MA1014 12/10/21

Mathematical Induction

Theorm : $\forall n \in \mathbb{N}$, sum of 1st odd numbers $= n^2$

$P(n)$ $1 + 3 + 5 + \dots (2n-1) = n^2$

$n = 0$ $0 = 0^2$ $n = 1$ $1 = 1^2$ $n = 2$ $1 + 3 = 2^2$

Induction : Base case ($n=0$ or $n=1$)

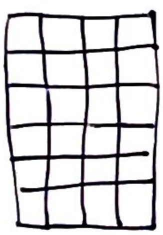
Inductive hypothesis : assume $P(k)$ is true

Inductive step : prove that $P(k) \Rightarrow P(k+1)$ $\forall k$

IH : $1 + 3 + 5 + \dots (2k-1) = k^2$

$\Rightarrow \underline{1 + 3 + \dots} + (2k+1)$
$= \underline{k^2} + (2k+1)$

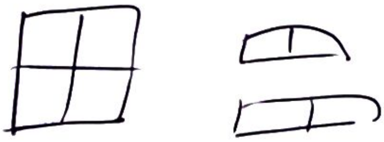
By induction $P(n)$ $\forall n \in \mathbb{N}$

Q 

chocolate bar
3×5 pieces
How many cuts to seperate all 15?

m pieces acroses n along
$m \times n$ bar

e.g  1×2 · 1 cut

 3 cuts

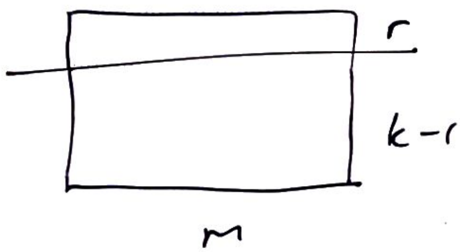
Theorm you break $mn-1$ times

Proof Induction on n (STRONG)

Base case $n=1$

$m \times 1 = m \times 1 - 1$ cuts

suppose it's true for all $m \times n$ bars whenever $n < k$

$m \times k$ 

1 cut

$m \times r$ pieces $mr-1$ cuts

$m(k-r)$ pieces $m(k-r)$ cuts

<u>assuming true for smaller numbers $n<k$</u>

$1 + mr - 1 + mk - mr - 1 = mk - 1$

so IH $\Rightarrow$ true for $n=k$ also so true for $\forall n$

Induction $(p(0), p(k) \Rightarrow p(k+1)\ \forall k) \Rightarrow P(n)\ \forall n$

strong induction $(P(0), P(n)\ \forall n < k \Rightarrow P(k)) = P(n)\ \forall n$

or $(P(0) P(1) P(k), P(k+1) \Rightarrow P(k+2)) \Rightarrow P(n)\ \forall n$
2 base case

eg F_n 0 1 1 2 3 5 8 13

n 1 2 3 4 5 6 7 8

$F_0 = 0 \quad F_1 = 1 \quad F_{n+2} = F_n + F_{n+1}$

<u>Theorm</u> $F_n^2 + F_{n+1}^2 = F_{2n+1} \quad \forall n$

<u>theorem</u> $F_n F_m + F_{n+1} F_{m+1} = F_{m+n+1} \quad \forall n, m \in \mathbb{N} \quad (m = n)$