MA1114 1/12/21

(abstract) Subspaces

Definition 4.8

Let $(V, +, \cdot)$ be a vector space

A subset $W \subseteq V$ is a subspace

if $(W, +, \cdot)$ is a vector space

in this case, write $W \leq V$

Examples

$\{0\} \leq V$, $V \leq V$

$\{ \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix} \mid x_1, x_2 \in \mathbb{R} \} \in \mathbb{R}^3$

$\mathbb{R}$: ———— vector space

Subspaces of $\mathbb{R}$:

$\{0\}$, $\mathbb{R}$

suppose $W \leq \mathbb{R}$ and $W \neq \{0\}$

$\Rightarrow$ there exists $0 \neq w \in W$

$\Rightarrow \frac{1}{w} \cdot w = 1 \in W$ by SMO

$\Rightarrow x \cdot 1 = x$ $x \in W$ for all $x \in \mathbb{R}$ $\Rightarrow W = \mathbb{R}$

How to check a subset is a subspace?

<u>Proposition</u> Suppose $(V, +, \cdot)$ is a vector space $W \subseteq V$ is a subspace if and only if:

- $\underline{0} \in W$
- $u + v \in W \ \forall u, v \in W$
- $\lambda \cdot v \ \forall \lambda \in \mathbb{R}, v \in W$

<u>Proof</u> "$\Leftarrow$"

VA0 ✓
VA1 ✓
VA2 $-v = (-1) \cdot v \in W \quad v \in W$ ✓
VA3-4, SM 0-4 ✓

"$\Rightarrow$" clear, except why is zero vector same?

$w \in W \Rightarrow 0 = 0 \cdot w = 0_W \cdot 0_V = 0_W$ since $w \in V$
$\Rightarrow 0_V = 0_W$

<u>Exercise</u>
Show that $\{A \in M_{n,n}(\mathbb{R}) \mid A^T = A\}$ is a subspace of $M_{n,n}(\mathbb{R})$?
(use propositions)