MA1014 1/11/21

Rules for Differentiation

f is differentiable at $x = c$

if $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exist

<u>Theorm</u> suppose $f(x)$, $g(x)$ are differentiable at $x = c$

then i) $(f+g)'(c) = f'(c) + g'(c)$

ii) $(k \cdot f)'(c) = k \cdot f'(c)$

iii) $(f \cdot g)'(c) = f'(c) \cdot g(c) + f(c) g(c)$

(induction $\Rightarrow \frac{d}{dx}(x^n) = nx^{n-1}$)

Proof of (iii) Use limit laws

$f(x) \cdot g(x) - f(c) \cdot g(c)$

$= f(x) \cdot g(x) - f(c) \cdot g(x) + f(c) \cdot g(x) - f(c) \cdot g(c)$

$\frac{f(x) g(x) - f(c) g(x)}{x - c}$ $(f \cdot g)' \neq f' g'$

$= \frac{f(x) g(x) - f(c) g(x)}{x - c} + \frac{f(c) g(x) - f(c) g(c)}{x - c}$

As $x \to c$ $f'(c) g(c) + f(c) \cdot g'(c)$