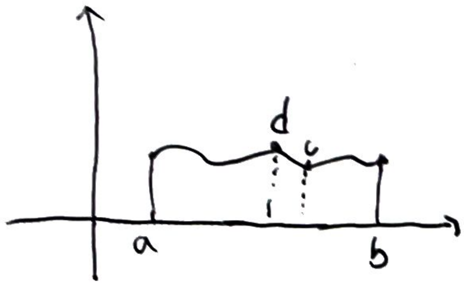
MA1014 4/11/21

Applications of Derivatives (Rolles & MVTs)

If $f:[a,b] \to \mathbb{R}$ is differentiation on (a,b)

then

<u>Rolles Theorm</u> If <u>$f(a)=f(b)$</u> then $\exists c \in (a,b)$ s.t. <u>$f'(c)=0$</u>



i.e the function has a turning point

<u>Proof</u> ① If f is constant

$f(x)=f(a)=f(b)\ \forall x$

$\Rightarrow f'(c)=0 \quad \forall c$

② If f is not constant

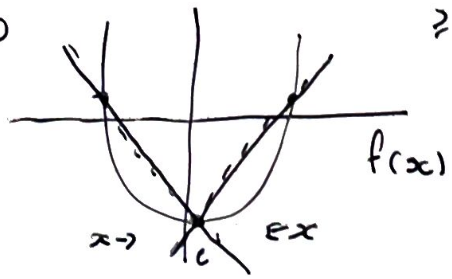
Extreme value theorm (f continuous)

f attains its bounds $\min f(c)$ & $\max f(d)$

we need to prove $f'(c)=0$

$$\lim_{x\to c^-} \frac{f(x)-f(c)}{x-c} = \lim_{x\to c^+} \frac{f(x)-f(c)}{x-c}$$

$\leq 0 \qquad\qquad \geq 0$



so $f'(c)=0$

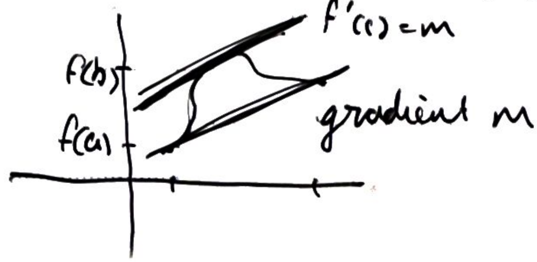
Mean Value Theorm:

similar to Rolles Theorm

Suppose $f : [a, b] \to \mathbb{R}$ cts & differentiable of (a,b)

we cannot assume $f(a) = f(b)$, but we let $m = \frac{f(b)-f(a)}{b-a}$

mean slope
average gradient



MVT $\exists\, c \in (a,b)$ such that $f'(c) = m = \frac{f(b)-f(a)}{b-a}$

Proof Rolle $\Leftrightarrow$ MVT

Given f, let $g(x) = f(x) - mx$

$g(b) = f(b) - mb$ f continuous $\Rightarrow$ g cts
$g(a) = f(a) - ma$ f differentiable $\Rightarrow$ g also

$g(b) - g(a) = f(b) - f(a) - m(b-a)$
$= \cancel{f(b)-f(a)} - \cancel{\frac{f(b)-f(a)}{b-a}}\,\cancel{(b-a)}$ so $g(a) = g(b)$

applying Rolles Theorm to g $\exists\, c \in (a,b)$ $g'(c) = 0$

$g'(c) = f'(c) - m = 0$ & we have $f'(c) = m$