MA1014 26/1/22

Integrals of Exp & Log.

We know $\frac{d}{dx}(e^x) = e^x$

$$\int e^x \, dx = e^x + c$$

Also $\int 2^x \, dx = \frac{1}{\ln 2} 2^x + c$

$$a^x = (e^{\ln a})^x = e^{\ln a \cdot x}$$

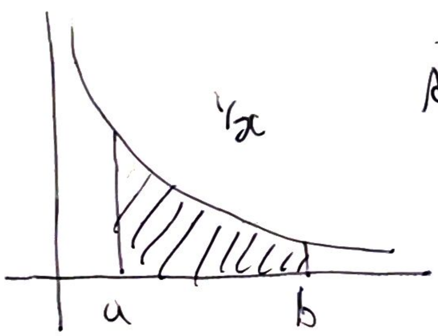
$$\frac{d}{dx} a^x = \ln a \cdot e^{\ln a \cdot x} = \ln a \cdot a^x$$

$$\int a^x \, dx = \frac{1}{\ln a} a^x + c$$

We know $\frac{d}{dx}(\ln x) = \frac{1}{x}$

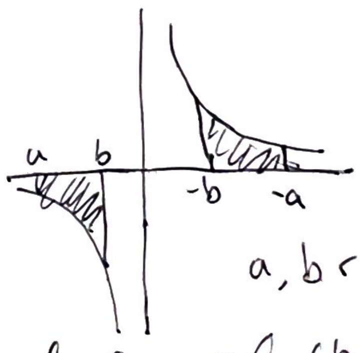
$$\int \frac{1}{x} \, dx = \ln x + c \qquad (x > 0)$$

$$\int_a^b \frac{1}{x} \, dx = \ln(b) - \ln(a) = \ln\left(\frac{b}{a}\right)$$



Area $\ln\left(\frac{b}{a}\right)$

$a, b > 0$
$b > a > 0$



$a, b < 0$

Area negative (below axis)

$$\int_a^b \frac{1}{x} \, dx = -\int_{-b}^{-a} \frac{1}{x} \, dx = -\ln\frac{-a}{-b} = \underline{\ln\left(b/a\right)}$$

$\int \frac{1}{x} dx = \ln|x|$

$\int_a^b \frac{1}{x} dx = \ln|b| - \ln|a| = \ln \frac{b}{a}$

if a, b have same signs

Integral by Substitution

we know $\frac{d}{dx} \ln(v(x)) = \frac{v'(x)}{v(x)}$

so $\int \frac{v'(x)}{v(x)} dx = \ln(v(x)) + c$

or $\ln|v(x)| + c$

$$\int \frac{dv}{v} = \ln(v) + c$$

<u>Examples</u> $v = \ln(x)$ $\frac{dv}{dx} = \frac{1}{x}$

$$\int \frac{(\ln(x))^n}{x} dx = \int v^n \, dv$$

$$= \begin{cases} \frac{v^{n+1}}{n+1} = \frac{(\ln(x))^{n+1}}{n+1} & n \neq -1 \\ \ln(v) = \ln(\ln(x)) & n = -1 \end{cases}$$

$$\int \frac{dx}{x \ln(x)} = \ln(\ln(x)) + c$$

$n = 1$ $\int \frac{2x}{(x^2+3)^n} dx = \int \frac{dv}{v^n} = \frac{v^{-n+1}}{-n+1} + c$

$v = x^2 + 3$, $dv = 2x\,dx$

but $\int \frac{2x}{x^2+3} dx = \underline{\ln(x^2+3) + c}$

$\int \frac{3x\,dx}{1+9x^2}$ $\leftarrow \frac{dv}{dx} dx$ $= \int \frac{\frac{1}{6} dv}{v} = \frac{1}{6} \ln v$

$\nwarrow v(x)$

$= \frac{1}{6} \ln(1+9x^2) + c$