Homework: Simplified Waves

10.3

Hayden Atchley

2 October 2023

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A uniform freeway link x_1x_3 is 6 km long. Node x_2 is the midpoint of x_1x_3. Table 1 gives data where D1, D2, and D3 are cumulative traffic counts at nodes x_1, x_2, and x_3, respectively. The flow-density relationship in Figure 1 applies uniformly on x_1x_3.

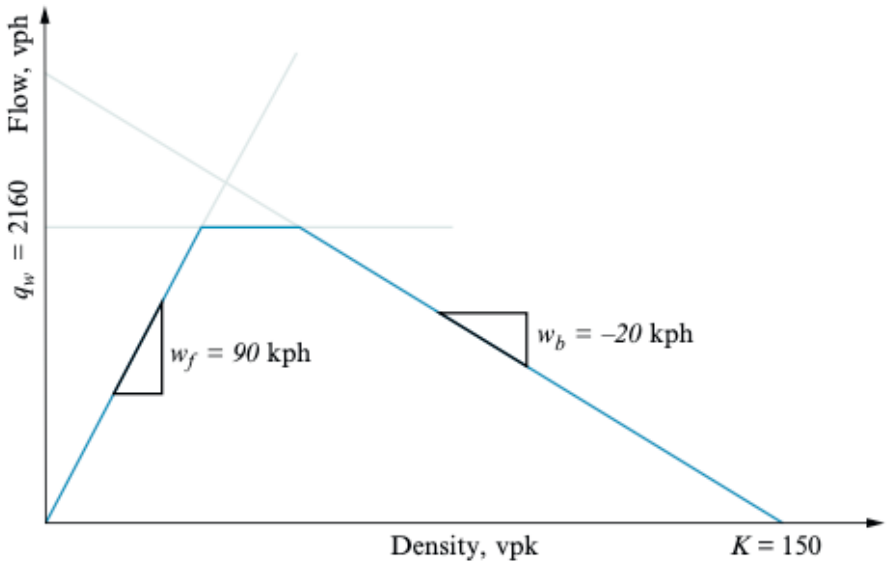

Figure 1: Flow-density relationship for freeway segment.

The cumulative vehicle count at x_2 at 9:01 is given by

$$N(9{:}01, x_2) = \min\left\{N^{\text{up}}(9{:}01, x_2), N^{\text{Q}}(9{:}01, x_2), N^{\text{dn}}(9{:}01, x_2)\right\}.$$

This equation is solved in three parts:

Table 1: Traffic Counts on Freeway Segment

Time	D1	D2	D3
08:50	1,965	1,950	1,551
08:51	1,970	1,957	1,555
08:52	1,978	1,961	1,560
08:53	1,982	1,968	1,566
08:54	1,987	1,973	1,571
08:55	1,991	1,980	1,578
08:56	1,996	1,984	1,583
08:57	2,000	1,990	1,589
08:58	2,008	1,996	1,594
08:59	2,012	2,000	1,600
09:00	2,018	2,005	1,605
09:01	2,024		1,610

$N^{\mathbf{up}}$

$$\begin{aligned} N^{\text{up}}(9{:}01, x_2) &= N\left(9{:}01 - \frac{x_2 - x_1}{w_f}, x_1\right) \\ &= N\left(9{:}01 - \frac{3 \text{ km}}{90 \text{ km/h}}, x_1\right) \\ &= N(9{:}01 - 0{:}02 = 8{:}59, x_1) \\ \text{(a)} \qquad &= 2012. \end{aligned}$$

$N^{\mathbf{Q}}$

$$\begin{aligned} N^{\text{Q}}(9{:}01, x_2) &= N(9{:}00, x_2) + q_m \Delta t \\ &= 2005 + 2160 \text{ vph} \times \frac{1 \text{ hr}}{60 \text{ min}} \\ &= 2005 + 36 \\ \text{(b)} \qquad &= 2041. \end{aligned}$$

$N^{\mathbf{dn}}$

Note that $N^{\text{dn}}(9{:}01, x_2) = N\left(t - \frac{x_3 - x_2}{w_b}, x_2\right) + K_2(x_3 - x_2)$.

$$\begin{aligned} K_2(x_3 - x_2) &= K \times 3 \text{ km} \\ &= 150 \times 3 \\ \text{(c)} \qquad &= 450. \end{aligned}$$

$$\begin{aligned} N\left(t - \frac{x_3 - x_2}{w_b}, x_2\right) &= N\left(9{:}01 - \frac{3 \text{ km}}{20 \text{ km/h}}, x_3\right) \\ &= N(9{:}01 - 0{:}09 = 8{:}52, x_3) \\ \text{(d)} \qquad &= 1560. \end{aligned}$$

$N^{\text{dn}}(9{:}01, x_2)$ is therefore equal to $450 + 1560 = 2010$.

N_2

$N(9{:}01, x_2)$ is then the minimum of $N^{\text{up}} = 2012$, $N^{\text{Q}} = 2041$, and $N^{\text{dn}} = 2010$, which is 2010. Therefore, (e) $N(9{:}01, x_2) = 2010$.