MA1061 3/11/21

Independance

If A, B are two events then what does A is independant of B mean?

It is natural to say so if

> knowing that A has occured has <u>no</u> effect on the probability of B.

In other words, "B is independant of A" if

$$P(B|A) = P(B) \quad \text{obviously assuming that } P(A)>0$$

Then using the definition of conditional probability and independance:

$$P(B|A) = \frac{P(A\cap B)}{P(A)} \Rightarrow P(A\cap B) = P(A)\,P(B)$$

<u>Definition</u> (Independance, rigorous def^n)

Events A and B are called <u>independant</u> (with respect to the probability P) if

$$P(A\cap B) = P(A)\,P(B) \qquad (6)$$

Equation 6 is often called the <u>multiplication formula for independants events</u>

note: if A and B are independant, so are A and $\bar{B}$, $\bar{A}$ and B, $\bar{A}$ and $\bar{B}$

Example

Suppose a card is drawn from a deck of cards (52). Let A be the event that a heart is drawn, and B the event that a Queen is drawn. Are A and B independant?

$\Omega = \{ \text{all of the cards} \}$

$P(\text{heart}) = 13/52 = 1/4$
$P(\text{queen}) = 4/52 = 1/13$

there is only one queen of hearts. Hence $P(A \cap B) = 1/52$

But $P(A)\,P(B) = \frac{1}{4} \times \frac{1}{13} = 1/52 = P(A \cap B)$

thus independant

Pairwise Independent Events

Definition

Events $A_1, \ldots, A_n$ are said to be independant (pairwise) if for any i, j, $1 \leq i < j \leq n$ we have

$$P(A_i \cap A_j) = P(A_i)\,P(A_j)$$

Definition (Mutually Independant Events)

Events $A_1, \ldots, A_n$, are said to be independant (mutually) if
$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \cdots P(A_{i_k})$,
for any collection of distinct events $A_{i_1}, A_{i_2}, \ldots, A_{i_k}$, of any size $k = 1, \ldots, n$

Example of mutually independant events

A certain football teams wins (W) with probability 0.6, loses (L) with probability 0.3, and ties (T) with probability 0.1. The team plays 3 games over the weekend.

> P(A) = the team wins at least twice and does not lose.

> P(B) = the teams wins, loses, and ties in some order

$$A = \{WWW,\ WWT,\ WTW,\ TWW\}$$

$$P(A) = P(WWW) + P(WWT) + P(WTW) + P(TWW)$$
$$= 0.6^3 + 0.6^2 \times 0.1 + 0.6^2 \times 0.1 + 0.6^2 \times 0.1$$
$$= 0.216 + 3(0.036)$$
$$= 0.324$$

$$B = \{WTL,\ WLT,\ LWT,\ LTW,\ TWL,\ TLW\}$$

$$0.6 \cdot 0.3 \cdot 0.1 = 0.018$$

hence $P(B) = 6(0.018)$
$= 0.108$

Pairwise Does Not Imply Mutually

$P(A \cap B) = 1/4 \quad P(A \cap C) = 1/4 \quad P(B \cap C) = 1/4$

thus $P(A \cap B \cap C) = 0 \neq P(A)\,P(B)\,P(C)$