MA1014 1/2/22

Summary of Methods for Indefinite & Improper Definite Integrals.

Antiderivatives. guess $F(x)$ with $F' = f \to \int f(x)\,dx = F(x) + C$

Substitution: $\int f'(v(x))\,v'(x)\,dx = f(v(x)) + C$

Parts: $\int u'(x)\,v(x)\,dx = u(x)v(x) - \int u(x)\,v'(x)\,dx$

Trig: $\int \sin^3 x \cos^{12} x\,dx = \int \sin(x)(1-\cos^2(x))^3 \cos^{12} x\,dx$

$v = \cos x$ $\quad = -\int (1-v^2)^3 v^{12}\,dv = \ldots$

Alg: Partial Fractions

$$\frac{2x^2+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad ?$$

$$\Rightarrow \int \frac{2x^2+3}{x(x-1)^2}\,dx = A\ln x + B\ln(x-1) - \frac{C}{x-1} + \text{constant}$$

Find A, B, C:

$$2x^2+3 = A(x-1)^2 + Bx(x-1) + Cx$$

$\Rightarrow$ $A = 3$ (constant, or $x=0$)
$C = 5$ ($x=1$)
$B = -1$ (x^2: $2 = A+B$)

Improper integrals $\int_a^b f(x)\,dx$ with a, or b, $= \infty$ or $-\infty$

or f not continuous at a or at b

Try $\lim_{\epsilon \to b^-} \int_a^\epsilon f(x)\,dx$?

Examples $\int_0^1 \frac{1}{x^p}\,dx$, $\int_1^\infty 1/x^p\,dx$

discontinuous $x=0$? ∞?

converges $\Leftrightarrow p<1$ converges $\Leftrightarrow p>1$

Look at $\int_0^1 1/x^p\,dx$ $\int_1^b 1/x^p\,dx$

& then $\lim_{a \to 0^+}$ $\lim_{b \to \infty}$

$\to p=1$: $[\ln x]_a^1 = \ln 1 - \ln a \to \infty$ as $a \to 0^+$ $\quad [\ln x]_1^b = \ln b - 0$

$\to p \neq 1$: $\left[\frac{1}{-p+1}\,\frac{1}{x^{p-1}}\right]_a^1$ $\quad \frac{1}{-p+1}\left[\frac{1}{x^{p-1}}\right]_1^b$

$= \frac{1}{-p+1}\left(\frac{1}{1^{p-1}} - \frac{1}{a^{p-1}}\right)$ $\quad \frac{1}{-p+1}\left(\frac{1}{b^{p-1}} - \frac{1}{1}\right)$

$\to$ if $p>1$ $\to \infty$ as $a \to 0^+$ $\quad \to \frac{1}{p-1}$

if $p<1$ $\to \frac{1}{1-p}$ converges $\quad \to \infty$ as $b \to \infty$