

The complete code, shown below, can also be found on [Github](#)

```
import numpy as np
from scipy.integrate import quadrature as integrate
from shape_functions import shape_functions
from jacobian import jacobian

#
# A: area
# k: hydraulic conductivity
# l: length
# alpha: alpha term associated with pipe permeability
# num_elements: number of elements
# num_element_nodes: number of nodes per element
# bc_essential: ( dict ) essential boundary conditions
# bc_natural: ( dict ) natural boundary conditions
#
def solve_fe( A, k, l, alpha, num_elements, num_element_nodes, bc_essential, bc_natural ):

    # Calculate number of nodes
    num_nodes = num_elements * ( num_element_nodes - 1 ) + 1

    # Get x-coordinate for each node
    x_coord = [ float(i) * ( float(l) / float( num_nodes - 1 ) ) for i in range( num_nodes ) ]

    # Get the shape functions
    N, dN, xi = shape_functions( num_element_nodes )

    # Create IEN and ID functions
    def IEN ( element_number, local_node_number ):
        return ( num_element_nodes - 1 ) * element_number + local_node_number

    # Create global arrays
    K = np.zeros( ( num_nodes, num_nodes ) )
    M = np.zeros( ( num_nodes, num_nodes ) )
    F = np.zeros( ( num_nodes, 1 ) )

    # Element loop
    for element in range( num_elements ):

        # Create local matrices
        ke = np.zeros( ( num_element_nodes, num_element_nodes ) )
        me = np.zeros( ( num_element_nodes, num_element_nodes ) )
        fe = np.zeros( ( num_element_nodes, 1 ) )

        # Get globals
        nodes = [ IEN( element, i ) for i in range( num_element_nodes ) ]
        x = [ x_coord[ node ] for node in nodes ]

        # Coordinate transformation and jacobian for this element
        J = jacobian( x[ 0 ], x[ len( x ) - 1 ] )

        # Node loop
        for row in range( num_element_nodes ):

            # Check for natural boundary condition contribution
```

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if nodes[ row ] in bc_natural:

    fe[ row, 0 ] -= N[ row ]( xi[ row ] ) * bc_natural[ nodes[ row ] ] * A

# Other node noop
for col in range( num_element_nodes ):

    # Integrate to calculate ke term
    _k = integrate( lambda _xi: ( dN[row]( _xi ) / J )*( dN[col]( _xi ) / J ),
                    xi[0],
                    xi[len(xi)-1] )[ 0 ]

    _m = integrate( lambda _xi: N[ row ]( _xi ) * N[ col ]( _xi ),
                    xi[0],
                    xi[len(xi)-1] )[ 0 ]

    # Add the term to ke, converting to local coordinates
    ke[ row, col ] = _k * A * k * J
    me[ row, col ] = _m * alpha * J

# Update global arrays with contributions from local
for row in range( num_element_nodes ):

    # Get the global row
    r = IEN( element, row )

    # Add local f to global F
    F[ r, 0 ] += fe[ row, 0 ]

    for col in range( num_element_nodes ):

        # Get the global column
        c = IEN( element, col )

        # Add the local k to global K
        K[ r, c ] += ke[ row, col ]
        M[ r, c ] += me[ row, col ]

# Apply essential boundary conditions using elimination method
K_mask = np.ones( K.shape, dtype=bool )
F_mask = np.ones( F.shape, dtype=bool )
dof = num_nodes - len( bc_essential )
for node, value in bc_essential.iteritems():

    # Mask row and column in the K matrix and M matrix
    K_mask[ node, : ] = False
    K_mask[ :, node ] = False

    # Mask row in the F array
    F_mask[ node, : ] = False

    # Subtract from the F array
    for row in range( num_nodes ):

        _k = K[ row, node ]
        _m = M[ row, node ]
        F[ row, 0 ] -= _k * value
        F[ row, 0 ] += _m * value

# Apply the mask and reshape
K = np.reshape( K[ K_mask ], ( dof, dof ) )
M = np.reshape( M[ K_mask ], ( dof, dof ) )
F = np.reshape( F[ F_mask ], ( dof, 1 ) )

```

```
# Solve for unknowns
d = np.linalg.solve( K-M, F )

# Place essential boundary condition values into solution
for node, value in bc_essential.iteritems():

    d = np.insert( d, node, value )

return d, x_coord
```

