A New Approach to the Electron Beam Dynamics in Undulators and Wigglers

Pascal ELLEAUME
European Synchrotron Radiation Facility
BP 220, F-38043 Grenoble
France

Abstract

A new approach to the dynamics of an electron in the field of an Insertion Device (ID) is presented. The Lorentz Force equation is integrated to second order in the inverse of the electron energy. For periodic IDs, a potential U(x,z) is derived which allows the computation of the angular kick experienced by any electron injected close or far away from the symmetry axis of the ID. This method addresses not only conventional IDs but any ID with periodic magnetic field. U(x,z) is deduced from 3D magnetostatic computation or can be easily measured. It can be tabled or fitted in order to be incorporated into tracking codes.

I. INTRODUCTION

A serious problem to be encountered in the operation of the third generation of synchrotron sources is the reduction of dynamic aperture and electron/positron beam lifetime when Insertion Devices (ID) are installed. Such effects have already been observed on SuperACO[1] . The perturbation may come from the insufficient vacuum inside the ID chamber, from the reduction of the physical aperture of the vacuum chamber or from the electron beam dynamics inside the highly non homogeneous magnetic field of the undulator or wiggler. When dealing with the dynamics of an electron in a magnetic field, it is convenient to expand the electron trajectory as a power of the inverse of the electron energy. At zero order, electrons follow a straight trajectory like in a drift space. At first order, the horizontal (vertical) trajectory is simply given by the integrals of the vertical (horizontal) magnetic field. Such integrals may depend on the transverse coordinate (x,x',z,z') at which the electron is injected inducing linear and non linear focusing. However field integrals can always be corrected by carefully manufacturing the ID. Magnet block pairing and shimming are the typical ways of correction[2] . The next contribution occurs at second order of the inverse of the electron energy. It is responsible for the well known vertical focusing of the conventional undulators. It scales proportional to the length of the ID and to the square of (B/E) where B is the typical magnetic field and E the electron energy. The exact computation of this second order effect is the object of this paper. This effect fully depends on the three dimensional nature of the ID magnetic field. In the following I shall briefly summarize the usual approach and then present the new one.

II. THE USUAL APPROACH

Let us define the coordinate system (x,z,s) where s is the longitudinal axis along which the electron propagates inside the ID. x and z are the transverse horizontal and vertical coordinates. In the usual approach, one models the three dimensional magnetic field of a conventional undulator by means of the following scalar potential:

$$
\begin{gathered}
V = \frac{B_0}{k_z} (\cos k_x x \cdot \sinh k_z z \cdot \cos k_0 s) \\
k_z^2 - k_x^2 = k_0^2 \\
\vec{B} = \vec{\nabla} V
\end{gathered}
\tag{1}
$$

It is easy to verify that the magnetic field given by (1) satisfies the Maxwell Equation in vacuum. The vertical magnetic field is sinusoidal with amplitude B_0 l on the symmetry axis (x=z=0). Various direction have been followed in order to derive the electron beam dynamics. One of them consists in deriving a Hamiltonian of the motion by successive canonical transformations[3] . Another less efficient approach consists in integrating the Lorentz Force Equation in such a field.

The quality of the results obtained essentially suffers from the approximation made by replacing the real undulator magnetic field by the analytical one (1) . Even though (1) is extremely simple and gives quite a good insight to the understanding of the three components of field present in a sinusoidal undulator, it suffers some drawbacks which are detailed below. The general magnetic field is periodic and contains not only the fundamental spatial period but also some harmonics which may make appreciable contributions if the electron propagates close to the surface of the magnet blocks. Conventional undulators have a vertical field strongly (weakly) varying with z (x) which is usually taken into account by using $k_x << k_z$. In the two dimensional case where k_x =0, (1) is exact for a pure sinusoidal undulator, however the modelization of the transverse contribution by a cos $k_x x$ term is very crude even though it is, in actual fact, of lower importance compared to the sinh $k_z z$ term. Finally the magnetic field of exotic undulators such as helical or elliptical devices is poorly or even not at all described by (1).

III. The New Approach

Starting from the original equations of motion

$$x'' = \frac{-\alpha}{\sqrt{1+x'^2+z'^2}}\left[z'B_s - (1+x'^2)B_z + x'z'B_x\right]$$
$$z'' = \frac{\alpha}{\sqrt{1+x'^2+z'^2}}\left[x'B_s - (1+z'^2)B_x + x'z'B_z\right] \tag{2}$$

where $\alpha = e/\gamma mc$ and $x' = dx/ds$. e is the electron charge, m its mass, c the speed of light and γmc^2 the total energy of the electrons. (2) can be solved by making a power expansion in α: the inverse of the electron energy. At first order in α, the solution is well known to be:

$$\frac{dx}{ds} = \alpha \int_{-\infty}^{s} B_z ds$$
$$\frac{dz}{ds} = -\alpha \int_{-\infty}^{s} B_x ds \tag{3}$$

At second order in α, the solution is more complex. However, one can show that within the following conditions:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{s} B_x ds'ds = 0 \qquad \int_{-\infty}^{\infty} B_x ds = 0$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{s} B_z ds'ds = 0 \qquad \int_{-\infty}^{\infty} B_z ds = 0 \tag{4}$$
$$\frac{dx}{ds}(-\infty) = \frac{dz}{ds}(-\infty) = 0$$

The angle after crossing the magnetic field region can be written as:

$$\frac{dx}{ds}(\infty) = -\frac{\alpha^2}{2}\int_{-\infty}^{\infty}\frac{\partial}{\partial x}\Phi(x,z,s)\,ds + o(\alpha^3)$$
$$\frac{dz}{ds}(\infty) = -\frac{\alpha^2}{2}\int_{-\infty}^{\infty}\frac{\partial}{\partial z}\Phi(x,z,s)\,ds + o(\alpha^3) \tag{5}$$
$$\Phi(x,z,s) = \left(\int_{-\infty}^{s} B_x ds\right)^2 + \left(\int_{-\infty}^{s} B_z ds\right)^2$$

(5) are the most important results. One way to phrase it is the following. If an electron is injected inside a region containing some magnetic field and if the integral and double integral of the magnetic field components transverse to the electron's unperturbed trajectory are zero then the only angular deviation experienced by the electron occurs at second order in a and is given by (5). If the angular kick expressed by (5) depends on the position x,z of the electron some focusing takes place. One can define three focal lengths F_x, F_z, F_c:

$$\frac{1}{F_x} = -\frac{\alpha^2}{2}\int_{-\infty}^{\infty}\frac{\partial^2}{\partial x^2}\Phi(x,z,s)\,ds$$
$$\frac{1}{F_z} = -\frac{\alpha^2}{2}\int_{-\infty}^{\infty}\frac{\partial^2}{\partial z^2}\Phi(x,z,s)\,ds \tag{6}$$
$$\frac{1}{F_c} = -\frac{\alpha^2}{2}\int_{-\infty}^{\infty}\frac{\partial^2}{\partial z\partial x}\Phi(x,z,s)\,ds$$

By further derivating (6), one generates terms describing some non linear behavior of the electron dynamics.

IV. Application to Insertion Devices

Insertion Devices such as undulator and wigglers have a magnetic field geometry which satisfies (4) except for some possible field errors. Consequently, if they do not present any field errors, they do not deflect nor focus the electrons at first order in the inverse of the electron energy. However they do deflect and focus the beam at second order as expressed by (5) and (6). A conventional undulator present a vertical periodic magnetic field, the field of which, satisfies the following condition on the symmetry axis:

$$B_x = \frac{\partial B_x}{\partial x} = \frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} = \frac{\partial B_z}{\partial z} = 0 \tag{7}$$

As a consequence of (5),(6) and (7), electrons injected on the symmetry axis are not deviated. Electrons injected close to this axis are deviated corresponding to the focusing:

$$\frac{1}{F_x}+\frac{1}{F_z}=\alpha^2\int_{-\infty}^{\infty}B_z^2\,ds$$

$$\frac{1}{F_z}=\alpha^2\left(\int_{-\infty}^{\infty}B_z^2\,ds-\int_{-\infty}^{\infty}\int_{-\infty}^{s}\frac{\partial^2 B_z}{\partial x^2}\,ds'\int_{-\infty}^{s}B_z\,ds'\,ds\right)\qquad(8)$$

$$\frac{1}{F_c}=0$$

This result is well known from existing theories. However (8) is a simple consequence of (5) which is much more general. New questions can be answer directly from (5) such as what is the angular deviation and focusing experienced by an electron injected away from the symmetry axis?, what happen for a helical undulator or any elliptical undulator?. Generally the field of Insertion Devices is periodic in the longitudinal coordinate s. Then the knowledge of the potential:

$$U(x,z)=\int_{1period}\Phi(x,z,s)\,ds\qquad(9)$$

is sufficient to predict the angular kick experienced by an electron injected along s at any transverse position x, z. Such a potential can be computed by some 3D magnetostatic code or directly measured in the laboratory where the ID is built. The following figure presents a 2D plot of U(x,z) computed[5] for an ESRF conventional undulator of 48 mm period, 20 mm gap and horizontal width of 70 mm. Taking the origin of x and z on the symmetry axis, U(x,z) is such that U(-x,z)=U(x,-z)=U(x,z) and the figure presents the variation of U for z between 0 and 9 mm and x between 0 and 42 mm (x=35 mm corresponds to the magnet block border).

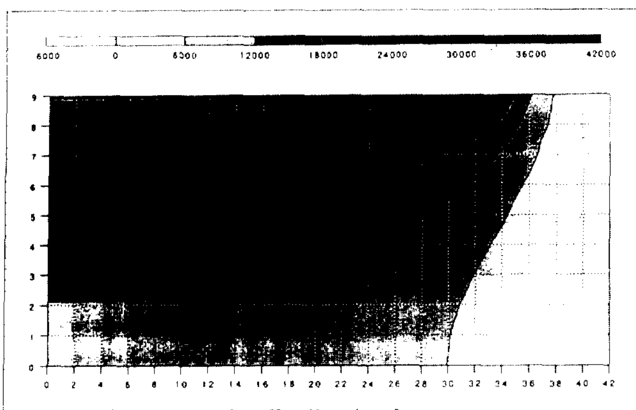

Figure 1. Potential U(x,z) in T^2 mm^3 for a conventional Vertical field undulator (period 48 mm, gap 20 mm, width of magnet block 70 mm, length 1.6 m).

The next figure presents U(x,z) in the same conditions as Figure 1. for a horizontal field undulator of a new type proposed by the author[6].

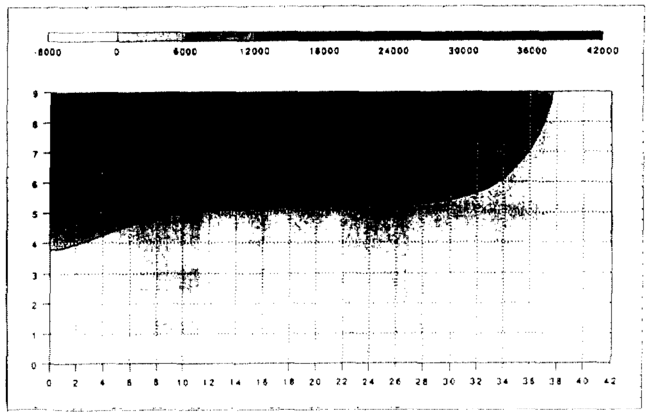

Figure 2. Potential U(x,z) in T^2 $mm^3$for an horizontal field undulator (period 48 mm, gap 20 mm, width of magnet block 70 mm, length 1.6 m).

Note that so far I have only considered electrons which are injected along s without any angle. The correct way of analyzing the dynamics of these electrons is to compute the potential with the field normal to the new tilted trajectory. Unfortunately, the field is then no longer exactly periodic and the simplification outlined above is not valid. An approximation consists in treating the trajectory as a ladder with constant x and z over each individual period of the field but varying from one period to the next. This approximation should be valid whenever the beta function of the electron beam is bigger than the period of the Insertion Device which is nearly always the case.

V. REFERENCES

[1] P. Brunelle, Beam Dynamics with many Undulators on Super-ACO, EPAC 90, proceedings of the 2nd European Particle Accelerator Conference, p1411 (1990).

[2] Status of ESRF Insertion Devices, J. Chavanne et al., this conference.

[3] L. Smith, Effect of Wiggler and Undulators on Beam Dynamics, ESG Tech Note-24, 1986. G. Wustefeld, J. Bahrdt, IEEE Particle Accelerator, San Francisco May 6-9 1991, p266 (1991).

[4] P. Elleaume, Beam Displacement and focusing induced by arbitrary field Insertion Devices, Report ESRF-SR/ID-89-33 (1989).

[5] J. Chavanne, P. Elleaume, RADIA Version 2.0, A software running under Wingz to compute Synchrotron Radiation and Magnetic fields, Report ESRF-SR/ID-90-46 (1990).

[6] P. Elleaume, A flexible Planar/Helical Undulator Design for Synchrotron Sources, Nucl. Instr. and Methods, A291 (1990) 371-377.