CHAPTER 14

General Motors Models

An anecdote has it that General Motors' CEO Charles Wilson once said "what's good for General Motors is good for America." It turned out that this statement was misquoted, and the true version dates back to 1953, when Wilson, who was appointed as the Secretary of Defense by President Eisenhower, was at his confirmation hearings before the Senate Armed Services Committee (source, Wikipedia):

> *During the hearings, when asked if as Secretary of Defense he could make a decision adverse to the interests of General Motors, Wilson answered affirmatively. But added that he could not conceive of such a situation "because for years I thought what was good for the country was good for General Motors and vice versa.*

While what Wilson actually said in this anecdote is not of great interest here, what is important is the role that General Motors played in the history of traffic flow theory. Back in the 1950s, General Motors sponsored a team of scientists in its research laboratories, from which pioneering work was done that broke the ground for traffic flow theory. At the foremost of such efforts was the family of General Motors models (referred to as the GM models hereafter).

14.1 DEVELOPMENT OF GM MODELS

GM models [55, 56] assume that a driver's control maneuver is a result of not only external stimuli such as the dynamics of the subject vehicle and its leading vehicle, but also the driver's sensitivity. Hence such a relationship can be expressed as

$$Response = f\,(sensitivity,\ stimuli).$$

When formulating the above relationship, GM researchers chose the subject vehicle's acceleration (deceleration is negative acceleration) produced after a reaction time, $\ddot{x}_i(t+\tau)$, as the response (see Figure 14.1). The consideration of stimuli and sensitivity evolved over time and resulted in a family of models.

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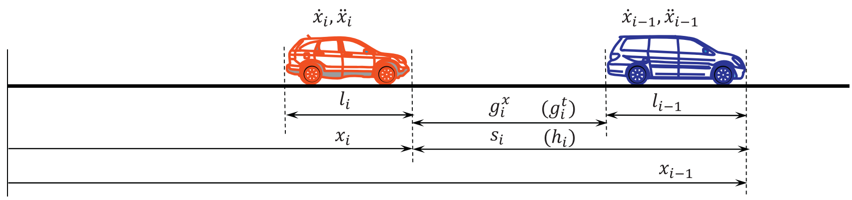

Figure 14.1 A car-following scenario.

14.1.1 GM1

Originally, General Motors researchers observed that drivers responded to the relative speed between the subject vehicle i and its leading vehicle $i-1$, $\dot{x}_{i-1}(t) - \dot{x}_i(t)$. If sensitivity is treated as a coefficient that is multiplicative to the stimulus, the subject driver's operational control can be formulated as

$$\ddot{x}_i(t+\tau_i) = \alpha[\dot{x}_{i-1}(t) - \dot{x}_i(t)]. \tag{14.1}$$

The above model is the first-generation model which can be used to interpret some car-following phenomena effectively. For example, when the subject vehicle approaches its leading vehicle (e.g., $\dot{x}_i(t) = 120\,\text{km/h}$ and $\dot{x}_{i-1}(t) = 100\,\text{km/h}$), the relative speed is negative, and hence the driver will decelerate since $\ddot{x}_i(t+\tau_i) < 0$ (assuming that α is a positive constant). In contrast, if the subject vehicle is falling behind its leading vehicle (e.g., $\dot{x}_i(t) = 100\,\text{km/h}$ and $\dot{x}_{i-1}(t) = 120\,\text{km/h}$), the subject vehicle will accelerate because the relative speed now becomes positive. However, the model has difficulty distinguishing scenarios with large and small car-following distances. For example, the model predicts the same deceleration response to the following two scenarios:

- Scenario 1: $\dot{x}_i(t) = 120\,\text{km/h}$, $\dot{x}_{i-1}(t) = 100\,\text{km/h}$, and $s_i(t) = 50\,\text{m}$
- Scenario 2: $\dot{x}_i(t) = 120\,\text{km/h}$, $\dot{x}_{i-1}(t) = 100\,\text{km/h}$, and $s_i(t) = 5000\,\text{m}$

Since both scenarios have a speed difference of $-20\,\text{km/h}$, intuitively, the subject driver in scenario 1 would brake much harder than the driver in scenario 2 because the former is facing an imminent collision.

14.1.2 GM2

The effect of spacing motivated General Motors researchers to choose different sensitivity coefficients, and hence the second-generation model resulted:

$$\ddot{x}_i(t+\tau_i) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} [\dot{x}_{i-1}(t) - \dot{x}_i(t)]. \tag{14.2}$$

Field experiments revealed that the sensitivity coefficient α ranges between 0.17 and 0.74. In GM2, a high sensitivity value α_1 is chosen when the two vehicles are close, while a low sensitivity value α_2 is employed when the two vehicles are far apart.

14.1.3 GM3

The effect of spacing was partially address in GM2 because one has to frequently calibrate the sensitivity coefficient depending on car-following distances. The inconvenience seemed to suggest that spacing should be explicitly included in the model, which led to the formulation of the third-generation model:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]}. \tag{14.3}$$

Although the issue of spacing has been suppressed, another problem pops up. The model is unable to predict any difference between the following scenarios:

- Scenario 1: In downtown Amherst, one vehicle is following another at a spacing of 100 m with speeds $\dot{x}_i(t) = 30\,\text{km/h}$, $\dot{x}_{i-1}(t) = 10\,\text{km/h}$.
- Scenario 2: On Interstate 90, one vehicle is following another at a spacing of 100 m with speeds $\dot{x}_i(t) = 130\,\text{km/h}$, $\dot{x}_{i-1}(t) = 110\,\text{km/h}$.

The subject driver on Interstate 90 is certainly under a great deal of pressure to maintain safety during car following because, at such a high speed, a moment's lapse of attention would result in a catastrophe. In contrast, the subject driver in downtown Amherst should have peace of mind because, if something goes wrong, he or she can always slam on the brake to stop the vehicle. Hence, our daily driving experiences suggest that the response in scenario 2 be greater than that in scenario 1. However, GM3 predicts no difference because in both scenarios the speed difference is 20 km/h and the spacing is 100 m.

14.1.4 GM4

GM3's inability to differentiate high-speed and low-speed car-following scenarios motivated General Motors researchers to further explore unexplained factors that can be extracted from the sensitivity coefficient. Interestingly, a dimension analysis reveals that the sensitivity coefficient has the same unit as frequency (i.e., 1/s) in GM1 and GM2 and the same unit as speed (i.e., m/s) in GM3. Since there is a need to explicitly consider speed as a stimulus, it seems ideal to extract speed from the sensitivity coefficient, leaving the remainder as a new, dimensionless coefficient. This gives rise to the fourth-generation model:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i(t+\tau_i)[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]}. \tag{14.4}$$

14.1.5 GM5

To generalize the results of the above GM models and, as becomes clear later, to facilitate finding "the bridge" between microscopic and macroscopic models, a generic form of GM models is proposed as the fifth model:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_i(t+\tau_i)]^m}{[x_{i-1}(t) - x_i(t)]^l}[\dot{x}_{i-1}(t) - \dot{x}_i(t)], \tag{14.5}$$

where x_i, $\dot{x}_i$, and $\ddot{x}_i$ are the displacement, speed, and acceleration of the subject vehicle i, and similar notation applies to its leader $i-1$. τ is the perception-reaction time that applies to all drivers, α is a dimensionless sensitivity coefficient, and m and l are speed and spacing exponents, respectively.

14.2 MICROSCOPIC BENCHMARKING

The following segment of code implements GM4, a full-bloom model in the family. At time step j, the displacement, speed, and acceleration of each vehicle are updated:

```
FOR i = 1:I
  v(j,i) = max([0, v(j-1,i) + a(j,i) * dt]);
  d_v = v(j,i-1) - v(j,i);
  x(j,i) = x(j-1,i) + v(j,i) * dt;
  d_x(j,i) = x(j,i-1) - x(j,i);
  delay = ceil(tau_i/dt);
  a(j + delay, i) = alpha * v(j,i) * d_v(j,i) / d_x(j,i);
END
```

where x, v, and a are displacement, speed, and acceleration, respectively; i is the vehicle ID, $i \in \{1,2, \ldots,I\}$; j is the time step, $j \in \{1,2, \ldots, J\}$; τ_i is the perception-reaction time of driver i; and Δt is the simulation time step.

Microscopic benchmarking refers to the scenario presented in Section 12.3.1 and the benchmarking result of GM4 is plotted in Figure 14.2, which is further elaborated as follows:

- Start-up: the model is unable to start a vehicle from standstill. Therefore, an external logic has to be imposed to assign an initial speed $\dot{x}_i(0)$ to the subject vehicle i. Note that the initial speed $\dot{x}_i(0)$ has to be set at the desired speed v_i. Otherwise, vehicle i will not be able to reach that speed by itself. See Figure 14.2 when $t > 0$ s.
- Speedup: rather than speeding up vehicle i as drivers normally do in the real world, the model predicts a deceleration by driver i even though its leading vehicle $i-1$ is thousands of meters ahead. See Figure 14.2 when $0\,\text{s} < t < 100\,\text{s}$.
- Free flow: the model predicts that vehicle i is unable to attain the free-flow condition by itself unless it is set to do so by an external logic. As long as it follows a slower leader, the model constantly decelerates the vehicle until it adopts the leader's speed. See Figure 14.2 when $0\,\text{s} < t < 100\,\text{s}$.
- Cutoff: when the third vehicle suddenly takes over as the new leader 40 m ahead at 24 m/s, the model predicts a sudden acceleration, while in the real world drivers may or may not decelerate the vehicle. See Figure 14.2 around $t = 100$ s.
- Following: the model is able to adopt the leader's speed and follow the leader at a reasonable distance. See Figure 14.2 when $100\,\text{s} < t < 200\,\text{s}$.
- Stop and go: the model predicts that vehicle i will gradually but surely collide with its leader while maintaining a speed, regardless of how low the speed is. When the leader resumes motion, vehicle i will be stuck there because of its infinitesimally low speed. See Figure 14.2 when $200\,\text{s} \geq t \leq 300\,\text{s}$.
- Trailing: vehicle i is stuck and fails to catch up with its speeding leader, unless another external logic brings it out of being stuck. See Figure 14.2 when $300\,\text{s} < t < 400\,\text{s}$. However, once the vehicle resumes motion, it will be tempted to catch up with its speeding leader and will adopt the leader's speed. Such an effect is not shown in Figure 14.2.
- Approaching: the simulation fails to be reasonable beyond $t = 300$ s.
- Stopping: the simulation fails to be reasonable beyond $t = 300$ s.

The above benchmarking is based on the set of parameters in Table 14.1, and the outcome may differ for a different set of parameters.

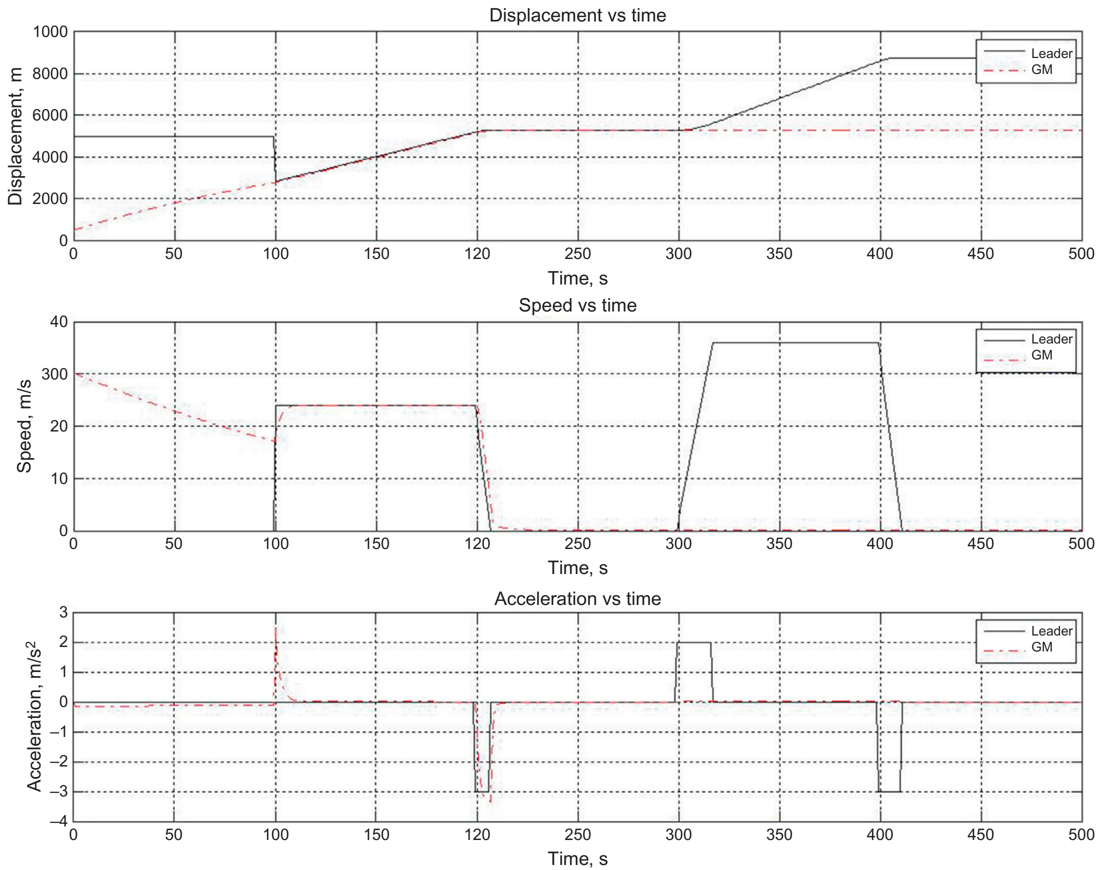

Figure 14.2 Microscopic benchmarking of GM4.

Table 14.1 Microscopic benchmarking parameters of GM4

τ_i	α	–
1.0 s	0.8	–
$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
467 m	30 m/s	0 m/s^2

14.3 MICROSCOPIC-MACROSCOPIC BRIDGE

As mentioned in Chapter 13, the relation between microscopic and macroscopic models is always of great interest because such a relation offers a "bridge" to connect microscopic and macroscopic worlds. This section is specifically devoted to such a purpose. It appears that GM5 is ideal to serve as a unifying factor that pulls together some of the microscopic and macroscopic/equilibrium models in the early history of traffic flow theory. For convenience, GM5 is repeated below:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_i(t+\tau_i)]^m}{[x_{i-1}(t) - x_i(t)]^l}[\dot{x}_{i-1}(t) - \dot{x}_i(t)]. \tag{14.6}$$

In addition, those early equilibrium models are listed in Table 14.2.

14.3.1 Greenberg Model

If one chooses $m = 0$ and $l = 1$, GM5 reduces to GM3:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t). - x_i(t)]} \tag{14.7}$$

Table 14.2 Single-regime models

Authors	Model	Parameters
Greenshields [9]	$v = v_\mathrm{f}\left(1 - \frac{k}{k_\mathrm{j}}\right)$	v_f, k_j
Greenberg [10]	$v = v_\mathrm{m}\ln\left(\frac{k_\mathrm{j}}{k}\right)$	v_m, k_j
Underwood [11]	$v = v_\mathrm{f}\mathrm{e}^{-\frac{k}{k_\mathrm{m}}}$	v_f, k_m
Drake et al. [12]	$v = v_\mathrm{f}\mathrm{e}^{-\frac{1}{2}\left(\frac{k}{k_\mathrm{m}}\right)^2}$	v_f, k_m
Drew [13]	$v = v_\mathrm{f}\left[1 - \left(\frac{k}{k_\mathrm{j}}\right)^{n+\frac{1}{2}}\right]$	v_f, k_j, n
Pipes [14] and Munjal [15]	$v = v_\mathrm{f}\left[1 - \left(\frac{k}{k_\mathrm{j}}\right)^{n}\right]$	v_f, k_j, n

v_f is free-flow speed, k_j is jam density, v_m is optimal speed, k_m is optimal density, and n is an exponent.

It can be proved that, by integration, this microscopic car-following model can be transformed into the Greenberg model, of which the following is a revised version:

$$\begin{cases} q = v_{\mathrm{f}}k & \text{when } 0 \leq k < k_{\mathrm{c}}, \\ q = v_{\mathrm{m}}\ln\frac{k_{\mathrm{j}}}{k}k & \text{when } k_{\mathrm{c}} \leq k \leq k_{\mathrm{j}}. \end{cases} \tag{14.8}$$

The purpose of the revision to avoid the infinite free-flow speed problem in the Greenberg model based on the following observation. It is known that under zero to light traffic conditions, there is enough room to allow drivers to maintain their desired speed, and hence free-flow speed v_{f} can be sustained up to a density called the critical density k_{c}. As traffic density continues to increase, traffic speed begins to drop until it reaches zero, when the density becomes maximum—that is, k_{j}.

The above flow-density relationship is plotted as the solid curve in Figure 14.3. For easy reference, the underlying speed-density curve is plotted as a dashed line in the background to show how it relates to the flow-density curve.

The above relation between the Greenberg model and GM3 suggests that one might be able to relate some of the existing equilibrium traffic flow models to GM5 by aggregating or integrating this model with varying speed and spacing exponents. The following constitutes some additional examples.

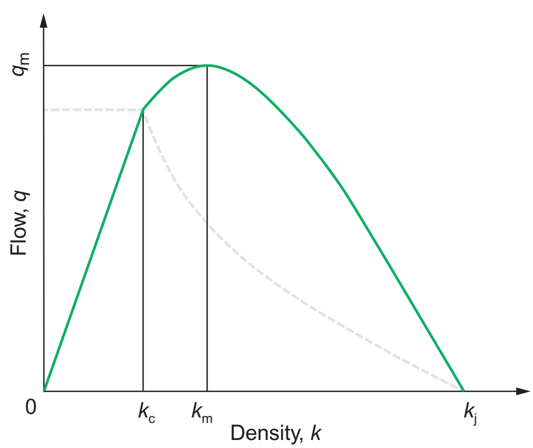

Figure 14.3 Flow-density relationship.

14.3.2 Greenshields Model

Setting $l = 2$ and $m = 0$ in GM5 (Equation 14.5) yields

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]^2}, \tag{14.9}$$

where all variables are defined as before. It can be proved that this microscopic model can be transformed into the Greenshields model:

$$v = v_\mathrm{f} - \frac{v_\mathrm{f}}{k_\mathrm{j}} k. \tag{14.10}$$

14.3.3 Underwood Model

Setting $l = 2$ and $m = 1$ in GM5 yields

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i(t+\tau_i)[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]^2}. \tag{14.11}$$

It can be proved that this microscopic model can be transformed into the Underwood model:

$$v = v_\mathrm{f} \mathrm{e}^{-k/k_\mathrm{m}}. \tag{14.12}$$

14.3.4 Model of Drake et al. (Northwestern Model)

Setting $l = 3$ and $m = 1$ in GM5 yields

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i(t+\tau_i)[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]^3}. \tag{14.13}$$

It can be proved that this microscopic model can be transformed into the model of Drake et al.:

$$v = v_\mathrm{f} \mathrm{e}^{-\frac{1}{2}\left(\frac{k}{k_\mathrm{m}}\right)^2}. \tag{14.14}$$

14.3.5 Pipes-Munjal Model

Setting $l = n + 1$ and $m = 0$ in GM5 yields

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]^{n+1}}. \tag{14.15}$$

It can be proved that this microscopic model can be transformed into the Pipes-Munjal model:

$$v = v_\mathrm{f} \left[1 - \left(\frac{k}{k_\mathrm{j}} \right)^n \right]. \tag{14.16}$$

14.3.6 Drew Model

Since the Drew model and the Pipes-Munjal model are exactly the same except for their exponent, one only needs to replace n with $n + \frac{1}{2}$ in the above derivation to obtain the Drew model. Hence, the Drew model corresponds to GM5 with $l = n + 1.5$ and $m = 0$.

14.3.7 Summary of the Bridge

Summarizing the above, we can draw a diagram that relates the models discussed above to GM5. Figure 14.4 serves such a purpose, with the speed exponent m of GM5 on the horizontal axis and the spacing exponent l of GM5 on the vertical axis. Macroscopic equilibrium models are labeled in red and microscopic car-following models are labeled in blue. Circles on the grid denote models and their corresponding m and l combination in relation to GM5.

The Pipes and Forbes models are actually a special case of GM1:

$$\dddot{x}_i(t + \tau_i) = \alpha[\dot{x}_{i-1}(t) - \dot{x}_i(t)].$$

Integrating both sides yields

$$\dot{x}_i(t + \tau_i) = \alpha[x_{i-1}(t) - x_i(t)] + C = \alpha s_i(t) + C.$$

If one chooses $\alpha = \frac{l_i}{4.47}$ and $C = l_{i-1}$, one obtains the Pipes model, while $\alpha = \tau_i$ and $C = l_i$ leads to the Pipes model.

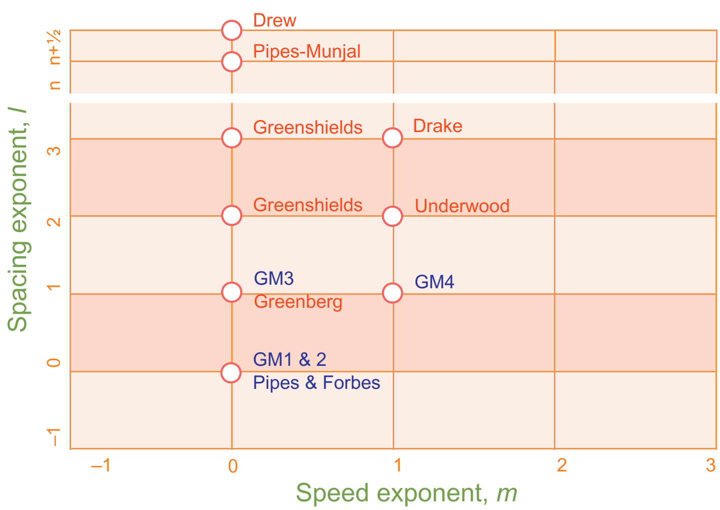

Figure 14.4 Microscopic-macroscopic bridge.

14.4 MACROSCOPIC BENCHMARKING

Macroscopic benchmarking refers to the scenario presented in Section 12.3.2. Fundamental diagrams implied by GM models and their associated equilibrium models are presented in Figure 14.5 against empirical observations. It can be seen that these fundamental diagrams achieve varying success in fitting empirical data, but none of them fit the data reasonably well in the entire range of density. For example, the Greenshields model overpredicts speed (and hence flow) in the majority of the density range except for the free-flow (i.e., low-density) condition; the Greenberg model has a problem fitting the data under the free-flow condition; the Underwood model, perhaps the best among the models, underestimates speed at low densities and overestimates speed at high densities, and capacity occurs at much lower speed than it ought to; the model of Drake et al. (Northwestern model) has a flow-density curve that is convex in the high-density range; the Drew and Pipes-Munjal models, which are essentially the same but are shown slightly differently to avoid complete overlap, share the same problem as the Greenshields model but to a lesser extent.

The above benchmarking is based on the set of parameters in Table 14.3, and the outcome may differ for a different set of parameters

14.5 LIMITATIONS OF GM MODELS

As seminal work in the early history, GM models spawned and inspired numerous research efforts that have shaped today's traffic flow theory, and thus the importance of this work cannot be underestimated. Meanwhile,

Table 14.3 Macroscopic benchmarking parameters of models associated with GM models

Greenshields	v_f 30 m/s	k_j 1/6 vehicles/m	– –
Greenberg	v_m 10.7 m/s	k_j 1/6 vehicles/m	k_c 0.01 vehicles/m
Underwood	v_f 30 m/s	k_m 0.05 vehicles/m	– –
Drake et al. (Northwestern)	v_f 30 m/s	k_m 0.04 vehicles/m	n 2
Drew	v_f 30 m/s	k_j 1/6 vehicles/m	n 0.1
Pipes and Munjal	v_f 30 m/s	k_j 1/6 vehicles/m	n 0.5

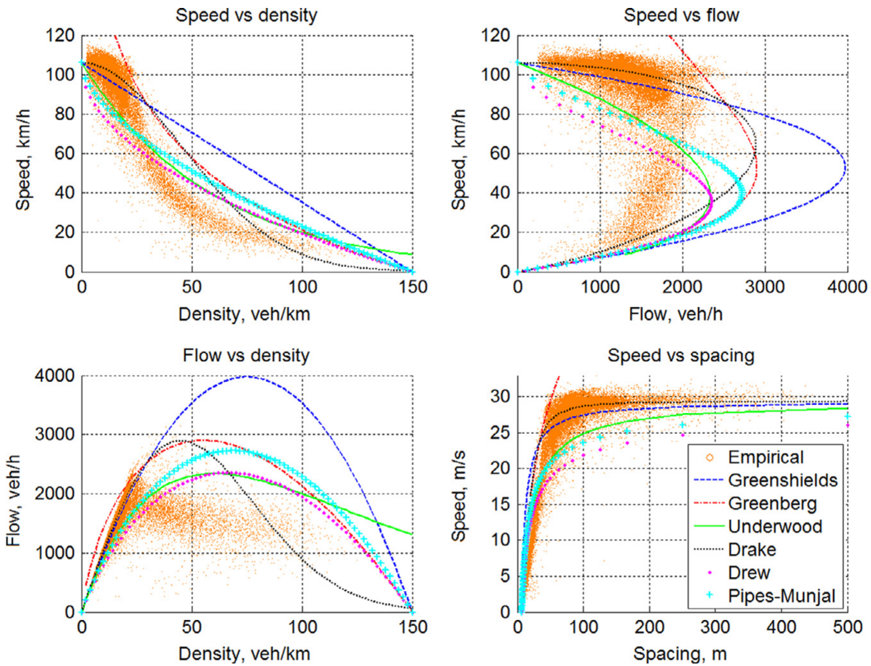

Figure 14.5 Fundamental diagrams implied by GM models and associated models.

GM models suffer from some limitations, which are presented below with use of GM4 as an example.

Universal car following

On the one hand, GM4 is mathematically attractive since it has only one equation that covers all situations. On the other hand, such a one-regime property stipulates universal car following, which is not realistic. For example, the model predicts that a vehicle in Atlanta must be following another vehicle in Boston even though they are over 1000 km apart.

Attraction as a mechanism of motion

If one compares GM4 (Equation 14.4) against Newton's law of universal gravitation (Equation 14.17) and Coulomb's law (Equation 14.18), one finds that they are strikingly similar to each other.

$$F = G\frac{m_1 m_2}{r^2}, \tag{14.17}$$

where F is the force between two masses, G is the gravitational constant, m_1 is the first mass, m_2 is the second mass, and r is the distance between the masses.

$$F = k_e\,\frac{q_1 q_2}{r^2}, \tag{14.18}$$

where F is the electrostatic force between two point charges (like charges repel each other and opposite charges attract each other), q_1 is the first point charge, q_2 is the second point charge, r is the distance between the two point charges, and k_e is a proportionality constant.

Therefore, GM4 can be interpreted as equivalent to Coulomb's law as follows. Vehicle i will be repelled by its leader $i-1$ when vehicle i is approaching vehicle $i-1$ at a higher speed, while vehicle i will be attracted to vehicle $i-1$ should vehicle i fall behind at a slower speed. Though the first half of the reasoning seems to make some sense, the second half does not. For example, what if the subject vehicle does not have a leader—for example, the first vehicle to enter the highway. Then the subject vehicle could not start because there would be no vehicle to pull it forward. Even if the subject vehicle is following a leader and the gap between them is increasing, it does not feel as if the subject vehicle is attracted to the leader. Actually, the subject vehicle speeds up because one would like to achieve the desired speed.

Slow start

According to GM4, a vehicle at a stopped position is unable to start. This is because the vehicle's speed at the current step ($\dot{x}_i(t) = 0$) determines its acceleration in the next step ($\ddot{x}_i(t+\tau_i) = 0$) (see Equation 14.4). Therefore, the vehicle has to maintain a nonzero speed at any time in order to avoid being trapped. As such, the model fails to apply when a vehicle is stopped by a red light at an intersection or is completely blocked by another vehicle on a highway. Otherwise, the subject vehicle has to slow down to an infinitesimal speed rather than to a complete stop in order to avoid being trapped. When the light turns green or the leading vehicle resumes motion, the subject vehicle will take a long time to get up to speed. This is because the vehicle's infinitesimal speed results in a weak attraction, which is the only mechanism to accelerate the vehicle. Figure 14.6 illustrates such a scenario, where a noticeable gap results between the first vehicle and the second vehicle.

An intimate pair

According to GM4, two vehicles can get arbitrarily close to each other as long as they are traveling at the same speed, which is certainly not true—no

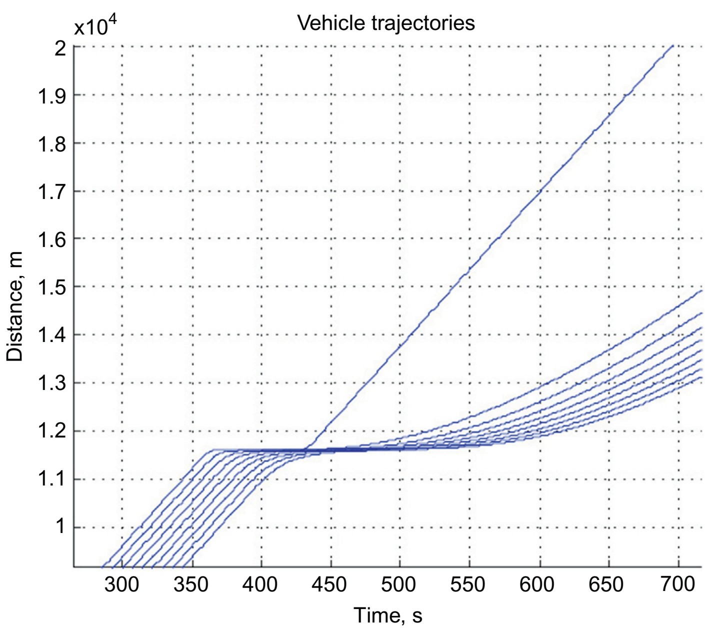

Figure 14.6 GM4 slow start.

one would dare follow another 1 inch apart at 120 km/h! The reason why GM4 allows such a ridiculous car-following distance is because, regardless of how close the two vehicles are, the model predicts no response as long as the two vehicles are moving at the same speed.

PROBLEMS

1. Prove that if one chooses $m = 0$ and $l = 0$, GM5 integrates to the Pipes/Forbes model.

2. Prove that if one chooses $m = 0$ and $l = 1$, GM5 integrates to the Greenberg model.

3. Prove that if one chooses $m = 0$ and $l = 2$, GM5 integrates to the Greenshields model.

4. Prove that if one chooses $m = 1$ and $l = 2$, GM5 integrates to the Underwood model.

5. Prove that if one chooses $m = 1$ and $l = 3$, GM5 integrates to the model of Drake et al. (Northwestern model).

6. Prove that if one chooses $m = 0$ and $l = n + 1$, GM5 integrates to the Pipes-Munjal model.

7. Prove that if one chooses $m = 0$ and $l = n + 1.5$, GM5 integrates to the Drew model.

8. Perform a one-step simulation based on the following conditions: Two cars are traveling in the same lane on a freeway. The length of both vehicles is $l_{i-1} = l_i = 6\,\text{m}$. Lane change is not considered in this problem. At time t, the leading vehicle $i - 1$ is traveling at a speed of $\dot{x}_{i-1}(t) = 72\,\text{km/h}$ and the following vehicle i is traveling at a speed of $\dot{x}_i(t) = 108\,\text{km/h}$. The spacing between the two vehicles (measured from front bumper to front bumper) is $s_i(t) = 40\,\text{m}$. The perception-reaction time of the following driver is $\tau_i = 1.5\,\text{s}$.

   a. Use GM1 to predict the deceleration that the following driver will adopt after a perception-reaction time. Assume the sensitivity factor is 0.5 1/s .

   b. Use GM2 to predict the acceleration that the following driver will adopt after a perception-reaction time.

   c. Use common sense to decide which sensitivity factor to use.

   d. Use GM3 to predict the acceleration that the following driver will adopt after a perception-reaction time. Assume the sensitivity factor is 10 m/s .

e. Use GM4 to predict the acceleration that the following driver will adopt after a perception-reaction time. Assume vehicles keep their speeds unchanged until after a perception-reaction time and the sensitivity factor is 0.5 .

f. Use GM5 to predict the acceleration that the following driver will adopt after a perception-reaction time. Assume vehicles keep their speeds unchanged until after a perception-reaction time, a sensitivity factor α of 0.5, $l = 2$, and $m = 2$.