Homework: GM Car Following

14.2, 14.8

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14.2

General Motors has developed several car following models. Of these, GM5 is given as in Equation 1.

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_i(t+\tau_i)]^m}{[x_{i-1}(t) - x_i(t)]^l} \times [\dot{x}_{i-1}(t) - \dot{x}_i(t)] \tag{1}$$

If $m = 0$ and $l = 1$, we can show that GM5 integrates to the Greenberg model (given by $v = v_m \ln(k_j/k)$):

$$\begin{aligned}
\ddot{x}_i(t+\tau_i) &= \alpha \frac{[\dot{x}_i(t+\tau_i)]^0}{[x_{i-1}(t) - x_i(t)]^1} \times [\dot{x}_{i-1}(t) - \dot{x}_i(t)] \\
&= \alpha \frac{1}{x_{i-1}(t) - x_i(t)} \times [\dot{x}_{i-1}(t) - \dot{x}_i(t)] \\
&= \alpha \frac{\dot{x}_{i-1}(t) - \dot{x}_i(t)}{x_{i-1}(t) - x_i(t)} \\
&= \alpha \frac{\Delta \dot{x}}{\Delta x} \\
\int \ddot{x}\ dt = \alpha \int \frac{\dot{x}}{x}\ dt &= \alpha \int \frac{dx/dt}{x}\ dt = \alpha \int \frac{dx}{x}.
\end{aligned}$$

Then, if v is flow speed and $k = 1/x$ is vehicle density:

$$\begin{aligned}
\int \ddot{x}\ dt &= \alpha \int \frac{dx}{x} \\
v &= \alpha \ln\left(\frac{1}{k}\right) + C.
\end{aligned}$$

If $v = 0$, then $k = k_j$, the jam density:

$$\begin{aligned}
0 &= \alpha \ln\left(\frac{1}{k_j}\right) + C \\
C &= -\alpha \ln\left(\frac{1}{k_j}\right) \\
C &= \alpha \ln(k_j).
\end{aligned}$$

This then gives us

$$\begin{aligned}
v &= \alpha \ln\left(\frac{1}{k}\right) + \alpha \ln(k_j) \\
&= \alpha \ln\left(\frac{k_j}{k}\right).
\end{aligned} \tag{2}$$

In order to find α, we must first derive the flow-density equation from Equation 2:

$$q = kv = k\alpha \ln\left(\frac{k_j}{k}\right).$$

q_m, the maximum flow rate, is found by maximizing this equation with respect to k (with k_m equal to the density at q_m):

$$\begin{aligned}
\frac{dq}{dk} = 0 &= \frac{d}{dk}\left[k\alpha \ln\left(\frac{k_j}{k}\right)\right] \\
0 &= \alpha \frac{d}{dk}[k \times (\ln k_j - \ln k)] \\
0 &= \alpha\left[\ln\left(\frac{k_j}{k_m}\right) + k_m \frac{-1}{k_m}\right] = \alpha\left[\ln\left(\frac{k_j}{k_m}\right) - 1\right].
\end{aligned}$$

Assuming $\alpha \neq 0$:

$$\begin{aligned}
0 &= \ln\left(\frac{k_j}{k_m}\right) - 1 \\
1 &= \ln\left(\frac{k_j}{k_m}\right) \\
k_m &= \frac{k_j}{e}.
\end{aligned}$$

Defining v_m as the speed at q_m, the above and Equation 2 gives us:

$$\begin{aligned}
v_m &= \alpha \ln\left(\frac{k_j}{k_m}\right) \\
v_m &= \alpha \ln\left(\frac{k_j}{k_j/e}\right) \\
v_m &= \alpha.
\end{aligned}$$

Then:

$$v = v_m \ln\left(\frac{k_j}{k}\right),$$

which is the Greenberg model.

14.8

a

Using GM1 ($\ddot{x}(t+\tau_i) = \alpha[\dot{x}_{i-1}(t) - \dot{x}_i(t)]$), with $\alpha = 0.51\ \mathrm{s}^{-1}$, $\dot{x}_{i-1}(t) = 72$ kph $= 20$ m/s, $\dot{x}_i(t) = 108$ kph $= 30$ m/s, and $\tau_i = 1.5$ s, the acceleration that driver i will adopt after a perception-reaction time is:

$$\begin{aligned}\ddot{x}(t+1.5) &= \frac{0.51}{\mathrm{s}}[30-20]\ \mathrm{m/s}\\ &= -5.1\mathrm{m/s}^2.\end{aligned}$$

b (and c)

Using GM2 ($\ddot{x}(t+\tau_i) = \begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}[\dot{x}_{i-1}(t) - \dot{x}_i(t)]$), with $\alpha = 0.74$ gives:

$$\begin{aligned}\ddot{x}(t+1.5) &= \frac{0.74}{\mathrm{s}}[30-20]\ \mathrm{m/s}\\ &= -7.4\mathrm{m/s}^2.\end{aligned}$$

d

Using GM3, with vehicle length $l_{i-1} = l_i = 6$ m, vehicle spacing (front bumper to front bumper) $s_i = 40$ m, and therefore $x_{i-1}(t) - x_i(t) = 40 - 6 = 34$ m, and $\alpha = 10$ m/s:

$$\begin{aligned}\ddot{x}(t+1.5) &= 10\frac{-10\ \mathrm{m/s}}{34\ \mathrm{m}}\\ &= -2.94\ \mathrm{m/s}^2\end{aligned}$$

e

GM4, with $\alpha = 0.5$:

$$\begin{aligned}\ddot{x}_i(t+1.5) &= 0.5 \times \frac{30 \times (20-30)}{34}\\ &= -4.41\ \mathrm{m/s}^2\end{aligned}$$

f

GM5, with $\alpha = 0.5$, $l = 2$, and $m = 2$:

$$\begin{aligned}\ddot{x}_i(t+1.5) &= 0.5 \times \frac{30^2 \times (20-30)}{34^2} \\ &= -3.89 \text{ m/s}^2\end{aligned}$$