MA1061 19/10/21

Properties of Probability and Space

Probability Space

we can say that the triple:

$$(\Omega, \mathcal{A}, P), \text{ where}$$

$\Omega = \{\omega_1, \ldots, \omega_n\}$ is the sample space
$\mathcal{A}$ = is the σ - algebra of subsets of Ω
P is a probability on $\mathcal{A}$

is a probablistic model or a probability space

Example

A card is selected at random from an ordinary deck of 52 cards. We consider the following events:

$$A = \{\text{heart}\}, \text{ and } B = \{\text{face card}\}$$

Set up a probability space and find the probability $P(A)$, $P(B)$, $P(A \cap B)$ & $P(A \cup B)$

$\Omega = \{\omega_1, \ldots \omega_{52}\}$ $P(\omega_1) = \ldots = P(\omega_{52})$

$\sum_{i=1}^{52} P(\omega_i) = 52\, P(\omega_1) = 1$ probability space $= (\Omega, \mathcal{A}, P)$

outcome	ω_1	ω_2	$\cdots$	ω_{52}
probability	$1/52$	$1/52$		$1/52$

$P(A) = \sum_{\{i : w_i \in A\}} P(w_i)$ for all $A \in \mathcal{A}$

$A = \{\text{Hearts}\}$ has 13 elements

thus

$$P(A) = \sum_{\{i : w_i \in A\}} (P(w_i)) = 13/52 = 1/4$$

$B = \{\text{face card}\}$ has 12 elements and thus $P(B) = 12/52$
$= 3/13$

$A \cap B = \{\text{face card AND a heart}\}$ and thus

$A \cap B = \{JH, QH, KH\}$, 3 elements

$P(A \cap B) = 3/52$

$A \cup B = \{\text{face card OR a heart}\}$, 22 elements

hence $P(A \cup B) = 22/52 = 11/26$

$$P(A) + P(B) = 13/52 + 12/52 = 25/52 \neq P(A \cup B)$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Properties of Probability

Obviously $P(\emptyset) = 0$ $P(\Omega) = 1$

we also have

$$P(A \setminus B) = P(A) - P(A \cap B),$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B), \quad (\text{addition rule})$$

If $A \cap B = \emptyset$, then

$$P(A \cup B) = P(A) + P(B)$$

$$P(\bar{A}) = 1 - P(A), \quad (\text{complement rule})$$