2.1 Method of Moments

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$X_1, \ldots, X_n$ - soem random variables, all have the same distribution.
P_θ - a collection of probability distribution
Each collection, called parametric family is indexed by a parameter (or a vectors or parameters) *.e.g*:

for normal distributions the parameter is $\theta = (M, \sum)$

M - the measure of cetral tendency
$\sum$ - measure of variance

for Poisson Distributions the parameter is θ is Λ

set of all possible vaules of the
parameter - parametric space, Ω_θ

For a particular realisation, when $\theta = \theta$, the distribution of oberservations is denoted by P_θ and expectation E_θ

If value of the parameter $\rightarrow$ propose a model, or a fmaily of models.

Let $X =$ observed data obtained from a sample, $\chi = X$

The members $P_\theta(x) = f_\chi(X, \theta)$ of the parametric family are disrtibutions over the space χ, where $\theta \in \Omega_\theta$ is unknown.

Statistic is the estimation $\hat{\theta}$ that can be calculated from the sampling X, e.g *sampling mean* and *sampling variance* etc.

Definition:

The problem of point estimator is to determine statistics $g_i(X_1, \ldots, X_n)$, $i = 1, \ldots, k$ (where k is the dimension of θ), which can be used to eliminate the value of each of the parameters $\theta = (\theta_1, \ldots, \theta_k)$ based on observed sample data from the population.

These statistics are called estimators $\hat{\theta}_i$ for the parameters, where $\hat{\theta}_i = g_i(X_1, \ldots, X_n)$, $i = 1, \ldots, k$

The values calculated from these statistics using particular sample data values are called **estimates** for the parameters.

$\boxed{\text{the estimators are random variable}}$

Three methods of estimation (most popular):

- the method of moments,
- the method of maximum liklihood,
- Baye's method

Criteria for choosing a desired point estimator:

- unbiasedness
- efficiency (minimal variance)
- sufficiency
- consistency

Definition:

Let W be any random variable with $p.d.f$ $f_W(w)$. For any positive integer;

1. **The** $r-th$ **moment of** W **about the origin**, μr, is given by
$$\mu r = E(W^r),$$
provided $\int_{-\infty}^{\infty} |w|^r \times fw(w)dw < \infty$.
(When r = 1, the subscript is usually omitted, i.e. $\mu r = \mu$)

2. **The** $r-th$ **moment of** W **about the mean**, $\mu' r$, is given by
$$\mu' r = E((W-\mu)^r),$$
provided fitness conditions of part 1 hold.

3. **The** $r-th$ **standardized moment**, $\tilde{\mu_r}$, is a moment that is normalised, typically by the normal standard deviation σ^r
$$\tilde{\mu_r} = \frac{E((W-\mu)^r)}{\sigma^r}$$