MA10061 8/11/21

Discrete Random Variable & Independance

Outcomes vs Random Variables

the outcomes / elementary events are often not numerical; e.g. Heads, Tails.

sometimes we wish to assign / associate a specific number to each outcome.

example: in a game by tossing a coin, assign 1 to head and 0 to tail.

example: tossing three coins, the outcomes are

$$\{HHH, HHT, HTH, THH, THH, THT, HTT, TTT\}$$

we may assign the number of tails to each outcome; thus we get the concept of a random variable

Definition of a Random Variable

A discrete random variable is a function defined on a discrete sample space Ω with values in $\mathbb{R}$, that is a function

$$X : \Omega \to \mathbb{R}$$

Example

Tossing a coin. Sample space $\Omega = \{H, T\}$. We can define a random variable $X : \Omega \to R$ as follows:

$$X(\omega) = \begin{cases} 5 & \text{if } \omega = H \\ -3 & \text{if } \omega = T \end{cases}$$

Random Variable as a Measurement.

Example.

In two tosses of a coin with sample space $\Omega = \{HH, HT, TH, TT\}$, we can define a random variable X as follows.

ω	HH	HT	TH	TT
$X(\omega)$	2	1	1	0

what does X measure? number of heads, not 1-to-one

$\Omega = \{\omega : \omega = (i, j), i, j = 1, 2, \ldots, 6\}$

Let X, Y be random variables on the sample space Ω. Then $X+Y$, $X+k$, kX, and XY are random variables on Ω as defined as follows

① $(X+Y)(\omega) = X(\omega) + Y(\omega)$;

② $(kX)(\omega) = kX(\omega)$;

③ $(X+k)(\omega) = X\omega + k$;

④ $(XY)(\omega) = X(\omega)Y(\omega)$

where k is a real number.

More generally, sums and products of random variables, for any polynomial, exponential, or continuous function $h(t)$, we define $h(x)$ to be random variable on Ω defined by

$$[h(X)](\omega) = h[X(\omega)]$$

Indicator

A special type of random variable:

example

Let $(\Omega, \mathcal{A})$ be a measurable space. For an event $A \in \mathcal{A}$, define the function

$$1_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases}$$

Properties of Indicators

① $1_\emptyset(\omega) = 0 \ \forall \omega \in \Omega$;

② $1_\Omega(\omega) = 1 \ \forall \omega \in \Omega$;

③ $1_{A \cup B} = 1_A + 1_B$ if A and B are disjoint;

④ $1_{A \cap B} = 1_A 1_B$;

⑤ $1_{A \cup B} = 1_A + 1_B - 1_{A \cap B}$;

⑥ $1_A + 1_{\bar{A}} = 1 \ \forall \omega$