Exercises

(1) Let T be the linear operator on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

  (a) Calculate the eigenvectors and corresponding eigenvectors

  (b) Using the eigenvectors as your new basis, compute the linear operator on your new basis

  (c) Check that the transformed operator is a diagonal matrix

  A bit of a computation exercise but an interesting result that can be generalised

(2) Let V be the vector space of all functions from $\mathbb{R}$ into $\mathbb{R}$ which are continuous, i.e., the space of continuous real-valued functions on the real line. Let T be the linear operator on V defined by

$$(Tf)(x) = \int_0^\infty f(t)dt$$

Show that T has no eigenvalues *Hint: Start with $Tf = \lambda f$*