MA1114 6/10/21

Vector Space $\mathbb{R}^n$

Definition 1.16

A vector in $\mathbb{R}^n$ is a column vector, with n entries $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$, $x_1, x_2 \dots x_n \in \mathbb{R}$

e.g. $n=3 \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\mathbb{R}^n = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \middle| \, x_1 \in \mathbb{R} \text{ for all } 1 \leq x \leq n \right\}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{pmatrix}$$

$$\lambda \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \lambda x_1 \\ \lambda x_2 \\ \vdots \\ \lambda x_n \end{pmatrix}$$

$$\underline{0} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \in \mathbb{R}^n \quad (\text{some } n \in \mathbb{N})$$

Definition 1.18

A <u>linear combination</u> of vectors $u, v \in \mathbb{R}^n$ is an expression of the form $\lambda u + \mu v$, $\lambda, \mu \in \mathbb{R}$

<u>Examples</u>

$u + v \quad (\lambda, \mu = 1)$

$\underline{0} \quad (\lambda, \mu = 0)$

In fact, we can take linear combination of 3, 4, 5 ... vectors

e.g. $u, v, w \in \mathbb{R}^n$

$\lambda u + \mu v + \nu w$ is a linear combination of u, v, w

$(\lambda, \mu, w \in \mathbb{R})$

Example

$P = \left\{ \lambda \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix} \middle| \lambda, \mu \in \mathbb{R} \right\}$

is $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \in P$? $\lambda = 2$, $\mu = 1$

Proposition 1.21 (Properties of vectors addition and scalar mult)

Any vectors $u, v, w \in \mathbb{R}^n$ and $\lambda, \mu \in \mathbb{R}$

VA0 $u + v \in \mathbb{R}^n$
VA1 $v + \underline{0} = v = \underline{0} + v$
VA2 $\exists$ "$-v$" $\in \mathbb{R}$ with $v + -v = \underline{0} = (-v) + v$
VA3 $(u + v) + w = u + (v + w)$
VA4 $u + v = v + u$

SM0 $\lambda \cdot v \in \mathbb{R}^n$
SM1 $1 \cdot v = v$
SM2 $\lambda \cdot (\mu v) = (\lambda \mu) \cdot v$
SM3 $(\lambda + \mu) \cdot v = \lambda v + \mu v$
SM4 $\lambda (u + v) = \lambda u + \lambda v$