Derivatives of a Vector Functions. Tangent Line and Normal Plane

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Derivates of Vector Functions

The derivative of a vector function of one variable is given by

$$\underline{r}'(t) = (x'(t), y'(t), z'(t)) = \tfrac{dr}{dt}(t)$$

we differentiate each coordinte function serperately!

Geometrically it is the tangent of the curve at the point $r(t)$.

Chain Rules

Suppose a curve C has a representation through the vector function:

$$r(t) = (x(t), y(t), z(t)) \quad t \in [a, bs]$$

Suppose also that the variable t can be re-parameterized through a variable s, i.e. $t = t(s)$ $s \in [c, d]$

$$\tfrac{dr}{ds}(t(s)) = \tfrac{dr}{dt}(t)\tfrac{dt}{ds}(s)$$

What is the derivative of $f(\underline{r}(t))$ with respect to t?

$$\tfrac{d}{dt}f(\underline{r}(t)) = \tfrac{\partial}{\partial x}f(\underline{r}(t))\tfrac{dx(t)}{dt} + \tfrac{\partial}{\partial y}f(\underline{r}(t))\tfrac{dy(t)}{dt} + \tfrac{\partial}{\partial z}f(\underline{r}(t))\tfrac{dz(t)}{dt}$$

Tangent Line

Equations for the tangent to the curve at point (x_0, y_0, z_0):

$$\boxed{x = x_0 + a_1 t,\ y = y_0 + a_2 t,\ z = z_0 + a_3 t}$$

where $\underline{a}$ is the tangent vector at point (x_0, y_0, z_0): $a_1 = x'(t_0)$, $a_2 = y'(t_0)$, $a_3 = z'(t_0)$

$$\text{or } \boxed{\tfrac{x-x_0}{a_1} = \tfrac{y-y_0}{a_2} = \tfrac{z-z_0}{a_3}}$$

Normal Plane

$$\boxed{(x - x_0)a_1 + (y - y_0)a_2 + (z - z_0)a_3 = 0}$$

where $\underline{a}$ is the **tangent** vector to the curve at point (x_0, y_0, z_0)