2.1 Examples in Method of Moments

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Example 1:

Let the distribution of X be $N(\mu, \sigma^2)$.

1. For a given sample of size n, use the method of moments to estimate μ and σ^2
2. The following data (3 d.p.) were generated from a normal distribution with the mean 2 and s.d. of 15.

3.163, 1.883, 3.252, -0.099, -0.653, 0.057,
4.098, 1.670, 1.396, 2.322, 1.838, 3.024, 2.706,
3.830, 3.349, -0.230, 1.496, 0.231, 2.987

true mean? and true variance?
Solution:

A $\mu = E(x), \sigma^2 = E(x^2) - \mu^2 \Rightarrow E(x^2) = \sigma^2 + \mu^2$
$m_1 = \frac{1}{n}\sum_{i=1}^{n} x_i, m_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 \Rightarrow m_1 = E(x)$ & $m_2 = E(x^2)$
$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x} \quad \sigma^2 = m_2 - \bar{x}^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

B let $X = \{$all the data from 2.$\}$

$\mu = \bar{x} = 2.005 \quad \sigma^2 = 2.1 \Rightarrow \sigma = 1.45$

$\mu = 2 \quad \sigma = 1.5$

$\hat{\mu} \approx \mu \quad \hat{\sigma} < \sigma$

Example 2:

Let $X_1, X_2, \ldots, X_n$ be a sample of 1 i.d. random variables with $p.d.f.$

$$f_x(x) = \begin{cases} \theta_x{}^{\theta-1} & , 0 < x \leq 1,\ \theta > 0 \\ 0 & \text{, otherwise} \end{cases} \tag{1}$$

1. Use the method of moments to estimate θ
2. For the following observations of X calculate the method of moments estimate for θ:

0.3,0.5,0.8,0.6,0.4,0.4,0.5,0.8,0.6,0.3

Solution:

A $\mu_1 = \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x\theta x^{\theta-1}dx = \frac{\theta}{\theta-1}x^{\theta+1}|_0^1 = \frac{\theta}{\theta+1}$

$m_1 = \frac{1}{n}\sum_{i-1}^{n} X_i = \bar{X} \Rightarrow \mu_i \Leftrightarrow \frac{\theta}{\theta+1} = \bar{x}$

$\Leftrightarrow \theta = (\theta+1)\bar{x}$

$\boxed{\hat{\theta} = \frac{\bar{x}}{1-\bar{x}}}$ MME

B $X = \{0.3, 0.5, 0.8, 0.6, 0.4, 0.4, 0.5, 0.8, 0.6, 0.3\}$

$\bar{X} = \frac{1}{10}(0.3 + 0.5 + 0.8 + 0.6 + 0.4 + 0.4 + 0.5 + 0.8 + 0.6 + 0.3) = \frac{5.2}{10} = 0.52$