Homework: Shockwaves and Wave Solutions

8.1, 8.3, 8.6

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8.1

Figure 1 shows an annotated flow-density curve with the following: (a) $v_f = 115$ kph; (b) $k_j = 135$ v/km; (c) $q_m = 2100$ vph, $k_m = 25$ v/km, $v_m = 84$ kph; (d) $q_A = 1400$ vph, $k_A = 12$ v/km, $v_A = 117$ kph; (e) $q_B=740 $vph, $k_B = 100$ v/km, $v_B = 7$ kph; (f) $w_f = 115$ kph, $w_j = -21$ kph, $w_m = 0$ kph, $w_a = 115$ kph, $w_B = -21$ kph; (g) $U_{AB} = -7.5$ kph.

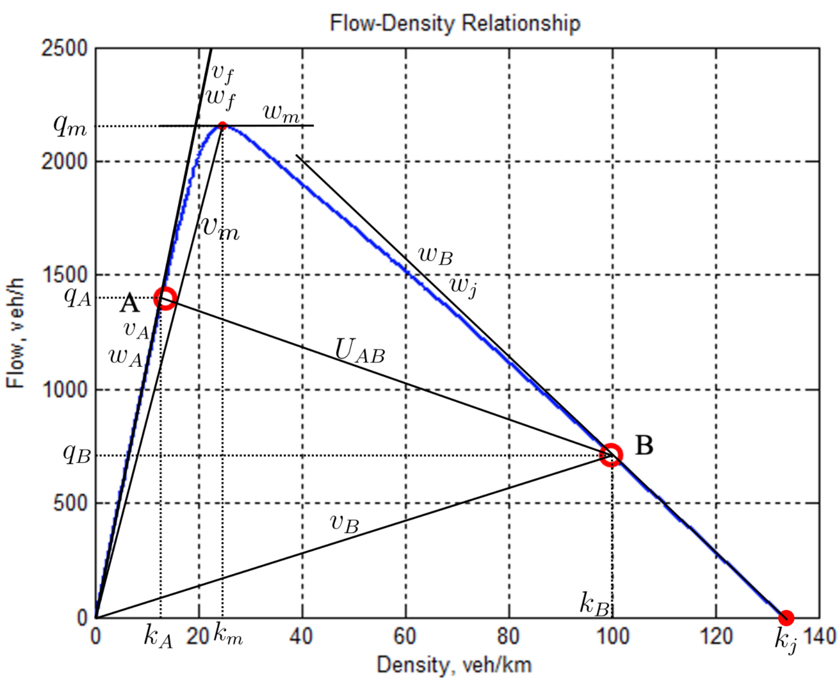

Figure 1: Annotated flow-density relationship.

8.3

Traffic on a 16-km stretch of Interstate 90 is observing the flow-density relationship as seen in Figure 1. The traffic is initially operating at condition B. At 07:00PM the demand upstream of the midpoint of this segment drops and traffic begins to operate at condition A. See Figure 2 for a time-space diagram depicting this. At 07:30PM the traffic (b) 2km upstream and (a) 2km downstream of the midpoint are both operating under condition B. (c) The queue will reach the upstream end of this segment when the shockwave AB has traveled 8 km. Because the speed of the shockwave U_{AB} is 7.5 kph, this will happen after 64 minutes, or at 08:04PM.

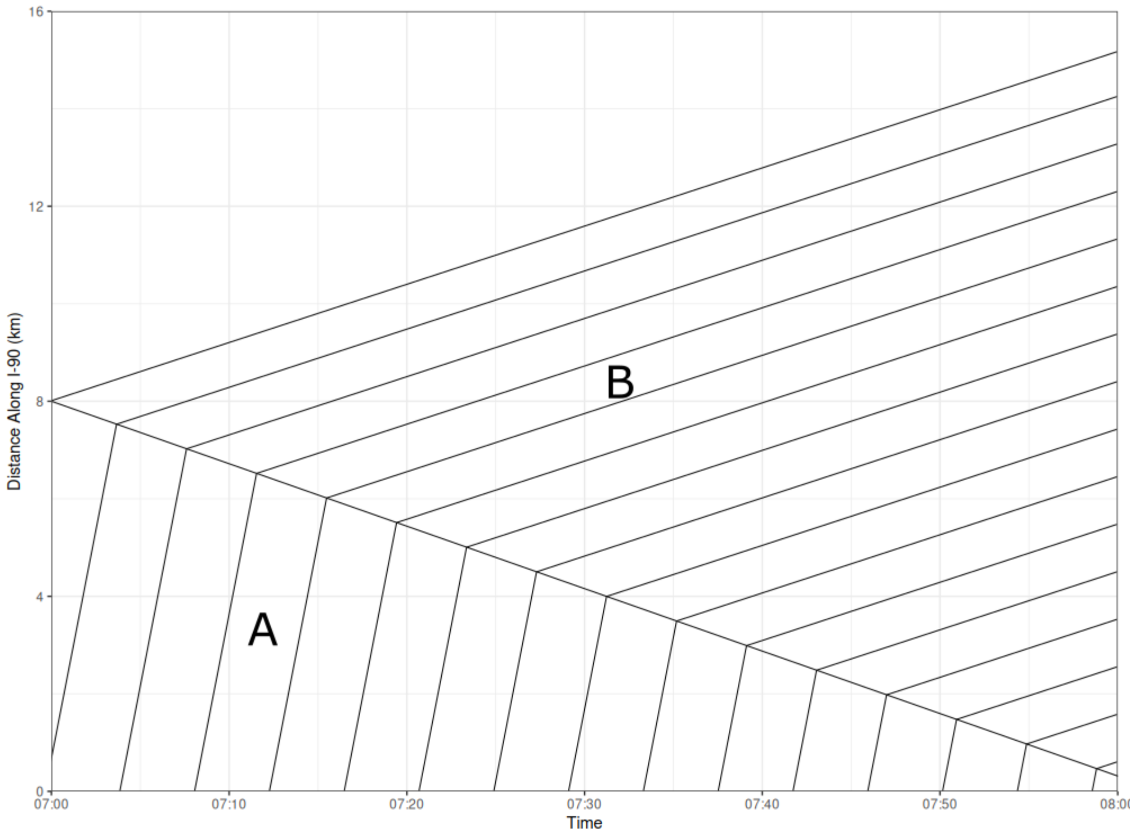

Figure 2: Time-space diagram of Interstate 90 segment.

Table 1: Traffic Conditions on I-91

Condition	Description	q (vph)	k (v/km)	v (kph)
A	Arrival Flow	2,000	40	50.0
D	Queued Flow	0	200	0.0
C	Capacity Flow	2,200	60	36.7
E	Reduced Flow	1,100	145	7.6

8.6

An intelligent transportation system problem. On Wednesday at 9:00 a.m., there is an accident on northbound Interstate 91. The traffic operation center (TOC) has to decide how to clean up the accident. After collecting information and communicating with highway patrol and emergency operator, the TOC determines that there are two alternatives:

1. Completely shut Interstate 91 for 10 minutes, clean it up, and then reopen Interstate 91 for normal operation.
2. Partially open Interstate 91 at reduced capacity, but the cleanup requires 30 minutes before normal operation can resume.

Table 1 gives the flow, density, and speed for each relevant traffic condition on this segment of the I-91.

Figure 3 shows a time-space diagram for alternative 1, where the interstate is closed for 10 minutes.

The point (t, d) where the shockwaves AD and CD intersect is the solution to the equation $d = U_{AD}t = U_{CD}(t - \frac{10}{60})$. Given that $U_{AD} = -12.5$ and $U_{CD} = -15.7$, we find that $(t, d) = (0.81, 10)$.

Figure 4 shows a time-space diagram for alternative 2, where the flow on the interstate is reduced for 30 minutes.

The point (t, d) where the shockwaves AE and CE intersect is the solution to the equation $d = U_{AE}t = U_{CE}(t - \frac{30}{60})$. Given that $U_{AE} = -8.6$ and $U_{CE} = -12.9$, we find that $(t, d) = (1.48, 13)$.

As it turns out, the total queue length will be shorter for alternative 1, where the interstate is closed for 10 minutes.

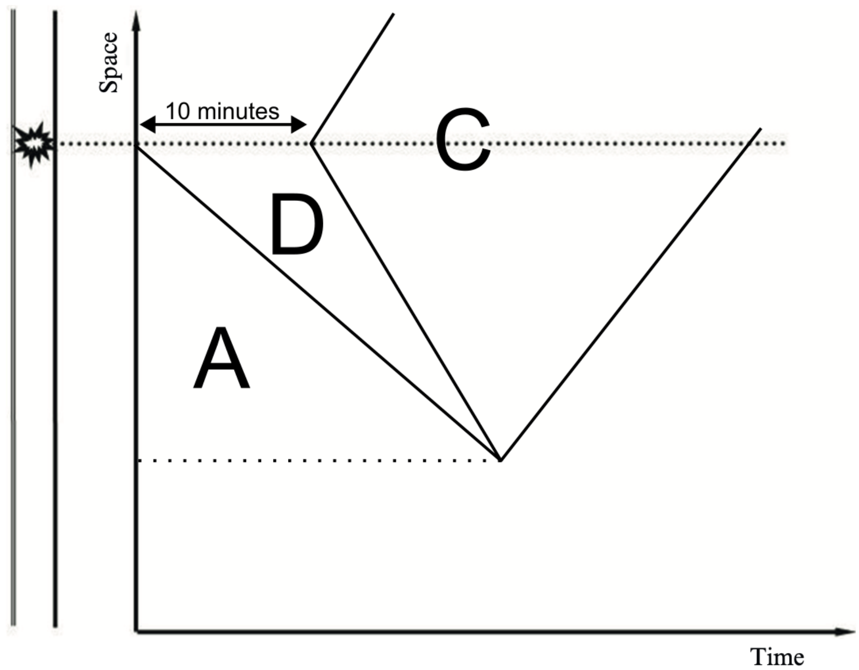

Figure 3: Time-space diagram of closing I-91 for 10 minutes.

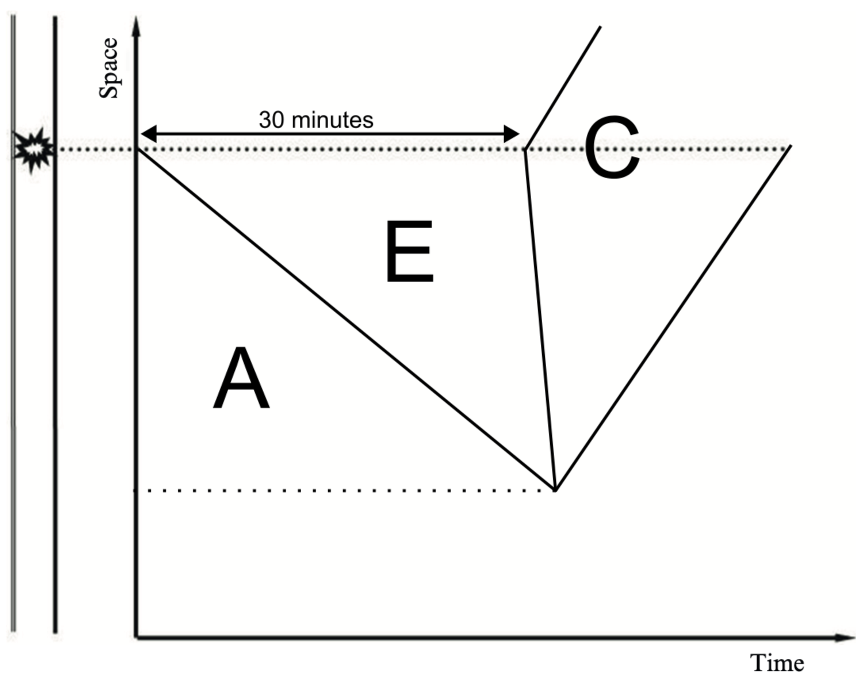

Figure 4: Time-space diagram of reducing flow on I-91 for 30 minutes.