MA1014 3/11/21

Implicit Differentiation & Higher Derivative

Chain Rule Implicit definition of y

$y=f(x)$ with $\underbrace{y^5 x}\, + x^3 + \sin(x) = 0$

$y' = f'(x)$?

$\frac{dy}{dx}$?

$\frac{d}{dx}(y^5)\cdot x + y^5 \frac{d}{dx}(x) + 3x^2 + \cos(x) = 0$

product rule

$5y^4 \frac{dy}{dx} x + y^5 \frac{dy}{dx} + 3x^2 + \cos(x) = 0$

$\frac{dy}{dx}(5y^4 + y^5) + 3x^2 + \cos(x) = 0$

$\frac{dy}{dx} = -\frac{3x^2 + \cos(x)}{5y^4 + y^5}$

Higher Derivatives

$f(x) = y = x^3 + 7x^2 \qquad \overset{f'(x)}{\frac{dy}{dx}} = 3x^2 + 14x$

$\frac{d}{dx}\left(\frac{d}{dx}y\right) = \frac{d^2y}{dx^2} = f''(x) = 6x + 14$

$\frac{d^3y}{dx^3} = f'''(x) = 6$

$\frac{d^4y}{dx^4} = f''''(x) = f^{(4)}(x) = 0$

$f(x) = \sin(x)$ $\quad f'(x) = \cos(x)$

$f''(x) = -\sin(x)$ $\quad f'''(x) = -\cos(x)$

$f''''(x) = +\sin(x) = f(x) \quad = f^{(100)}(x)$

n larger than degree of polynomial $f(x)$

$f^{(n)}(x) = 0$

$f(x) = x^5, 5x^4, 20x^3, 60x^2, 120x^1, 120, \underline{0}$

$u(x) \cdot v(x) \quad (u+v)' = u \cdot v' + u' \cdot v$

$(u+v)'' = u' \cdot v' + u'' \cdot v + uv'' + u' \cdot v'$
$= u'' \cdot v + 2u'v' + uv''$

$(u+v)''' = u'''v + 3u''v' + 3u'v'' + uv'''$

$u = x^5 \quad V = \cos(x) \quad \frac{d^n}{dx^n}(u \cdot v)$

$n = 1, 2, 3, 4, 5$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Binomial coefficients in Pascals triangle

$(u \cdot v)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} u^{(k)} v^{(n-k)}$

Examples

What are the equations of the tangent and normal lines to the circle of radius one at the point

$$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Solution

$$x^2 + y^2 = 1$$

$$2x + 2y\frac{dy}{dx} = 0$$

so $\dfrac{dy}{dx} = -\dfrac{2x}{2y} = -\dfrac{x}{y} = -\sqrt{3}$

Tangent $y = -\sqrt{3}\,x + C_1$

Normal $y = \dfrac{1}{\sqrt{3}}x + C_2$

$$\frac{1}{2} = -\sqrt{3}\,\frac{\sqrt{3}}{2} + C_1 \iff \frac{1}{2} = -\frac{3}{2} + C_1 \iff C_1 = 2$$

$$\frac{1}{2} = \frac{1}{\sqrt{3}}\,\frac{\sqrt{3}}{2} + C_2 \iff C_2 = 0$$