Homework: Queueing Theory

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A deterministic queueing theory is one in which constant behavior is assumed. For example, an arrival rate of 60 individuals per hour in a deterministic theory means that each minute exactly one individual will arrive. A stochastic theory, on the other hand, involves an element of randomness, i.e. there is a distribution which behavior follows. There will be e.g. an *average* arrival rate, but the exact arrival times are somewhat random.

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A freeway on-ramp has a toll booth which takes 20 seconds to service a vehicle, for a capacity rate of 180 vehicles per hour. The peak period flow on this on-ramp is 100 vehicles per hour.

This system is modeled as an M/M/1 Markov model, and solving this gives the following: The average number of vehicles in the system (including any vehicles being served) is 1.25 and the average wait time (before being served) is 0.0069 hours or 25 seconds. Additionally, the probability of a queue 3 vehicles long or longer $P(\geq 4)$ (note that the queue does not include any vehicles currently being served) is given by $1 - \sum_{n=0}^{3} P(n)$. $P(0)$, $P(1)$, $P(2)$ and $P(3)$ are given by 0.44, 0.25, 0.14, and 0.08, respectively, and so the probability of there being three or more vehicles in the queue is 0.1.

An additional toll booth is being considered, and this proposed scenario is modeled with an M/M/c (or M/M/N) Markov model where $c = 2$ (or $N = 2$). Solving this model gives an individual expected wait time of 1.7 seconds, which is 23.3 seconds faster than the existing scenario. At a regional value of time of \$10/hr, this saves 6.5¢ per vehicle. At an arrival rate of 100 vehicles per hour, this saves \$6.48 per hour. Since the toll operator wage is \$25/hr, this additional toll booth does not make much economic sense.

Additionally, the probability of three or more vehicles in the queue $P(\geq 4)$ is calculated as above to give $P(\geq 4) = 1 - \sum_{n=0}^{3} P(n) = 1 - 0.991 = 0.009$.