MA1114 21/3/22

Complex Eigenvalues

Fundamental Theorem of Algebra

If $p(t) \in \mathbb{C}[t]$ ← polynomials with $\mathbb{C}$ coefficients

Then $p(t) = \overbrace{\prod_{i=1}^{\text{degree of } p(t)}}^{\text{product}} (t-\lambda_i)$ for some $\lambda_i \in \mathbb{C}$

Proof omitted - uses complex analysis and topology

Proposition

If $p(t) \in \mathbb{R}[t]$ (all coefficients real)

Then for all $z \in \mathbb{C}$ $p(z) = 0$

$\Rightarrow p(\bar{z}) = 0$ i.e roots comes in conjugate pairs

Example

$p(t) = t^2 + t + 1$

$t = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$\Rightarrow$ two roots : $\dfrac{-1+\sqrt{-3}}{2}$, $\dfrac{-1-\sqrt{-3}}{2}$

$= \dfrac{-1}{2} + i\dfrac{\sqrt{3}}{2}$, $-\dfrac{1}{2} - \dfrac{i\sqrt{3}}{2}$

Proof

$p(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t^1 + a_0$ with $a_i \in \mathbb{R}$

if $z \in \mathbb{C}$ is a root then $p(z) = 0$

$\Rightarrow \underline{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z^1 + a_0 = 0}$

$\Rightarrow a_n z^n + a_{n-1} z^{n} + \cdots + a_1 z^1 + a_0 = 0$

$\Rightarrow \overline{a_n z^n} + \overline{a_{n-1} z^{n-1}} + \cdots + \overline{a_1 z^1} + a_0 = 0$

$\Rightarrow \overline{a_n}\, \overline{z^n} + \overline{a_{n-1}}\, \overline{z^{n-1}} + \cdots + \overline{a_1}\, \overline{z^1} + \overline{a_0} = 0$

$\Rightarrow p(\bar{z}) = 0$

$\Rightarrow \bar{z}$ is a root.