MA1114 6/10/21

Lines & Planes in $\mathbb{R}^2$ & $\mathbb{R}^3$

$u, v \in \mathbb{R}^2$ or $\mathbb{R}^3 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \, x, y, z \in \mathbb{R}^3 \right\}$

$\qquad = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| \, x, y \in \mathbb{R}^2 \right\}$

A line is a set of form $\{ u + \lambda v \mid \lambda \in \mathbb{R} \}$

Definition 1.11

Let $u, v, w \in \mathbb{R}^3$, suppose that v and w do not go in the same direction.

A plane is a set of the form $\{ u + \lambda v + \mu w \mid \lambda, \mu \in \mathbb{R} \}$

fixed point λv & μw direction vectors

Example

- $\left\{ \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \middle| \, \lambda, \mu \in \mathbb{R} \right\}$ this is the x, y plane
- $\left\{ \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \middle| \, \lambda, \mu \in \mathbb{R} \right\}$

$\left\{ \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix} \middle| \, \lambda \in \mathbb{R} \right\} = \left\{ \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix} \middle| \, \lambda \in \mathbb{R} \right\}$

<u>CAUTION</u> lines may "look like" planes

e.g. $\left\{ \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \middle| \, \lambda, \mu \in \mathbb{R} \right\} = \left\{ (\lambda + 3\mu) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \middle| \, \lambda, \mu \in \mathbb{R} \right\}$

$\qquad = \left\{ \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \middle| \, \lambda \in \mathbb{R} \right\}$

Definition 1.13

Two vectors u, v are <u>parallel</u> if $u = \lambda v$ for some $\lambda \in \mathbb{R}$
(or $v = \lambda u$ for some $\lambda \in \mathbb{R}$)

$\therefore$ we need, v, w <u>not</u> parallel in 1.11
"v and w 'independant'"

Exercise

Are these planes or lines? Do they go through origin?

① $\left\{ \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \middle\vert\, \lambda, \mu \in \mathbb{R} \right\}$	plane	origin
② $\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} \pi \\ 1 \\ 6 \end{pmatrix} \middle\vert\, \lambda, \mu \in \mathbb{R} \right\}$	plane	$\times$
③ $\left\{ \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 60 \\ 20 \\ 50 \end{pmatrix} + \mu \begin{pmatrix} 12 \\ 2 \\ 1 \end{pmatrix} \middle\vert\, \lambda, \mu \in \mathbb{R} \right\}$	plane	$\times$

Planes which go through the origin <u>closed</u> under scalar multiplication and vector addition.

What does that mean?

$P = \{ u + \lambda v + \mu w \mid \lambda, \mu \in \mathbb{R} \}$

- closed under scalar multiplication: $p \in P,\ q \in \mathbb{R} \Rightarrow p \cdot a \in P$
- closed under vectors addition $p, q \in P \Rightarrow p + q \in P$