MA1014 1/2/22

Differential Equations

Solve $\frac{dy}{dx} = 5e^{2x+1}$

i.e. find y, to make this true

ORDER 1 $y = \int 5e^{2x+1}\,dx = \frac{5}{2}e^{2x+1} + \underline{c}$

Solve $\frac{d^2y}{dx^2} = -y$ <u>Homogenous</u>

ORDER 2 $y = A\cos x + B\sin x$

<u>two</u> arbitary constants.

$y' = -A\sin x + B\cos x$

$y'' = -A\cos x - B\sin x = -y$

Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2$

ORDER 2
$$\left.\begin{array}{l} y = e^{mx} + k \\ y' = me^{mx} \\ \rightarrow\ y'' = m^2e^{mx} \end{array}\right\} e^{mx}(m^2-3m+2)+2k = 2$$

Need $k=1$, $m^2-3m+2=0$

$(m-1)(m-2)=0$

$m=1,\ m=2$

$y = Ae^{x} + Be^{2x} + 1$

In general a differential equation of order n

will have n arbitary constants in the solution

Particular solutions : either by

- initial values $y(0) = 3$, $y'(0) = 1$

- boundary values $y(1) = 6$, $y \to 2$ as $x \to \infty$

Initial Value Theorem (IVP)

$$\begin{cases} y' = xe^{-x^2/2} & \Rightarrow y = -e^{-x^2/2} + c \\ y(0) = 1 & \Rightarrow c = 2 \end{cases}$$

Methods : $y' = f(x) = y = \int f(x)\,dx$

seprable D.E.s

$$y' = f(x,y) = g(x) \cdot h(x)$$

IVP $\begin{cases} y' = x^2 e^{x-y} = x^2 e^x \cdot e^{-y} \\ y(0) = \ln 2 \end{cases}$

$$\frac{dy}{dx} = g(x) \cdot h(y)$$

$$\int \frac{dy}{h(y)} = \int g(x)\,dx$$

For the example

$$\int \frac{dy}{e^{-y}} = \int x^2 e^x \, dx$$

$$\int e^y \, dy = \int x^2 e^x \, dx$$

$$e^y = x^2 e^x - \int 2x e^x \, dx$$

$$= x^2 e^x - \left(2x e^x - \int 2 e^x \, dx\right)$$

$$= (x^2 - 2x + 2) e^x \cancel{+c}$$

$x = 0$

$y = \ln 2$ $\quad 2 = 2 + c \Rightarrow c = 0$

$$\underline{y = \ln(x^2 - 2x + 2) + x}$$