MA1061 18/10/21

Probability Introduction

The mathematical analysis of random events, in other words of empirical phenomena which:

- Do not have <u>deterministic regularity</u>, meaning that distinct observations do not necessarily yield the same outcomes;

- Do posses some <u>statistical regularity</u>, in that the frequency of the distinct outcomes demonstrates some statistical stability.

<u>A classical example</u>

"Fair" toss of "unbiased" coin: Head or tail?

- Impossible to predict outcome of each toss with certainty
- Even when repeated many times no apparent regularity
- However, after the large number of "independant" experiments, "heads" should appear with a frequency approaching $\frac{1}{2}$
- This gives a possible quantitative estimate of "randomness"
- It would be reasonable to assign probability $\frac{1}{2}$ to the event "heads".
- Generally speaking, a probability is a number associated with or assigned to a <u>set</u> in order to measure it in some sense

- So probability is closely related to set theory

Preliminary set theory

A *set* is a well-defined list or description of objects

e.g. $\{1,3,5\}$, the integers $\mathbb{Z}$, even integers $\{b \in \mathbb{Z} : b \text{ is even}\}$

$\omega \in A$: ω is an element of A
$A \subseteq B$: A is a subset of B
universal set Ω
empty set $\emptyset$

Set operations:

union - $A \cup B = \{\omega \in \Omega : \omega \in A \text{ or } \omega \in B\}$

intersection - $A \cap B = \{\omega \in \Omega : \omega \in A \text{ and } \omega \in B\}$

complement - $\bar{A} = \{\omega \in \Omega : \omega \notin A\}$

difference - $A \setminus B = \{\omega \in \Omega : \omega \in A \text{ and } \omega \notin B\}$

De Morgan's Rule

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

Collection of sets: $\mathcal{A} = \{A_1, A_2, \dots, A_i\}$ where $A_1, A_2, \dots, A_i \subseteq \Omega$

σ - algebra

A collection $\mathcal{A}$ of sets of Ω is called a σ - algebra if:

① $\mathcal{A}$ is non-empty; ② $\mathcal{A}$ is closed under complement; ③ $\mathcal{A}$ is closed under countable union

Probability Space with Finite Numbers of Outcomes

- Consider an experiment with N possible outcomes which are enumerated as $\omega_1, \quad , \omega_N$

- We call $\omega_1, \dots, \omega_N$ <u>elementary events</u>, or <u>sample points</u>, while the set

$$\Omega = \{\omega_1, \dots, \omega_N\}$$

is called the <u>space of elemtary events</u> or <u>sample space</u>

Events

An <u>event</u> A is a set of outcomes or, in other words, a subset of the sample space Ω . i.e,

$$A \subseteq \Omega$$

<u>Examples</u> Toss three coins "at least two heads appear"

$$A = \{HHH, HHT, HTH, THH\} \subseteq \Omega$$

Toss a die: 'odd' = $\{1, 3, 5\}$

Toss 2 dice: $\Omega = \{(i,j) : i, j = 1, 2, \dots, 6\}$
'total score is 3' is given by the subset

$$A = \{(1,2), (2,1)\}$$