Homework: Traffic Flow

2.4, 2.7, 3.1

Hayden Atchley

13 September 2023

2.4

Over the course of 2 minutes, 5 vehicles passed a point with speeds of 30, 45, 20, 36 and 40 km/h. The flow q is given by $q = N/T$, where N is the number of vehicles and T the total time. Therefore, the flow is $q = 5/2 = 2.5$ vehicles per minute or $q = 5/2 \times 60 = 150$ vehicles per hour. The time-mean speed v_t is the arithmetic mean of the spot speeds, or $v_t = 34.2$ km/h. The space-mean speed v_s is the harmonic mean of the spot speeds, or $v_s = 31.6$ km/h. The time-mean speed is greater.

2.7

Figure 1 shows the output of a 6-foot long loop detector. The x-axis indicates seconds, and the numbers above each bump indicate the on and off times in 1/60 of a second (e.g. `32-45` indicates that the detector turned on at 32/60 seconds and off at 45/60 seconds).

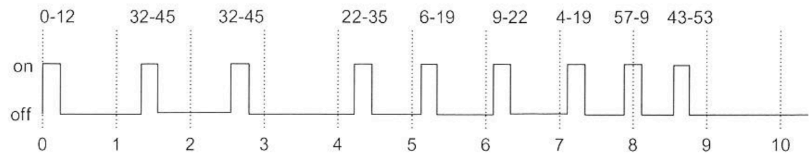

Figure 1: Loop detector on/off data.

(a) There were 9 vehicles over the 10-second observation window, so the hourly flow rate is $9/10 \times 3600 = 3240$ vehicles per hour.

(b) Occupancy is defined as the ratio of detector "on" time to the total observation time: $o = \frac{\sum_{i=1}^{N} (\xi_i = t_i^{\text{OFF}} - t_i^{\text{ON}})}{T}$. Table 1 gives the on and off times of each vehicle (converted to seconds), and the total time each vehicle triggered the detector. From this, the occupancy is determined to be $\frac{\sum \xi_i = 1.9}{T = 10} = 0.19$

Table 1: Vehicle Detector Times

Vehicle Number	Detector On Time (s)	Detector Off Time (s)	Activated Time (s)
1	0.00	0.20	0.20
2	1.53	1.75	0.22
3	2.53	2.75	0.22
4	4.37	4.58	0.22
5	5.10	5.32	0.22
6	6.15	6.37	0.22
7	7.07	7.32	0.25
8	7.95	8.15	0.20
9	8.72	8.88	0.17

(c) The speed is given by the length of the detector and vehicle divided by the time the detector was active, i.e. $\dot{x} = \frac{l_{car}+l_{loop}}{\xi}$. Table 2 shows this information. From here, v_t and v_s are calculated as before, so $v_t = 68.53$ mph and $v_s = 67.82$ mph.

Table 2: Vehicle Speeds

Vehicle Number	Activated Time (s)	Detector Length (ft)	Vehicle Length (ft)	Vehicle Speed (mph)
1	0.20	6	15	71.6
2	0.22	6	15	66.1
3	0.22	6	15	66.1
4	0.22	6	15	66.1
5	0.22	6	15	66.1
6	0.22	6	15	66.1
7	0.25	6	15	57.3
8	0.20	6	15	71.6
9	0.17	6	15	85.9

(d) The relationship between density (k), flow (q), and speed (v) is given by definition as $q = k \times v_s$. This requires, however, an accurate v_s, which from point data such as this detector requires an assumption about vehicle length. We were given a uniform vehicle length of 15 feet, so this relationship will hold. The density is therefore $k = q/v_s = 3240/67.82 = 47.8$ vehicles per mile.

(e) Estimating the speed from the $q = k \times v_s$ relationship will give an average speed of 67.8 mph. This is consistent with the space-mean-speed, since that is the speed used in part (d) (and the speed used in the relationship equation above).

3.1

Figure 2 shows a time-space diagram for several vehicles. The shaded area is of interest, bounded by $t = 5$s and $t = 15$s, and $x = 0.2$km and $x = 0.5$km.

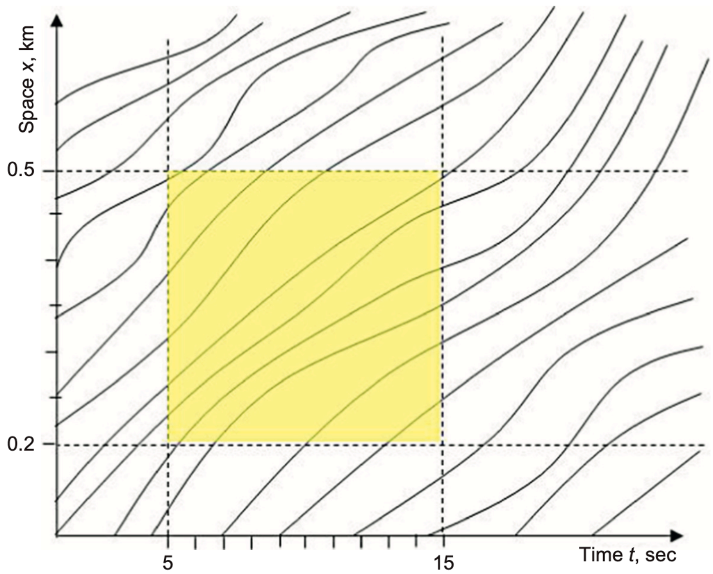

Figure 2: Time-space diagram showing bounding box of interest.

(a) Denoting the vehicle whose path most closely intersects the origin $(t = 0, x = 0)$ as i, the vehicle in front of i as $i + 1$, and the vehicle behind i as $i - 1$, the vehicles that traverse the bounding box are $i + 5$ through $i - 4$, inclusive.

(b,c,d,e) Table 3 shows the time and location that each vehicle enters and exits the bounding box, as well as the total distance and duration each vehicle traveled in the bounding box. This table also contains each vehicle's average speed while in the bounding box, given as the total distance each vehicle travels in the bounding box divided by the time each vehicle is in the box. The (arithmetic) mean speed of these vehicles is 80.8 km/h.

(f) Using the generalized definitions $q = \mathrm{d}(A)/|A|$, $k = \mathrm{t}(A)/|A|$, and $v_s = \mathrm{d}(A)/\mathrm{t}(A)$, where $\mathrm{d}(A)$ and $\mathrm{t}(A)$ are the total distance and time (respectively) traveled by vehicles in the bounding box A, and $|A|$ is the area of A (in this case $(15 - 5) \times (0.5 - 0.2) = 3$ km$\times$s), we get the following:

$$q = \frac{1.25 \text{ veh} \times \text{km}}{3 \text{ km} \times \text{s}} \times 3600 \ \frac{\text{sec}}{\text{hr}} = 1506 \text{ vph},$$

$$k = \frac{56.75 \text{ veh} \times \text{sec}}{3 \text{ km} \times \text{s}} = 18.9 \text{ veh/km},$$
$$\text{and } v_s = \frac{1.25 \text{ veh} \times \text{km}}{56.75 \text{ veh} \times \text{s}} \times 3600 \frac{\text{sec}}{\text{hr}} = 79.6 \text{ km/h}.$$

Table 3: Vehicle Entry and Exit of Bounding Box

Vehicle	Distance (km)			Time (s)			Vehicle Speed (km/h)
	Entry	Exit	Total	Entry	Exit	Total	
$i+5$	0.49	0.50	0.01	5.0	5.8	0.8	45
$i+4$	0.45	0.50	0.05	5.0	6.8	1.8	103
$i+3$	0.39	0.50	0.11	5.0	8.5	3.5	113
$i+2$	0.32	0.50	0.17	5.0	10.8	5.8	109
$i+1$	0.26	0.49	0.23	5.0	15.0	10.0	83
i	0.22	0.45	0.22	5.0	15.0	10.0	81
$i-1$	0.20	0.38	0.17	5.5	15.0	9.5	66
$i-2$	0.20	0.33	0.13	6.5	15.0	8.5	55
$i-3$	0.20	0.31	0.11	10.1	15.0	4.9	81
$i-4$	0.20	0.24	0.04	13.0	15.0	2.0	72

(g) The general relationship between speed, density, and flow is $q = k \times v_s$. Using the values obtained in (f) to determine if this holds:

$$1506 \stackrel{?}{=} 18.9 \times 79.6$$
$$1506 = 1506$$

The relationship holds here. However, taking the harmonic mean of the speeds in Table 3 gives $v_s = 74.7$ km/h, which differs from the result in (f). Therefore the relationship $q = k \times v_s$ would not hold with this value of v_s. The relationship is true by definition only if the values q, k, and v_s are defined as in part (f).

(h) Restricting our observations to $x = 0.2$km, there are 4 vehicles that pass this mark in the 10-second window from $t = 5$s to $t = 15$s. The flow is therefore $q = 4/10 = 0.4$ veh/sec = 1440 vph.

(i) There are 6 vehicles that are in the bounding box at $t = 5$s, and the bounding box has a length of 0.3km, so the density is $k = 6/0.3 = 20$ veh/km.

(j) Using values of flow, density, and speed from (h), (i), and (e) respectively, the relationship $q = k \times v$ does not hold:

$$
\begin{aligned}
1440 &\stackrel{?}{=} 20 \times 80.8 \\
1440 &\neq 1616
\end{aligned}
$$

This is because the values of q and k are taken from different measurement sources, and the average speed in (e) is not necessarily consistent with either of these sources.