MA1014 8/2/22

Finding y_p

__Examples.__

$\lambda^2 - 2\lambda - 3 = 0$

$(\lambda-3)(\lambda+1) = 0$

$\lambda = 3, -1$

$y_H = c_1 e^{3x} + c_2 e^{-x}$

① $y'' - 2y' - 3y = x$

② $y'' - 2y' - 3y = e^{4x}$

③ $y'' - 2y' - 3y = e^{x}$

④ $y'' - 2y' - 3y = e^{3x}$

⑤ $y'' - 2y' - 3y = \text{sen}\, 2x$ $\qquad y_p = A \,\text{sen}\, 2x$

⑥ $y'' + 4y = \text{sen}\, 2x$

⑦ $y'' + y' = 1$

⑧ $y'' + y' = x$

$y'' + y' = 0$

$\lambda^2 + \lambda = 0$

$\lambda(\lambda+1) = 0$

$\lambda = 0, -1$

① $y_p = Ax + B$ $\qquad y_p'' - 2y_p' - 3y_p = -3Ax - 3B - 2A$

$= x$?

$y_p = -\frac{1}{3}x + \frac{2}{9} \Rightarrow$ — " — $= x$

$y = c_1 e^{3x} + c_2 e^{-x} - \frac{1}{3}x + \frac{2}{9}$

Guess: polynomial of same degree

(7) Guess $y_p = A$ constant

$$y_p'' + y_p' = 0 \qquad \times$$

$y_p = Ax + B \qquad y_p' = A \qquad y_p'' = 0$

$$A + 0 = 1$$
$$A = 1$$

$y_p = x + B$

(8) $y_p'' + y_p' = x \qquad$ Guess $y_p = Ax + B$

$0 + A = x \qquad \times$

$y_p = Ax^2 + Bx + C \qquad = \frac{1}{2}x^2 - x + c$

$y_p'' + y_p' = x$

$2A + 2Ax + B = x + 0$

$A = \frac{1}{2} \quad B = -1$

$y_H = c_1 e^{0x} + c_2 e^{-1x} = \underline{c_1 + e^{-x}}$

---

(9) $y'' + y' + x + 1 \qquad y_H = c_1 + c_2 e^{-x}$

as $\lambda = 0$ or -1

$$\left.\begin{aligned} y_p &= Ax^2 + Bx \\ y_p' &= 2Ax + B \\ y_p'' &= 2A \end{aligned}\right\}$$

$2Ax = x \implies A = \frac{1}{2}$

$2A + B = 1 \implies B = 0$

$y_p = \frac{1}{2}x^2$

$y = c_1 + c_2 e^{-x} + \frac{1}{2}x^2$

② $y'' - 2y' - 3y = e^{4x}$

$y_H = C_1 e^{-x} + C_2 e^{3x}$

Guess $y_p = Ae^{4x}$

$y_p'' - 2y_p' - 3y_p = (16 - 2\times 4 - 3)Ae^{4x}$

$5Ae^{4x} = e^{4x} \Rightarrow A = 1/5$

$y = C_1 e^{-x} + Ce^{3x} + 1/5 e^{4x}$

③ $y'' - 2y' - 3y = e^{x}$

$y_p = Ae^{x}$ $\quad (1-2-3)Ae^{x} = e^{x}$

$A = -1/4$

$y = C_1 e^{-x} + C_2 e^{3x} - 1/4 e^{x}$

④ $y'' - 2y' - 3y = e^{3x}$

$y_H = C_1 e^{-x} + C_2 e^{3x}$

Guess $y_p = Axe^{3x}$

$y_p' = 3Axe^{3x} + Ae^{3x}$ (product rule)

$-3y_p = (-3Ax)e^{3x}$

$-2y_p' = (-6Ax - 2A)e^{3x}$

$y_p'' = (9Ax + 6A)e^{3x}$

$\overline{0Ax + 4A\ e^{3x}}$

$y_p = 1/4 xe^{3x}$ works! Gen. Solution $y = C_1 e^{-x} + C_2 e^{3x} + 1/4 xe^{3x}$

⑥ $y'' + 4y = \text{sen } 2x$

Guess $y_p = A\,\text{sen } 2x + B\cos 2x$ no good because: X

$p(\lambda) = \lambda^2 + 4 = 0 \Leftrightarrow \lambda = \pm 2i$

$y_H = C_1 \cos 2x + C_2\,\text{sen } 2x$

Guess $y_p = Ax\,\text{sen } 2x + Bx\cos 2x$

$y_p' = A\,\text{sen } 2x + 2Ax\cos 2x$
$\quad B\cos 2x - 2Bx\,\text{sen } 2x$

$y_p' = (A - 2B)\,\text{sen } 2x + (B + 2Ax)\cos 2x$

$y_p'' = -2B\,\text{sen } 2x + 2(A - 2Bx)\cos 2x$
$\quad 2A\cos 2x - 2(B + 2Ax)\,\text{sen}(2x)$

$= (-4B - \cancel{4Ax})\,\text{sen } 2x$

y_p''

$(4A - \cancel{4Bx})\cos 2x$

$+4y_p \quad +$

$\quad \cancel{+4Ax}\ \text{sen } 2x$
$\quad \cancel{+4Bx}\ \cos 2x$

$\text{sen } 2x$

$-4B = 1 \qquad 4A = 0$

$B = -1/4 \qquad A = 0 \qquad y_p = -1/4\, x\cos 2x$

$y = C_1\cos 2x + C_2\,\text{sen } 2x - 1/4\, x\cos 2x$