Polar, cylindrical and spherical Coordinates

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Polar Coordinates:

Hence

$$x = rcos(\theta),\ y = rsin(\theta)$$

So we have the change of variables

$$(x, y) \to (r, \theta) \text{ with } 0 \leq \theta < 2\pi$$

Useful Formulas:

$$r = \sqrt{x^2 + y^2},\ \theta = arctan(\tfrac{y}{x})$$

Important Conclusion: Polar coordinates are suitable when the problem have cirular symmetry

In general, let

$$x = rcos(\theta),\ y = rsin(\theta) \text{ with } 0 \leq \theta < 2\pi$$

Then a function $g(r, \theta)$ is a composition of $f(x, y)$, with $x(r, \theta) = rcos(\theta)$ and $y(r, \theta) = rsin(\theta)$, i.e.,

$$g(r, \theta) = f(x(r, \theta), y(r, \theta))$$

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Cylindrical Coordinates:

We can extend the idea of polar coordinates in 3 dimensions. Consider the chance of coordinates:

$$(x, y, z) \to (r, \theta, z)$$

with

$$x = rcos(\theta),\ y = rsin(\theta),\ z = z \text{ with } 0 \leq \theta < 2\pi$$

Then a cylinder with axis of symmetry the z-axis and radius 1 can be represented as

r=1

A function $f(z, y, z)$ can be written as a function

$$g(r, \theta, z) = f(rcos(\theta), rsin(\theta), z)$$

in cylindrical coordinates.

Spherical Coordinates:

We can extend the idea of polar coordinates in 3 dimensions, in yet another way than cylindrical coordinates. Consider the chance of coordinates:

$$(x, y, z) \to (r, \theta, \phi)$$

with

$$x = rcos(\theta)sin(\phi),\ x = rsin(\theta)sin(\phi),\ z = rcos(\phi)$$

with

$$0 \leq \theta < 2\pi \text{ AND } 0 \leq \phi \leq \pi$$

A sphere with the centre at $(0, 0, 0)$ and radius 1 can be represented as

$$r = 1$$

Important Idea: Suitable coordinates for problems

A function $f(x, y, z)$ can be written as a function

$$g(r, \theta, \phi) = f(rcos(\theta)sin(\phi), rsin(\theta)sin(\phi), rcos(\phi))$$

In spherical coordinates