MA1014 11/10/21

Elementary Functions

Basic functions $f: \mathbb{R} \to \mathbb{R}$

$f(x) = \tan(x)$ x in radians

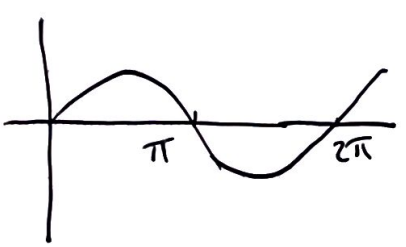
Except $x = \pi/2$ $360° = 2\pi$

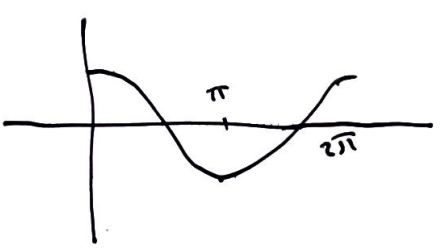
$x = \frac{2n+1}{2}\pi$ $\tan: (-\frac{\pi}{2}, \frac{\pi}{2}) \Rightarrow \mathbb{R}$

$\tan^{-1}: \mathbb{R} \Rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

$x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

$\tan(x) = \frac{\sin(x)}{\cos(x)}$





Domain $(f) = \{x \mid x \text{ is defined}\}$

Domain $(\tan) = \{x \in \mathbb{R} \mid x \neq \frac{2n+1}{2}\pi,\ n \in \mathbb{Z}\}$

Range $(f) = \{y \mid y = f(x) \text{ for some } x\}$

Examples $f(x) = \frac{7x-27}{x-4}$

Domain $(f) = \mathbb{R} - \{4\}$
$= (-\infty, 4) \cup (4, \infty)$

Range $(f) = \mathbb{R} - \{7\}$

$$f(x) = \frac{7x-28}{x-4} + \frac{1}{x-4}$$

$$= \frac{7(x-4)}{x-4} + \frac{1}{x-4}$$

$$= 7 + \frac{1}{x-4}$$

Definition A function $f : x \to y$

is

one-to-one (1-1) $\Leftrightarrow$ if $f(x) = f(x')$ then $x = x'$
injective mono

onto
surjective (a surjection) $\Leftrightarrow$ Range $(f) = y$

bijective (a bijection) $\Leftrightarrow$ 1-1 & onto

Example $f: x \to x$ $\quad f: \mathbb{R} \to \mathbb{R}$

$f(x) = x$ $\quad 3 \to 3$

$7\frac{1}{2} \to 7\frac{1}{2}$

Remark we often talk about inverse functions

$f: X \to Y$ inverse $f^{-1}: Y \to X$

$f^{-1}(f(x)) = x$ $\quad f(f^{-1}(y)) = y$

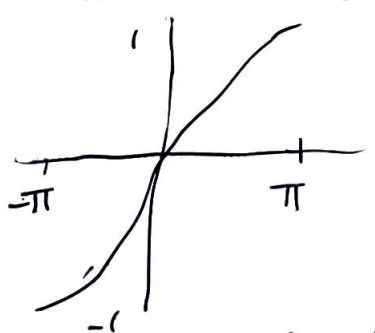
$f^{-1} \cdot f = id_X$ $\quad f \cdot f^{-1} = id_Y$

Important points f invertible

Examples $\sin: \mathbb{R} \to \mathbb{R}$

$\sin^{-1}$ is the inverse of a $\sin: (-\pi, \pi] \to [-1, 1]$

which is 1-1 and onto



Domain $[-\pi, \pi]$ Range $[-1, 1]$