2.1 Method of Moment Likelihood

Steven Cheung
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Definition:

Let $f(X_1, \ldots, X_n, \theta),\ \theta \in \Omega_\theta \subset \mathbb{R}^k$, be the joint probability mass (or density) function function of n random variables $X_1, \ldots, X_n$ with sample values $x_1, \ldots, x_n$. The likelihood function of the sample is given by

$$L(\theta, x_1, \ldots, x_n) = h(x_1, \ldots, x_n),\ [L = (\theta), \text{ in a breifer notation}]$$

we emphasize that L is a function of θ for fixed sample values

If $X_1, \ldots, X_n$ are discrete $i.i.d$ random variables with probability mass function $p(x, \theta)$, then the likelihood function is given by

$$\begin{aligned} L(\theta) &= P(X_1 = x_1, \ldots, X_n = x_n) \\ &= \Pi_{i=1}^n P(X_i = x_i)\ = \Pi_{i=1}^n p(x_i, \theta) \end{aligned}$$

and in a continuous case, if the probability density function is $f(x, \theta)$,. then the likelihood function is

$$L(\theta) = \Pi_{i=1}^n f(x_i, \theta)$$

Definition:

Let $L(\theta) = \Pi_{i=1}^n p(x_i, \theta)$ (or $L(\theta) = \Pi_{i=1}^n f(x_i, \theta)$) be the likelihood function. Corresponding to random sample $x_1, \ldots, x_n$ drawn from the discrete $p.d.f$ $p(x, \theta)$ (or from the continuous $p.d.f$ $p(x, \theta)$), where θ is an unknown parameter. Let $\hat{p}$ be a value of the parameter such that $L(\hat{\theta}) \geq L(\theta)$ for all possible values of the parameter θ. Then $\hat{\theta}_l$ is called a **maximum likelihood estimate** of θ.

The **Maximum Likelihood Estimates** of θ is

$$L(\hat{\theta}_l, x_1, \ldots, x_n) = max_\theta \in \Omega_\theta L(\theta, x_1, \ldots, x_n)$$

where Ω_θ is the set of possible values of parameter θ

Procedure to Find Maximum Likelihood Estimate(s) (MLEs):

1. Define the likelihood function, $L(\theta)$
2. Often it is easier to take the natural logarithm of $L(\theta)$, $ln(L(\theta))$
3. When applicable, differentiate $ln(L(\theta))$ with respect to θ, and then equate the derivative to 0
4. Solve for the parameter θ to obtain $\hat{\theta}$
5. Check whether it is a maximiser of global maximiser $\hat{\theta}_l$