MA1114 18/10/21

Matrix Multiplication ctd.

Example

$$\left[\begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\right]^T$$

$$AB = \begin{bmatrix} -1 & 6 \\ 1 & 9 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 6 & 9 \end{bmatrix} = (AB)^T$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}^T \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 6 & 9 \end{bmatrix} = (AB)^T$$

Proof (proposition 1.62)

$(AB)^T$ is an $n \times m$ matrix

$B^T A^T$

$$(AB)_{ji} = \sum_{k=1}^{r} A_{jk} B_{ki} = \sum_{k=1}^{r} B_{ki} A_{jk}$$

$$= \sum_{k=1}^{r} (B^T)_{ki} (A^T)_{jk}$$

$$= [(B^T)(A^T)]_{ij}$$

multiplication as a composition of functions

Proposition 1.66

$A \in M_{mr} \qquad B \in M_{rn}$

$T_A : \mathbb{R}^r \to \mathbb{R}^m$

$T_B : \mathbb{R}^n \to \mathbb{R}^r$

suppose we define
$(T_A \circ T_B)(x) := (T_B(x))\, T_A$

$T_A \circ T_B : \mathbb{R}^n \to \mathbb{R}^m$

$T_{AB} : \mathbb{R}^n \to \mathbb{R}^m$

$T_{AB} = T_A \circ T_B$

Proof

$x \in \mathbb{R}^n$

$T_{AB}(x) = AB_x = A(B_x)$ associativity of matrix multn.
$= T_A(T_B(x))$

Proposition 1.67 (matrix multiplication is linear)

$A, A' \in M_{mr} \quad B, B' \in M_{rn} \quad \lambda, \mu \in \mathbb{R}$

1. $A(\lambda B + \mu B') = \lambda AB + \mu AB'$

2. $B(\lambda A + \mu A') = \lambda AB + \mu A'B$

Proof Same size

① $(A(\lambda B + \mu B'))_{ij} = \sum_{k=1}^{r} A_{jk} (\lambda B + \mu B')_{kj}$

$= \sum_{k=1}^{r} A_{jk} (\lambda B_{kj} + \mu B'_{kj})$

$= \sum_{k=1}^{r} \lambda A_{ik} B_{kj} + \mu A_{ik} B'_{kj}$

$\lambda \sum_{k=1}^{r} A_{ik} B_{kj} + \mu \sum_{k=1}^{r} A_{ik} B'_{kj}$

$\lambda (AB)_{ij} + \mu (AB')_{ij}$

$\lambda AB_{ij} + \mu AB'_{ij}$

Corollary 1.68 corollory - a statement that follows quickly and easily from a propⁿ

(matrix transformation is linear)

$A \in M_{mr}$ $u, v \in \mathbb{R}^r$ $\lambda, \mu \in \mathbb{R}$

$T_A(\lambda u + \mu + v) = \lambda T_A(u) + \mu T_A(v)$
linear combination
of u & v

Proof
$T_A(\lambda u + \mu v) = A(\lambda u + \mu v)$ by previous proposition with $n=1$ $B=u$, $B'=v$

this is $\lambda Au + \mu Av$

$\lambda(T_A(u)) + \mu(T_A(v))$