MA1014 6/10/21

Least Upper Bound

Def:

An upper bound for S is any numbers M which no element of S are bigger than

$\forall x \in S,\ x \leq M$

similar def. of lower bound M for S: $\forall x \in S \quad x \geq M$

Example

$\underset{S}{[0,1]} \subseteq \mathbb{R}$ upper $M = 1000$

$\forall\ 0 \leq x \leq 1,\ x \leq 1000$

So we could find a better upper bound, $M := 2$ or best $M = 1$

A best upper bound for S is an upper bound which no other upper bounds are smaller than

M is a L.U.B for S if

U.B (1) $x \leq M \quad \forall x \in S$

L (2) if $x \leq \bar{m} \quad \forall x \in S$ then $M \leq \bar{m}$

Similar def. for Greatest lower bound

$M =$ GLB if (1) $x \geq M \quad \forall x \in S$

(2) if $x \geq \bar{m} \quad \forall x \in S$ then $M \geq \bar{m}$

Example

$(0, \infty)$ not bounded above ($\nexists$ any upper bound)

Bounded below: $-2, -1, 0, -\frac{1}{2}$ lower bound

0 = greatest lower bound

$$[1, \sqrt{2}) = \{x \in \mathbb{R} : 1 \leq x < \sqrt{2}\}$$
$$= \{x \in \mathbb{R} : 1 \leq x^2 < 2\}$$

Bounded $x > 0$

least upper bound axiom - Any non-empty set of $\mathbb{R}$ $S \subseteq \mathbb{R}$
then if S ($S \neq \phi$) is bound above it has a least upper bound

$\mathbb{R}$ satisfies this axiom

Any bounded above non-empty $S \subseteq \mathbb{R}$ has LUB

$\{x \in \mathbb{R} : x > 0, x^2 < 2\}$ GLB $= 0$ LUB $= \sqrt{2}$

But $\sqrt{2}$ is not a rational number

so only GLB $= 0$ and no LUB