MA1114 11/10/21

Vector Space $\mathbb{R}^n$ ctd.

Definition 1.22

A line in $\mathbb{R}^n$ is a set $\{u+\lambda\cdot v \mid \lambda\in\mathbb{R}\}$ for $u, v\in\mathbb{R}^n$ $v\neq 0$

Definition 1.23

$u, v\in\mathbb{R}^n$ are parallel if $u=\lambda v$ or $v=\lambda u$ for some $\lambda\in\mathbb{R}$

Definition 1.24:

A plane in $\mathbb{R}^n$ is a set $\{u+\lambda v+\mu w \mid \lambda, \mu\in\mathbb{R}\}$ for some $u, v, w\in\mathbb{R}^n$, $v, w\neq 0$, v, w not parallel

Defintion 1.26:

A subspace of $\mathbb{R}^n$ is a subset of $\mathbb{R}^n$, satisfying VA0-4 and SM0-4

Question:

How to check a subset S is a subspace?

VA3, VA4, SM1, SM2, SM3, SM4 ✓

VA0 closed under addition VA1 $\underline{0}\in S$ VA2 $v\in S$ $-v\in S$ SM0

Definition, Proposition 1.28

A subset S of $\mathbb{R}^n$ is a subspace $\Leftrightarrow$
- $\underline{0} \in S$
- $u+v \in S$ for $u, v \in S$
- $\lambda \cdot v \in S$ for all $v \in S$, $\lambda \in \mathbb{R}$

Proof

VA2 follows from

$$-V := (-1)v \in V \text{ by SM0}$$

defined equal to

$$\begin{aligned} v+(-v) = v+(-1)\cdot v &= 1(v)\cdot(-1)v \\ &= v(1\cdot(-1)) \\ &= v(0) \\ &= \underline{0} \text{ by SM3} \end{aligned}$$

Example 1.29

A line $L \overset{\text{subset}}{\subseteq} \mathbb{R}^n$ is a subspace $\Leftrightarrow$ $\underline{0} \in L$

A plane $P \subseteq \mathbb{R}^n$ " . . . . $\underline{0} \in L$

$$L = \{u + \lambda v \mid \lambda \in \mathbb{R}\} \quad u, v \in \mathbb{R}^n$$

$$u = \underline{0} \Rightarrow L = \{\lambda \mid \lambda \in \mathbb{R}\} \quad \underline{0} \in L$$

$$\lambda v + \mu u = (\lambda + \mu) v \in L$$

$$S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_1 - x_2 \\ 3x_3 \end{pmatrix} \middle| x_1, x_2, x_3 \in \mathbb{R} \right\}$$

check this is a subspace

$\underline{0} \in S$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_1 - x_2 \\ 3x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_1 - y_2 \\ 3y_3 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \\ x - x_2 + y_1 - y_2 \\ 3x_3 + 3y_3 \end{pmatrix}$$

$$z_1 = x_1 + y \quad \text{for } 1 \leq x \leq 3$$

$$= \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_1 - z_3 \\ 3z_1 \end{pmatrix} \in S$$

Fact

- The subspaces of $\mathbb{R}^2$: - lines that go through $\underline{0}$
  - $\{\underline{0}\}$
  - $\mathbb{R}^2$

$\mathbb{R}^3$: (same as $\mathbb{R}^2$)
- planes through $\underline{0}$