Vector Functions

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Until now we have been concerned with functions defined on $\mathbb{R}^n$, giving values in $\mathbb{R}$. These are called **real** or **scalar functions**. But it is useful in applications to consider functions that give **vectors** as values.

Problem: Suppose we want to describe the motion of a particle in space we define a function $r : \mathbb{R} \to \mathbb{R}^3$, say,

$$\underline{r}(t) = (x(t), y(t), z(t))$$

where $\underline{r}(t)$ is the distance of the particle from the origin at the time instance t. We call such functions **vector functions**.

3 dimensional motion $r(t) = (cos(t), sin(t), t)$
spiraling up, like a slinky

Recall:

The equations
$$x = x_0 + a_1 t$$
$$y = y_0 + a_2 t$$
$$z = z_0 + a_3 t$$
determine the straight **line**(parametric representation)
$$-\infty < t < \infty$$

$\overrightarrow{a} = (a_1, a_2, a_3)$ is the vector, which is parallel to the line

Parametrisations

Curves on a plane or in space can be described through vector functions of one variable

A curve can have **more than one** parametrisations!