MA1014 5/10/21

Interval & Absolute Value

Absolute value of $a \in \mathbb{R}$ is defined by

$$|a| = \max\{a, -a\} = \begin{cases} a, & a \geqslant 0 \\ -a, & a < 0 \end{cases}$$

$|-a| = |a|$ $\quad$ $|a| = 0 \iff a = 0$

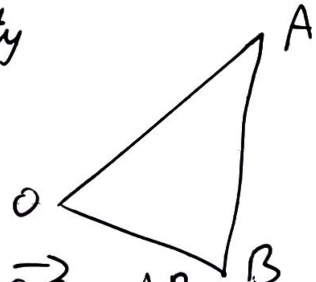
$|ab| = |a||b|$,

$|a| = +\sqrt{a^2}$

Proof $a^2 \geqslant 0$ so $|a^2| = a$

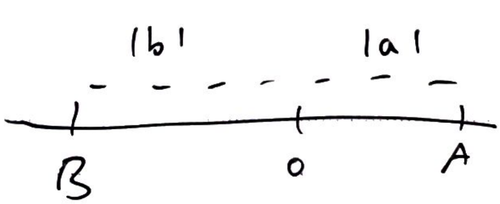
$$|a^2| = |a| \cdot |a| = |a|^2$$
$$|a|^2 = a^2 \Rightarrow |a| = +\sqrt{a^2}$$

Triangle inequality



distance $\vec{OA} + \vec{OB} = AB$

think $|a|$ distance from OA



$|a+b| \leq |a+b|$

Proof $\quad |x| = +\sqrt{x^2}$

$$|a+b| = \sqrt{(a+b)^2}$$

$$|a| = \sqrt{a^2}$$
$$|b| = \sqrt{b^2}$$

$$\left(\sqrt{a^2} + \sqrt{b^2}\right)^2 = a^2 + b^2 + 2\sqrt{a^2}\sqrt{b^2}$$

$$|a+b|^2 = (a+b)^2 = a^2 + b^2 + 2ab$$

$$2|a||b| \geq 2ab$$

$$\left(\sqrt{a^2} + \sqrt{b^2}\right) \geq |a+b|^2$$

$$|a| + |b| \geq |a+b|$$