MA1014 12/10/21

Elementary Functions ctd.

Def

A function f is even if $f(-x) = f(x)$

f is odd if $f(-x) = -f(x)$

e.g. $f(x) = |x|$, $f(x) = x^2$, $f(x) = x^4$, $f(x) = x^{2n}$ $n \in \mathbb{N}, \mathbb{Z}$ — even

odd: x^3, x, x^5, x^{2n+1}

e.g. $f(x) = \sin(x)$

odd $\sin(-x) = -\sin(x)$

$\tan(x)$ is odd, $\cos(x)$ EVEN

write $f(x) = g(x) + h(x)$ g even, h odd.

$e = 2.7182818..$ $f(x) = e^x$ = exponential = $\exp(x)$

$\exp: \mathbb{R} \to \mathbb{R}^{>0} = (0, \infty)$

domain range

Inverse $\ln: (0, \infty) \to \mathbb{R}$

$e^{\ln(y)} = y$ $\ln(e^x) = x$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \text{odd}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{even}$$

$$e^x = \cosh(x) + \sinh(x)$$

Polynomial degree d

$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots a_d x^d$$

$$a_d \neq 0$$

$$f(x) = (a_0 + a_2 x^2 + \dots) + (a_1 x + a_3 x^3 + \dots)$$

Rational function $f(x) = \frac{p(x)}{q(x)}$ with p, q polynomials

$$f(x) = \frac{7x - 27}{x - 4}$$

$$\text{Domain}\left(\frac{p(x)}{q(x)}\right) = \mathbb{R} - \{x : q(x) = 0\}$$