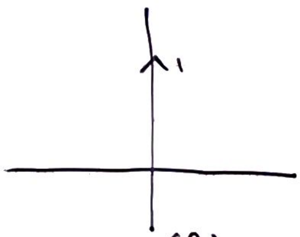
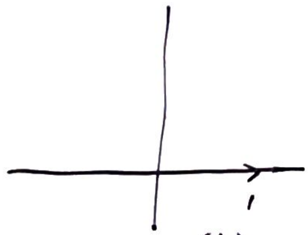
MA1114 5/10/21

Vectors in 2 & 3 Dimensions

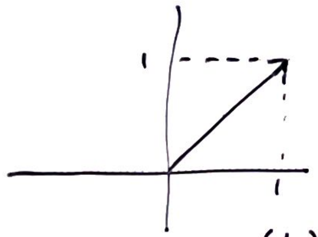
Vector Addition



$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

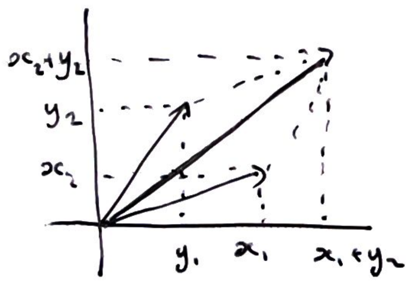


$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$



$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

ie to add vectors, concatenate them



let $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ be column vector

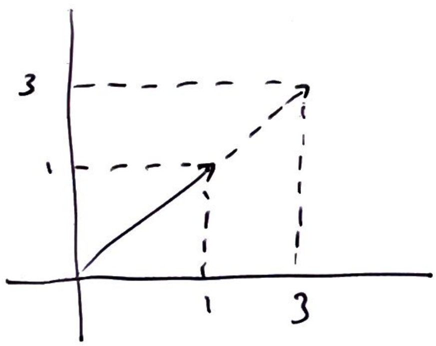
$$\begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix}$$

Definition 1.2

For vectors $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

we have $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix}$

Scaling vectors



$3 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

Definition 1.3

$$\lambda \in \mathbb{R}, \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$$

$$\lambda \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \lambda x_1 \\ \lambda x_2 \end{pmatrix}$$

e.g

1) $0 \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

2) $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

3) $8\begin{pmatrix} \sqrt{3} \\ \pi \end{pmatrix} + 6\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \pi \end{pmatrix} = \begin{pmatrix} 3\sqrt{3} \\ 3\pi \end{pmatrix} + \begin{pmatrix} -3\sqrt{3} \\ 6\pi \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 9\pi \end{pmatrix}$

Definition 1.6

A 3 dimensional vector has three real number entries

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad x, y, z \in \mathbb{R}$$

as defined before, we can add vectors

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix}$$

similary with scalar

$$\lambda \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda x \\ \lambda y \\ \lambda z \end{pmatrix}$$

Definition 1.7

A line in a $\mathbb{R}^2$ is a set of point

$$\left\{ \begin{pmatrix} a \\ b \end{pmatrix}, \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \middle| \lambda \in \mathbb{R} \right.$$

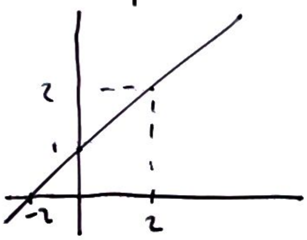
for some $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ fixed vectors

for $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix}$ is a point on a line $(\lambda = 0)$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is the direction of the line

Example



$$\left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix} \middle| \lambda \in \mathbb{R} \right\}$$

more similar definition : $y = mx + c$

in vector notation, $\left\{ \begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix} \middle| \lambda \in \mathbb{R} \right.$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \lambda \\ c + \lambda m \end{pmatrix} \Rightarrow y = c + \lambda m$$

e.g. consider the two lines $\rightarrow$

$$\left\{ \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix} \middle| \lambda \in \mathbb{R} \right\} = L_A$$

$$\left\{ \lambda \begin{pmatrix} 2 \\ 4 \end{pmatrix} \middle| \lambda \in \mathbb{R} \right\} = L_B$$

$p \in L_A \quad p = \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{\lambda}{2} \begin{pmatrix} 2 \\ 4 \end{pmatrix} \in L_B$

$p \in L_A \quad p = \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 2\lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix} \in L_B$

Warnings:

- Two lines which "appear" different may be different
- Show lines go through the origin

e.g Show that lines which go through the origin are closed under:

(a) vector addition

(b) scalar multiplication

- for (a) show $p, q \in L$ $\quad p + q \in L$
- for (b) show $p, q \in L$ and $\lambda \in \mathbb{R}$, $\lambda p \in L$

$$p = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad q = \mu \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

a) $$p + q = \begin{pmatrix} \lambda x_1 + \mu x_1 \\ \lambda x_2 + \mu x_2 \end{pmatrix} = \begin{pmatrix} x_1 (\lambda + \mu) \\ x_2 (\lambda + \mu) \end{pmatrix}$$

$$= (\lambda + \mu) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in L$$

b) $$\mu \cdot p = \begin{pmatrix} \lambda x_1 \mu \\ \lambda x_2 \mu \end{pmatrix} = \lambda \mu \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in L$$

Definition 1.10

let $u, v \in \mathbb{R}^2$ or $\mathbb{R}^3$ be vectors. A line is a set of form with $v \neq 0$

$$\{ u + \lambda v \mid \lambda \in \mathbb{R} \}$$