CHAPTER 16

More Single-Regime Models

This chapter presents a few more car-following models, including the Newell nonlinear model, the Newell simplified model, the intelligent driver model (IDM), and the Van Aerde model. Except for the Newell simplified model, these models not only capture the essence of car-following behavior but also aggregate to sound macroscopic behavior.

16.1 NEWELL NONLINEAR MODEL

Newell actually proposed two car-following models, one in 1961, which will be referred to as the "Newell nonlinear" car-following model [58], and the other in 2002, which will be referred to as the "Newell simplified" car-following model [59] hereafter . The Newell nonlinear car-following model takes the following form:

$$\dot{x}_i(t+\tau_i) = v_i(1 - \mathrm{e}^{-\frac{\lambda_i}{v_i}(s_i(t)-l_i)}), \qquad (16.1)$$

where $\dot{x}_i(t)$ is the speed of the vehicle with ID i at time t, τ_i is driver i's perception-reaction time, v_i is driver i's desired speed, λ_i is a parameter associated with driver i (i.e., the slope of driver i's speed-spacing curve evaluated at $\dot{x}_i = 0$), $s_i = x_{i-1} - x_i$ is the spacing between vehicle i and its leader $i-1$, and l_i is the minimum value of s_i, which can be viewed as the nominal vehicle length. Note that, in microscopic modeling, the driver and his/her vehicle is considered as a single unit and treated as a particle. Therefore, the "driver", the "vehicle", the "unit", and the "particle" are the same thing and used interchangeably according to the context throughout the book.

16.1.1 Properties of the Newell Nonlinear Model

Newell acknowledged that "no motivation for this choice is proposed other than the claim that it has approximately the correct shape and is reasonably simple." This acknowledgment seems to tell us two things:

1. Unlike the Pipes, Forbes, and Gipps models, which are derived from driving experiences such as safety rules, this model does not seem to be

http://dx.doi.org/10.1016/B978-0-12-804134-5.00016-7

based on driving experiences, but seems to be based rather on a discovery after some contemplation and empirical studies.

2. If there were something behind the contemplation, it might have been *the correct shape*—the model leads to an equilibrium speed-density curve that resembles field observations.

Under equilibrium conditions, Equation 16.1 reduces to the following speed-density relationship:

$$v = v_{\mathrm{f}}\left(1 - \mathrm{e}^{-\frac{\lambda}{v_{\mathrm{f}}}\left(\frac{1}{k}-\frac{1}{k_{\mathrm{j}}}\right)}\right). \tag{16.2}$$

where v is traffic speed, which is aggregated from vehicle speed $\dot{x}_i$, v_{f} is free-flow speed, which is aggregated from v_i, λ is a parameter aggregated from λ_i, k is traffic density, which is the reciprocal of average spacing s, which, in turn, is aggregated from spacing s_i, and k_{j} is jam density, which is the reciprocal of average vehicle length l, which, in turn, is aggregated from nominal vehicle length l_i.

16.1.2 Benchmarking

Microscopic benchmarking refers to the scenario presented in Section 12.3.1 and macroscopic benchmarking refers to the scenario presented in Section 12.3.2.

Microscopic Benchmarking

The benchmarking result of the Newell nonlinear model is plotted in Figure 16.1. The performance of the Newell nonlinear model is summarized as follows:

- Start-up: The model is able to start a vehicle up from standstill. See Figure 16.1 when $t > 0$ s.
- Speedup: The model allows the vehicle speed to jump from 0 to 30 m/s in one time step, resulting in an acceleration of 30 m/s^2. This is unrealistic, so an external logic has to be imposed to limit the maximum acceleration. Note that simply setting a limiting acceleration would result in an unrealistic acceleration profile (e.g., the vehicle may attain maximum acceleration at high speeds). Therefore, a more realistic acceleration logic is necessary. However, with this addition, the Newell nonlinear model ceases to be a steady-state model, and instead becomes a dynamic model. See Figure 16.1 when $0\,\mathrm{s} < t < 100\,\mathrm{s}$.
- Free flow: The model is able to reach and settle at the desired speed under free-flow conditions. See Figure 16.1 when $0\,\mathrm{s} < t < 100\,\mathrm{s}$.

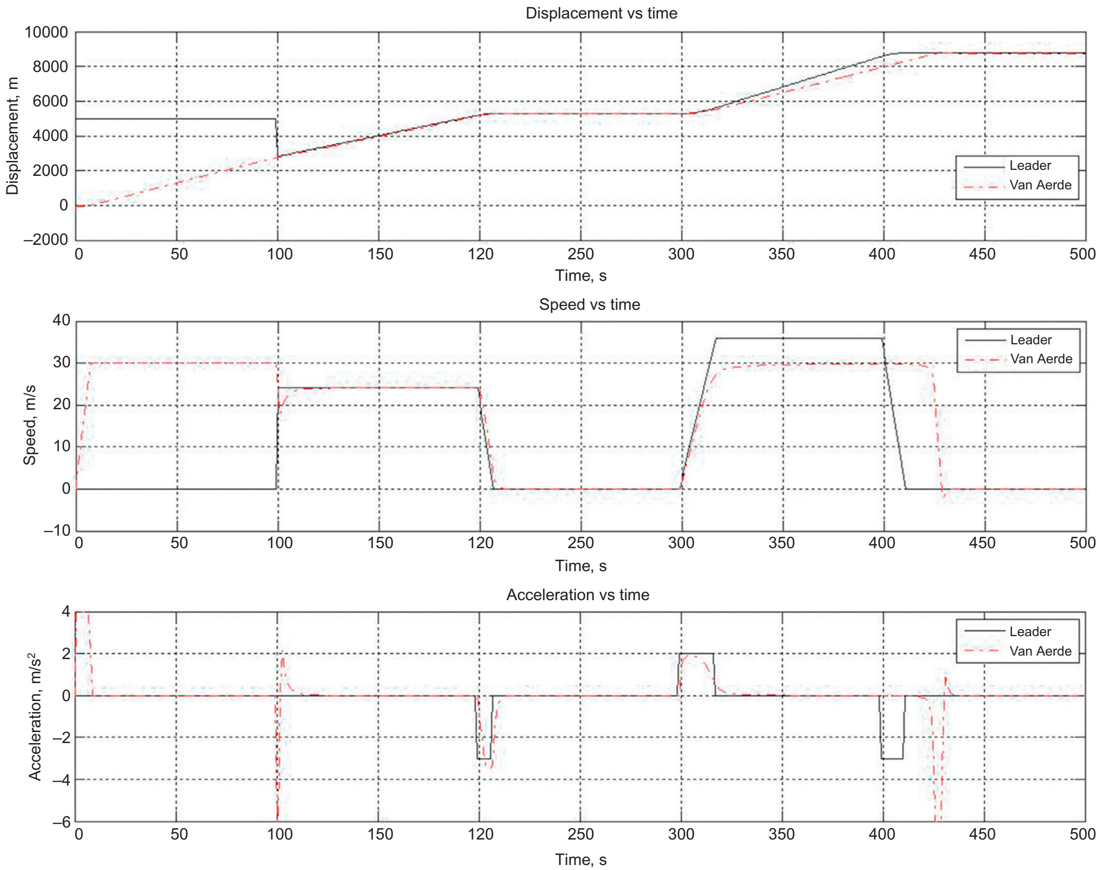

Figure 16.1 Microscopic benchmarking of the Newell nonlinear model.

Table 16.1 Microscopic benchmarking parameters of the Newell nonlinear model

l_i	v_i	τ_i	λ	–
6 m	30 m/s	1.0 s	7.9	–
A_i	B_i	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
4.0 m/s^2	6.0 m/s^2	−97 m	0 m/s	0 m/s^2

- Cutoff: By itself, the Newell nonlinear model would predict a deceleration of about $-184.6\,\text{m/s}^2$ when the third vehicle cuts in and an acceleration of $182.9\,\text{m/s}^2$ in the next time step. This is a very unrealistic jerking, so an external logic has to be imposed to limit the maximum acceleration and deceleration. Hence, the same argument as for speedup applies here. See Figure 16.1 around $t = 100\,\text{s}$ after these external conditions have bene incorporated.
- Following: The model is able to adopt the leader's speed and follow the leader at a reasonable distance. See Figure 16.1 when $100\,\text{s} < t < 200\,\text{s}$.
- Stop and go: The model is able to stop the vehicle safely behind its leader and start the vehicle moving when the leader departs. See Figure 16.1 when $200\,\text{s} \geq t \leq 300\,\text{s}$.
- Trailing: The model is able to speed up normally without being tempted to speed up by its speeding leader. See Figure 16.1 when $300\,\text{s} < t < 400\,\text{s}$.
- Approaching: With the above external logic on limiting deceleration, the model is able to decelerate properly when approaching a stationary vehicle. See Figure 16.1 when $400\,\text{s} \geq t < 420\,\text{s}$.
- Stopping: The model is able to stop the vehicle safely behind the stationary vehicle. See Figure 16.1 when $t \geq 420\,\text{s}$.

The above benchmarking is based on the set of parameters in Table 16.1, and the outcome may differ for a different set of parameters.

Macroscopic Benchmarking

The fundamental diagram implied by the Newell nonlinear model is presented in Figure 16.2, where the model parameters are adopted from Newell's original paper.

The Newell nonlinear model indeed exhibits the correct shape that resembles field observations in the entire density range, as claimed by Newell. First, the model meets the boundary conditions at $(k = 0, v = v_f)$ and $(k = k_j, v = 0)$. Second, the flow-density exhibits a concave shape, and the fitting quality is reasonably good given that only three parameters are employed.

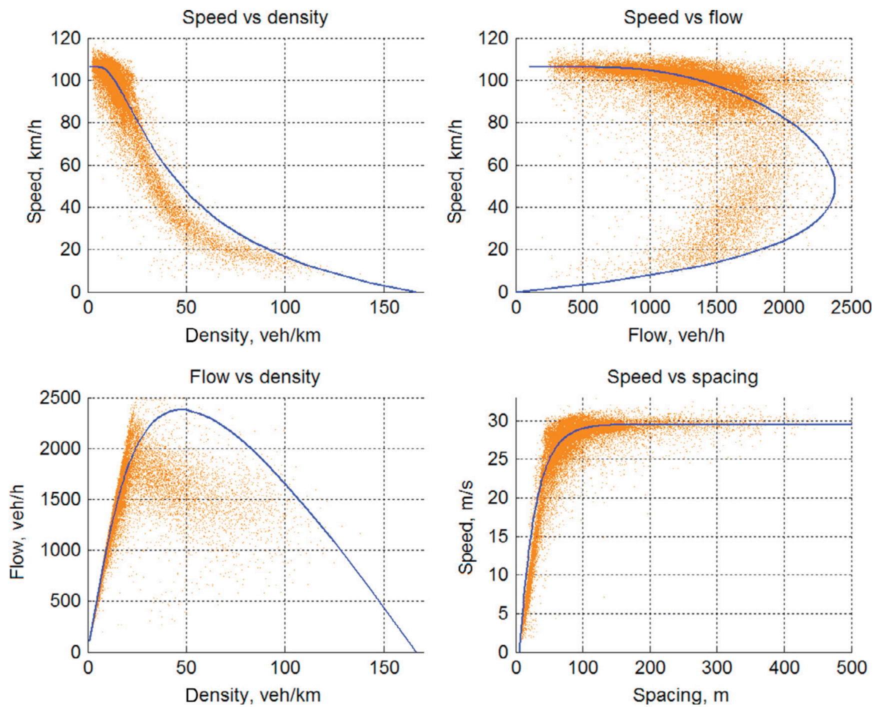

Figure 16.2 Fundamental diagram implied by the Newell nonlinear model.

Table 16.2 Macroscopic benchmarking parameters of the Newell nonlinear model

v_f	k_j	λ
29.5 m/s	0.2 vehicles/m	0.8

The above benchmarking is based on the set of parameters in Table 16.2, and the outcome may differ for different set of parameters.

16.2 NEWELL SIMPLIFIED MODEL

After about 40 years, Newell published a simplified car-following model [59]. This is indeed a very simple model because one does not need to worry about safety rules, speed choices, and acceleration responses. What one needs to do is simply to translate the leading vehicle's trajectory. For example, if vehicle $i-1$'s trajectory $x_i(t)$ is given in the right panel in Figure 16.3, vehicle i's trajectory can be directly determined by the following equation:

$$x_i(t+\tau_i) = x_{i-1}(t) - l_i. \tag{16.3}$$

Graphically, this means translating trajectory $x_{i-1}(t)$ to the right by a horizontal distance of τ_i and then downward by a vertical distance of l_i—that is, one can squeeze a rectangle with dimensions $\tau_i \times l_i$ between the two trajectories. From the speed-spacing relationship in the right panel in Figure 16.3, it becomes clear that the physical meaning of l_i is the minimum value of the spacing—that is, the nominal vehicle length—and τ_i is the reciprocal of the tangent to the speed-spacing relationship drawn at point $(0, l_i)$. Evidence shows that τ_i can most likely be interpreted as the perception-reaction time of driver i. Figure 16.3 also reveals that the spacing between the two vehicles is

$$s_i(t) = x_{i-1}(t) - x_i(t). \tag{16.4}$$

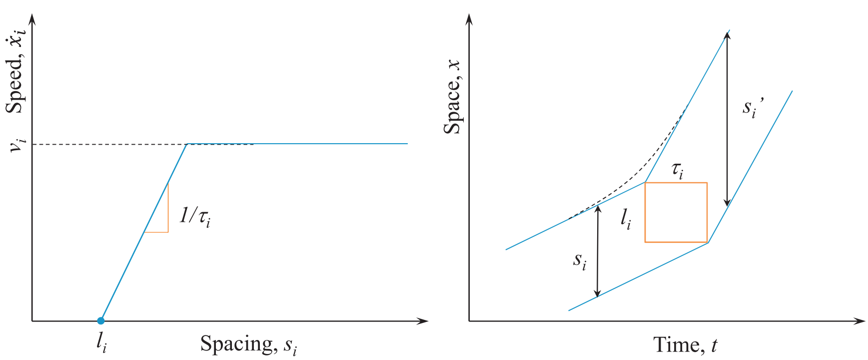

Figure 16.3 Newell simplified car-following model.

In addition, the locations of vehicle i at time t and $t+\tau_i$ can be related as

$$x_i(t+\tau_i) = x_i(t) + \dot{x}_i(t)\tau_i. \tag{16.5}$$

Combining the above three equations, we get

$$s_i(t) = \dot{x}_i(t)\tau_i + l_i. \tag{16.6}$$

This is the same as the Pipes/Forbes model (by taking the minimum spacing), which, in turn, is equivalent to GM1. Since the Newell simplified model is essentially the Pipes/Forbes model, the properties and benchmarking of the latter apply to the former.

16.3 INTELLIGENT DRIVER MODEL

The intelligent driver model (IDM) [60, 61] is expressed as a superposition of the follower i's acceleration term and a deceleration term which depends on the desired spacing s_i^*:

$$\ddot{x}_i(t+\tau_i) = A_i\left[1 - \left(\frac{\dot{x}_i}{v_i}\right)^{\delta} - \left(\frac{s_i^*}{s_i}\right)^2\right], \tag{16.7}$$

where $\ddot{x}_i$ is driver i's acceleration, A_i is driver i's maximum acceleration when starting from standstill, δ is the acceleration exponent, $s_i = x_{i-1} - x_i$ is the spacing between vehicle i and its leader $i-1$, and the desired spacing s_i^* is a function of speed $\dot{x}_i$ and relative speed $(\dot{x}_i - \dot{x}_{i-1})$:

$$s_i^* = s_0 + s_1\sqrt{\frac{\dot{x}_i}{v_i}} + T_i\dot{x}_i + \frac{\dot{x}_i[\dot{x}_i - \dot{x}_{i-1}]}{2\sqrt{g_i b_i}}, \tag{16.8}$$

where s_0, s_i, and T_i are parameters.

16.3.1 Properties of the IDM

Under equilibrium conditions, Equation 16.7 reduces to the following density-speed relationship:

$$k = \frac{1}{(s_0 + vT)\left[1 - \left(\frac{v}{v_f}\right)^{\delta}\right]^{-1/2}}. \tag{16.9}$$

If one further assumes that $s_0 = s_1 = 0$ and $\delta = 1$, a special case of Equation 16.9 results:

$$v = \frac{(s-L)^2}{2v_f T^2}[-1 + \sqrt{1 + \frac{4T^2 v_f^2}{(s-L)^2}}], \tag{16.10}$$

where T is the average safe time headway, $s = 1/k$ is the average spacing, and k is traffic density.

16.3.2 Benchmarking

Microscopic benchmarking refers to the scenario presented in Section 12.3.1 and macroscopic benchmarking refers to the scenario presented in Section 12.3.2.

Microscopic Benchmarking

The benchmarking result of the IDM is plotted in Figure 16.4. The performance of the IDM is summarized as follows:

- Start-up: The model is able to start the vehicle up from standstill. See Figure 16.4 when $t > 0$ s.
- Speedup: The model is able to speed the vehicle up realistically to its desired speed. See Figure 16.4 when $0\,\text{s} < t < 100\,\text{s}$.
- Free flow: The model is able to reach and settle at the desired speed under free-flow conditions. See Figure 16.4 when $0\,\text{s} < t < 100\,\text{s}$.
- Cutoff: The model retains control and responds reasonably when a vehicle cuts in in front. See Figure 16.4 around $t = 100\,\text{s}$.
- Following: The model is able to adopt the leader's speed and follow the leader at a reasonable distance. See Figure 16.4 when $100\,\text{s} < t < 200\,\text{s}$.
- Stop and go: The model exhibits some oscillation in acceleration, stopping behind the leading vehicle. The model is able to start moving when the leader departs. See Figure 16.4 when $200\,\text{s} \geq t \leq 300\,\text{s}$.
- Trailing: The model is able to speed up normally without being tempted to speed up by its speeding leader. See Figure 16.4 when $300\,\text{s} < t < 400\,\text{s}$.
- Approaching: The model is able to decelerate properly when approaching a stationary vehicle. See Figure 16.4 when $400\,\text{s} \geq t < 420\,\text{s}$.
- Stopping: The model is able to stop the vehicle safely behind the stationary vehicle. See Figure 16.4 when $t \geq 420\,\text{s}$.

The above benchmarking is based on the set of parameters in Table 16.3, and the outcome may differ for a different set of parameters.

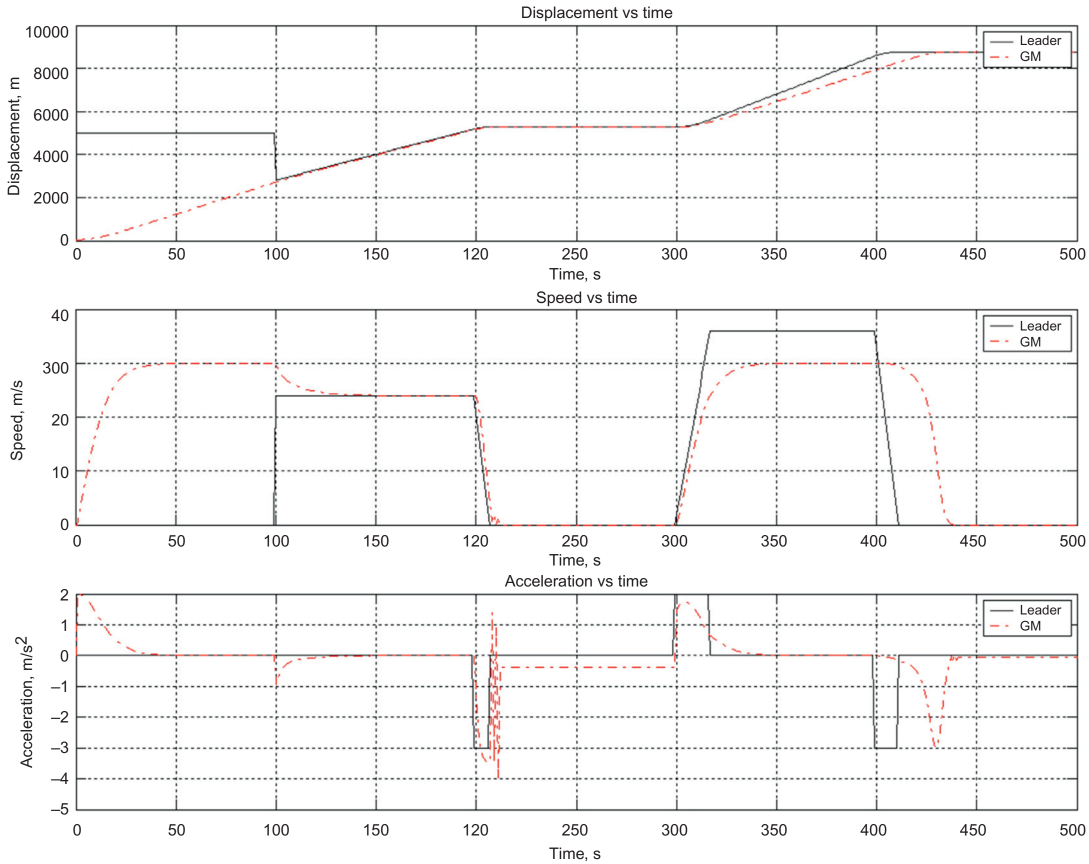

Figure 16.4 Microscopic benchmarking of the IDM.

Table 16.3 Microscopic benchmarking parameters of the IDM

l_i	v_i	τ_i	δ	s_0
6 m	30 m/s	1.0 s	2	2 m
A_i	b_i	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
2.0 m/s^2	4.0 m/s^2	39.5 m	0 m/s	0 m/s^2

Table 16.4 Macroscopic benchmarking parameters of the IDM

v_f	T	δ	s_0
29.5 m/s	1.7 s	15	4 m

Macroscopic Benchmarking

The fundamental diagram implied by the IDM, in particular Equation 16.9, is presented in Figure 16.5. The model employs four parameters and exhibits a desirable shape with good fitting quality.

The above benchmarking is based on the set of parameters in Table 16.4, and the outcome may differ for a different set of parameters.

16.4 VAN AERDE MODEL

The Van Aerde car-following model [62, 63] combines the Pipes model [52] and the Greenshields model [9] into a single equation:

$$s_i = c_1 + c_3\dot{x}_i + c_2/(v_f - \dot{x}_i), \tag{16.11}$$

where

$$\begin{cases} c_1 & = \frac{v_\mathrm{f}}{k_\mathrm{j} v_\mathrm{m}^2}(2v_\mathrm{m} - v_\mathrm{f}), \\ c_2 & = \frac{v_\mathrm{f}}{k_\mathrm{j} v_\mathrm{m}^2}(v_\mathrm{f} - v_\mathrm{m})^2, \\ c_3 & = \frac{1}{q_\mathrm{m}} - \frac{v_\mathrm{f}}{k_\mathrm{j} v_\mathrm{m}^2}, \end{cases} \tag{16.12}$$

where v_f is the free-flow speed of the roadway facility, k_j is the jam density, and v_m is the optimal speed at capacity q_m.

16.4.1 Properties of the Van Aerde Model

Under equilibrium conditions, Equation 16.11 reduces to the following density-speed relationship:

$$k = \frac{1}{c_1 + c_3 v + c_2/(v_f - v)}, \tag{16.13}$$

where all variables are as defined before.

16.4.2 Benchmarking

Microscopic Benchmarking

The benchmarking result of the Van Aerde model is plotted in Figure 16.6. The performance of the Van Aerde model is summarized as follows:

- Start-up: The model is able to start the vehicle up from standstill. See Figure 16.6 when $t > 0\,\text{s}$.
- Speedup: The same argument as in the corresponding part for the Newell nonlinear car-following model applies here. See Figure 16.6 when $0\,\text{s} < t < 100\,\text{s}$.
- Free flow: The model is able to reach and settle at the desired speed under free-flow conditions. See Figure 16.6 when $0\,\text{s} < t < 100\,\text{s}$.
- Cutoff: The same argument as in the corresponding part for the Newell nonlinear car-following model applies here. See Figure 16.6 around $t = 100\,\text{s}$.
- Following: The model is able to adopt the leader's speed and follow the leader at a reasonable distance. See Figure 16.6 when $100\,\text{s} < t < 200\,\text{s}$.
- Stop and go: The model is able to stop the vehicle safely behind its leader and start the vehicle moving when the leader departs. See Figure 16.6 when $200\,\text{s} \geq t \leq 300\,\text{s}$.
- Trailing: The model is able to speed up normally without being tempted to speed up by its speeding leader. See Figure 16.6 when $300\,\text{s} < t < 400\,\text{s}$.
- Approaching: With limiting deceleration, the model is able to decelerate properly when approaching a stationary vehicle. See Figure 16.6 when $400\,\text{s} \geq t < 420\,\text{s}$.
- Stopping: The model is able to stop the vehicle safely behind the stationary vehicle. See Figure 16.6 when $t \geq 420\,\text{s}$.

The above benchmarking is based on the set of parameters in Table 16.5, and the outcome may differ for a different set of parameters.

Macroscopic Benchmarking

The fundamental diagram implied by the Van Aerde model is presented in Figure 16.7. The model employs four parameters and exhibits a desirable shape with good fitting quality.

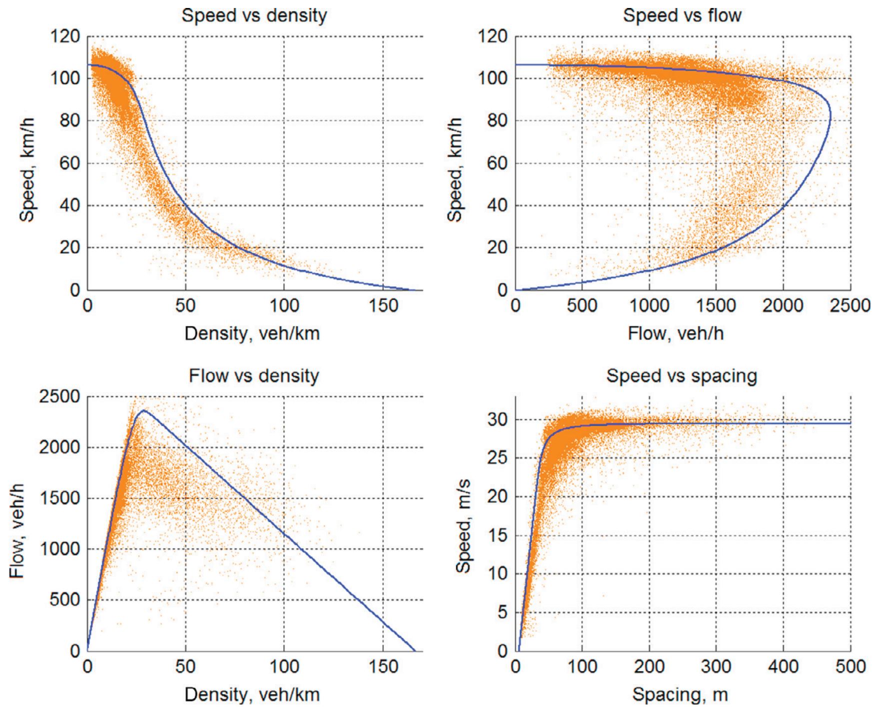

Figure 16.5 Fundamental diagram implied by the IDM.

Table 16.5 Microscopic benchmarking parameters of the Van Aerde model

k_j	v_f	τ_i	v_m	q_m
1/6 vehicles/m	30 m/s	1.0 s	25 m/s	1800 vehicles/h
$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$		
-99.4 m	0 m/s	0 m/s^2		

Table 16.6 Macroscopic benchmarking parameters of the Van Aerde model

v_f	k_j	v_m	q_m
29.5 m/s	0.25 vehicles/m	20 m/s	1950 vehicles/h

The above benchmarking is based on the set of parameters in Table 16.6, and the outcome may differ for a under different set of parameters.

PROBLEMS

1. Read the capacity condition (q_m, k_m, v_m) off the Newell nonlinear model in Figure 16.2, which is generated with the following parameters: $v_f = 29.5$ m/s, $k_j = 0.2$ vehicles per meter, and $\lambda = 0.8$ 1/s. What capacity condition does the "cloud" (i.e., empirical data) tell you? Comment on how realistic the Newell nonlinear model is when compared with the empirical data.
2. The plot at the bottom right of Figure 16.2 depicts the speed-spacing relationship of the Newell nonlinear model. The curve starts at a point where the spacing is $s = 5$ m and the speed is $v = 0$ m/s. Then the curve runs upward with a slope of $\lambda = 0.8$ 1/s.
   a. Assume that this portion of the curve is linear, and establish the underlying linear equation $s = f(v)$.
   b. Assume a vehicle length of $l = 5$ m and perception-reaction time $\tau = 1.25$ s. What is the underlying space-speed relationship—that is, $s = g(v)$—according to the Forbes model?
   c. How do you compare the above two models?
   d. What would you say about the physical meaning of parameter λ?
3. Show that Van Aerde model is a combination of the Pipes model and the Greenshields model.
4. Vehicle B is following vehicle A according to the Newell simplified car-following model with parameters $\tau = 2$ s and $l = 5$ m.

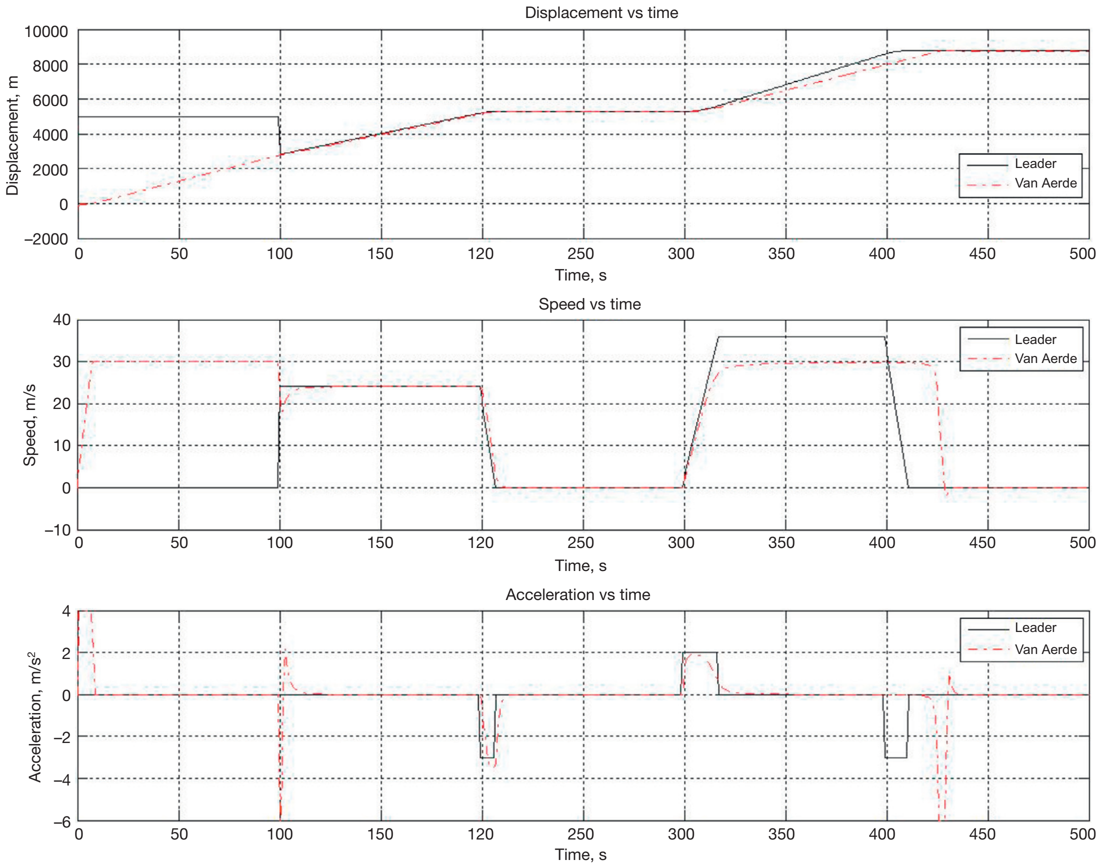

Figure 16.6 Microscopic benchmarking of the Van Aerde model.

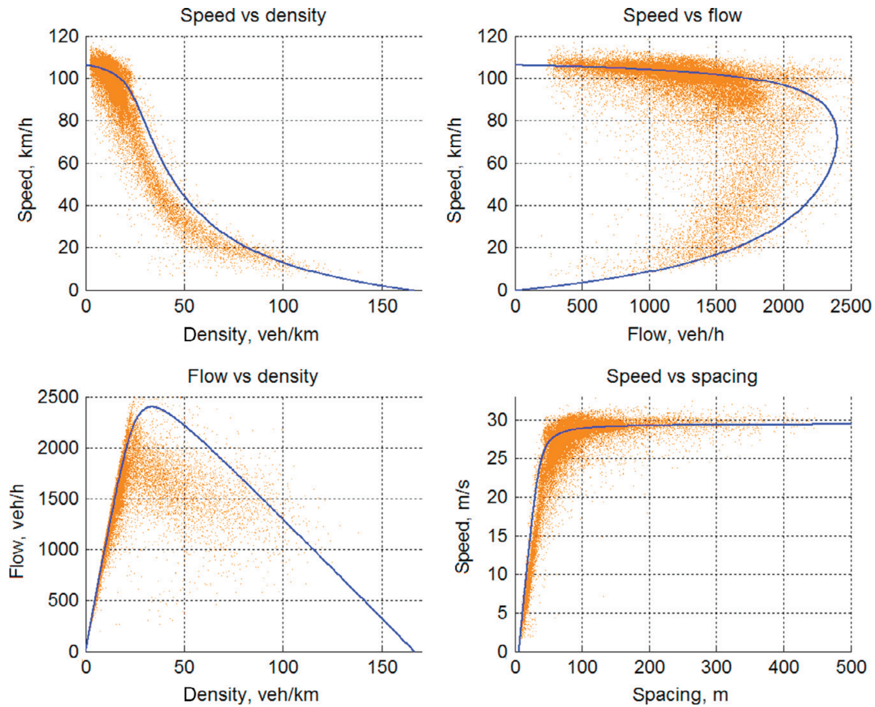

Figure 16.7 Fundamental diagram implied by the Van Aerde model.

   a. Write the underlying car-following model.
   b. Assume that vehicle A's trajectory is described by the following equation: $x_A\ (t) = \sqrt{t} - 10$, where $t > 0$. Determine vehicle B's trajectory.
   c. At time $t = 16$, find the spacing between vehicles A and B.

5. Derive the corresponding density-speed relationship $k = K(v)$ from the IDM under he assumption that $s^* = s_0 + T_i\dot{x}_i$.

6. Further assume $s_0 = 0$ and $\delta = 1$, and derive the corresponding speed-spacing relationship $v = V(s)$ from the above density-speed model.

7. Derive the capacity condition of the Van Aerde model.