2.2 Properties of Point Estimators - Unbiasedness

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Desired Properties of Point Estimators:

- unbiasedness

- efficiency (minimal variance)

- sufficiency

- consistency

Definition:

A point Estimator $\hat{\theta}$ is called an <u>unbiased estimator</u> of the parameter θ if $E(\hat{\theta}) = \theta$ for all possible values of θ. Otherwise $\hat{\theta}$ is said to be <u>biased</u>.

Furthermore, the bias of $\hat{\theta}$ is given

$$B(\hat{\theta}, \theta) = Bias(\hat{\theta}, \theta) = E(\hat{\theta}) - \theta$$

Theorem:

The mean of a random sample $\bar{X}$ is an unbiased estimator of the population mean μ.

Proof:

Let $X_1, \ldots, X_n$ be random variables with mean μ. Then the sample mean is
$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} n\mu = \mu$
Hence, $\bar{X}$ is an unbiased estimator of μ.