Tangent and Normal Lines

tangent has gradient = derivative

$f(x) = x \sin x$
$f'(x) = 1 \cdot \sin x + x \cos x$

What is the <u>tangent line</u> at the origin

$f'(0) = 1 \cdot \sin(0) + 0 \cdot \cos(0) = 0$
$\qquad y = 0$

$f'(x) = 2x + 2 \qquad f'(-3) = -6 + 2 = \underline{-4}$
$\qquad y = -4x + c$
$x = -3 \quad y = f(-3) = 12 + c$
$\qquad\qquad 9 - 6 = 12 + c \quad \Rightarrow \quad c = -9$
$\qquad\qquad y = -4x - 9$

<u>General Formula</u>

Tangent line to the curve
$\qquad\qquad y = f(x)$
at some point $x = c$
is $y = \underbrace{f'(c)}_{\text{gradient } m} x + (f(c) - f'(c)c)$ ✓
goes through $(c, f(c))$ ✓

Normal Line

gradient $-\frac{1}{m}$

perpendicular.

$$f(x) = x^2 + 2x$$

at $x = -3$

Normal line has gradient $-\frac{1}{-4} = \frac{1}{4}$

$$y = \frac{1}{4}x + 3 + \frac{3}{4} \quad \Rightarrow \quad y = \frac{1}{4} + \frac{15}{4}$$

$$x = -3, \ y = 3$$

normal
$$y = \frac{1}{4}x + \frac{15}{4}$$

$$y = -4x - 1$$
tangent