Homework: Engine Modeling

19.2–19.6

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The car I chose for this problem set is a 2005 Chevrolet Corvette C6 Z06. This is a 6-speed manual transmission car with a 427.8 in^3 V8 engine. An example of this car is shown in Figure 1.

Figure 1: A 2005 Chevrolet Corvette C6 Z06 racing at the Motor Speedway of the South in the 2005 Piston Cup final.

19.2

The peak power of the C6 is 504 hp at 6300 rpm, and the peak torque is 467 lb-ft at 4800 rpm. This is 3.76×10^5 watts at 660 radians/sec and 633 N·m at 503 radians/sec, respectively.

This gives:

$$\begin{aligned}
C_1 &= \frac{P_{\mathrm{max}}}{\omega_{\mathrm{p}}} &= \frac{3.76 \times 10^5 \text{ W}}{660 \text{ rad/s}} &= 570 \text{ J} \\
C_2 &= \frac{P_{\mathrm{max}}}{\omega_{\mathrm{p}}^2} & &= 0.863 \text{ J} \cdot \text{s} \\
C_3 &= -\frac{P_{\mathrm{max}}}{\omega_{\mathrm{p}}^3} & &= -0.00131 \text{ J} \cdot \text{s}^2.
\end{aligned}$$

The equation for power is then:

$$\begin{aligned}
P &= \sum_{i=1}^{3} C_i \omega^i \\
&= 570 \text{ J} \times \omega + 0.863 \text{ J} \cdot \text{s} \times \omega^2 + -0.00131 \text{ J} \cdot \text{s}^2 \times \omega^3.
\end{aligned}$$

19.3

The equation for torque is power divided by engine speed:

$$\begin{aligned}
\Gamma &= \sum_{i=1}^{3} C_i \omega^{i-1} \\
&= 570 \text{ N} \cdot \text{m} + 0.863 \text{ N} \cdot \text{m} \cdot \text{s} \times \omega + -0.00131 \text{ N} \cdot \text{m} \cdot \text{s}^2 \times \omega.
\end{aligned}$$

This equation has a maximum at (330 rad/s, 712 N·m), or (3150 rpm, 525 lb-ft). This is very different from the specified (4800 rpm, 467 lb-ft) for the engine.

19.4

The power and torque equations for Model II are given by:

$$\begin{aligned}
\Gamma &= C_1 + C_2(\omega - \omega_{\mathrm{t}})^2 \\
P &= C_3\omega + C_2(\omega - \omega_{\mathrm{t}})^2\omega,
\end{aligned}$$

where

$$C_1 = \Gamma_{\mathrm{max}}$$

$$C_2 = -\frac{P_{\mathrm{max}}}{2\omega_{\mathrm{p}}^2(\omega_{\mathrm{p}} - \omega_{\mathrm{t}})}$$

$$C_3 = \frac{P_{\mathrm{max}}}{2\omega_{\mathrm{p}}^2}(3\omega_{\mathrm{p}} - \omega_{\mathrm{t}}).$$

Solving these gives $C_1 = 633$ J, $C_2 = -0.0027$ J·s^2, and $C_3 = 637$ J. Then:

$$P = 637 \text{ J} \times \omega + -0.0027 \text{ J} \cdot \text{s}^2 \times \omega \left(\omega - 503 \ \frac{\text{rad}}{\text{s}}\right)^2$$

and

$$\Gamma = 633 \text{ N} \cdot \text{m} + -0.0027 \text{ N} \cdot \text{m} \cdot \text{s}^2 \times \left(\omega - 503 \ \frac{\text{rad}}{\text{s}}\right)^2.$$

19.5

Maximizing the equations from Model II (problem 19.4) gives a maximum power of 3.76×10^5 W at 660 rad/s or 504 hp at 6300 rpm, and a maximum torque of 633 N·m at 503 rad/s or 467 lb-ft at 4800 rpm. These are both identical to the manufacturer-specified values. This makes sense because the values of $C_{1,2,3}$ were defined in a way to fix the maximum of the model parabolas at the specified points.

19.6

Model III gives the power equation as follows:

$$P = \lambda E_{\mathrm{f}}\eta \left[A\frac{V_{\mathrm{e}}\omega_{\mathrm{e}}p_0}{4\pi A R_{\mathrm{a}} T_0}\left(1 + \frac{V_{\mathrm{e}}^2\omega_{\mathrm{e}}^2(k-1)}{32\pi^2 A^2 k R_{\mathrm{a}} T_0}\right)^{\frac{k+1}{2(k-1)}}\right] - \alpha P_{\mathrm{max}} \mathrm{e}^{\frac{\beta(\omega - \omega_{\mathrm{p}})}{\omega_{\mathrm{p}}}}.$$

The torque equation is the power equation divided by engine speed:

$$\Gamma = \lambda E_{\mathrm{f}}\eta \left[A\frac{V_{\mathrm{e}}p_0}{4\pi A R_{\mathrm{a}} T_0}\left(1 + \frac{V_{\mathrm{e}}^2\omega_{\mathrm{e}}^2(k-1)}{32\pi^2 A^2 k R_{\mathrm{a}} T_0}\right)^{\frac{k+1}{2(k-1)}}\right] - \frac{1}{\omega}\alpha P_{\mathrm{max}} \mathrm{e}^{\frac{\beta(\omega - \omega_{\mathrm{p}})}{\omega_{\mathrm{p}}}}.$$

The variables used in these equations are defined as follows (value assumptions for this exercise are also listed):

$$\begin{aligned}
\lambda &= \text{stoichiometric air-fuel ratio} && = 0.068, \\
E_\mathrm{f} &= \text{fuel energy density} && = 46.9\ \mathrm{MJ/kg}, \\
\eta &= \text{engine thermal efficiency} && = 0.29, \\
A &= \text{intake cross-sectional area,} && = \pi(70\ \mathrm{mm})^2/4 = 0.003848\mathrm{m}^2 \\
V_\mathrm{e} &= \text{engine displacement,} && \\
\omega_\mathrm{e} &= \text{engine speed in rad/s,} && \\
p_0 &= \text{stagnation pressure} && = 101.325\ \mathrm{kPa}, \\
R_\mathrm{a} & && = 287\ \mathrm{N{\cdot}m/kg/K}, \\
T_0 &= \text{stagnation temperature} && = 293.15\ \mathrm{K}, \\
k &= \text{specific heat ratio} && = 1.407, \\
P_\mathrm{max} &= \text{maximum engine power,} && \\
\omega &= \text{engine speed,} && \\
\omega_\mathrm{p} &= \text{engine speed at max power,} && \\
\alpha, \beta &= \text{calibration constants} && = 0.15 \text{ and } 10, \text{ respectively.}
\end{aligned}$$

The 2005 Chevrolet Corvette C6 Z06 has an engine displacement $V_\mathrm{e} = 7$ L, with $P_\mathrm{max} = 504$ hp and $\omega_\mathrm{p} = 6300$ rpm.

All three models are shown graphically in Figures 2 and 3.

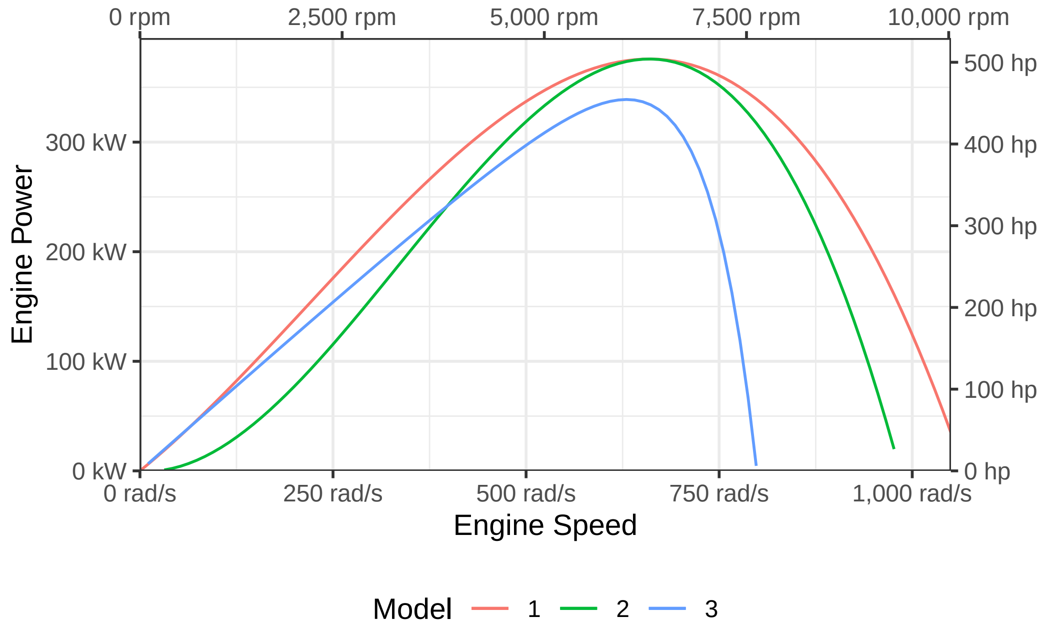

Figure 2: Predicted 2005 Chevrolet Corvette C6 Z06 engine power as a function of engine speed.

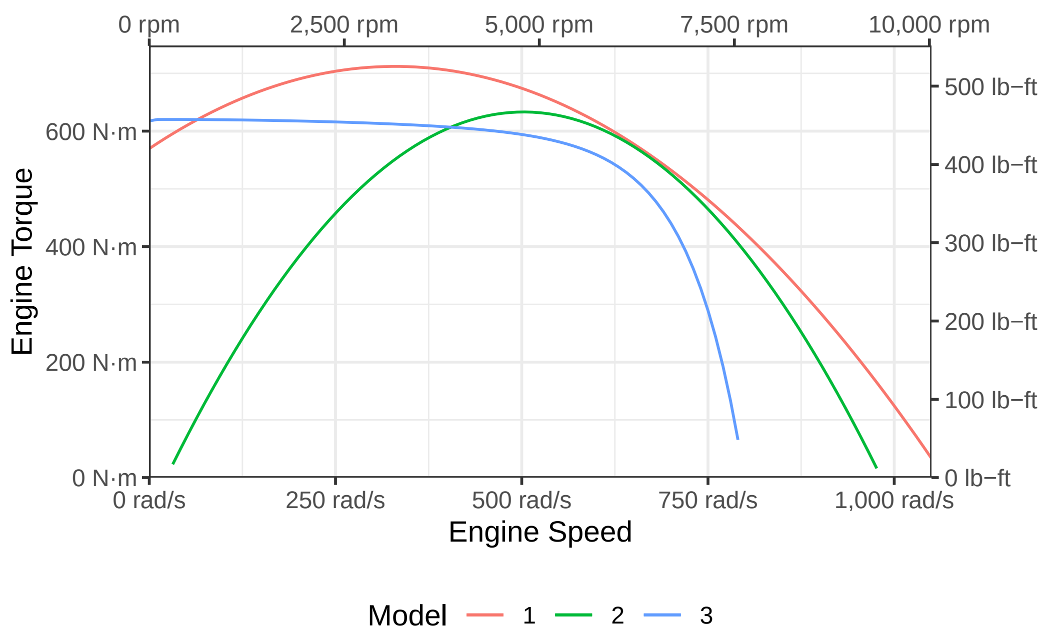

Figure 3: Predicted 2005 Chevrolet Corvette C6 Z06 engine torque as a function of engine speed.