CHAPTER 2

Traffic Flow Characteristics I

According to their reporting mechanisms, traffic sensors can be classified into three categories: mobile sensors, point sensors, and space sensors. A mobile sensor resides in a vehicle, moves along with the vehicle, and logs the location of this particular vehicle over time. A point sensor sits at a fixed location on a roadway, sees the passage of vehicles above or under it, and reports traffic data only at this particular location over time. A space sensor flies in the sky, observes traffic on a stretch of road, and records the positions of vehicles at an instant of time over this particular stretch of road. It is interesting to see what traffic data reported by these sensors look like and, further, how traffic flow characteristics are determined from these data.

2.1 MOBILE SENSOR DATA

Let us start with mobile sensors. If a vehicle is equipped with a global positioning system (GPS) device, the device can report the vehicle's position as time progresses. Since GPS signals typically come once every second (i.e., at a frequency of 1 Hz), the GPS data may look similar to the data in Table 2.1, where the vehicle's longitudinal x and lateral y displacements are relative to the vehicle's position at 09:00:00.

Figure 2.1 shows the scenario in which a vehicle (with an on-board GPS device and ID number i) is moving on a roadway (drawn on the left) and the associated time-space diagram (drawn on the right). Every circle represents a GPS reading (only x is shown). If one connects these circles, the **trajectory** of this vehicle is obtained—that is, the location of the vehicle as a function of time: $x_i = x_i(t)$. It is easy to calculate the **speed** of the vehicle, $\dot{x}_i$, from the GPS data as illustrated in Figure 2.1:

$$\dot{x}_i = \frac{\Delta x}{\Delta t}.$$

If the vehicle's trajectory is known and smooth, we can determine $\dot{x}_i$ by taking the first derivative of the trajectory:

$$\dot{x}_i = \frac{\mathrm{d}x}{\mathrm{d}t}.$$

Traffic Flow Theory
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Table 2.1 GPS data

Time	***x* (feet)**	***y* (feet)**	***x* (m)**	***y* (m)**
09:00:00	0	0	0.0	0.0
09:00:01	3	0	0.9	0.0
09:00:02	5	0	1.5	0.0
09:00:03	7	0	2.1	0.0
09:00:04	10	1	3.0	0.3
09:00:05	15	4	4.6	1.2
09:00:06	18	9	5.5	2.7
09:00:07	21	12	6.4	3.7
09:00:08	23	12	7.0	3.7
09:00:09	27	12	8.2	3.7
09:00:10	30	12	30	12

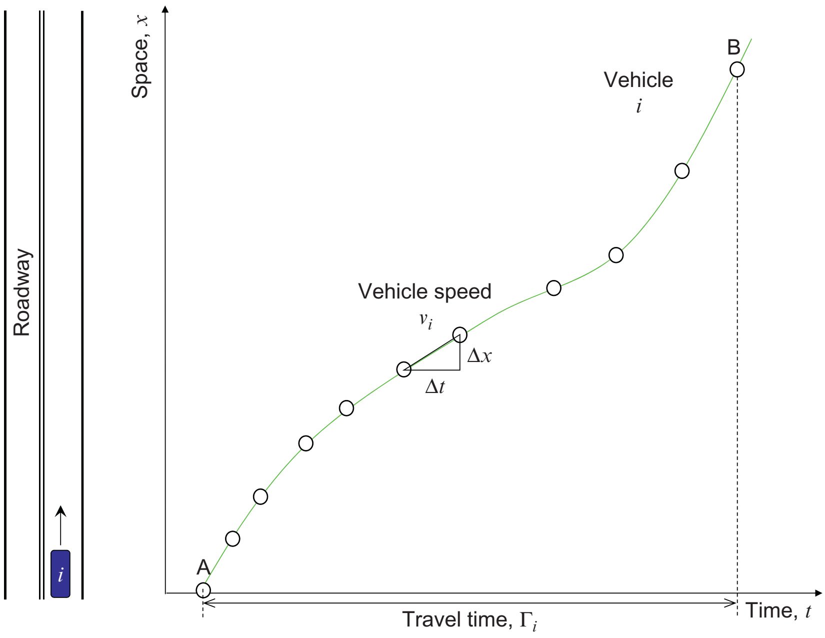


Figure 2.1 A trajectory.

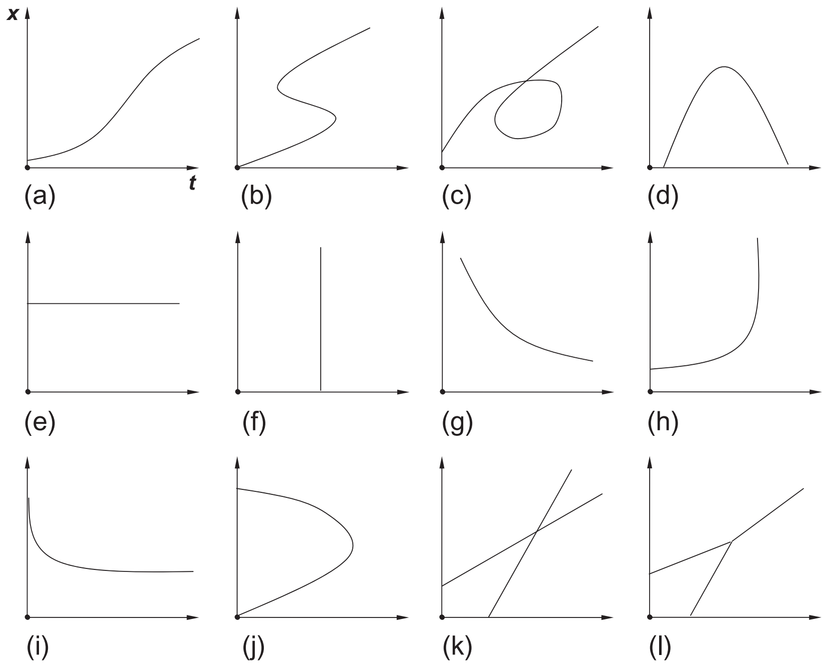


Figure 2.2 Vehicle trajectories.

The vehicle's **travel time**, t_i, between two points A and B can be directly read from the trajectory:

$$\Gamma_i = t_i^{\mathrm{B}} - t_i^{\mathrm{A}}.$$

Figure 2.2 illustrates some hypothetical vehicle trajectories, some of which are valid (i.e., trajectories that make sense), while some are not. Test yourself and see if you are able to identify which trajectories are valid and understand how these vehicles move. Plot (a) is valid, and it shows a vehicle moving in the positive x direction over time. Plot (b) is not a valid trajectory for the following reasons. If one draws a vertical line, it may intersect the trajectory several times. This means that at an instant of time the vehicle can appear at multiple locations simultaneously, which is impossible. For the same reason, plots (c) and (j) are not valid either. Plot (d) is valid, and the trajectory suggests that the vehicle first moves forward (i.e., in the positive x direction) and then, at some point in time, reverses. Plot (e) is valid, and simply suggests that the vehicle does not move (maybe parked). Plot (f) is impossible because it suggests an infinite speed (i.e., the tangent of the trajectory). Plot (g) is a valid since the vehicle just moves backward at a time-varying speed. Plot (h) is likely but very unusual because the vehicle first moves at reasonable speeds and then almost flies at the end. Plot (i) is valid, and the vehicle gradually comes to a stop. Plot (k) can be interpreted in two ways: one is a two-lane scenario where a fast vehicle overtakes a slow vehicle; the other is a one-lane scenario where the fast vehicle collides with the slow vehicle and they exchange momentum. Plot (l) suggests that a fast vehicle catches up with a slow vehicle and then they move as a single unit thereafter.

2.2 POINT SENSOR DATA

If a point sensor (such as a loop detector or a video camera) is installed on the road at location x, this sensor will be able to observe vehicles passing above or under it. In a time-space diagram as illustrated in Figure 2.3, each vehicle will be counted (e.g., the tick marks) at this location. For example, during an observation period T, a total of N vehicles are counted by the sensor. N is referred to as the **traffic count**, which can be converted to the hourly equivalent rate of flow (referred as "**flow**" q hereafter) as follows:

$$q = \frac{N}{T}.$$

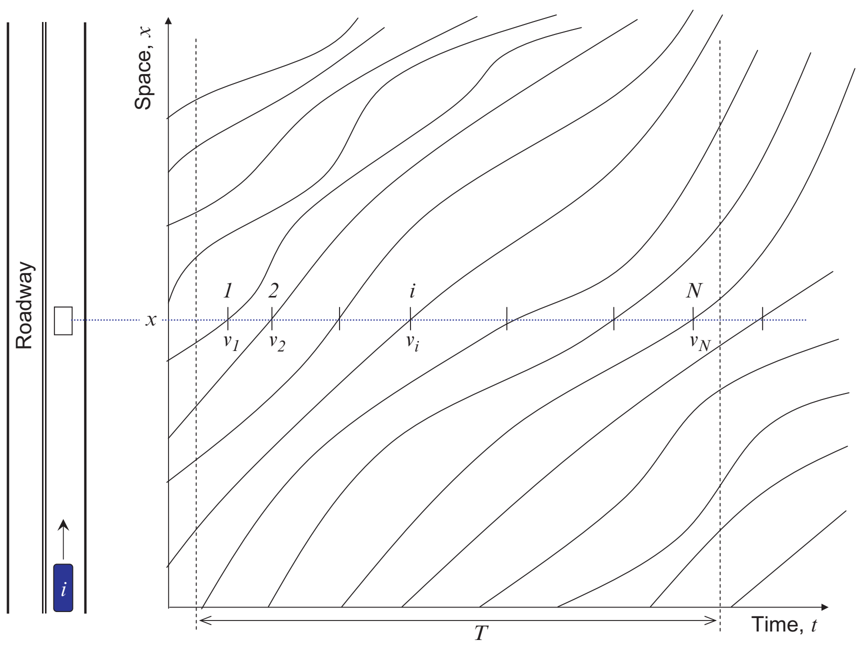


Figure 2.3 Point sensor data.

Headway h_i is defined as the temporal separation between two consecutive vehicles, and can be determined as:

$$h_i = t_i - t_{i-1}.$$

If one ignores the error due to incomplete headways of the first and last vehicles, the observation duration T can be expressed as

$$T = \sum_{i=1}^{N} h_i.$$

Both vehicles and point sensors have physical dimensions. If the sizes of the vehicles and sensors are taken into consideration, more information can be obtained from the time-space diagram (see Figure 2.4).

When a vehicle's front bumper enters the detection zone of a loop detector, a detection signal will be generated in the detector according to electromagnetism. When the vehicle's rear bumper exits the detection zone, the signal will drop. For an illustration of this effect, see the lower plot above

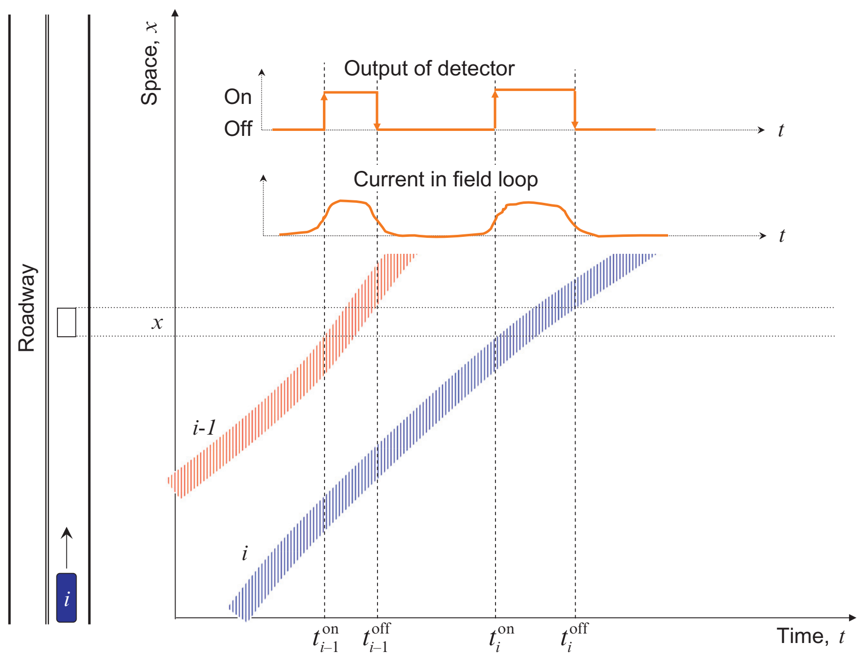


Figure 2.4 Loop detector data.

the two trajectories in Figure 2.4. If a threshold is set properly, the loop detector outputs two states: "on" when a vehicle is above the loop and "off" when the loop detects no vehicle over it. When the loop outputs "on," the loop is said to be "busy." With the above setup, let us revisit some of the traffic flow characteristics discussed above and determine more characteristics:

Traffic count N: Since the on state consists of an upward transition and a downward transition of the detector output, one need only count either the upward transition or the downward transition consistently over all vehicles in order to obtain the traffic count.

Headway h_i: If one chooses reference points on all vehicles consistently (e.g., front bumpers), the headway between vehicles $i-1$ and i can be calculated as $h_i = t_i^{\mathrm{on}} - t_{i-1}^{\mathrm{on}}$ and the time gap between them is $t_i^{\mathrm{on}} - t_{i-1}^{\mathrm{off}}$.

On time ξ_i: The duration from the moment when a vehicle's front bumper enters the detection zone to the moment when the vehicle's rear bumper exits the detection zone: $\xi_i = t_i^{\mathrm{off}} - t_i^{\mathrm{on}}$.

Vehicle speed $\dot{x}_i$: During the on time, vehicle i travels a distance of $d + l_i$ where d is the width of the loop (typically 6 feet or 1.8 m for small loops) and l_i is the length of the vehicle. Hence, the vehicle's instantaneous speed can be determined as

$$\dot{x}_i = \frac{d + l_i}{\xi_i} = \frac{d + l_i}{t_i^{\mathrm{off}} - t_i^{\mathrm{on}}}.$$

Occupancy o: In traffic flow theory, occupancy is defined as the percentage of time when a loop is busy—that is, when the loop detects vehicles above it. Hence, if the observation period is T, during which N vehicles are detected, the total on time is $\sum_{i=1}^{N} \xi_i$ and the occupancy is determined as

$$o = \frac{\sum_{i=1}^{N} \xi_i}{T}.$$

Time-mean speed v_{t}: If one averages vehicle speeds observed at a point of roadway, one obtains a mean speed in the time domain, and hence such a mean speed is termed "time-mean speed."

$$v_{\mathrm{t}} = \frac{1}{N} \sum_{i=1}^{N} \dot{x}_i \text{...in the time domain.}$$

Interested readers are referred to [1], where there is detailed discussion of how various traffic flow characteristics are measured and calculated as well as how errors inherent in point sensors are introduced.

2.3 SPACE SENSOR DATA

If one takes aerial photos of a roadway from a helicopter, one is able to locate vehicles in each of these snapshots. For example, Figure 2.5 illustrates a snapshot taken at time t where vehicles are labeled as triangles. Some space-related traffic flow characteristics can be determined from these aerial photos:

Spacing s_i is defined as the spatial separation between two consecutive vehicles and can be determined as

$$s_i = x_{i-1} - x_i.$$

Density k is defined as the number of vehicles observed on a unit length of road and can be determined as

$$k = \frac{N}{L},$$

where L is the length of the stretch of road under observation and N is number of vehicles observed on this stretch of road. If one ignores the

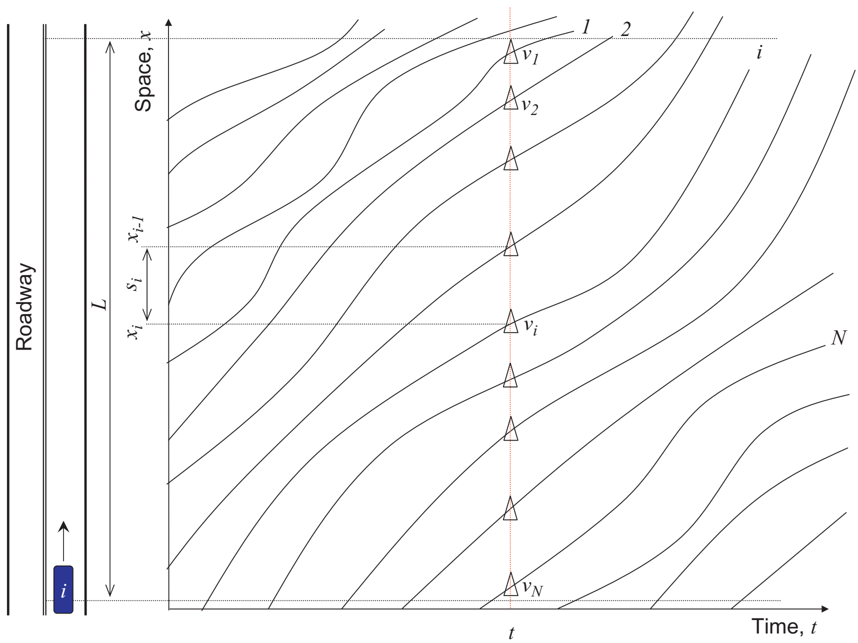


Figure 2.5 A snapshot of roadway.

error due to incomplete spacings of the first and last vehicles, the length of roadway L can be expressed as

$$L = \sum_{i=1}^{N} s_i.$$

Unfortunately, one is unable to determine the **vehicle speed** v_i from a single snapshot, but with two snapshots (at t_1 and t_2, respectively), one is able to compare vehicle locations and find the distance traversed by each vehicle—that is, $\Delta x_i = x_i(t_2) - x_i(t_1)$. Since the time between the two snapshots $\Delta t = t_2 - t_1$ is known, the speed of each vehicle can be determined accordingly:

$$v_i(t) = \frac{\Delta x_i}{\Delta t}.$$

Space-mean speed v_s: If one averages vehicle speeds obtained from aerial photos, a mean speed in the space domain results, and hence such a mean speed is termed the "space-mean speed."

$$v_s = \frac{1}{N} \sum_{i=1}^{N} \dot{x}_i \ldots \text{ in the space domain.}$$

2.4 TIME-SPACE DIAGRAM AND CHARACTERISTICS

The discussion so far has covered the three types of sensors (mobile, point, and space sensors), data reported by these sensors, and traffic flow characteristics determined with use of these data. It is informative to put everything together and form a complete picture. Figure 2.6 shows a time-space diagram with vehicle trajectories where data reported by the three types of sensors are illustrated.

Table 2.2 relates traffic flow characteristics to sensor types. Three categories of traffic flow characteristics are presented: flux, speed, and concentration. The characteristics are considered at two levels of detail: *microscopic* characteristics are vehicle specific and hence all bear subscript i, and *macroscopic* characteristics are aggregated measures and the aggregation can be done over vehicles, time, or space.

2.5 RELATIONSHIPS AMONG CHARACTERISTICS

So far, traffic flow characteristics have been introduced with the aid of field observations. It is interesting to investigate further the relationships among these traffic flow characteristics.

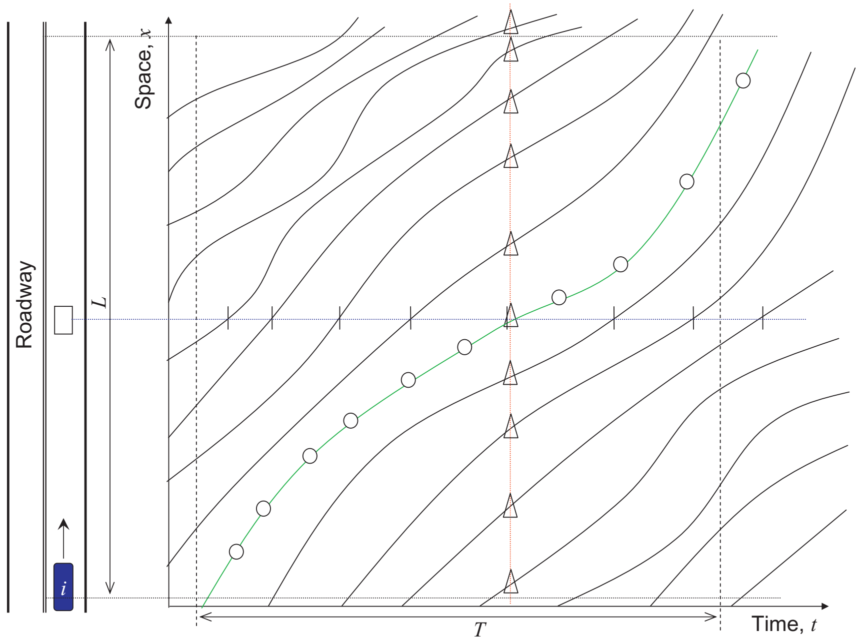


Figure 2.6 Time-space diagram and three types of sensors.

Table 2.2 Sensors and traffic flow characteristics

Category	Sensors	Microscopic characteristics	Macroscopic characteristics
	Mobile	–	–
Flux	Point	h_i	N, q
	Space	–	–
	Mobile	$\dot{x}_i$	–
Speed	Point	$\dot{x}_i$	v_t
	Space	$\dot{x}_i$	v_s
	Mobile	–	–
Concentration	Point	ξ_i	o
	Space	s_i	N, k

2.5.1 Flow, Speed, and Density

By definition, the following relationship holds as an *identity*:

$$q = k \times v_s;$$

that is, flow q is the product of density k and space-mean speed v_s.

2.5.2 Flow and Headway

From the above discussion, it follows that

$$q = \frac{N}{T},$$

$$T = \sum_{i=1}^{n} h_i,$$

$$q = \frac{N}{\sum_{i=1}^{n} h_i} = \frac{1}{\frac{1}{N}\sum_{i=1}^{n} h_i} = \frac{1}{h}.$$

Therefore, flow q is the reciprocal of average headway h. For example, a flow of 1200 vehicles per hour suggests an average headway of

$$\frac{1}{1200 \text{ vehicles per hour}} = \frac{3600\,\text{s/h}}{1200 \text{ vehicles per hour}} = 3\,\text{s}.$$

2.5.3 Density and Spacing

Similarly,

$$k = \frac{N}{L},$$

$$L = \sum_{i=1}^{n} s_i,$$

$$k = \frac{N}{\sum_{i=1}^{n} s_i} = \frac{1}{\frac{1}{N}\sum_{i=1}^{n} s_i} = \frac{1}{s}.$$

Therefore, density k is the reciprocal of average spacing s. For example, a density of 40 vehicles per mile (or 25 vehicles per kilometer) suggests an average spacing of

$$\frac{1}{40 \text{ vehicles per mile}} = \frac{5280 \text{ feet per mile}}{40 \text{ vehicles per mile}} = 132 \text{ feet}$$

or

$$\frac{1}{25 \text{ vehicles per kilometer}} = \frac{1000\,\text{m/km}}{25 \text{ vehicles per kilometer}} = 40\text{m}.$$

2.5.4 Time-Mean Speed and Space-Mean Speed

As discussed before, time-mean speed is vehicle speed averaged in the time domain, whereas space-mean speed is vehicle speed averaged in the space domain. Below we give an example that illustrates the difference between the two mean traffic speeds. Consider two lanes of traffic which is perfectly controlled so that there are only two streams of traffic: fast vehicles all travel at 60 miles per hour (or 96 km/h) in the inner lane and slow vehicles all move at 30 miles per hour (or 48 km/h) in the outer lane. Traffic flow in each lane is 1200 vehicles per hour, and lane change is prohibited. What is the time-mean speed and space-mean speed of traffic in both lanes?

Calculation of space-mean speed is straightforward, one simply averages the speed of vehicles observed on the road (see Figure 2.7). In 1 mile of the road, one observes a total of 60 vehicles, of which 20 vehicles are in the inner lane (1200 vehicles per hour/60 miles per hour) and 40 vehicles in the outer lane (1200 vehicles per hour/30 miles per hour). Therefore, space-mean speed is determined as

$$v_s = \frac{20 \times 60\,\text{mi/h} + 40 \times 30\,\text{mi/h}}{60} = 40\,\text{mi/h}.$$

Or in 1 km of the road, one observes a total of 37.5 vehicles, of which 12.5 vehicles are in the inner lane (1200 vehicles per hour/96 km/h) and 25 vehicles in the outer lane (1200 vehicles per hour/48 km/h). Therefore, space-mean speed is determined as

$$v_s = \frac{12.5 \times 96\,\text{km/h} + 25 \times 48\,\text{km/h}}{37.5} = 64\,\text{km/h}.$$

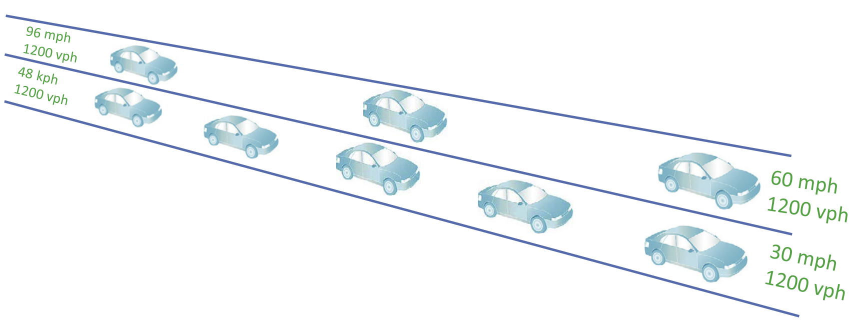


Figure 2.7 Time-mean speed versus space-mean speed. mph, miles per hour; vph, vehicles per hour.

For time-mean speed, one has to imagine a hypothetical observer standing at the roadside watching vehicles passing in front of him or her. As a result, the observer records 2400 vehicles in 1 h, of which 1200 vehicles are in the inner lane and 1200 vehicles are in the outer lane. Hence, by definition, time-mean speed is

$$v_\mathrm{t} = \frac{1200 \times 60\,\mathrm{mi/h} + 1200 \times 30\,\mathrm{mi/h}}{2400} = 45\,\mathrm{mi/h},$$

or

$$v_\mathrm{t} = \frac{1200 \times 96\,\mathrm{km/h} + 1200 \times 48\,\mathrm{km/h}}{2400} = 72\,\mathrm{km/h}.$$

Obviously, the results show that the two mean speeds are not equal. Wardrop [2] demonstrated that the following relationship between time-mean speed and space-mean speed always holds:

$$v_\mathrm{t} = v_\mathrm{s} + \frac{\sigma^2}{v_\mathrm{s}},$$

where σ is the variance of vehicle speeds. It can be seen that time-mean speed v_t is always greater than or equal to space-mean speed v_s and they are equal only if the traffic is uniform—that is, all vehicles are traveling at the same speed ($\sigma = 0$).

Note that, in the above example, fast vehicles are overrepresented in the time-mean speed, with a fast to slow ratio of 1:1, while in reality the correct ratio is 1:2, which is the case in the calculation of space-mean speed. It can be further demonstrated that the space-mean speed is an unbiased estimate of the true traffic mean speed, while the time-mean speed is not.

2.5.5 Occupancy and Density

The following is reproduced from Ref. [3, Chapter 2]:

$$\begin{aligned}
o &= \frac{1}{T}\sum_{i=1}^{N}\tau_i = \frac{1}{T}\sum_{i=1}^{N}\frac{d+l_i}{\dot{x}_i} \approx \frac{d+l}{T}\sum_{i=1}^{N}\frac{1}{\dot{x}_i}(\text{assume} l_i \to l)\\
&= (d+l)\frac{1}{T}\sum_{i=1}^{N}\frac{1}{\dot{x}_i} = (d+l)\left(\frac{N}{T}\right)\left(\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\dot{x}_i}\right) = (d+l)q\frac{1}{v_\mathrm{s}}\\
&= (d+l)k = c_k k.
\end{aligned}$$

The approximately equal sign is based on the assumption of uniform vehicle length, $l_i = l$. With such an assumption, occupancy o is proportional to density k, and the proportion coefficient c_k is the sum of loop width d and uniform vehicle length l.

2.6 DESIRED TRAFFIC FLOW CHARACTERISTICS

Control and optimization of traffic operations rely on an accurate understanding of traffic flow conditions, which in turn comes from field data collection.

Though the three types of sensors have their relative merits in terms of traffic data collection, they are practically very different, especially in terms of large-scale applications on a regular basis. Mobile sensors are not practical because not every vehicle is equipped with a GPS device. Though some vehicles may have GPS navigation systems or GPS-enabled cell phones, they are generally not intended for logging vehicle trajectories. Even if every vehicle had a GPS device and it were turned on to log vehicle trajectories, it would be prohibitive to make every driver comply with data extraction, let alone processing the data to generate a time-space diagram like that in Figure 2.6. Space sensors are not suitable for applications on a regular basis. Think about the cost of hiring a helicopter flying over a road to observe traffic 24 hours a day and 7 days a week, not to mention the complexity of and time spent extracting traffic data from the huge number of aerial photos. Therefore, the only type of sensor that is feasible for automatic, regular, and large-scale applications is a point sensor such as a loop detector or a video camera (see Chapter 1 for details).

Traffic flow characteristics are not equally attractive when traffic control and management is concerned. For example, space-mean speed is preferred over time-mean speed as an unbiased estimate of the true mean traffic speed. In addition, space-mean speed is required in the identity $q = k \times v_s$ to calculate density or flow. Density is preferred over occupancy as a measure of traffic concentration. For example, the *Highway Capacity Manual* uses density as the measure of effectiveness to determine the level of service on freeways and multilane highways.

Hence, we have the following dilemma. On the one hand, space-based traffic flow characteristics such as space-mean speed and density are preferred. Therefore, space sensors are called for to provide measures of these traffic flow characteristics. On the other hand, space sensors are prohibitive to deploy on a large scale on a regular basis, while point sensors are widespread (most intelligent transportation systems use point sensors), but report less attractive traffic flow characteristics such as time-mean speed and occupancy. Therefore, such a dilemma inevitably results in the estimation of space-based characteristics from point sensor data.

2.6.1 Determining Space-Mean Speed from Point Sensor Data

If individual vehicle speeds ($\dot{x}_i, i = 1, 2, \ldots, N$) are available from a point sensor, these speeds can be used to determine space-mean speed as follows:

$$v_\mathrm{s} = \frac{1}{\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\dot{x}_i}}.$$

This mean is called the harmonic mean, in contrast to the arithmetic mean, which is the time-mean speed:

$$v_\mathrm{t} = \frac{1}{N}\sum_{i=1}^{N}\dot{x}_i.$$

Unfortunately, many point sensor systems log only aggregated measures such as time-mean speed. In these systems, individual vehicle speeds are measured, but they are discarded after aggregation. As such, one has to resort to time-mean speed as a surrogate for space-mean speed, though one needs to recognize their difference, which might be considerable in some cases.

2.6.2 Determining Density from Point Sensor Data

Point sensor systems report occupancy, but not density. Using the above relationship between occupancy and density, one may be able to estimate density from occupancy:

$$k = \frac{o}{d + l},$$

though the reader must be cautioned about the implicit assumption of uniform vehicle length, which might be a strong one in some cases.

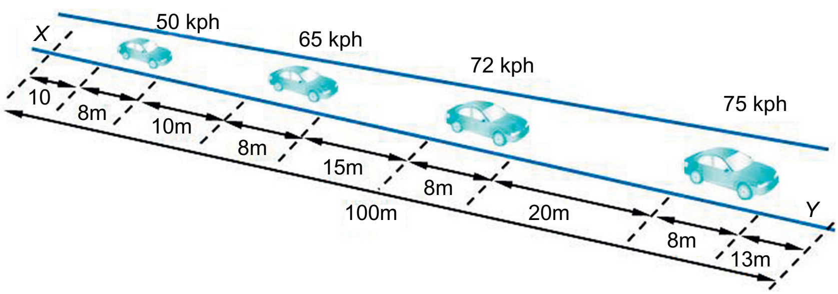
If a point sensor system time-stamps the passage of vehicles at two locations on the road with no vehicle appearing or exiting in between (e.g., a tunnel), it is possible to construct a curve showing the cumulative number of vehicles as a function of time at each location. Hence, density can be read directly from the cumulative curves. Interested readers are referred to [4] for further details.

Below are a few additional ways to calculate density k (not necessarily from point sensor data):

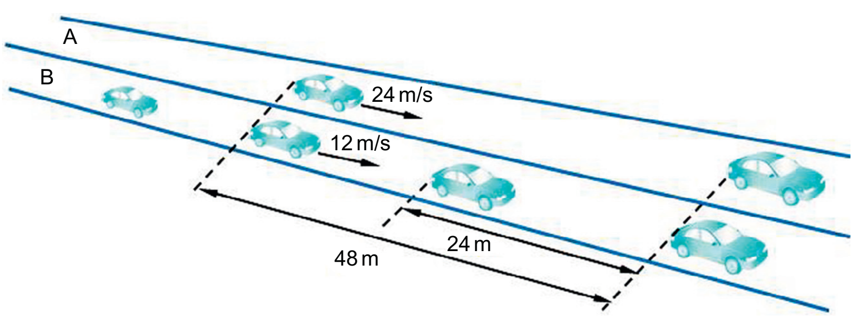
- $k = \frac{1}{s}$ if the average spacing s is known.
- $k = \frac{q}{v_s}$. For point sensor data, replacing v_s with v_t sometimes yields more accurate k than that estimated from occupancy.
- Estimate density from travel times with the Kalman filter technique [5].

PROBLEMS

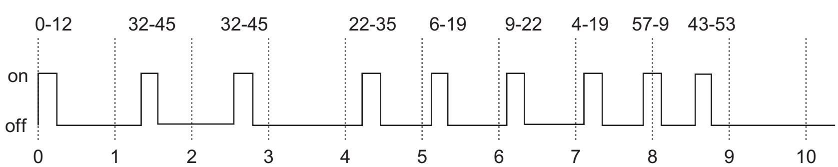
1. In a highway segment XY, an observer standing at location X counted four vehicles passing in front of him in 20 s. Their speeds are labeled in the figure. Find the flow, density, time-mean speed, and space-mean speed in this scenario.



2. The figure below illustrates two streams of uniform traffic in two lanes. In lane A, all vehicles are traveling at 24 m/s with a spacing of 48 m, while in lane B all vehicles are traveling at 12 m/s with a spacing of 24 m. Find the time-mean speed of all vehicles in the two lanes.



3. A traffic engineer counted vehicles on Route 9 and found that, on average, a vehicle passed in front of her at a rate of one every 5 s. If vehicles keep coming at this rate, what would be the equivalent hourly flow rate?
4. A roadside observer reports that five vehicles passed in the past 2 min. Their speeds are 30, 45, 20, 36, and 40 km/h respectively. Find the flow in vehicles per hour, time-mean speed, and space-mean speed. Which is greater, time-mean speed or space-mean speed?
5. For problem 2, find the space-mean speed.
6. Vehicle time headways and spacings were measured at a point along a highway, from a single lane, over the course of 1 h. The average values were calculated as 2.5 s per vehicle for the headway and 50 m per vehicle for the spacing. Calculate the average speed of the traffic.
7. A loop detector has recorded the information shown in the figure. The loop width is 6 feet. Each gate represents the brief period when a vehicle is in passing over the loop. "On" means a vehicle is in the detection zone, while "off" means no vehicle is in the detection zone. The numbers above each gate represent the duration of each gate. For example, "0–12" means a vehicle enters the detection zone at 0/60th second and exits at 12/60th second. The number of seconds is labeled at the bottom of the figure—for example, 1, 2, and 3 mean the first, second, and third seconds, respectively. Assume the vehicle length is uniformly 15 feet, and determine the following from the figure:



   a. Vehicle count during the observation period and the equivalent hourly flow rate
   b. Occupancy during the observation period
   c. Time-mean speed and space-mean speed
   d. Traffic density
   e. Estimate the speed from the speed-flow-density relationship and compare the result with that for (c). Is your estimated speed the time-mean speed or the space-mean speed?

8. The figure below shows aerial photos of a segment of Interstate 90. The two snapshots were taken 0.5 s apart and the scale of the ruler is 1:10 m. Using the section measured by the ruler and focusing on the middle lane traffic, find the vehicle spacings, traffic density, vehicle displacements, space-mean speed, and flow.