Homework: Traffic Flow

2.4, 2.7, 3.1

Hayden Atchley

12 September 2023

2.4

Over the course of 2 minutes, 5 vehicles passed a point with speeds of 30, 45, 20, 36 and 40 km/h. The flow q is given by $q = N/T$, where N is the number of vehicles and T the total time. Therefore, the flow is $q = 5/2 = 2.5$ vehicles per minute or $q = 5/2 * 60 = 150$ vehicles per hour. The time-mean speed v_t is the arithmetic mean of the spot speeds, or $v_t = 34.2$ km/h. The space-mean speed v_s is the harmonic mean of the spot speeds, or $v_s = 31.58$ km/h. The time-mean speed is greater.

2.7

Figure 1 shows the output of a 6-foot long loop detector. The x-axis indicates seconds, and the numbers above each bump indicate the on and off times in 1/60 of a second (e.g. `32-45` indicates that the detector turned on at 32/60 seconds and off at 45/60 seconds).

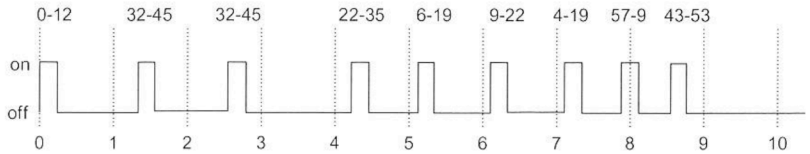

Figure 1: Loop detector on/off data.

(a) There were 9 vehicles over the 10-second observation window, so the hourly flow rate is $9/10 * 3600 = 3240$ vehicles per hour.

Table 1: Vehicle Detector Times

Vehicle Number	Detector On Time (s)	Detector Off Time (s)	Total Activated Time
1	0.00	0.20	0.20
2	1.53	1.75	0.22
3	2.53	2.75	0.22
4	4.37	4.58	0.22
5	5.10	5.32	0.22
6	6.15	6.37	0.22
7	7.07	7.32	0.25
8	7.95	8.15	0.20
9	8.72	8.88	0.17

(b) Occupancy is defined as the ratio of detector "on" time to the total observation time: $o = \frac{\sum_{i=1}^{N}(\xi_i = t_i^{\text{OFF}} - t_i^{\text{ON}})}{T}$. Table 1 gives the on and off times of each vehicle (converted to seconds), and the total time each vehicle triggered the detector. From this, the occupancy is determined to be $\frac{\sum \xi_i = 1.9}{T=10} = 0.19$

(c) The speed is given by the length of the detector and vehicle divided by the time the detector was active, i.e. $\dot{x} = \frac{l_{car}+l_{loop}}{\xi}$. **?@tbl-speeds** shows this information. From here, v_t and v_s are calculated as before, so $v_t = 68.53$ mph and $v_s = 67.82$ mph.

(d) The relationship between density (k), flow (q), and speed (v) is given by definition as $q = k \times v_s$. This requires, however, an accurate v_s, which from point data such as this detector requires an assumption about vehicle length. We were given a uniform vehicle length of 15 feet, so this relationship will hold. The density is therefore $k = q/v_s = 3240/67.82 = 47.77$ vehicles per mile.

(e) Estimating the speed from the $q = k \times v_s$ relationship will give an average speed of 67.82 mph. This is consistent with the space-mean-speed, since that is the speed used in part (d) (and the speed used in the relationship equation above).

3.1

?@fig-time-space shows a time-space diagram for several vehicles. The shaded area shows an area of interest, bounded by $t = 5$s and $t = 15$s, and $x = 0.2$km and $x = 0.5$km.

(a) Denoting the vehicle whose path most closely intersects the origin as i, the vehicle in front of i as $i+1$, and the vehicle behind i as $i-1$, the vehicles that traverse the bounding box are $i + 5$ through $i - 4$, inclusive.