MA1014 5/10/21

1.3 Ordered Fields

Field Axioms

name:	additive	multiplicative
identity	$\forall x \in F,\ x + 0 = x$	$x \cdot 1 = x$
inverse	$\forall x \in F, \exists (-x),\ x + (-x) = 0$	$\exists x^{-1}\quad x \cdot x^{-1} = 1$
associativity	$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$
commutativity	$x + y = y + x$	$xy = yx$
distributivity	$\forall x, y, z \in F$	$x(y + z) = xy + xz$

e.g $0x = x0 = 0$

let $y = 0x$

then $y + y = 0x + 0x = x(0 + 0)$ Dist

$= x \cdot 0 = y$

$(y + y) + (-y) = y + (y + (-y)) = y + 0$

$= y$ Assoc

$y + (-y) = 0 \qquad \therefore y = 0$

e.g 1.1 d) $xy=0 \iff x=0 \quad y=0$

$\Rightarrow$ suppose $xy=0$

If $y \neq 0$ then $\exists y^{-1} : y \cdot y^{-1} = 1$

So $x = x \times 1 = x \cdot (y \cdot y^{-1}) = (xy) \cdot y^{-1}$
$= 0 \cdot y^{-1} = 0$ by part (c)

$\Leftarrow$ If $x=0$ or $y=0$ $\quad xy=0$

by part 1.1 (c)

Order Axioms

Total order $\quad$ Either $a<b$, $b<a$ or $a=b$

Transitivity $\quad a<b \cap b<c$ then $a<c$ or $a<b<c$

Compatability $\quad a<b \quad a+c<b+c$, $a<b \quad c>0 \quad ac<bc$

Consequenses of Order Axioms:

Put $a=0$ $\quad 0<b$, $c>0 \Rightarrow 0<bc$
So if : $a>0 \quad a^2 = a \cdot a \quad >0$
$a<0 \quad a+(-a) < 0 + (-a)$
$0 < -a$
So $-a>0$ & $a^2 = (-a) \cdot (-a) > 0$
$a \neq 0 \Rightarrow a^2 > 0$, $a^2 \geq \quad \forall a$