CHAPTER 23

The Unified Diagram

Using the field theory and the longitudinal control model presented in the previous two chapters as a framework, one can conveniently relate traffic flow models to each other. As such, a unified perspective can be cast on traffic flow modeling with bridges not only within but also between microscopic and macroscopic levels.[1]

23.1 MOTIVATION

Half a century ago, Newell [58] proposed a nonlinear car-following model of the following form:

$$\dot{x}_i(t+\tau_i) = v_i\left(1 - \mathrm{e}^{-\frac{\lambda_i}{v_i}(s_{ij}(t)-l_i)}\right), \tag{23.1}$$

where $\dot{x}_i(t)$ is the speed of the vehicle with ID i at time t, τ_i is driver i's perception-reaction time, v_i is driver i's desired speed, λ_i is a parameter associated with driver i (i.e., the slope of driver i's speed-spacing curve evaluated at $\dot{x}_i = 0$), s_{ij} is the spacing between vehicle i and its leader bearing ID j, and l_i is the minimum value of s_{ij}. Newell acknowledged that "no motivation for this choice is proposed other than the claim that it has approximately the correct shape and is reasonably simple."

It would be interesting to interpret the Newell model and furnish it with a possible motivation (this section). In doing so, we find that the interpretation gives rise to a broader picture that can be used to relate some existing traffic flow models to each other (Section 23.2). As such, a unified perspective can be cast on traffic flow modeling with bridges not only within but also between microscopic and macroscopic levels (Section 23.3).

Without further delay, the Newell model can be slightly rearranged as follows:

$$1 - \frac{\dot{x}_i(t+\tau_i)}{v_i} - \mathrm{e}^{\frac{l_i - s_{ij}(t)}{v_i/\lambda_i}} = 0 \tag{23.2}$$

[1] This chapter is reproduced from Ref. [129].

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The above equation is, in turn, a special case of the following equation when vehicle i's acceleration $\ddot{x}_i$ is zero:

$$\ddot{x}_i(t + \tau_i) = g_i \left[1 - \frac{\dot{x}_i(t)}{v_i} - e^{\frac{l_i - s_{ij}(t)}{v_i/\lambda_i}} \right], \tag{23.3}$$

where g_i is a positive, nonzero parameter associated with vehicle i. Equation (23.3) is a *dynamic* car-following model which describes the acceleration performance of vehicle i, whereas Equation (23.2) is a *steady-state* version of the dynamic model since the former describes the speed choice of driver i in the steady state—that is, when acceleration is not considered ($\ddot{x}_i = 0$).

Steady-state and dynamic car-following models are both widely applied and successful in microscopic traffic flow simulation. However, dynamic models appear to be more desirable in modeling driver operational control (e.g., car following) if the following two issues are concerned. The first pertains to human factors. Though a driver may have a speed choice in mind (e.g., "I wish to travel at 113 km/h (or 70 miles per hour)"), such a goal is achieved over time, during which time the driver's operational control at each instant is based on acceleration (e.g., "I need to speed up or slow down in order to get to the target speed"), which naturally results from the driver's operation on the gas and brake pedals. The second pertains to physics. The acceleration of an object can change abruptly, whereas its speed profile has to be smooth. For example, when a driver steps on the brake pedal and keeps the foot there to bring the vehicle to a stop, a deceleration is constantly applied until the vehicle stops, at which moment the deceleration suddenly disappears. In contrast, the speed profile of the vehicle has to be smooth—that is, starting from its initial speed and continuously decreasing to zero. As another example, when a subject vehicle is being cut off from its leader (because of a third vehicle squeezing in between), the sudden change of spacing may result in a steady-state model model to suggest an unattainable speed in response (at which point, an extra, *dynamic* constraint on limiting acceleration has to be introduced which exceeds the scope of steady-state modeling). In contrast, a dynamic model works directly on acceleration, and even though limiting acceleration may be involved, it is still within the scope of dynamic modeling.

We can further rearrange Equation (23.3) as follows by multiplying both sides by vehicle mass m_i:

$$m_i \ddot{x}_i(t + \tau_i) = m_i g_i - m_i g_i \frac{\dot{x}_i(t)}{v_i} - m_i g_i e^{\frac{l_i - s_{ij}(t)}{v_i/\lambda_i}}. \tag{23.4}$$

One immediately recognizes that the above equation takes the form of Newton's second law of motion:

$$\sum F_i = G_i - R_i - F_i^j, \tag{23.5}$$

where $\sum F_i = m_i\ddot{x}_i(t+\tau_i)$, $G_i = m_i g_i$, $R_i = G_i\frac{\dot{x}_i(t)}{v_i}$, and $F_i^j = G_i e^{\frac{l_i - s_{ij}(t)}{v_i/\lambda_i}}$. Therefore, Equation (23.4) can be interpreted as an application of Newton's second law of motion in driver operational control. The acceleration of a driver–vehicle unit i is the result of "forces" acting on the unit, and these forces can be further interpreted as follows. The term G_i functions as the driving force, which is analogous to the gravity and is determined as the product of vehicle mass m_i and the acceleration of roadway gravity g_i. The term R_i is like a resistance: the faster the vehicle travels, the greater the resistance is. In addition, the resistance balances the gravity when the driver's desired speed is achieved. The term F_i^j can be interpreted as a repelling (vehicle interaction) force from leading vehicle j depending on the spacing s_{ij} between the two vehicles. Since this is a noncontact force, it is an action at a distance as if it were mediated by a "field."

23.2 A BROADER PERSPECTIVE

Extending the above discussion, we find it appropriate to interpret the driver's operational control using the concept of a field. More specifically, the driving environment perceived by a driver can be represented as a field, in which objects (such as roadways and other vehicles) are each represented as a component field and their superposition represents the overall hazard that the subject driver tries to avoid. Hence, the aim of modeling of vehicle motion is to seek the least hazardous route by navigating through the field along its valley. A field theory of such a nature was introduced in Chapter 21. Only major results of the field theory are reproduced below for easy reference.

23.2.1 Overview of the Field Theory

The generic form of the field theory is

$$\begin{cases} m_i\ddot{x}_i(t+\tau_i) &= \gamma_i^0[G_i(t) - R_i(t)] + \gamma(\alpha_i^j)\frac{\partial U_{i,x}}{\partial x}, \\ m_i\ddot{y}_i(t+\tau_i) &= -\gamma(\alpha_i^k)\frac{\partial U_{i,y}}{\partial y}. \end{cases} \tag{23.6}$$

Readers are referred to Chapter 21 for the derivation of the results and the notation presented in this subsection. A special case of the theory, which is referred to as the longitudinal control model, can take the following form after some simplifications:

$$\ddot{x}_i(t+\tau_i) = g_i\left[1 - \left(\frac{\dot{x}_i(t)}{v_i}\right)^{\delta} - \mathrm{e}^{\frac{s_{ij}(t)^* - s_{ij}(t)}{Z}}\right] \tag{23.7}$$

or

$$\ddot{x}_i(t+\tau_i) = g_i\left[1 - \left(\frac{\dot{x}_i(t)}{v_i}\right) - \mathrm{e}^{\frac{s_{ij}(t)^* - s_{ij}(t)}{s_{ij}(t)^*}}\right] \tag{23.8}$$

if one chooses $\delta = 1$ and $Z = s_{ij}(t)^*$. The desired spacing $s_{ij}(t)^*$ is motivated by safety rules and can take many forms, of which two examples are provided:

$$s^*_{ij}(t) = \dot{x}_i(t)\tau_i + l_j, \tag{23.9}$$

$$s^*_{ij}(t) = \frac{\dot{x}_i^2(t)}{2b_i} + \dot{x}_i(t)\tau_i - \frac{\dot{x}_j^2(t)}{2B_j} + l_j. \tag{23.10}$$

Aggregating Equation 23.8 over vehicles by assuming steady-state conditions yields the following equilibrium speed-density relationship:

$$v = v_{\mathrm{f}}\,[1 - \mathrm{e}^{1-\frac{k^*}{k}}]. \tag{23.11}$$

A more specific form is

$$v = v_{\mathrm{f}}\left[1 - \mathrm{e}^{1-\frac{1}{k(\gamma v^2+\tau v+l)}}\right] \tag{23.12}$$

or

$$k = \frac{1}{(\gamma v^2 + \tau v + l)\left[1 - \ln\left(1 - \frac{v}{v_{\mathrm{f}}}\right)\right]}. \tag{23.13}$$

23.2.2 Relating Microscopic Car-Following Models

Following the rationale in Section 23.1, it turns out that the field theory can be used as a framework to relate traffic flow models to each other at both the microscopic level and the macroscopic level.

Newell Model

We return to the Newell nonlinear car-following model. Comparison of Equations 23.1 and 23.7 reveals that the former results if one chooses to (1) apply the steady-state condition—that is, $\ddot{x}_i(t+\tau_i) = 0$; (2) set $Z = v_i/\lambda_i$; (3) let $s^*_{ij}(t) = l_i$; and (4) use $\dot{x}_i(t)$ as the response variable and apply a time delay τ_i.

Further, the physical meaning of parameter λ_i in Equation 23.1 is the tangent of the speed-spacing curve (Figure 1 in Newell's original paper) evaluated when the speed is zero. This parameter can be interpreted as the reciprocal of the perception-reaction time (i.e., $\lambda_i = 1/\tau_i$) as implied by Newell's Figure 1 and the numerical values in his Figure 2). In contrast, this tangent is evaluated as $1/(\tau + l/v_f)$ in the longitudinal control model. With this understanding, the vehicle interaction force F_i^j suggested by the Newell model can be interpreted as the negative exponential of the gap $(s_{ij}(t) - l_i)$ between the subject vehicle i and its leader j scaled down by the distance $(v_i\tau_i)$ traversed by vehicle i at the desired speed v_i during one perception-reaction time τ_i.

The field theory is related to other microscopic car-following models as follows.

Forbes Model

The Forbes model [53, 54, 66] is based on the following safety rule: the time gap between a vehicle and its leader should always be equal to or greater than the reaction time τ_i. This model can be admitted into the longitudinal control model (Equation 21.15) as a means to determine the *desired spacing* $s_{ij}^*(t)$, which is formulated in Equation 23.9.

General Motors Models

The family of General Motors models (GM models) [55, 56] is generically formulated in its fifth model (GM5):

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i^m(t+\tau_i)[\dot{x}_j(t) - \dot{x}_i(t)]}{[x_j(t) - x_i(t)]^l}. \tag{23.14}$$

If one chooses $m = l = 1$, Equation 23.14 reduces to the fourth-generation GM model (GM4):

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i(t+\tau_i)[\dot{x}_j(t) - \dot{x}_i(t)]}{[x_j(t) - x_i(t)]}, \tag{23.15}$$

where x_i, $\dot{x}_i$, $\ddot{x}_i$, and τ_i are the displacement, speed, acceleration, and perception-reaction time of the subject vehicle i, respectively; similar notation applies to its leader j; α is a dimensionless coefficient. In relation to the field theory, GM4 considers only the vehicle interaction force F_i^j and ignores the unsatisfied desire for mobility $(G_i - R_i)$ (see Chapter 21 for details). Rather than translating intrusion exponentially to vehicle interaction force as in the longitudinal control model (Equation 21.15), F_i^j

in GM4 mimics Coulomb's law in electrostatics. More specifically, GM4 views vehicle i as a particle which carries a moving coordinate with electric charge equivalent to its speed $\dot{x}_i$ and vehicle j as another particle which moves relative to vehicle i with charge equivalent to their relative speed $[\dot{x}_j(t) - \dot{x}_i(t)]$. The magnitude of the interaction force is proportional to the product of the two charges and inversely proportional to their distance. According to Equation 23.15, vehicle i is attracted to (or repelled by) vehicle j if the latter travels faster (or slower) than the former.

Gipps Model

The Gipps model [57] consists of a system of two inequalities with one governing the free-flow regime and the other governing the car-following regime.

The free-flow inequality reproduced below is a result of fitting empirical observations, and its function is to accelerate a vehicle from its initial speed asymptotically toward its desired speed without oscillation:

$$\dot{x}_i(t+\tau_i) = \dot{x}_i(t) + 2.5 g_i \tau_i \left(1 - \frac{\dot{x}_i(t)}{v_i}\right)\sqrt{0.025 + \frac{\dot{x}_i(t)}{v_i}}. \tag{23.16}$$

We can rewrite the above equation in the following differential form after considering the time difference τ:

$$\ddot{x}_i(t+\tau_i) \approx \frac{\dot{x}_i(t+\tau_i) - \dot{x}_i(t)}{\tau_i} = g_i' \left(1 - \frac{\dot{x}_i(t)}{v_i}\right), \tag{23.17}$$

where $g_i' = 2.5 g_i \sqrt{0.025 + \frac{\dot{x}_i(t)}{v_i}}$. Note that Equation 23.17 is actually the *unsatisfied desire for mobility* term in Equation 21.15 when the vehicle interaction term disappears.

The Gipps car-following model is derived from the following safety rule: at any moment, a driver i should leave sufficient distance behind the leader j such that driver i has enough room to respond and decelerate at a rate of $b_i > 0$ to a safe stop behind j should the leader apply an emergency brake ($B_j > 0$). The scenario is illustrated in Figure 23.1 and the model reproduced below:

$$s_{ij}^*(t) \geq \frac{\dot{x}_i(t+\tau_i)^2}{2b_i} + \frac{\tau_i}{2}[\dot{x}_i(t) + \dot{x}_i(t+\tau_i)] + \dot{x}_i(t+\tau_i)\theta - \frac{\dot{x}_j^2}{2B_j} + L. \tag{23.18}$$

The astute reader has recognized that the above model describes the *desired spacing*, which follows exactly the same safety rule used to derive

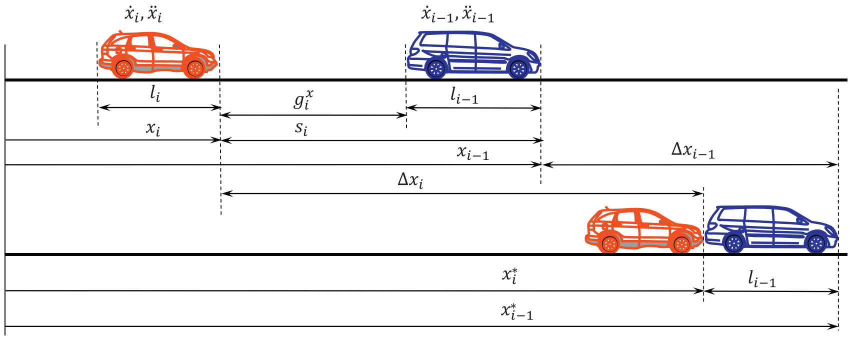

Figure 23.1 The Gipps model.

Equation 23.10, which is slightly modified from and simpler than the above model. Of course, one could opt to use this model in place of Equation 23.10 to apply the longitudinal control model.

Note that the Gipps model has been identified as "overly safe" because of the rather conservative safety rule and the additional safety margin $\dot{x}_i(t+\tau_i)\theta$. Consequently, excessive car-following distances result, and the model significantly underestimates highway capacity. In reality, though these safety measures make sense, drivers tend to use them as a good rule of thumb but frequently follow other vehicles closer than the desired spacing.

Intelligent Driver Model

The intelligent driver model (IDM) [60, 61] is expressed as a superposition of the follower i's acceleration term and a deceleration term which depends on the desired spacing s_{ij}^*:

$$\ddot{x}_i(t+\tau_i) = g_i\left[1 - \left(\frac{\dot{x}_i}{v_i}\right)^\delta - (\frac{s_{ij}^*}{s_{ij}})^2\right], \tag{23.19}$$

where δ is the acceleration exponent, $s_{ij} = x_j - x_i$ is the spacing between vehicle i and its leader j, and the desired spacing s_{ij}^* is a function of speed $\dot{x}_i$ and relative speed $(\dot{x}_i - \dot{x}_j)$: $s_{ij}^* = s_0 + s_1\sqrt{\dot{x}_i/v_i} + T_i\dot{x}_i + \dot{x}_i[\dot{x}_i - \dot{x}_j]/[2\sqrt{g_i b_i}]$, where s_0, s_1, b_i, and T_i are parameters. Compared with Equation 21.15, the IDM strikingly resembles the longitudinal control model. From the perspective of the field theory, the IDM relates the interaction F_i^j between vehicle i and its leader j to the squared ratio of the desired spacing s_{ij}^* to the actual spacing s_{ij}. In addition, the IDM has its own safety rule to determine s_{ij}^* which is conveniently admittable to the longitudinal control model.

Van Aerde Model

The Van Aerde car-following model [62, 63] combines the Pipes model [52] and the Greenshields model [9] into a single equation:

$$s_{ij} = c_1 + c_3\dot{x}_i + c_2/(v_f - \dot{x}_i), \tag{23.20}$$

where $c_1 = v_{\mathrm{f}}(2v_{\mathrm{m}} - v_{\mathrm{f}})/(k_{\mathrm{j}}v_{\mathrm{m}}^2)$, $c_2 = v_{\mathrm{f}}(v_{\mathrm{f}} - v_{\mathrm{m}})^2/(k_{\mathrm{j}}v_{\mathrm{m}}^2)$, $c_3 = 1/q_{\mathrm{m}} - v_{\mathrm{f}}/(k_{\mathrm{j}}v_{\mathrm{m}}^2)$, where v_{f} is the free-flow speed of the roadway facility, k_{j} is the jam density, and v_{m} is the optimal speed occurring at capacity q_{m}.

The Van Aerde model constitutes yet another safety rule which can be related to the longitudinal control model as the desired spacing s_{ij}^*.

CARSIM Model

The CARSIM model [65] consists of a set of acceleration algorithms (reproduced below to be consistent in notation):

A1: Vehicle i is moving but has not yet reached its desired speed v_i. Depending on vehicle i's initial speed and the urgency of the task, the acceleration rate is found by entering the data in Tables 1 and 2 in Ref. [65].

A2: Vehicle i has reached its desired speed v_i. No specific algorithm is provided except that the driver will try to reach v_i as fast as possible while satisfying all safety and operational constraints.

A3: Vehicle i was stopped and has to start from standstill. A maximum acceleration rate is applied constrained by a noncollision requirement after a response delay τ_i.

A4: Vehicle i is in car-following mode with its leader j. A_4 is determined by satisfying the following safety rule: vehicle i should leave a nonnegative gap ($s_{ij} - l_j \geq 0$) from leader j should vehicle i be advanced one time increment Δt: $s_{ij}(t) = x_j(t) - x_i(t + \Delta t) \geq l_j$, where $x_i(t + \Delta t) = x_i(t) + \dot{x}_i\Delta t - 0.5A_4\Delta t^2$ and the other variables are as defined before.

A5: Vehicle i in car-following mode is subject to a noncollision constraint which is reinforced by considering the desired spacing: $s_{ij}^*(t) = x_j(t) - x_i(t+\Delta t) \geq \max\{\dot{x}_i(t+\Delta t)\tau_i + l_j \text{or} \dot{x}_i(t+\Delta t)\tau_i + [\dot{x}_i(t+\Delta t)]^2/(2B_i) - [\dot{x}_j(t)]^2/(2B_j) + l_j\}$, where $\dot{x}_i(t + \Delta t) = \dot{x}_i(t) + A_5\Delta t$, and B_i and B_j are the maximum deceleration rate of i and j, respectively. The astute reader immediately recognizes that the first choice of the right-hand side follows the rationale of the Forbes model [53, 54, 66] and the second choice is similar to that of the Gipps model [57] if driver i is willing to apply an emergency brake (i.e., $b_i = B_i$) as well.

The CARSIM model is compatible with the longitudinal control model. A3 results when $\dot{x}_i$ is set to zero in Equation 21.15. A1 is obtained when the vehicle interaction term (i.e., F_i^j) becomes zero. As vehicle i speeds up, Equation 21.15 predicts that the actual acceleration decreases, which is reflected in lookup Tables 1 and 2 in Refs. [65]. A3 is found when $\dot{x}_i$ is equal to v_i in Equation 21.15. A4 and A5 are related to the longitudinal control model through safety rules which are the same in both models except for a slight implementation difference.

Psychophysical Model

The model developed by Wiedemann [64] is a typical psychophysical model whose principle is depicted in Figure 23.2. The rough curve ABCDEF is a trajectory of the vehicle operation condition in the Δx–$\Delta\dot{x}$ plane. Δx is the spacing between the subject vehicle i and its leader j—that is, $\Delta x = s_{ij}$—and $\Delta\dot{x}$ is their speed difference $\dot{x}_i - \dot{x}_j$. Starting with point A, vehicle i moves freely if it is not impeded by leader j, which is slower but far ahead. Hence, $\Delta\dot{x}$ remains approximately constant and Δx keeps decreasing. The free-flow state continues up to point B, where the trajectory intersects the

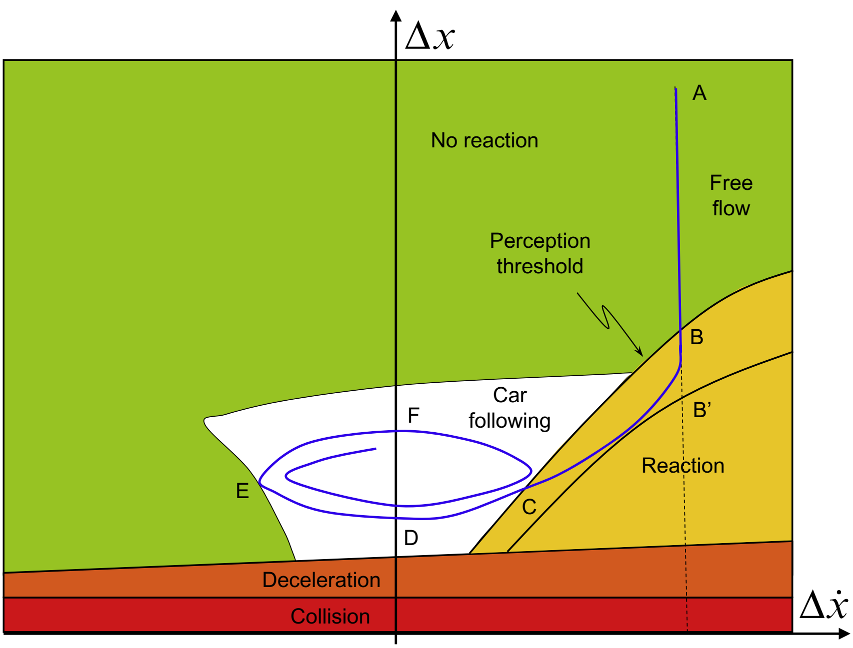

Figure 23.2 The Wiedmann model.

perception threshold. After point B, vehicle i begins to approach vehicle j. In response, driver i reduces his or her speed while Δx keeps decreasing. The approaching regime continues up to point C, where the two vehicles become sufficiently close and their speed difference is small. After point C, the two vehicles are in the car-following regime. As driver i tries to adapt to vehicle j's speed, the gap closes. Driver i stops decelerating when the two vehicles are moving at the same speed and their distance remains constant. If driver i temporarily loses attention (e.g., talking on a cell phone) and slows down, the gap begins to open until the driver realizes that he or she is falling behind. Consequently, the driver tries to catch up, and hence the gap closes again. If driver i overshoots, he or she may be reminded to back up again. Therefore, the trajectory of driver i oscillates within a unconscious reaction region (the white region) in the Δx–$\Delta \dot{x}$ plane.

Though the psychophysical model is not directly contained in or derived from the field theory as the above-mentioned models are, the effect of the former can be reproduced by the latter. For example, the follower i is in the free-flow regime when the leader j is far ahead. As i moves close to j, the former will ride up onto j's potential field, and hence perceive a repelling force F_i^j. This signifies the beginning of the approaching regime. As F_i^j increases, $\dot{x}_i$ adapts to $\dot{x}_j$. Sooner or later, i will find an equilibrium position around the desired spacing s_{ij}^* where the unsatisfied desire for mobility balances the vehicle interaction force. At this point, vehicle i enters the car-following regime. As i's directional responsiveness (i.e., γ_i) drifts over time, the vehicle may oscillate around the equilibrium position unconsciously, as predicted by Equation 23.6.

Rule-Based Model

The model developed by Kosonen [67] is a representative of rule-based models, and it is reproduced below to be consistent in notation:

1. `NO SPEED CHANGE`

   `Keep the present speed (default case).`

2. `ACCELERATE IF` [$\dot{x}_i < v_i$] `and` [$t - t_{\text{last}} > T_{\text{acc}}(\dot{x}_i)$]

   `The current speed` $\dot{x}_i$ `is less than the desired speed` v_i `and the time elapsed since the last acceleration` t_{last} `is more than` T_{acc}.

3. `NO ACCELERATION IF` [$s_{ij} < s_{\min}(\dot{x}_i, \dot{x}_j) + w_{\text{stab}}(\dot{x}_i, \dot{x}_j)$]

   `The distance from obstacle` s_{ij} `is less than the minimum safe distance` $s_{\min}$ `plus the width of the stable area` w_{stab}.

4. `SLOW DOWN IF` [$s_{ij} < s_{\min}(\dot{x}_i, \dot{x}_j)$]

   `The distance from the obstacle` D_{obs} `is less than the minimum safe distance` $s_{\min}$.

5. `DO NOT SLOW DOWN IF` [$\dot{x}_i < \dot{x}_j$] `or` [$t - t_{\text{last}} < T_{\text{maxdec}}$]

   `Own speed is less than obstacle speed or maximum deceleration rate is exceeded.`

6. `GOTO ZERO IF` [$s_{ij} < 0$] `and (Obstacle = physical)`

   `Distance to physical obstacle is below zero (i.e., collision).`

At each time step, the motion of vehicle i is checked against the above rules one by one. A later rule always supersedes any earlier rules should there be a conflict. Similarly to the situation in the psychophysical model, the above rule-based model is not directly contained in or derived from the field theory. However, the effect of the rule-based model can be reproduced as well if one is willing to fuzzify the field theory. For example, after fuzzification and discretization, the desired spacing s^*_{ij} can be decomposed into two portions $s_{\min}$ and w_{stab} (see Figure 23.3) to mimic the original setup in Ref. [67]. Therefore, vehicle i does nothing by default if it has reached its desired speed and the road is free (i.e., rule 1 above). If vehicle i's desire for mobility has not been fully satisfied (i.e., $\dot{x}_i < v_i$), it will accelerate (rule 2). If i approaches j and is within w_{stab}, i will not accelerate (rule 3). Vehicle i needs to decelerate if it intrudes into $s_{\min}$ (rule 4). There is no need for i to decelerate if it becomes slower than j (rule 5). Vehicle i will stop if it collides with j, which is ensured by the steep potential field when the vehicles touch (rule 6).

23.2.3 Relating Macroscopic Equilibrium Models

Under equilibrium condition, vehicles move in a uniform manner and hence lose their identifies: $\tau_i \to \tau$, $\dot{x}_i = \dot{x}_j \to v$, $v_i = v_j \to v_f$, $\ddot{x}_i = \ddot{x}_j = 0$, $s_{ij} \to s = \frac{1}{k}$, $s^*_{ij} \to s^* = \frac{1}{k^*}$, $b_i = b_j \to b$, $B_i = B_j \to B$, and $l_i = l_j \to \frac{1}{k_j}$, where the right arrow means "aggregate to" and items before the arrow are

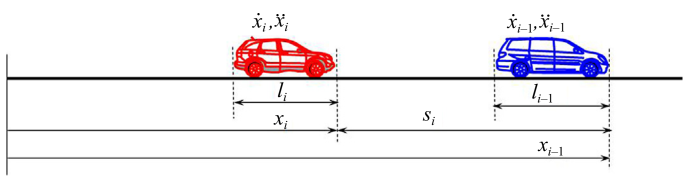

Figure 23.3 The Kosonen model.

microscopic variables and those after the arrow are macroscopic variables. With this notation, the microscopic longitudinal control model (Equation 21.15) translates to its macroscopic counterpart (Equation 23.11 or more specifically Equations 23.12 and 23.13), which depicts the equilibrium speed-density relationship. The field theory and the longitudinal control model are related to existing equilibrium models as follows.

Newell Model (Macroscopic)

If we apply the above notation, the Newell car-following model translates to its macroscopic counterpart of the following form:

$$v = v_{\mathrm{f}}[1 - \mathrm{e}^{\frac{\lambda}{v_{\mathrm{f}}}\frac{1}{k_{\mathrm{j}}}(1-\frac{k_{\mathrm{j}}}{k})}]. \tag{23.21}$$

Notice the close resemblance between Equations 23.21 and 23.11. In addition, through its microscopic counterpart, the above model's connection to the longitudinal control model was discussed in Section 23.2.2.

Van Aerde Model (Macroscopic)

The equilibrium counterpart of the Van Aerde model can be written as

$$k = \frac{1}{c_1 + c_3 v + c_2/(v_{\mathrm{f}} - v)} m \tag{23.22}$$

where all variables are as defined before. Through its microscopic counterpart, the above model is connected to the longitudinal control model as discussed in Section 23.2.2.

IDM (Macroscopic)

Under equilibrium conditions, a special macroscopic case was derived from the IDM [60, 61]:

$$v = \frac{(s-L)^2}{2v_{\mathrm{f}}T^2}\left[-1 + \sqrt{1 + \frac{4T^2 v_{\mathrm{f}}^2}{(s-L)^2}}\right], \tag{23.23}$$

where T is average safe time headway and $s = 1/k$ is average spacing, where k is traffic density.

Pipes-Munjal Model

The Pipes-Munjal model [15] takes the following form:

$$v = v_{\mathrm{f}}\left[1 - \left(\frac{k}{k_{\mathrm{j}}}\right)^n\right], \tag{23.24}$$

where n is a coefficient and the other variables are as defined before. In the reverse direction (i.e., from macroscopic to microscopic[1]), the above model seems to suggest a microscopic basis of roughly the following form:

$$\ddot{x}_i = g_i \left[1 - \frac{\dot{x}_i}{v_i} - \left(\frac{l}{s_{ij}} \right)^n \right]. \tag{23.25}$$

Note that the microscopic basis may take many other forms and the above form is only one of the possibilities. With the above equation, it becomes clear that the Pipes-Munjal model can be derived from the field theory if one chooses the vehicle interaction force F_i^j of the form $(\frac{L}{s_{ij}})^n$. A similar technique can be applied to other equilibrium models in an effort to restore their microscopic basis from the perspective of the field theory.

Drew Model

The Drew model [13] takes the following form:

$$v = v_{\mathrm{f}} \left[1 - \left(\frac{k}{k_{\mathrm{j}}} \right)^{n+\frac{1}{2}} \right], \tag{23.26}$$

where all variables are as defined before. If we repeat the above technique and replace n with $n + \frac{1}{2}$, the suggested microscopic basis is

$$\ddot{x}_i = g_i \left[1 - \frac{\dot{x}_i}{v_i} - \left(\frac{l}{s_{ij}} \right)^{n+\frac{1}{2}} \right], \tag{23.27}$$

which we can derive from the field theory by choosing $F_i^j = g_i(\frac{l}{s_{ij}})^{n+\frac{1}{2}}$.

Model of Wang et al.

Wang et al. [130] recently proposed a stochastic equilibrium model whose three-parameter deterministic version takes the form

$$v = \frac{v_{\mathrm{f}}}{1 + \mathrm{e}^{\frac{k - k_{\mathrm{c}}}{\theta}}}, \tag{23.28}$$

where k_{c} is the critical density (i.e., the density after which speed drop becomes noticeable as density increases from 0 to k_{j}) and θ is a coefficient. The microscopic basis of the model could be

[1] The same technique was used to derive the Van Aerde car-following model (microscopic) from the Greenshields model (macroscopic).

$$\ddot{x}_i = g_i\left[1 - \frac{\dot{x}_i}{v_i} - \left(1 - \frac{1}{1 + \mathrm{e}^{\frac{1}{\theta}\left(\frac{1}{s_{ij}} - \frac{1}{s^{\mathrm{c}}}\right)}}\right)\right] \tag{23.29}$$

where $s^{\mathrm{c}} = 1/k_{\mathrm{c}}$ is the critical spacing (i.e., average spacing at critical density). According to the field theory, one need only choose $F_i^j = g_i[1 - 1/(1 + \mathrm{e}^{\frac{1}{\theta}(\frac{1}{s_{ij}} - \frac{1}{s^{\mathrm{c}}})})]$ to obtain the model of Wang et al.

Model of del Castillo and Benítez

Del Castillo and Benítez [118, 131] proposed a family of exponential generating functions which can be represented as

$$f(\lambda) = \mathrm{e}^{1-(1+\frac{\lambda}{n})^n}, \tag{23.30}$$

where λ is called the "equivalent spacing," which is a function of density k, and n is a parameter. Setting $n = 1$ and $n \to \infty$ results in the following two special cases, respectively:

$$v = v_{\mathrm{f}}\left[1 - \mathrm{e}^{\frac{|C_{\mathrm{j}}|}{v_{\mathrm{f}}}(1-\frac{k_{\mathrm{j}}}{k})}\right] \tag{23.31}$$

and

$$v = v_{\mathrm{f}}\left[1 - \mathrm{e}^{1-\mathrm{e}^{\frac{|C_{\mathrm{j}}|}{v_{\mathrm{f}}}\left(\frac{k_{\mathrm{j}}}{k}-1\right)}}\right], \tag{23.32}$$

where C_{j} is the kinematic wave speed at the jam density and the other variables are as defined before. Equation 23.32 is referred to as the "maximum sensitivity curve." Equation 23.31 takes a form similar to the Newell model and the longitudinal control model. If one chooses $|C_{\mathrm{j}}| = \lambda/k_{\mathrm{j}}$, Equation 23.31 becomes the Newell model, and hence is connected to the longitudinal control model. If the conjecture that $\lambda = 1/\tau$ is true, $|C_{\mathrm{j}}| = \lambda/k_{\mathrm{j}} = l/\tau$, which is the speed required to traverse a nominal vehicle length L (i.e., a vehicle length plus some buffer space) during one perception-reaction time τ. l typically ranges from 5 to 10 m and τ is around 1 s. This yields $|C_{\mathrm{j}}|$ around 5-10 m/s or 11-22 miles per hour, which agrees well with the numbers provided in Ref. [118].

Note that the above two special cases are derived from the exponential family of speed-density curves, which represent a much broader set of models than the Newell model. In addition, the family of speed-density curves can be represented generically as

$$v = v_f[1 - e^{\psi(k)}], \tag{23.33}$$

where $\psi(k)$ is a generic function and admits the corresponding terms in Equations 23.31 and 23.32. From the perspective of the field theory, the model of del Castillo and Benítez seems to suggest a vehicle interaction force F_i^j proportional to $e^{\psi(1/s_{ij})}$.

GM-associated Models

In addition, the family of equilibrium models, including the models of Greenshields [9], Greenberg [10], Underwood [11], and Drake et al. [12], which are associated with GM models, was discussed in Chapter 14.

23.3 THE UNIFIED DIAGRAM

To summarize the discussion above, a unified perspective can be cast on these traffic flow models. Such a perspective is presented as a diagram in Figure 23.4.

23.3.1 Description of the Unified Diagram

The diagram consists of three panes. The left pane contains *picoscopic* models, which are able to represent vehicle motion in longitudinal x, lateral y, and vertical z directions on a three-dimensional surface. The field theory formulated in Equation 23.6 belongs to this category. The middle pane has *microscopic* car-following models, which describe only vehicle motion in the longitudinal x direction. In this category, models which describe vehicle motion based on acceleration are grouped as "dynamic" models, such as GM models, while those describing vehicle motion based on speed choices are grouped as "steady-state" models, such as the Newell model. The right pane includes *macroscopic* models, which describe equilibrium speed-density relationships. The connecting lines show which models are related. The numbers on these lines, which are explained below, indicate where the bridges between models are discussed in the text. For example, connection 10 indicates the relation between the microscopic longitudinal control model (Equation 21.15) and the Newell nonlinear car-following model (Equation 23.1).

23.3.2 Connections in the Unified Diagram

This subsection refers the connection numbers in Figure 23.4 to the proper locations in the text where the nature of these connections is discussed.

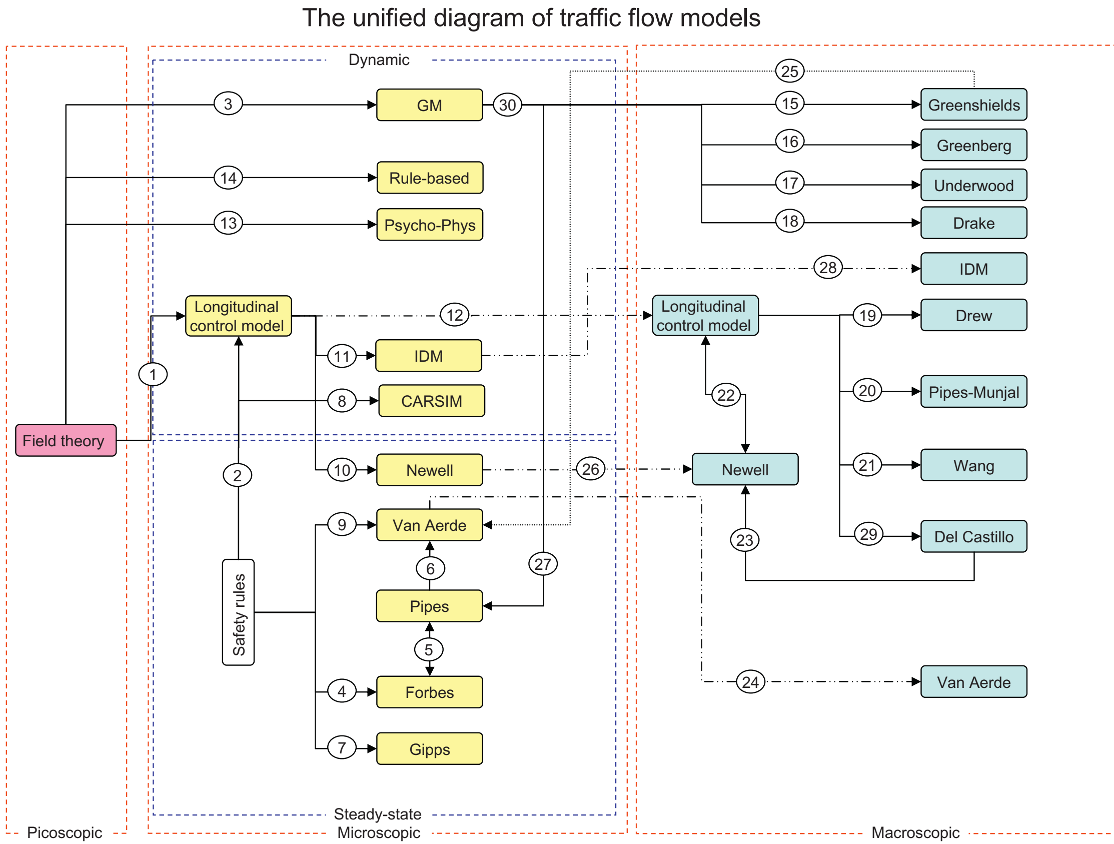

Figure 23.4 The united diagram.

- 1: See Section 23.2.1 for the longitudinal control model as a special case of the field theory.
- 2: See Section 23.2.1 for safety rules being admitted into the longitudinal control model.
- 3: See Section 23.2.2 for GM models as a special case of the field theory.
- 4: See Sections 23.2.1 and 23.2.2 for the Forbes model as a safety rule.
- 5: See Chapter 14 for the equivalence between the Pipes model and the Forbes model.
- 6: See Section 23.2.2 for the Pipes model being admitted into the Van Aerde model.
- 7: See Sections 23.2.2 and § 23.2.1 for the Gipps model as a safety rule.
- 8: See Section 23.2.2 for the relation between the CARSIM model and the longitudinal control model as well as the safety rule in the CARSIM model.
- 9: See Section 23.2.2 for the Van Aerde model as a safety rule.
- 10: See Sections 23.1 and 23.2.2 for the Newell model as a special case of the longitudinal control model.
- 11: See Section 23.2.2 for the relation between the IDM and the longitudinal control model.
- 12: See Section 23.2.1 for the derivation of the macroscopic counterpart of the longitudinal control model.
- 13: See Section 23.2.2 for the relation between the psychophysical model and the longitudinal control model.
- 14: See Section 23.2.2 for the relation between the rule-based model and the longitudinal control model.
- 15: See Chapter 14 for the Greenshields model being derived from GM5.
- 16: See Chapter 14 for the Greenberg model being derived from GM5.
- 17: See Chapter 14 for the Underwood model being derived from GM5.
- 18: See Chapter 14 for the model of Drake et al. being derived from GM5.
- 19: See Section 23.2.3 for the relation between the model of Drew et al. and the longitudinal control model.
- 20: See Section 23.2.3 for the relation between the Pipes-Munjal model and the longitudinal control model.
- 21: See Section 23.2.3 for the relation between the model of Wang et al. and the longitudinal control model.
- 22: See Section 23.2.3 for the close resemblance between the Newell model (macroscopic) and the longitudinal control model.

- 23: See Section 23.2.3 for the equivalence between the Newell model (macroscopic) and one of the special cases derived from del Castillo and Benítez's family of exponential generating functions.
- 24: See [62, 63] for microscopic and macroscopic versions of the Van Aerde model.
- 25: See Section 23.2.2 for the Greenshields model being admitted into the Van Aerde model.
- 26: See Section 23.2.3 for the derivation of the macroscopic counterpart of the Newell car-following model.
- 27: See Chapter 14 for the Pipes model being derived from GM5.
- 28: See Section 23.2.3 for the macroscopic IDM being derived from its microscopic counterpart.
- #29: See § 23.2.3 for how the Field Theory is related to Del Castillo model.
- #30: See Chapter 14 for how May's original unifying effort fits into the larger Unified Diagram.

23.4 SUMMARY

Motivated by Newell's untold secret in his nonlinear car-following model and May's original unifying effort depicted in Figure 6.6 in Ref. [17], a broader unified perspective was cast on traffic flow modeling and a larger unified diagram was constructed.

The Newell model [58], after being rearranged slightly, gives rise to a mechanics model which involves noncontact forces, which, in turn, can be explained conveniently using the concept of a field. The field theory of this nature was presented in Chapter 21, and was concisely reproduced here for easy reference. With use of the field theory as a framework, existing traffic flow models can be related to each other, thereby providing a unified perspective to examine the coherence among these models.

Microscopic car-following models are related to the field theory by varation of its components, such as vehicle interaction force, desired spacing (via safety rules), and directional responsiveness. Even though some models are not directly contained in or derived from the field theory, their effects can be reproduced from the latter. When aggregated, many of these car-following models reduce to their macroscopic counterparts—that is, equilibrium speed-density relationships. Those macroscopic equilibrium models, which do not come with a proposed microscopic basis, fortunately contain information to deduce their microscopic nature, though which the connection to the field theory might be established.

To summarize the above analysis, a unified diagram was constructed which gathers together traffic flow models at the picoscopic, microscopic, and macroscopic levels with lines connecting related models. Each connection is denoted by a number which points to the discussion of the connection in this book.

PROBLEMS

1. Microscopic car-following models come in different flavors. One type of car-following model uses acceleration as the driver's control variable, and this sounds like instructing the driver to "speed up or slow down by x m/s^2 next." We call them *dynamic* models. Another type of car-following model employs speed as the control variable, which suggests that the driver ought to "bring the speed to y km/h next." We call them *steady-state* models. A third type of car-following model works on the position of the vehicle, which translates to asking the driver to "move your vehicle to position z m next." We call them *static* models. Search the car-following models and provide at least one example for each type.
2. When proposing his nonlinear car-following model, Newell acknowledged that "no motivation for this choice is proposed other than the claim that it has approximately the correct shape and is reasonably simple." Give your opinion on this modeling philosophy and discuss whether or not it is a good one to follow.
3. Many macroscopic equilibrium models have been proposed, and their flexibility to fit field observations varies depending partially on the number of parameters they employ. Provide at least three example equilibrium models that employ
   - **a.** two parameters,
   - **b.** three parameters, and
   - **c.** four parameters.