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# Chapter 8: Linear Maps

Recall from chapter 1

$A \in M_{m,n}(\mathbb{R})$ $\quad T_A : \mathbb{R}^n \mapsto \mathbb{R}^m$
$\quad\quad v \mapsto Av$

recall that $u, v \in \mathbb{R}^n$, $\lambda \in \mathbb{R}$

$$A(u+v) = Au + Av$$
$$A(\lambda u) = \lambda Au$$

In general, $T_A(u+v) = A(u+v)$
$$= A(u) + A(v)$$
$$= T_A(u) + T_A(v)$$

$$T_A(\lambda u) = A(\lambda u)$$
$$= \lambda(Au)$$
$$= \lambda T_A(u)$$

we say that $T_A$ is linear.

<u>Definition</u>

Let $V, W$ be vectorspaces. A function $T: U \to W$ is linear, if for all $u, v \in U$, $\lambda, \mu \in \mathbb{C}$, $T(\lambda u + \mu v) = \lambda T(u) + \mu T(v)$

$U$ is called the <u>domain</u>
$W$ " " " " <u>codomain</u>

If $A \in M_{mn}(\mathbb{C})$, $U = \mathbb{C}^n$ $\quad W = \mathbb{C}^m$, $T_A : \mathbb{C}^n \to \mathbb{C}^m$
$v \mapsto Av$ is linear.

check $\lambda, \mu \in \mathbb{C}$ $\quad u, v \in \mathbb{C}^n$

$$\begin{aligned} T_A(\lambda u + \mu v) &= A(\lambda u + \mu v) \\ &= A(\lambda u) + A(\mu v) \\ &= \lambda Au + \mu Av \\ &= \lambda T_A(u) + \mu T_A(v) \end{aligned}$$

__Example__ let $u, w$ be as above with $n = m$

$r \in \mathbb{C}$ $\quad \mathbb{C}^n \to \mathbb{C}^n$

$T_r : v \mapsto rv$ is linear for any $r \in \mathbb{C}$

$\lambda, \mu \in \mathbb{C}$, $\quad u, v \in \mathbb{C}^n$

$$\begin{aligned} T_r(\lambda u + \mu v) &= r(\lambda u + \mu v) \\ &= r\lambda u + r\mu v \\ &= \lambda r u + \mu r v \end{aligned}$$

In particular $T_0$ and $T_1$ are linear
$T_0$ is the 0 map (since $0v = 0$)

__Definition__

Let $V$ and $W$ be vector spaces. The __zero map__ is the map $0 : V \to W$
$v \mapsto 0$

The identity map is the map $1 : V \to V$
$v \mapsto v$

Exercise: check these are linear. If $m \geq n$, the projection $T : \mathbb{C}^m \to \mathbb{C}^n$

sends $\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$ to $\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$