

## CHAPTER 20

# Vehicle Modeling

Picoscopic modeling requires explicit models for vehicles that are separated from driver models. As an attempt in this direction, this chapter is devoted to the modeling of individual vehicle dynamics, using a driver's desired acceleration, deceleration, and steering as inputs to determine vehicle dynamic responses, including longitudinal acceleration, lateral acceleration, and yaw velocity. The vehicle model derived herein is called the dynamic interactive vehicle (DIV) model.<sup>1</sup>

### 20.1 OVERVIEW OF THE DIV MODEL

In automotive engineering, there is a wealth of literature discussing dynamic vehicle models. These models typically come with many degrees of freedom and high modeling fidelity. Typical to these models are their applications in vehicle design, handling, and stability, involving one or a few vehicles. Our interest is a dynamic vehicle model which is well suited for the simulation of a network of vehicles. Such an application involves a large number of interacting vehicles, yet demands a modeling fidelity beyond the microscopic level. On this note, those vehicle models in automotive engineering are overqualified given their complexity and high computation costs. Therefore, a DIV model with high computational efficiency and reasonable modeling fidelity is desirable.

The DIV model will be capable of accepting three inputs from its driver: throttle position, brake pedal position, and steering angle. The model will relate each input to a particular driver's desire and represent the desire on a scale of 0 to 1 for the throttle and brake positions, and on a scale from  $-1$  to  $1$  for the steering angle. Each of these inputs will then play a role in vehicle dynamics to produce vehicle motion. The following subsections present how the DIV model incorporates essential components of vehicle dynamics in order to faithfully model its motion. These components include the engine, the braking system, and the steering mechanism. Details of

<sup>1</sup>This chapter is reproduced from [106].

how the DIV model will account for effects due to rolling resistance, air resistance, and gravity are also presented in the following subsections.

## 20.2 MODELING LONGITUDINAL MOVEMENT

Forces in the longitudinal direction of the DIV model include the forces due to the engine and the braking system, rolling and aerodynamic resistances, and the force due to gravity. The equation of motion for such a vehicle can be derived by use of Newton's second law of motion:

$$\sum F = m\ddot{x} = F_e - F_b - R_a - R_r - R_g, \quad (20.1)$$

where  $m$  is the mass of the vehicle (kg),  $\ddot{x}$  is vehicle acceleration ( $\text{m/s}^2$ ),  $F_e$  is the tractive force produced by the engine (N),  $F_b$  is the force produced by the brake (N),  $R_a$  is aerodynamic resistance (N),  $R_r$  is rolling resistance (N), and  $R_g$  is grade resistance (N).

### 20.2.1 Modeling Acceleration Performance

The engine plays an important role in vehicle acceleration performance. Here we adopt the engine model recommended in Ref. [72] where engine power  $P$  and torque  $\Gamma$  are functions of engine speed  $\omega$ :

$$\Gamma = \Gamma_{\max} - \frac{P_{\max}}{2\omega_p^2(\omega_p - \omega_t)}(\omega - \omega_t)^2, \quad (20.2)$$

$$P = \frac{P_{\max}}{2\omega_p^2}(3\omega_p - \omega_t)\omega - \frac{P_{\max}}{2\omega_p^2(\omega_p - \omega_t)}(\omega - \omega_t)^2\omega, \quad (20.3)$$

where  $P_{\max}$  is the maximum engine power achieved at engine speed  $\omega_p$  and  $\Gamma_{\max}$  is the maximum engine torque achieved at engine speed  $\omega_t$ . The four parameters for a specific vehicle are publicly available on the Internet. This model automatically guarantees that  $\Gamma = \frac{P}{\omega}$ .

Using the torque being delivered to the wheel, we can calculate the engine force  $F_e$  produced by the engine to promote vehicle motion with the aid of the appropriate final transmission gear ratio  $N_{\text{fit}}$ , wheel radius  $r$ , and mechanical efficiency of the driveline  $\zeta$ :

$$F_e = \frac{\Gamma N_{\text{fit}} \zeta}{r}. \quad (20.4)$$

### 20.2.2 Modeling Braking Performance

The brake system is represented by equating the force applied to the brake pedal by the driver to the corresponding deceleration of the vehicle. This means of representing the braking ability of a vehicle is as a result of the work presented in Ref. [107]. The objective of this study was to define the brake characteristics within the space bounded by the relationship between brake pedal force and vehicle deceleration, which will lead to acceptable driver-vehicle performance. In essence, this study determined ergonomic properties for brake pedals that would give drivers the most effective control [108]. Therefore, using the results from this study, the DIV model will be able to account not only for the braking performance of the vehicle but also the manner in which the driver interacts with the brake system.

The results of the aforementioned study include several linear relationships which describe the force being applied to the brake pedal and the rate of deceleration of the vehicle. From these relationships, the DIV model will use the proportionality constant to provide optimal pedal force gain. This proportionality constant, 0.021 g/lb, corresponds to the maximum deceleration rate through minimal pedal force. Using this proportionality constant, we will use the following formulation in the DIV model to represent the brake system of a vehicle and the driver's interaction with that system:

$$F_b = d_b p_f W, \quad (20.5)$$

where  $F_b$  is brake force (N),  $d_b$  is the driver's desire to brake (0-1), and  $p_f$  is the pedal-force gain coefficient.

### 20.2.3 Modeling Aerodynamic Drag

Aerodynamic drag is another force that retards the motion of a vehicle. This force is dependent on atmospheric conditions, the frontal area of the vehicle,  $A_f$ , and the velocity at which the vehicle is traveling relative to the wind,  $v_r$ . The equation below further describes aerodynamic drag:

$$R_a = \frac{\rho}{2} C_D A_f v_r^2, \quad (20.6)$$

where  $\rho$  is the mass density of air (1.2041 kg/m<sup>3</sup>) and  $C_D$  is the coefficient of aerodynamic resistance.

### 20.2.4 Modeling Grade Resistance

The force due to gravity is mainly experienced when the vehicle is on an incline. The force due to gravity that is acting on the vehicle is calculated as

$$F_g = W \sin \theta \approx W \tan \theta = WG, \quad (20.7)$$

where  $\theta$  is the angle of the incline in radians and  $G$  is the grade of the incline with positive sign for upgrade and negative sign for downgrade.

## 20.3 MODELING LATERAL MOVEMENT

The structure used to represent the movement of the DIV model in the  $X$ - $Y$  plane was adapted from Ref. [109], which included the formulation of a kinematics framework and a dynamic framework to model a vehicle's motion in a two-dimensional space. The kinematics framework that was presented in Ref. [109] was chosen for the DIV model for two primary reasons: (1) all the pertinent dynamic properties of the vehicle have already been accounted for by other means in the DIV model, and (2) the ease of use with an accurate  $X$ - $Y$  position representation.

At the base of the kinematics framework for the two-dimensional representation of vehicle motion is the treatment of the vehicle as a nonholonomic system, which is a system whose state depends on the path taken in order to achieve it. In addition, nonholonomic constraints are employed under the assumption that there is no slippage at the wheels during a turn. The assumption that there is no slippage at the wheels is predominantly applicable to instances of high-speed cornering, as wheel slippage at low speeds is negligible. The general form of the nonholonomic constraint may be represented as

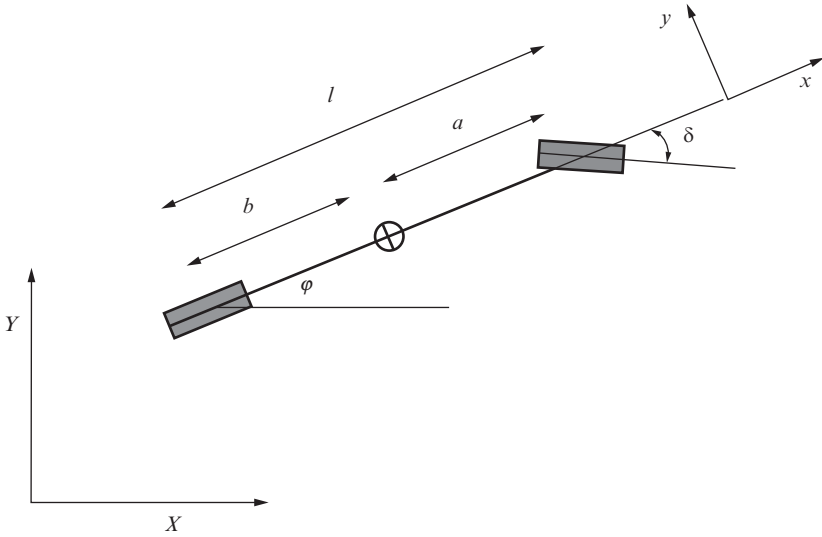
$$\dot{x} \sin(\phi) - \dot{y} \cos(\phi) = 0, \quad (20.8)$$

where  $\dot{x}$  and  $\dot{y}$  represent the velocities in the  $x$  and  $y$  directions of the vehicle coordinate system and  $\phi$  is the vehicle orientation with respect to the global  $X$ - $Y$  coordinate system. See Figure 20.1 for an illustration of the coordinate system being used and also for the definition of the variables that will be used in the development of the DIV model.

After a few more iterations of Equation 20.8, the velocity of the center of gravity with respect to the global coordinate system is defined as

$$\dot{X} = \dot{x} \cos(\phi) - \dot{y} \sin(\phi) \quad (20.9)$$

$$\dot{Y} = \dot{x} \sin(\phi) + \dot{y} \cos(\phi) \quad (20.10)$$



**Figure 20.1** DIV model in the  $X$ - $Y$  plane.

With Equations 20.9 and 20.10, the global position of the vehicle can now be determined. However, before these equations can be used, lateral velocity,  $\dot{y}$ , has to be defined. The definition of the Ackerman angle,  $\delta$ , also has to be introduced as this is the parameter that is responsible for changing the orientation of the vehicle.

$$\dot{y} = \dot{\phi}b \quad (20.11)$$

$$\dot{\phi} = \frac{\tan(\delta)}{l} \dot{x}, \quad (20.12)$$

and

$$\delta = d_t \frac{\pi N_s}{r_s}, \quad (20.13)$$

where  $d_t$  is the driver's desire to turn ( $-1$  to  $1$ ),  $N_s$  is the number of steering wheel revolutions, and  $r_s$  is the steer ratio (ratio of radians turned to the Ackerman angle)

## 20.4 MODEL CALIBRATION AND VALIDATION

A key feature of the DIV model is that it is meant to be easily calibrated. The calibration of the DIV model will entail the user providing the model with a few performance specifications of the vehicle being modeled. These

specifications will be assessable as they are available to the public via car manufactures and various organizations that offer tools to research a myriad of vehicles—for example, Cars.com. The vehicle performance specifications that the DIV model requires include the aerodynamic resistance coefficient, engine displacement, gear ratios, steer ratio, and the vehicle dimensions.

In addition to these specifications, the model also has a few variables relating to the environment that impact vehicle motion, including wind speed and the gradient of the roadway. Once the values of the vehicle performance specifications and the various values describing the surrounding environment have been entered into the DIV model, it will be able to replicate the motion of the vehicle.

In the validation of the DIV model, three standard performance tests were used to determine whether or not the DIV model is capable of successfully replicating the movement of the vehicle. These tests are typically conducted on vehicles to determine their capabilities of accelerating, braking, and handling. To test vehicle acceleration, the time for a vehicle to go from rest to 97 km/h (60 miles per hour) is recorded, as is the time it takes a vehicle to cover 402 m (a quarter of a mile). The Federal Motor Carrier Safety Administration dictates maximum allowable stopping distances from various speeds that all vehicle manufacturers must satisfy, standardizing vehicle braking. Finally, to measure how well a vehicle handles, the diameter of the circle traced by the vehicle's outer front wheel with the maximum steering angle is recorded.

For details of model calibration and validation, see Ref. [106].

## PROBLEMS

1. Conduct an Internet search and find the following information about the 2016 Volvo XC90 engine:
  - a. final drive axle ratio
  - b. first gear ratio
  - c. sixth gear ratio
  - d. tire size
  - e. base curb weight
2. It is known that the vehicle speed  $v$  (m/s) is related to the engine speed  $\omega$  (revolutions per minute) as follows:

$$v = \frac{\pi r}{30N_{ft}} \omega, \quad (20.14)$$

where  $r$  is the tire radius in meters and  $N_{ft}$  is the final transmission gear ratio, which is the product of the axle ratio and the gear ratio. Use the information from the previous Internet search and assume the vehicle is cruising at 30 m/s in sixth gear, and find the corresponding engine speed.

3. When a vehicle is starting up, it needs the maximum torque to generate engine force. Assume first gear is used and half of the maximum torque is available at start-up. Calculate the corresponding engine force for a 2016 Volvo XC90 engine assuming the mechanical efficiency of the driveline is 80%.
4. A 2016 Volvo XC90 engine has a drag coefficient  $C_F$  of 0.32 and a frontal area  $A_f$  of  $2.79 \text{ m}^2$ , and is traveling at 100 km/h. How much aerodynamic drag results if the air density  $\rho$  is  $1.20 \text{ kg/m}^3$ ?
5. The above-mentioned vehicle is running up a hill with a grade  $G$  of 5%. Calculate the grade resistance acting on the vehicle.
6. Assume that the above-mentioned vehicle is subject only to aerodynamic drag and grade resistance. Calculate the maximum acceleration available at start-up.