also receives input from roadways. These sources of information, together with vehicle properties, determine the vehicle's dynamic responses based on vehicle dynamic equations. Moving longitudinally and laterally, the vehicle constitutes part of the environment. Other vehicle dynamic responses such as speed, acceleration, and yaw rate are fed back to the driver to determine driving strategies in the next step. Thus, traffic operation is composed of movement and interaction of all vehicles in the environment.

The driver model can be formulated by applying the principles of the field theory. Basically, objects in a traffic system (e.g., roadways, vehicles, and traffic control devices) are perceived by a driver as component fields. The driver interacts with an object at a distance, and the interaction is mediated by the field associated with the object. The superposition of these component fields represents the overall hazard encountered by the driver. Hence, the driving strategy is to seek the least hazardous route by navigating through the field along its valley, and traffic flow consists of the motion and interaction of all vehicles. With this understanding, the driver model at the picoscopic scale is formulated as follows.

The driver's strategy of moving on roadways is to achieve gains (mobility and safety) and avoid losses (collisions and violation of traffic rules). Such a strategy can be represented as navigating through the valley of an overall

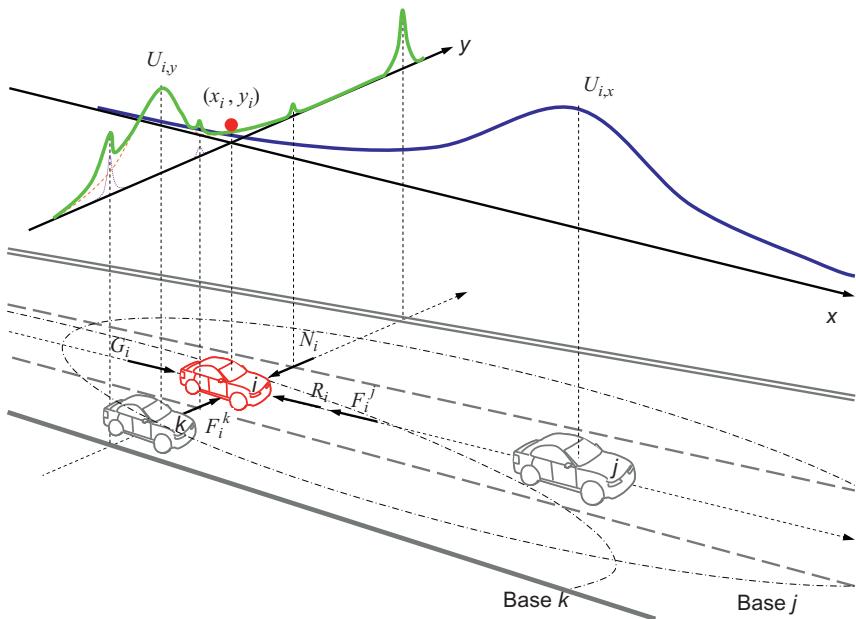


Figure 24.4 The illustration of a perceived field.

field U_i which consists of component fields such as those due to moving units U_i^B , roadways U_i^R , and traffic control devices U_i^C —that is,

$$U_i = U_i^B + U_i^R + U_i^C.$$

For example, Figure 24.4 illustrates two sections of the overall field, $U_{i,x}$ and $U_{i,y}$. The subject vehicle i is represented as a ball which rides on the tail of curve $U_{i,x}$ since the vehicle is within vehicle j 's field. Therefore, vehicle i is subject to a repelling force F_i^j which is derived from $U_{i,x}$ as

$$F_i^j = -\frac{\partial U_{i,x}}{\partial x}.$$

The effect of F_i^j is to push vehicle i back to keep a safe distance. By incorporating the driver's unsatisfied desire for mobility ($G_i - R_i$), we can determine the net force in the x direction as

$$m_i \ddot{x}_i = \sum F_{i,x} = G_i - R_i - F_i^j = (G_i - R_i) + \frac{\partial U_{i,x}}{\partial x}.$$

The section of U in the lateral γ direction, $U_{i,y}$ (the bold curve), is the sum of two components: the cross section of the field due to vehicle k (the dashed curve) and that due to the roadway field (the dotted curve). The former results in a repelling force F_i^k which makes driver i shy away from k and the latter generates a correction force N_i if i deviates from its lane center. Therefore, the net effect can be expressed as:

$$m_i \ddot{y}_i = \sum F_{i,y} = F_i^k - N_i = -\frac{\partial U_{i,y}}{\partial \gamma}.$$

By incorporating time t , driver i 's perception-reaction time τ_i , and driver i 's directional response γ , we can express the above equations as

$$\begin{aligned} m_i \ddot{x}_i(t + \tau_i) &= \sum \tilde{F}_{i,x}(t) = \gamma_i^0 [G_i(t) - R_i(t)] + \gamma(\alpha_i^j) \frac{\partial U_{i,x}}{\partial x}, \\ m_i \ddot{y}_i(t + \tau_i) &= \sum \tilde{F}_{i,y}(t) = -\gamma(\alpha_i^k) \frac{\partial U_{i,y}}{\partial \gamma}, \end{aligned}$$

where $\gamma_i^0 \in [0, 1]$ represents the driver's attention to unsatisfied desire for mobility (typically $\gamma_i^0 = 1$), and α_i^j , α_i^k , and α_i^N are viewing angles, which are also functions of time.

24.3.2 Microscopic Modeling

We can formulate the microscopic model by simplifying the above pico-scopic model as follows: (a) ignoring interactions inside a driver-vehicle unit, allowing it to be modeled as an active particle, (b) representing a driver's longitudinal and lateral control using separate but simpler models, (c) reducing the vehicle dynamic system to a particle, and (d) simplifying road surface and lanes to a collection of parallel lines.

Modeling Longitudinal Control

With the above simplifications, the three-dimensional potential field U in [Figure 24.4](#) reduces to a two-dimensional potential function. The upper part of [Figure 24.5](#) illustrates an example where driver i (the middle one) is traveling behind a leading vehicle j and is followed by a third vehicle p in the adjacent lane. The potential field U_i perceived by the driver is shaded in the lower part [Figure 24.5](#) and is represented by a curve in the upper part. Since the trailing vehicle in the adjacent lane does not affect the subject driver's longitudinal motion, the “stress” on the subject driver to keep a safe distance comes only from the leading vehicle and can be represented as

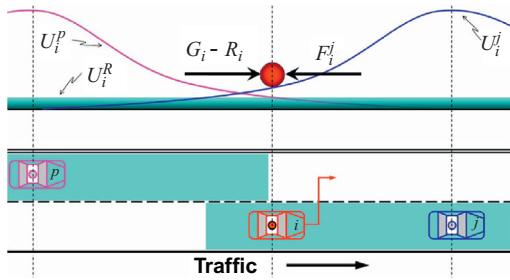


Figure 24.5 Microscopic modeling.

$$F_i^j = -\frac{\partial U_i^j}{\partial x}.$$

By incorporating roadway gravity G_i , roadway resistance R_i , and interaction between vehicles F_i^j , we can express the net force on i more specifically as

$$m_i \ddot{x}_i = G_i - R_i - F_i^j.$$

If one chooses proper functional forms for the above terms, special cases of the model can be obtained—for example, the longitudinal control model presented in Chapter 22:

$$\ddot{x}_i(t + \tau_i) = g_i \left[1 - \left(\frac{\dot{x}_i(t)}{v_i} \right) - e^{-\frac{s_{ij}(t)^* - s_{ij}(t)}{s_{ij}(t)^*}} \right], \quad (24.1)$$

$$s_{ij}^*(t) = x_{i-1}(t) - x_i(t) \geq \frac{\dot{x}_i^2(t)}{2b_i} + \dot{x}_i \tau_i - \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} + l_j,$$

where it is assumed that $G_i = m_i \times g_i$, $R_i = m_i \times (\frac{\dot{x}_i(t)}{v_i})$, and $F_i^j = m_i \times f(s_{ij}, s_{ij}(t)^*)$, where g_i is the maximum acceleration that driver i is willing to apply when starting from standstill, $\dot{x}_i(t)$ is the actual speed of vehicle i , v_i is the desired speed of driver i , $s_{ij} = x_j - x_i$ is the actual spacing between vehicles i and j , x_i is the position of vehicle i , x_j is the position of vehicle j , and s_{ij}^* is the desired spacing between vehicles i and j . l_j is the nominal length of vehicle j and is conveniently used as the spacing between two vehicles in jammed traffic. The difference $(s_{ij}^* - s_{ij})$ represents how far vehicle i intrudes beyond s_{ij}^* . The rationale for representing the interaction force F_i^j between vehicles i and j with an exponential function is to set the desired

spacing s_{ij}^* as a baseline, beyond which the intrusion by vehicle i is translated exponentially to the repelling force acting on the vehicle.

The desired spacing s_{ij}^* is derived according to the Gipps model [57]. More specifically, s_{ij}^* should allow vehicle i to stop behind its leading vehicle j after a perception-reaction time τ_i and a deceleration process at a comfortable level $b_i > 0$ should vehicle j apply an emergency brake at rate $B_j > 0$. Of course, the desired spacing can be derived on the basis of other safety rules if appropriate.

Modeling Lateral Control

The driver's lateral control concerns changing lanes to seek a speed gain or to use an exit. The shaded areas in the bottom part of Figure 24.5 can be interpreted as driver j and p 's personal spaces after the lane barrier has been accounted for. A lane change decision is reached whenever driver i intrudes into another driver's personal space. With such a decision, driver i begins to search for open spaces in adjacent lanes. In this particular case, an open space happens to be available in the left lane, barely allowing the center of vehicle i to move in. Consequently, the result of the gap-acceptance decision is to abruptly switch vehicle i to the left lane.

24.3.3 Mesoscopic Modeling

Mesoscopic modeling applies the principles of nonequilibrium statistical mechanics or kinetic theory to model traffic flow. Essential to the modeling is the determination of a distribution function $f(x, v, t)$ such that $f(x, v, t)dx dv$ denotes the probability of having a vehicle within space range $(x, x + dx)$ and speed range $(v, v + dv)$ at time t (see Figure 24.6). The time evolution of traffic flow is described by an evolution equation,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \frac{dx}{dt},$$

whose right-hand side is to be determined. Therefore, the central question is how to rigorously derive the evolution equation. This can be done by use of a procedure similar to that used to derive the Boltzmann equation [169, 170] from basic principles. The classical Boltzmann equation describes particles moving in a three-dimensional domain, so the first step is to reduce the three-dimensional case to a one-dimensional case which represents traffic moving on a unidirectional highway.

Existing models, in particular those based on Prigogine's work, are postulated. To derive the one-dimensional Boltzmann equation from basic

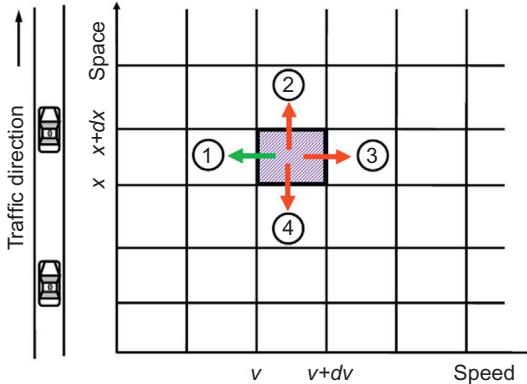


Figure 24.6 The x - v diagram.

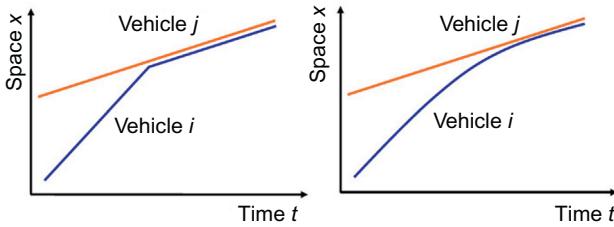


Figure 24.7 Car following.

principles, a sound understanding of the mechanism of traffic evolution is required. Existing models, including a derived model [165, 166], assumed that the mechanism is vehicle “collision.” For example, the fast follower i in the left panel in Figure 24.7 keeps its speed up to the collision point and then abruptly changes its speed. To be realistic, the speed change of vehicle i needs to be smooth as it approaches its leader j as illustrated in the right panel in Figure 24.7. This is possible only if car following is incorporated as the mechanism of particle interaction. As such, the longitudinal control model can be used to derive the one-dimensional Boltzmann equation and, thus, ensures micro-meso coupling.

The derivation of the one-dimensional Boltzmann equation starts from the application of the conservation law (e.g., vehicles entering and exiting the highlighted cell in Figure 24.6 should be conserved). Existing models considered only one direction (i.e., direction 1 below), in which vehicles exit the cell, and a similar treatment applies to vehicles entering the cell.

This approach causes modeling errors. Actually, vehicles may exit the cell in four directions: (1) vehicles slowed down (and hence exited the cell) because of a sluggish leader, (2) vehicles physically moved out of the cell, (3) vehicles accelerated because of an aggressive follower, and (4) vehicles reversed, which is unlikely. The opposite applies to vehicles entering the cell. Therefore, application of the law to include all directions is the correct approach. Since it is mathematically complicated to derive the one-dimensional Boltzmann equation, this chapter presents only potential directions of exploration, leaving the actual derivation to be addressed in future research.

Once the one-dimensional Boltzmann equation has been formulated, one may solve it using initial and boundary conditions to study how traffic evolves over time and space. However, solving the equation can be quite involved, as is the case for any classical Boltzmann equation. Fortunately, some important results can be inferred without fully solving the equation. For example, a hydrodynamical formulation, which is essential to macroscopic modeling, can be derived from the equation. In addition, the equation contains an equilibrium relationship between vehicle speed and traffic density which is also essential to macroscopic modeling. Such a relationship is analogous to the Maxwell-Boltzmann distribution (the distribution of molecular speed at different temperatures) which is the stationary (i.e., $\frac{\partial f}{\partial t} = 0$) solution to a classical Boltzmann equation.

24.3.4 Macroscopic Modeling

Macroscopic modeling applies the principles of fluid dynamics to model traffic flow as a one-dimensional compressible continuum fluid. While the above mesoscopic modeling describes the distribution of vehicles in a highway segment, macroscopic modeling represents only the average state. Therefore, traffic density $k(x, t)$ can be related to the distribution $f(x, v, t)$ as its zeroth moment $k(x, t) = \int f(x, v, t)dv$ and traffic speed as the first moment $u(x, t) = \frac{1}{k} \int vf(x, v, t)dv$. From this understanding, it becomes clear that it is feasible to derive a hydrodynamical formulation from the mesoscopic model. The one-dimensional Boltzmann equation discussed above can be expressed in a general form as

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = C,$$

where C denotes the rate of change of $f(x, v, t)$. Multiplying both sides of this equation by 1, v , and $\frac{1}{2}v^2$ and integrating it over v , we obtain hydrodynamical equations of mass, momentum, and energy conservation. The mass conservation equation

$$\frac{\partial k}{\partial t} + \frac{\partial(ku)}{\partial x} = \int C dv$$

is of particular interest because it describes the time evolution of traffic density $k(x, t)$. To solve the equation, a speed-density relationship must be introduced into the macroscopic model. This relationship can be derived from the mesoscopic model under stationary conditions or, alternatively, can be obtained directly from the microscopic model if equilibrium conditions are assumed. For example, the macroscopic version of the longitudinal control model is

$$v = v_f [1 - e^{1 - \frac{k^*}{k}}], \quad (24.2)$$

where $k^* = \frac{1}{\gamma v^2 + \tau v + l}$, v_f is the free-flow speed, $k_j = 1/l$, l is the bumper-to-bumper distance between vehicles when traffic is jammed, and τ is the average perception-reaction time of drivers.

Therefore, the macroscopic model consists of a system of equations including the hydrodynamical formulation and one of the above speed-density relationships:

$$\frac{\partial k}{\partial t} + \frac{\partial(ku)}{\partial x} = \int C dv,$$

$$v = V(k).$$

We can solve the system of equations graphically using the method of characteristics or numerically using a finite difference approach. A typical finite difference method is illustrated in [Figure 24.8](#), where one partitions the time-space domain into cells and keeps track of traffic flowing into and out of each cell [21, 27, 171].

24.4 SUMMARY

This chapter has presented a broad perspective on traffic flow modeling at four scales: picoscopic, microscopic, mesoscopic, and macroscopic, from the most to the least detailed level. Modeling objectives and model properties at each scale were discussed and existing efforts were reviewed.

To ensure modeling consistency and provide a microscopic basis for macroscopic models, it is critical to address the coupling among models

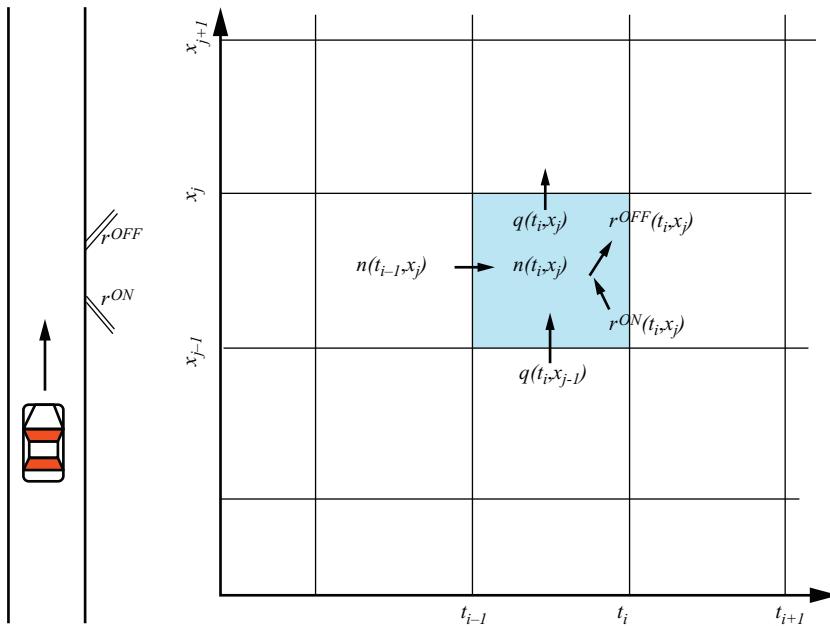


Figure 24.8 The finite difference method.

at different scales—that is, how less detailed models are derived from more detailed models and, conversely, how more detailed models are aggregated to less detailed models. With this understanding, a consistent modeling approach was proposed based on the field theory. Basically, in this approach, objects (e.g., roadways, vehicles, and traffic control devices) are perceived by the subject driver as component fields. The driver interacts with an object at a distance, and the interaction is mediated by the field associated with the object. In addition, the field may vary when perceived by different drivers depending on their characteristics, such as responsiveness and perception-reaction time. The superposition of these component fields represents the overall hazard encountered by the subject driver. Hence, the objective of the driver is to seek the least hazardous route by navigating through the field along its valley. Consequently, traffic flow is modeled as the motion and interaction of all vehicles.

Modeling strategies at each of the four scales were discussed. More specifically, the field theory serves as the basis of picoscopic modeling, which represents a driver-vehicle unit as a driver-vehicle-environment closed-loop control system. The system is able to capture vehicle motion in longitu-

dinal and lateral directions. The microscopic model is obtained from the picoscopic model by simplification of its driver–vehicle interactions, vehicle dynamics, and vehicle lateral motion. The mesoscopic model is derived from basic principles with use of the microscopic model as the mechanism of traffic evolution. The macroscopic model includes an evolution equation (which is derived by taking moments of the mesoscopic model) and an equilibrium speed–density relationship (which is the stationary solution to the mesoscopic model or is derived from the microscopic model directly). Therefore, the proposed approach ensures model coupling and modeling consistency. As such, consistent models derived from this approach are able to provide the theoretical foundation to develop the “zoomable” traffic simulation tool discussed in [Section 24.1](#).

PROBLEMS

1. This chapter discussed a spectrum of four modeling scales—namely, macroscopic, mesoscopic, microscopic, and picoscopic, from the least to the most detailed. Provide at least one example model at each scale.
2. Define each of the four modeling scales with their properties, such as state variables.
3. Discuss the issues that multiscale traffic flow modeling is currently facing in relation to existing models.