

MA1014 31/1/22

Other Integration Methods

$$1. \int x^7 dx = \frac{1}{8} x^8 + C$$

$$2. \int 5x^3 dx = \frac{5}{4} x^4 + C$$

$$3. \int (3x^5 + 7x^{10}) dx = \frac{1}{2} x^6 + \frac{7}{11} x^{11} + C$$

$$4. \int e^{3x+2} dx = \frac{1}{3} e^{3x+2} + C$$

$$5. \int \cos(2-5x) dx = -\frac{1}{5} \sin(2-5x) + C$$

$$6. \int \sqrt{3x-7} dx = \frac{2}{9} (3x-7)^{3/2} + C$$

$$7. \int \sec^2(2x+1) dx = \frac{1}{2} \tan(2x+1) + C$$

$$8. \int (5x^7 + e^{6-2x} + 23 + \frac{2}{x}) dx = \frac{5}{8} x^8 - \frac{1}{2} e^{6-2x} + 23x + 2 \ln|x| + C$$

$$9. \int \cos(7-11x) dx = -\frac{1}{11} \sin(7-11x) + C$$

$$10. \int \cos x \sin x dx = \frac{1}{2} \int_0^{\pi/4} \sin(2x) dx = -\frac{1}{4} [\cos 2x]_0^{\pi/4} = -\frac{1}{4} (0-1) = \frac{1}{4}$$

$$11. \int 2x e^{x^2} dx = \int \frac{dv}{dx} e^v dx = \int e^v dv = e^v = e^{x^2} + C$$

$v = x^2$

$$12. \int 6x^5 e^{x^6} dx = \int e^v dv = e^v = e^{x^6} + C$$

$v = x^6$

$$13. \int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} \cdot \cos x dx = \int \frac{1}{v} dv = \ln|v| + C$$

$v = \sin x \quad \frac{dv}{dx} = \cos x \quad = \ln|\sin x| + C$

$$14. \int \cos x e^{\sin x} dx = \int e^v dv = e^v + C = e^{\sin x} + C$$

$$15. \int \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx = \int v' e^v dv = e^v = e^{\sqrt{x}} + C$$

$$16. \int \cos x \sin^5 x dx = \int v^5 dv = \frac{1}{6} v^6 + C = \frac{1}{6} (\sin x)^6 + C$$

$$17. \int \sec^2 x \tan^7 x dx = \int v^7 dv = \frac{1}{8} v^8 + C = \frac{1}{8} \tan^8 x + C$$

$$18. \int e^x \sqrt{e^x + 1} dx = \int v^{\frac{1}{2}} dv = \frac{2}{3} v^{\frac{3}{2}} = \frac{2}{3} (e^x + 1)^{\frac{3}{2}} + C$$

$v = e^x + 1$

$$19. \int \ln x dx = \int 1 \cdot \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + C$$

$$20. \int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C = e^x (x - 1) + C$$

$$21. \int (\ln x)^2 dx = \int 1 \cdot (\ln x)^2 dx = x (\ln)^2 - \int x \cdot 2 \ln x \cdot \frac{1}{x} dx$$

$$= x (\ln)^2 - 2 \int \ln x dx$$

$$= x (\ln)^2 - 2x \ln x + 2x + C$$

$$22. \int x e^{2x} dx = x \cdot \frac{1}{2} e^{2x} - \int 1 \cdot \frac{1}{2} e^{2x} = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4}$$

$$23. \int \arctan(3x) dx = \int 1 \cdot \arctan(3x) dx$$

$$24. \int x^3 \ln x dx = \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^4 \cdot \frac{1}{x} dx = \frac{1}{4} x^4 \ln x - \frac{1}{4} \int x^3 dx$$

$$= \frac{1}{4} x^4 \ln x - \frac{x^4}{16} + C$$

$$25. \int \ln(3x) dx = x \ln(3x) - \int x \cdot \frac{1}{3x} dx = x \ln(3x) - \frac{1}{3} x + C$$

$$26. \int x \ln x dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

$$27. \int_1^{18} (x^3 + 10) 3x^2 dx, v = x^3 + 10 = \int_{11}^{18} v dv = \frac{1}{2} (18^2 - 11^2) = 101.5$$

$$28. \int_0^1 (x^6 + 6x)(x^5 + x) dx = \int_0^1 x^{11} + x^7 + 6x^6 + 6x^2 dx$$

$$= \left[\frac{1}{12} x^{12} + \frac{1}{8} x^8 + \frac{6}{7} x^7 + 2x^3 \right]_0^1$$

$$= \frac{1}{12} + \frac{1}{8} + \frac{6}{7} + 2 = 3 \frac{11}{168}$$

$$29. \int_1^4 (2x / (x^3 + 3)^2) dx$$

$$30. \int x^2 \sqrt{x^3 + 2} dx = \int \frac{1}{3} v^{1/2} dv = \frac{2}{9} v^{3/2} + C = \frac{2}{9} (x^3 + 2)^{3/2} + C$$

$$31. \int 4x^3 - 3 \cos x + \frac{7}{x} + 2 dx = x^4 - 3 \sin x - 7 \ln|x| + 2x + C$$

$$32. \int 3x^2 + e^{2x} - 11 + \cos x dx = x^3 + \frac{1}{2} e^{2x} - 11x + \sin x + C$$

$$33. \int \sec^2 x dx = \tan x + C$$

$$34. \int \frac{7}{1+x^2} dx = 7 \arctan(x) \quad \frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$35. \int 16x^7 - \sqrt{x} + \frac{3}{\sqrt{x}} dx = 2x^8 - \frac{2}{3} x^{3/2} + 6\sqrt{x} + C$$

$$36. \int 23 \sin x - \frac{2}{\sqrt{1-x^2}} dx = -23 \cos x - 2 \arcsin x + C \quad \frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$37. \int_1^2 \frac{x^2}{\sqrt{2x+1}} dx = \frac{1}{5} \int \frac{(v-1)^2}{\sqrt{v}} dv = \frac{2}{5} v^{5/2} + C = \frac{2}{5} (2x+1)^{5/2} + C \Big|_1^2 \\ = \frac{2}{5} (2(2)+1)^{5/2} - \frac{2}{5} (2(1)+1)^{5/2} \\ = 16.12529678.$$

$$38. \int_0^5 \frac{s}{3\sqrt{s+2}} ds =$$

$$39. \int_1^2 \frac{2x}{1+x^2} dx =$$

$$40. \int_0^\pi \cos(\theta + \pi/3) d\theta$$

$$41. \int \sin\left(\frac{\pi+x}{5}\right) dx$$

$$42. \int \frac{\sin 2x}{\sqrt{1+\cos 2x}} dx$$

$$43. \int_{\pi/4}^{\pi/3} \sin^2 \theta \cos \theta d\theta$$

$$44. \int_1^3 \frac{1}{r \ln r} dr =$$

$$45. \int \frac{\sin 2x}{1 + \cos^2 x} dx =$$

$$46. \int \frac{\sin 2x}{1 + \sin x} dx =$$

$$47. \int_0^1 z \sqrt{1-z^2} dz =$$

$$48. \int_1^2 \frac{\ln 2x}{x} dx =$$

$$49. \int_{\xi=0}^{\sqrt{2}} \xi (1 + 2\xi^2)^{10} d\xi$$

$$50. \int_2^3 \sin p (\cos 2p)^4 dp$$

$$51. \int x e^{-x^2} dx$$

$$52. \int \frac{e^{1/t}}{t^2} dt$$

Integration by Parts

$$\frac{d}{dx} x^2 e^x = 2x e^x + x^2 e^x$$

$$\frac{d}{dx} (x e^x) = 1 \cdot e^x + \underline{x e^x} \quad \text{product rule}$$

$$x e^x = \int (e^x + x e^x) dx$$

$$\begin{aligned} \int x e^x dx &= x e^x - \int e^x dx \\ &= x e^x - e^x + C \end{aligned}$$

$$\begin{aligned} 24 \int \underline{x^3} \underline{\ln x} dx &= \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^4 \cdot \frac{1}{x} dx \\ &= \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C \end{aligned}$$