MA1014 2/2/22

Integrating Factor Method

6.3 Linear First Order ODEs $\boxed{y' + p(x) y = q(x)}$

Special Case:

* If p is always zero:

$$y' = q(x) \quad \rightsquigarrow y = \int q \, dx + c$$

* If q is always zero

we seperate variables

$$\frac{dy}{dx} = -p(x) y$$

$$\rightsquigarrow \int \frac{dy}{y} = -\int p(x) + c$$

$\ln y =$

$$y = e^{-\int p(x) dx + c}$$

$k = e^c$

$$= k e^{-\int p(x) dx}$$

General situation

(*) $y' + py = q$

$p, q : (a,b) \to \mathbb{R}$ continuous

Integrating Factor Method

① $H(x) = \int p(x)\,dx \qquad H'(x) = p(x)$

② Multiply ④ by $e^{H(x)}$

$$e^{H(x)} y' + p(x) e^{H(x)} y = q(x) e^{H(x)}$$

LHS is $\quad (e^{H(x)} \cdot y)' = q(x) e^{H(x)}$

$$\rightsquigarrow e^{H(x)} \cdot y = \int q(x) e^{H(x)}\,dx$$

③ Find $\int q(x) e^{H(x)}\,dx$

④ So $y = e^{-H(x)} \cdot \left(\int q(x) e^{H(x)}\,dx + c \right)$

[⑤ simplify]

Example 6.17

p, q constants

$$y' + ay = b \qquad a, b \in \mathbb{R}$$

① $H(x) = \int a\,dx = ax$