CHAPTER 15

Gipps Model

The Pipes, Forbes, and General Motors models introduced in previous chapters are all single-regime models—that is, they have only one equation that applies to the entire driving process and do not consider different driving scenarios or regimes. On the positive side, such models are simple and mathematically attractive. On the flip side, however, their descriptive power is frequently of concern. For example, a driver may encounter different regimes such as start-up, speedup, free flow, cutoff, following, stop and go, trailing, approaching, and stopping. A one-equation model may or may not apply to all regimes. As such, multiregime models might be helpful in capturing different driving scenarios. This chapter introduces a model along this line—the Gipps model.

15.1 MODEL FORMULATION

Just like the Pipes and Forbes models, the Gipps model [57] is derived from a safe driving rule, perhaps a more realistic but conservative one. A driver typically employs a safety rule to evaluate if the current car-following situation is safe. For example,

- The Pipes rule stipulates that at each moment the driver needs to estimate his or her own speed (in miles per hour), divide it by 10, and multiply the quotient by a car length, and the result is the minimum gap that should be maintained. If the actual gap is less, one should fall back; otherwise, it is safe.
- The Forbes rule ensures safe headway. For example, a driver with a perception-reaction time of 3 s can use the following to ensure safety. When the vehicle in front passes a roadside utility pole, start counting "one thousand one, one thousand two, one thousand three." If the driver passes the pole before the counting is finished, a 3 s headway is not maintained; otherwise, it is safe.

Though the above two safety rules may sound reasonable to a certain degree, rarely do drivers in the real world drive in such a manner. Perhaps a more realistic safety rule is the following. "The driver of the following vehicle selects his speed to ensure that he can bring his vehicle to a safe stop

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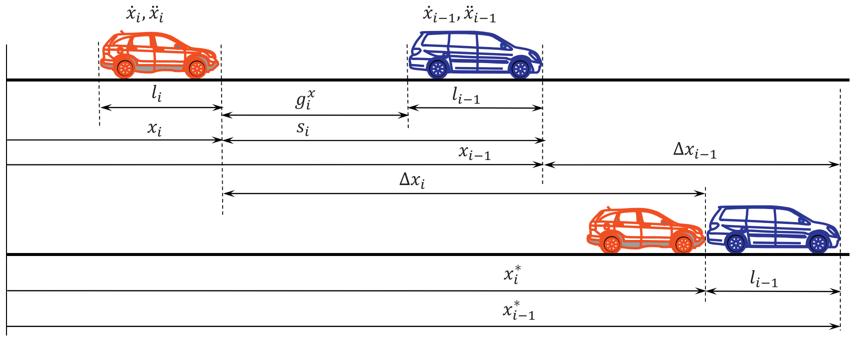

Figure 15.1 Gipps car-following scenario.

should the vehicle ahead come to a sudden stop" [57]. Put another way, at any moment the following driver should leave enough safe distance in front such that in case the leading vehicle commences an emergency brake, the subject driver has time to responde and decelerate to a stop behind the leading vehicle without a collision. The Gipps car-following model is based on such an assumption, and the scenario is depicted in Figure 15.1.

At time t, vehicle i is located at $x_i(t)$ and the leading vehicle $i-1$ is at $x_{i-1}(t)$. At this moment, vehicle $i-1$ at speed $\dot{x}_{i-1}(t)$ commences an emergency brake at a rate of B_{i-1}. Alerted by the braking light in front, driver i at speed $\dot{x}_i(t)$ goes through a perception-reaction process of duration τ_i, trying to understand the situation, evaluate potential options, and then decides to brake as well at a *tolerable* rate of b_i. Hence, the vehicle starts to decelerate from $\dot{x}_i(t+\tau_i)$ to a stop, with the most adverse situation being stopped right after vehicle $i-1$.

Therefore, the distance traveled by vehicle $i-1$ during its emergency brake is $-\frac{\dot{x}^2_{i-1}(t)}{2B_{i-1}}$ since B_{i-1} is negative, so the vehicle stops at location

$$x^*_{i-1} = x_{i-1}(t) - \frac{\dot{x}^2_{i-1}(t)}{2B_{i-1}}. \tag{15.1}$$

Meanwhile, vehicle i travels a distance of $\frac{\dot{x}_i(t)+\dot{x}_i(t+\tau_i)}{2}\tau_i$ during the perception-reaction time and then travels a braking distance of $-\frac{\dot{x}_i(t+\tau_i)^2}{2b_i}$. Hence, the vehicle stops at location

$$x^*_i = x_i(t) + \frac{\dot{x}_i(t) + \dot{x}_i(t+\tau_i)}{2}\tau_i - \frac{\dot{x}^2_i(t+\tau_i)}{2b_i}. \tag{15.2}$$

To be conservative, Gipps added one more buffer space term in the above equation:

$$x_i^* = x_i(t) + \frac{\dot{x}_i(t) + \dot{x}_i(t+\tau_i)}{2}\tau_i + \dot{x}_i(t+\tau_i)\theta - \frac{\dot{x}_i^2(t+\tau_i)}{2b_i}, \tag{15.3}$$

where θ is an extra buffer time appended to the perception–reaction time. To ensure safety, the following relationship must hold:

$$x_{i-1}^* - l_{i-1} \geq x_i^*. \tag{15.4}$$

Plugging in everything, we find the above inequality translates to

$$x_{i-1}(t) - \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} - l_{i-1} \geq x_i(t) + \frac{\dot{x}_i(t) + \dot{x}_i(t+\tau_i)}{2}\tau_i + \dot{x}_i(t+\tau_i)\theta - \frac{\dot{x}_i^2(t+\tau_i)}{2b_i}. \tag{15.5}$$

Note that the actual spacing is $s_i(t) = x_{i-1}(t) - x_i(t)$, so the safe spacing is

$$s_i(t) \geq \frac{\dot{x}_i(t) + \dot{x}_i(t+\tau_i)}{2}\tau_i + \dot{x}_i(t+\tau_i)\theta - \frac{\dot{x}_i^2(t+\tau_i)}{2b_i} + \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} + l_{i-1}. \tag{15.6}$$

Though the above inequality can serve the need of a safety check, the driver needs a basis to determine what to do next in order to achieve the safety goal. Hence, it is necessary to identify which variables in the above inequality are the inputs and which variable is the output. Our daily driving experience suggests the following: $s_i(t)$ and l_{i-1} can be visually estimated; $\dot{x}_i(t)$ and $\dot{x}_{i-1}(t)$ are measurable from the speedometer or motion relative to the roadside; τ_i, θ, b_i, and B_{i-1} are internal to the driver and hence are implicitly known. As such, these variables can be treated as the inputs, while the only output in the above inequality is $\dot{x}_i(t+\tau_i)$, which is the target speed that the driver tries to achieve next. Therefore, finding the output translates to solving the following quadratic inequality:

$$-\frac{1}{2b_i}\dot{x}_i^2(t+\tau_i) + \left(\frac{\tau_i}{2} + \theta\right)\dot{x}_i(t+\tau_i) + \frac{\dot{x}_i(t)\tau_i}{2} + \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} + l_{i-1} - s_i(t) \leq 0. \tag{15.7}$$

The roots of the above quadratic equation are

$$\dot{x}_i(t+\tau_i) = b_i\left(\frac{\tau_i}{2} + \theta\right) \pm \sqrt{b_i^2\left(\frac{\tau_i}{2} + \theta\right)^2 - b_i\left[-\dot{x}_i(t)\tau_i - \frac{\dot{x}_{i-1}^2(t)}{B_{i-1}} - 2l_{i-1} + 2s_i(t)\right]}. \tag{15.8}$$

Let $\theta = \tau_i/2$ as suggested by Gipps. Then,

$$\dot{x}_i(t+\tau_i) = -b_i\tau_i \pm \sqrt{b_i^2\tau_i^2 - b_i[-\dot{x}_i(t)\tau_i - \frac{\dot{x}_{i-1}^2(t)}{B_{i-1}} - 2l_{i-1} + 2s_i(t)]}. \tag{15.9}$$

Consider the signs of the roots and that speed is a positive value, then the solution to inequality (15.7) is

$$0 \le \dot{x}_i(t+\tau_i) \le -b_i\tau_i + \sqrt{b_i^2\tau_i^2 - b_i[-\dot{x}_i(t)\tau_i - \frac{\dot{x}_{i-1}^2(t)}{B_{i-1}} - 2l_{i-1} + 2s_i(t)]}. \tag{15.10}$$

The above derivation has formulated the Gipps model in the car-following regime. In the free-flow regime—that is,, vehicle i is not blocked by a leading vehicle, Gipps suggests the following speed choice:

$$0 \le \dot{x}_i(t+\tau_i) \le \dot{x}_i(t) + 2.5A_i\tau_i(1 - \frac{\dot{x}_i(t)}{v_i})\sqrt{0.025 + \frac{\dot{x}_i(t)}{v_i}}, \tag{15.11}$$

where A_i is the maximum acceleration that driver i is willing to apply and v_i is the desirable speed that driver i is willing to travel at whenever possible. Unlike inequality (15.10), which is derived from a safety rule, this speed choice model is an empirical one obtained from fitting vehicle experimental data. It basically accelerates/decelerates the vehicle to the desirable speed without causing oscillation.

The above two speed choices may cause a little confusion when they are applied to vehicle control because one has to constantly decide for one of them. For example, one should choose (15.10) in the case of car following and (15.11) in the case of free flow. However, what is the cutoff point between free flow and car following? To resolve this confusion, Gipps suggests that there is no need to make a distinction. Under any situation, one just needs to compute the two speeds and choose the lower one—that is,

$$\dot{x}_i(t+\tau_i) = \min\begin{cases} \dot{x}_i(t) + 2.5A_i\tau_i(1 - \frac{\dot{x}_i(t)}{v_i})\sqrt{0.025 + \frac{\dot{x}_i(t)}{v_i}} & \text{(free flow)}, \\ -b_i\tau_i + \sqrt{b_i^2\tau_i^2 - b_i[\dot{x}_i(t)\tau_i - \frac{\dot{x}_{i-1}^2(t)}{B_{i-1}} + 2l_{i-1} - 2s_i(t)]} & \text{(car following)}. \end{cases} \tag{15.12}$$

15.2 PROPERTIES OF THE GIPPS MODEL

As usual, the macroscopic property of the Gipps model under equilibrium conditions is of primary interest. To simplify the analysis, the Gipps safety rule presented in (15.6) can be simplified as follows if one ignores the speed change during the perception-reaction process and the additional buffer time θ:

$$s_i(t) \geq \dot{x}_i(t)\tau_i - \frac{\dot{x}_i^2(t)}{2b_i} + \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} + l_{i-1}. \tag{15.13}$$

Setting both sides equal and rearranging terms yields

$$s_i(t) = \dot{x}_i(t)\tau_i - \frac{\dot{x}_i^2(t)}{2b_i} + \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} + l_{i-1}. \tag{15.14}$$

Under equilibrium conditions, the above car-following model leads to the following speed-density relationship:

$$\frac{1}{k} = \left(-\frac{1}{2b} + \frac{1}{2B}\right) v^2 + \tau v + l \tag{15.15}$$

or

$$\frac{1}{k} = \gamma v^2 + \tau v + l, \tag{15.16}$$

where k is traffic density, $\gamma = -\frac{1}{2b} + \frac{1}{2B}$, $b < 0$ is the average tolerable braking rate, $B < 0$ is the average emergency braking rate, v is the average traffic speed, τ is the average perception-reaction time, and l is the average nominal vehicle length. The corresponding flow-speed relationship is

$$q = \frac{v}{\gamma v^2 + \tau v + l}. \tag{15.17}$$

To find the capacity, one takes the first derivative of flow q with respect to v and sets the result to zero:

$$\left.\frac{\mathrm{d}q}{\mathrm{d}v}\right|_{v_\mathrm{m}} = -\left.\frac{\gamma - \frac{l}{v^2}}{(\gamma v + \tau' + \frac{l}{v})^2}\right|_{v_\mathrm{m}} = 0. \tag{15.18}$$

Solving the equation yields

$$v_\mathrm{m} = \sqrt{\frac{l}{\gamma}}, \tag{15.19}$$

and correspondingly,

$$q_m = \frac{1}{2\sqrt{\gamma l} + \tau}. \tag{15.20}$$

From a check of the second derivative of q at v_m, it turns out that q_m is indeed the maximum value of q.

15.3 BENCHMARKING

The microscopic benchmarking refers to the scenario presented in Section 12.3.1 and the macroscopic benchmarking refers to the scenario presented in Section 12.3.2.

15.3.1 Microscopic Benchmarking

The benchmarking result of the Gipps model is plotted in Figure 15.2. The performance of the Gipps model is summarized as follows:

- Start-up: the model is able to start the vehicle up from standstill. See Figure 15.2 when $t > 0$ s.
- Speedup: the model is able to speed the vehicle up realistically to its desired speed. See Figure 15.2 when $0\,\text{s} < t < 100\,\text{s}$.
- Free flow: the model is able to reach and settle at the desired speed under free-flow conditions. See Figure 15.2 when $0\,\text{s} < t < 100\,\text{s}$.
- Cutoff: the model overdecelerates slightly, which causes a small oscillation in speed, but in general the model retains control and responds reasonably when a vehicle cuts in in front. See Figure 15.2 around $t = 100\,\text{s}$.
- Following: the model is able to adopt the leader's speed and follow the leader at a reasonable distance. See Figure 15.2 when $100\,\text{s} < t < 200\,\text{s}$.
- Stop and go: the model is able to stop the vehicle safely behind its leader and start the vehicle moving when the leader departs. See Figure 15.2 when $200\,\text{s} \geq t \leq 300\,\text{s}$.
- Trailing: the model is able to speed up normally without being tempted to speed up by its speeding leader. See Figure 15.2 when $300\,\text{s} < t < 400\,\text{s}$.
- Approaching: the model is able to decelerate properly when approaching a stationary vehicle at a distance. See Figure 15.2 when $400\,\text{s} \geq t < 420\,\text{s}$.
- Stopping: the model is able to stop the vehicle safely behind the stationary vehicle. See Figure 15.2 when $t \geq 420\,\text{s}$.

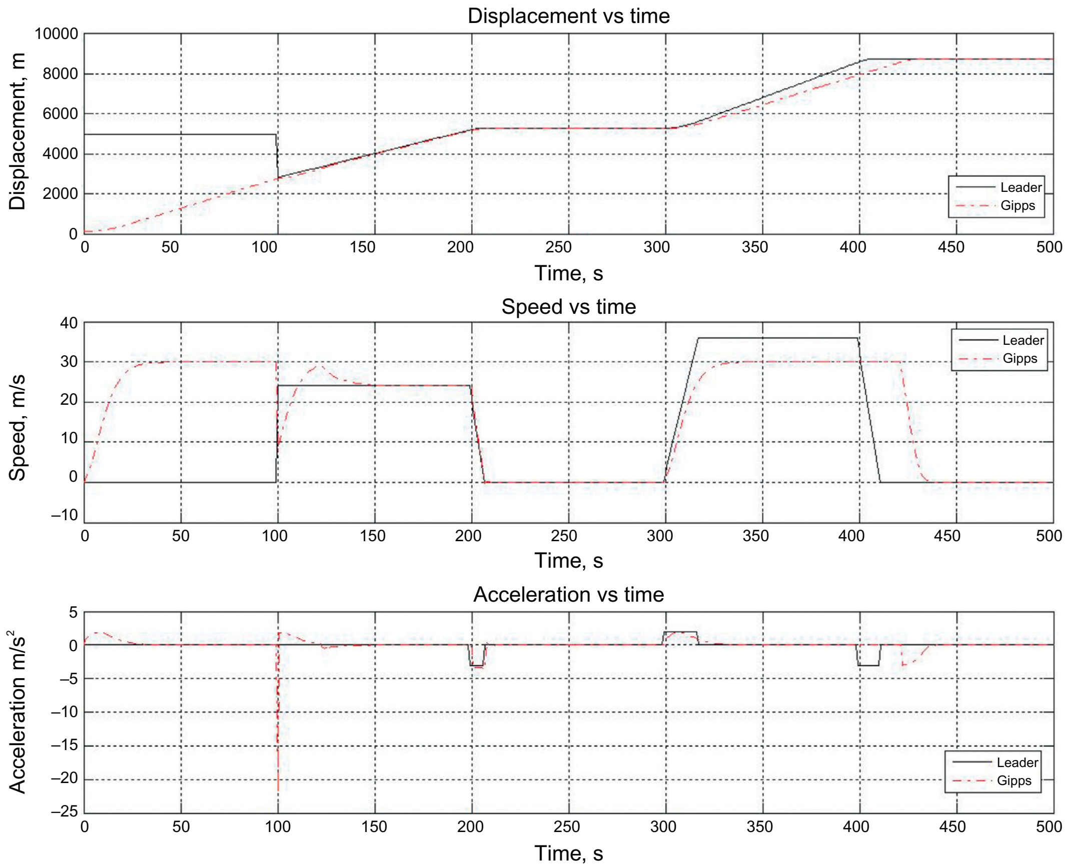

Figure 15.2 Microscopic benchmarking of the Gipps model.

Table 15.1 Microscopic benchmarking parameters of the Gipps model

l_i	v_i	τ_i	b_i	
6 m	30 m/s	1.0 s	$-3.4\,\mathrm{m/s^2}$	
A_i	B_{i-1}	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
$1.7\,\mathrm{m/s^2}$	$-6.0\,\mathrm{m/s^2}$	120 m	0 m/s	$0\,\mathrm{m/s^2}$

The above benchmarking is based on the set of parameters in Table 15.1, and the outcome may differ for a different set of parameters.

15.3.2 Macroscopic Benchmarking

The fundamental diagram implied by the Gipps model is plotted in Figure 15.3 against empirical observations. The model parameters are set the same as suggested in the original paper. It can be seen that the model fits empirical

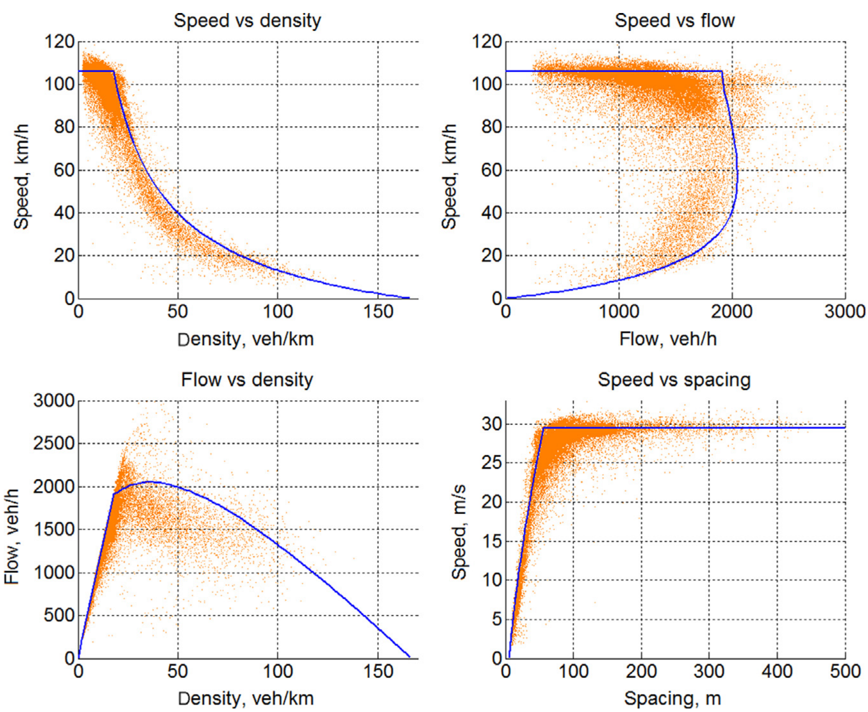

Figure 15.3 Fundamental diagram implied by the Gipps model.

Table 15.2 Macroscopic benchmarking parameters of the Gipps model

b	B	τ	l
$-3.0\,\mathrm{m/s^2}$	$-3.5\,\mathrm{m/s^2}$	$1\,\mathrm{s}$	$6.5\,\mathrm{m}$

data reasonably well except for free-flow conditions (i.e., in the low-density range). In addition, the speed at capacity predicted by the Gipps model is much lower than it should be.

The above benchmarking is based on the set of parameters in Table 15.2, and the outcome may differ for a different set of parameters.

PROBLEMS

1. Compare the Forbes model and the Gipps model and explain the difference in their modeling philosophy.
2. The following figure is the result of macroscopic benchmarking of the Gipps model with the following model formulation and parameters:

$$\frac{1}{k} = \left(-\frac{1}{2b} + \frac{1}{2B}\right) v^2 + \tau v + l,$$

   where tolerable deceleration $b = -3.0\,\mathrm{m/s^2}$, emergency braking rate $B = -3.5\,\mathrm{m/s^2}$, perception-reaction time $\tau = 1.0$ sm and nominal vehicle length $l = 6.5\,\mathrm{m}$.

   Find the capacity condition $(q_\mathrm{m}, k_\mathrm{m}, v_\mathrm{m})$ of the model and comment on your result.
3. Perform a one-step simulation based on the following conditions: Two cars are traveling in the same lane on a freeway. The length of both vehicles is $l_{i-1} = l_i = 6\,\mathrm{m}$. Lane change is not considered in this problem. At time t, the leading vehicle $i - 1$ is traveling at a speed of $\dot{x}_{i-1}(t) = 72\,\mathrm{km/h}$ and the following vehicle i is traveling at a speed of $\dot{x}_i(t) = 108\,\mathrm{km/h}$. The spacing between the two vehicles (measured from front bumper to front bumper) is $s_i(t) = 40\,\mathrm{m}$. The perception-reaction time of the following driver is $\tau_i = 1.5\,\mathrm{s}$. Use the Gipps model to predict the speed that the following driver will adopt after a perception-reaction time. Assume the model parameters take the same values as those in Section 15.3.1.

4. Building on the above problem, at time t, a third vehicle at speed 108 km/h cuts in between the two vehicles. The spacing between the third vehicle and the following vehicle is 15 m. Use the Gipps model to compute the speed that the following driver needs to adopt after a perception-reaction time.