

# Dynamical Estimation of State-of-Health of Batteries by Using Adaptive Observer

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**Abstract**—The battery performance relative to the initial conditions such as usable capacity, thermal behavior, and electrical response are correlated to its State-of-Health (SOH) that determines its remaining life cycles. The most direct and simple way for estimating the SOH is to measure or estimate the electrical characteristics of the battery. Such the characteristics can be modeled by equivalent circuit that is mostly composed of a series resistor and parallel RC networks. It implies the parametric estimation of the equivalent circuit model can facilitate the determination of the SOH. In this study, an adaptive observer incorporated with estimator for unobservable RC state is devised to accurately estimate the relevant parameters. A simulation is presented to validate the effectiveness of the proposed algorithm. This proposed scheme can be applied to different types of batteries, and dynamically monitor the parameters relating to the SOH.

**Index Terms**—Batteries, state-of-health, adaptive observer, equivalent circuit model, battery management system

## I. INTRODUCTION

Various types of batteries have been widely used in portable devices and vehicles. They are designed for the concerns of energy, power, size, and cost to meet the requirements of the products. In the last decade, the energy and power of the batteries required in electric vehicles have been remarkably extended for achieving high traction power and long traveling range, e.g. Ni-MH and Li-ion batteries. Particularly, the batteries used in the pure electric vehicles take the manufacturing cost up to 30-40% overall. Therefore, the reliability and durability are much crucial to such the cost-effective applications. To this end, Battery Management System (BMS) has been developed to provide the functions such as protection, estimation, and diagnosis for battery modules or packs. The State-of-Health (SOH) defined to indicate the degradation of battery performance is essential to the BMS for the possibility to reduce the fade rate so as to extend the life cycles.

The estimations for SOH have attracted much attention in recent years. The methodologies can be categorized into two groups: one is to depend on the operating history of the battery

[7], [8] and the other is to search the sensitive parameters with the strong correlation to the performance fades [5], [9], [10]. In this study, we focus on the methodology to precisely estimate the parameters in the electrical battery model for barely using the information of its working voltage and current. To this end, an adaptive observer based on the Equivalent Circuit Model (ECM) is employed to achieve the model identification. Furthermore, an estimator is also conceived to estimate the unmeasurable state for enhancing the convergent rates. The Lyapunov stability theory is utilized to facilitate the development and also assure the parametric convergence. The proposed method can access to any SOH estimation method developed based on the electrical models.

The paper is organized as follows. In Section II, an equivalent circuit model parameterized by passive elements is introduced to characterize the electrical performance of the batteries. The parametric values correlate with the battery response can be used to estimate the SOH. In Section III, a scheme based on adaptive observer is devised by using the Lyapunov stability theory. The convergent rate of the parameters is enhanced with the additional observer together with an auxiliary estimator. In Section IV, a simulation is presented to validate the effectiveness of the proposed system on the parametric estimation. Finally, the conclusions are given.

## II. MATHEMATICAL BATTERY MODEL

Many battery models have been developed via mathematical approaches in either macroscopic or microscopic prospects so as to characterize its electrochemical behaviors such as chemical reaction as well as power generation. It is intuitive to employ the electrochemical models mostly based on partial differential equations in the battery systems for estimating the effective capacity, voltage variation, and heat generation rate [3], [6]. They can accurately simulate the nonlinear dynamics of the battery subject to charge or discharge operations. However, such models are generally complex and entailed numerical methods to solve the initial and boundary condition problems. To avoid the drawback and ease the computation, electrical models regard the battery as a lumped system has been widely introduced to catch the electrical characteristics such as current and voltage relation [1], [4]. The inherent phenomena as heat generation, leakage current, and impedance depends on frequency as well as the State-of-Charge (SOC) can be performed by the models. In

addition, they provide very close prediction of battery response within the order 5% error. Due to their acceptable accuracy and convenient analysis, they have been widely applied on modeling the charge and discharge behavior of the battery in terms of voltage and current. These merits make them particularly applicable on the indicators of battery status, SOC, and SOH for example.

The electrical model is typically described by a circuit composed of passive components such as resistors, inductances, and capacitors. Those elements are configured to match the frequency response of the battery impedance. As the result, the architecture of the model is usually the circuit of a resistor and RC ladders in series connection. The resistances are typically modeled as nonlinear functions with respect to the SOC for capturing both the battery transient and steady performance. One of Equivalent Circuit Models (ECM) is shown in Fig. 1 [2] where a series resistor  $R_s$ , RC network ( $R_T$  and  $C_T$ ), and open circuit voltage source  $v_{oc}$ , depending on the SOC, are connected to predict the particular behavior of the battery in different load pattern. The left-hand side circuit is introduced to estimate runtime and the SOC based on capacitor  $C_{Capacity}$  that stores all battery energy. Due to its architecture simplicity and system observability, it is applied in the SOH study by using an approach of the parametric estimation.

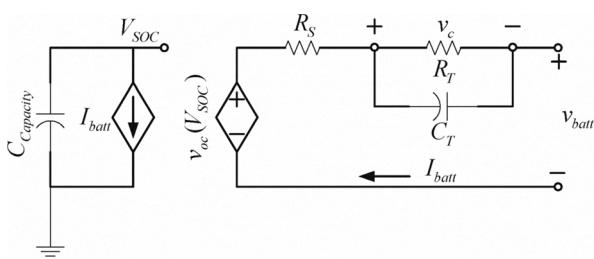


Figure 1. Simplified equivalent circuit model for Li-ion battery.

The ECM shown in Fig. 1 can be described by Kirchhoff's law as

$$\dot{v}_c = -\frac{1}{C_T R_T} v_c + \frac{1}{C_T} I_{batt} \quad (1)$$

$$v_{batt} = v_{oc} - R_S I_{batt} - v_c \quad (2)$$

where  $v_{batt}$  and  $I_{batt}$  are the load voltage and current, respectively, are measurable. However,  $v_c$  is voltage cross RC network and cannot be measured.  $v_{oc}$  is the open circuit voltage of the battery and can be expressed as

$$v_{oc} = f(V_{SOC}) \quad (3)$$

$$\dot{V}_{SOC} = \frac{1}{C_{Capacity}} I_{batt} \quad (4)$$

where  $V_{SOC}$  is the voltage of the capacitor in the left-hand side circuit. The function  $f(\cdot)$  nonlinearly maps  $V_{SOC}$  to  $v_{oc}$  can be obtained empirically in laboratory stage. In this paper, all the

information in the left-hand circuit is assumed to be known, including  $f(\cdot)$  and its derivative, so that we just need to handle (1) and (2).

### III. DESIGN OF ADAPTIVE OBSERVER

To estimate the impedance of the battery, it is required to know the parameters of the ECM. Recall from (1) and (2) that  $R_s$  can be extracted out straightforward and  $R_T$  as well as  $C_T$  can be estimated by any identification or adaptive approach if only if  $v_c$  could be known. Unfortunately, it is impossible in this case since  $v_c$  is not measurable. To tackle this,  $v_c$  should be eliminated from (1) and (2) for using available information  $v_{batt}$  and  $I_{batt}$  instead. To this end, let differentiate (2) on both sides we have

$$\begin{aligned} \dot{v}_{batt} &= \dot{v}_{oc} - R_s \dot{I}_{batt} - \dot{v}_c = \dot{v}_{oc} - R_s \dot{I}_{batt} + \frac{1}{C_T R_T} v_c - \frac{1}{C_T} I_{batt} \\ &= \dot{v}_{oc} - R_s \dot{I}_{batt} - \frac{1}{C_T} I_{batt} + \frac{1}{C_T R_T} (v_{oc} - R_s I_{batt} - v_{batt}) \\ &= \dot{v}_{oc} - R_s \dot{I}_{batt} - \frac{1}{C_T R_T} (R_T + R_s) I_{batt} + \frac{1}{C_T R_T} (v_{oc} - v_{batt}) \\ &= \dot{v}_{oc} - \theta_1 \dot{I}_{batt} - \theta_2 I_{batt} + \theta_3 (v_{oc} - v_{batt}) \end{aligned} \quad (5)$$

where  $\theta_1 = R_s$ ,  $\theta_2 = \frac{1}{C_T R_T} (R_T + R_s)$ , and  $\theta_3 = \frac{1}{C_T R_T}$  are the new parameters in terms of the battery variables. Note that the dynamics of  $v_{batt}$  now only depends on the signal  $v_{oc}$ ,  $\dot{v}_{oc}$ ,  $I_{batt}$ ,  $\dot{I}_{batt}$ , and  $v_{batt}$  where  $\dot{I}_{batt}$  and  $\dot{v}_{oc}$  are the derivative of  $I_{batt}$  and  $v_{oc}$ , respectively, with respect to time.  $\dot{v}_{oc}$  can be evaluated by using (3) and (4) such that

$$\dot{v}_{oc} = \frac{\partial v_{oc}}{\partial V_{SOC}} \frac{dV_{SOC}}{dt} = \frac{f'(V_{SOC})}{C_{Capacity}} I_{batt}. \quad (6)$$

According to (5), an observer for  $v_{batt}$  is given as

$$\dot{\hat{v}}_{batt} = \dot{v}_{oc} - \hat{\theta}_1 \dot{I}_{batt} - \hat{\theta}_2 I_{batt} + \hat{\theta}_3 (v_{oc} - \hat{v}_{batt}) + u_b \quad (7)$$

where  $\hat{v}_{batt}$  is the estimation for  $v_{batt}$  and  $\hat{\theta}_i$ ,  $i = 1, 2, 3$  is the estimated value of  $\theta_i$ ,  $i = 1, 2, 3$ .  $u_b$  is the control input to force  $\hat{v}_{batt}$  to track the trajectory of  $v_{batt}$ . Define the estimated error as

$$\tilde{v}_{batt} = v_{batt} - \hat{v}_{batt} \quad (8)$$

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i, i = 1, 2, 3. \quad (9)$$

The design of the observer is to make both  $\tilde{v}_{batt}$  and  $\tilde{\theta}_i$  convergent to zero. To achieve this, Lyapunov stability theory is employed to guarantee the convergence of the state and parameters.

Choose Lyapunov candidate function as

$$V_1 = \frac{1}{2} \tilde{v}_{batt}^2 + \frac{1}{2} \tilde{\theta}^T \Sigma \tilde{\theta} \quad (10)$$

where  $\Sigma$  is positive definite matrix and the vector  $\theta$  is defined as

$$\theta^T = [\theta_1 \ \theta_2 \ \theta_3].$$

Differentiating (10) along the trajectory of the system and substituting (5) and (7) into it, it follows

$$\begin{aligned} \dot{V}_1 &= \dot{\tilde{v}}_{batt} \tilde{v}_{batt} - \dot{\tilde{\theta}}^T \Sigma \tilde{\theta} \\ &= -\tilde{\theta}_1 \dot{I}_{batt} \tilde{v}_{batt} - \tilde{\theta}_2 I_{batt} \tilde{v}_{batt} + \tilde{\theta}_3 (v_{oc} - v_{batt}) \tilde{v}_{batt} \\ &\quad - u_b \tilde{v}_{batt} - \dot{\tilde{\theta}}^T \Sigma \tilde{\theta} \\ &= -\lambda_b \tilde{v}_{batt}^2 < 0 \end{aligned} \quad (11)$$

provided that

$$u_b = \lambda_b \tilde{v}_{batt}, \quad (12)$$

$$\hat{\dot{\theta}} = \begin{bmatrix} \dot{\hat{\theta}}_1 \\ \dot{\hat{\theta}}_2 \\ \dot{\hat{\theta}}_3 \end{bmatrix} = \begin{bmatrix} -g_1 \dot{I}_{batt} \tilde{v}_{batt} \\ -g_2 I_{batt} \tilde{v}_{batt} \\ g_3 (v_{oc} - v_{batt}) \tilde{v}_{batt} \end{bmatrix} \quad (13)$$

where  $\lambda_b > 0$  and  $\Sigma = diag\left(\frac{1}{g_i} > 0, i = 1, 2, 3\right)$ . In (11), we

have the assumption

$$\dot{\tilde{\theta}} = \dot{\theta} - \hat{\dot{\theta}} = -\dot{\hat{\theta}} \quad (14)$$

that is valid for the constant or slow-varying values. Since the parameters in the ECM vary slowly with the status of the SOC in most operation region, excluding SOC under 10%, (14) is practically hold in the battery systems.

Recall that (11) barely guarantees the exponential convergence of  $\tilde{v}_{batt}$  since we cancelled the unknown term  $\tilde{\theta}_i, i = 1, 2, 3$  from it. Hence, it is desired to have persistent excitation input vector

$$\dot{x}^T = [\dot{I}_{batt} \ I_{batt} \ v_{oc} - v_{batt}] \quad (15)$$

to make sure the convergence of the parameters. It can be seen from (7) and (15) that the parameter  $\hat{R}_s = \hat{\theta}_1$  depends on  $\dot{I}_{batt}$  that is much sensitive comparing to the ones upon  $\hat{\theta}_2$  and  $\hat{\theta}_3$ . As the consequence, it would converge more quickly than the others. To enhance the convergent rate of  $\hat{C}_T$  and  $\hat{R}_T$ , the second observer for  $v_c$  is introduced as

$$\dot{\hat{v}}_c = -\hat{\phi}_1 \hat{v}_c + \hat{\phi}_2 I_{batt} \quad (16)$$

together with an estimator for  $v_c$  given by

$$\hat{v}_c = v_{oc} - \hat{R}_s I_{batt} - v_{batt} \quad (17)$$

In (16),  $\hat{v}_c$  is the estimated  $\hat{v}_c$ .  $\hat{\phi}_1$  and  $\hat{\phi}_2$  are the estimated  $\phi_1 = \frac{1}{C_T R_T}$  and  $\phi_2 = \frac{1}{C_T}$ , respectively. It can be noticed that  $\phi_1$  has the same definition with  $\theta_3$ . However, they correlate with different signal to yield dissimilar convergent trajectories and rates. The idea is based on the fact that  $\hat{v}_c$  would exactly track the trajectory of  $v_c$  when  $\hat{R}_s$  approaches its actual value sooner. To find the adaptive algorithm for (16), let define Lyapunov function

$$V_2 = \frac{1}{2} \tilde{v}_c^2 + \frac{1}{2} \tilde{\phi}^T \Lambda \tilde{\phi} \quad (18)$$

where  $\tilde{v}_c = \hat{v}_c - \hat{v}_c$  and  $\tilde{\phi}^T = [\phi_1 - \hat{\phi}_1 \ \phi_2 - \hat{\phi}_2]$  are the estimated error for  $\hat{v}_c$  and  $\phi^T = [\phi_1 \ \phi_2]$ , respectively.  $\Lambda$  is positive definite matrix. The derivative of (18) along the trajectory of the system yields

$$\begin{aligned} \dot{V}_2 &= \dot{\tilde{v}}_c \tilde{v}_c - \frac{1}{2} \dot{\tilde{\phi}}^T \Lambda \tilde{\phi} = -R_s \tilde{v}_c^2 - \tilde{\phi}_1 \hat{v}_c \tilde{v}_c + \tilde{\phi}_2 \hat{v}_c I_{batt} - \frac{1}{2} \dot{\tilde{\phi}}^T \Lambda \tilde{\phi} \\ &= -R_s \tilde{v}_c^2 < 0 \end{aligned} \quad (19)$$

provided that

$$\dot{\hat{\phi}}_1 = \begin{bmatrix} \dot{\hat{\phi}}_1 \\ \dot{\hat{\phi}}_2 \end{bmatrix} = \begin{bmatrix} -P_1 \tilde{v}_c \hat{v}_c \\ P_2 \tilde{v}_c I_{batt} \end{bmatrix} \quad (20)$$

where  $\Lambda = diag\left(\frac{1}{P_i} > 0, i = 1, 2\right)$ . Here we exploit the similar assumption in (14) on  $\dot{\hat{\phi}}$ . Owing to  $R_s$  is always positive, (19) is hold in any battery operating condition. Finally, the battery parameters in Fig. 1 are evaluated by

$$\begin{bmatrix} \hat{R}_s \\ \hat{R}_T \\ \hat{C}_T \end{bmatrix} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\phi}_2 \hat{\theta}_1^{-1} \\ \hat{\phi}_2^{-1} \end{bmatrix}. \quad (21)$$

According to (11) and (19), all the estimated error will be attenuated to zero and  $v_c$  can be accurately tracked by the estimator as long as  $\hat{R}_s$  would converge to its actual value, which guarantees the convergences of the parameters in the equivalent circuit model. To sum up, the proposed adaptive scheme of the adaptive observer and estimator are depicted in Fig. 2.

#### IV. SIMULATION RESULTS

A simulation built in Simulink is presented to validate the proposed scheme. The battery model depicted in Fig. 1 is used to model a practical battery system. Without loss of generality, the open-circuit voltage and the associated parameters of battery could be regarded as constant in a short-time simulation. Their actual values set to be  $v_{oc} = 12.6$  volt,  $R_s = 0.1$  ohm,  $R_T = 0.05$  ohm, and  $C_T = 50$  farad. In the simulation, a

smoothed random signal as shown in Fig. 3(a) is imposed on the battery output side to represent load pattern. The resulting output current is shown in Fig. 3(b) where the positive and negative current indicates the discharge and charge mode of the battery, respectively. The estimated load voltage error is depicted in Fig. 3(c). It is shown that the error approached to zero within the first 10 seconds. The actual and estimated voltage cross the capacitor  $C_T$  is plotted in Fig. 3(d). It can be found that the estimated trajectory of the estimator coincides with the actual one at early 10 sec, implying the fast convergence of  $R_S$ . On the other hand, the observer estimated error tends to zero after 75 sec.

According to the parameters in the observer, the evaluation for  $R_S$ ,  $R_T$ , and  $C_T$  are shown in Fig. 4. It is shown that  $R_S$  approaches to the actual value at the first 10 sec, reflecting to the rate illustrated in Fig. 3(d), and its steady state is bounded within 1% error. Due to the observer for  $R_T$  and  $C_T$  strongly depends on the exactness of estimated  $R_S$ , they converge to the actual values relatively slow. It can be seen that their steady states are both bounded within 10% error.

## V. CONCLUSIONS

Based on the ECM, an adaptive observer incorporated with the estimator is developed to dynamically estimate the parameters and enhance the convergent rate. The Lyapunov stability theory is employed to guarantee the convergence of the adaptive observer scheme. A simulation for a battery subject to the bounded random load voltage is presented to verify the effectiveness of the proposed algorithm. It is shown that the series resistance approaches the actual value very fast with very small bounded error. The parameters for the parallel RC network is demonstrated to gradually tend to actual values with the acceptable bounded error after the estimator well tracks the actual trajectory. The estimated parameters can be used to evaluate the SOH of the batteries. Such the evaluations partly depend on the relation between the electrical parameters and the SOH empirically developed in the laboratory phase. The devised scheme is able to be implemented on an individual circuit or be integrated on the BMS. In addition, it can be

applied to any type of the batteries and account for the deviations of the battery packs in the production and modulation.

## ACKNOWLEDGMENT

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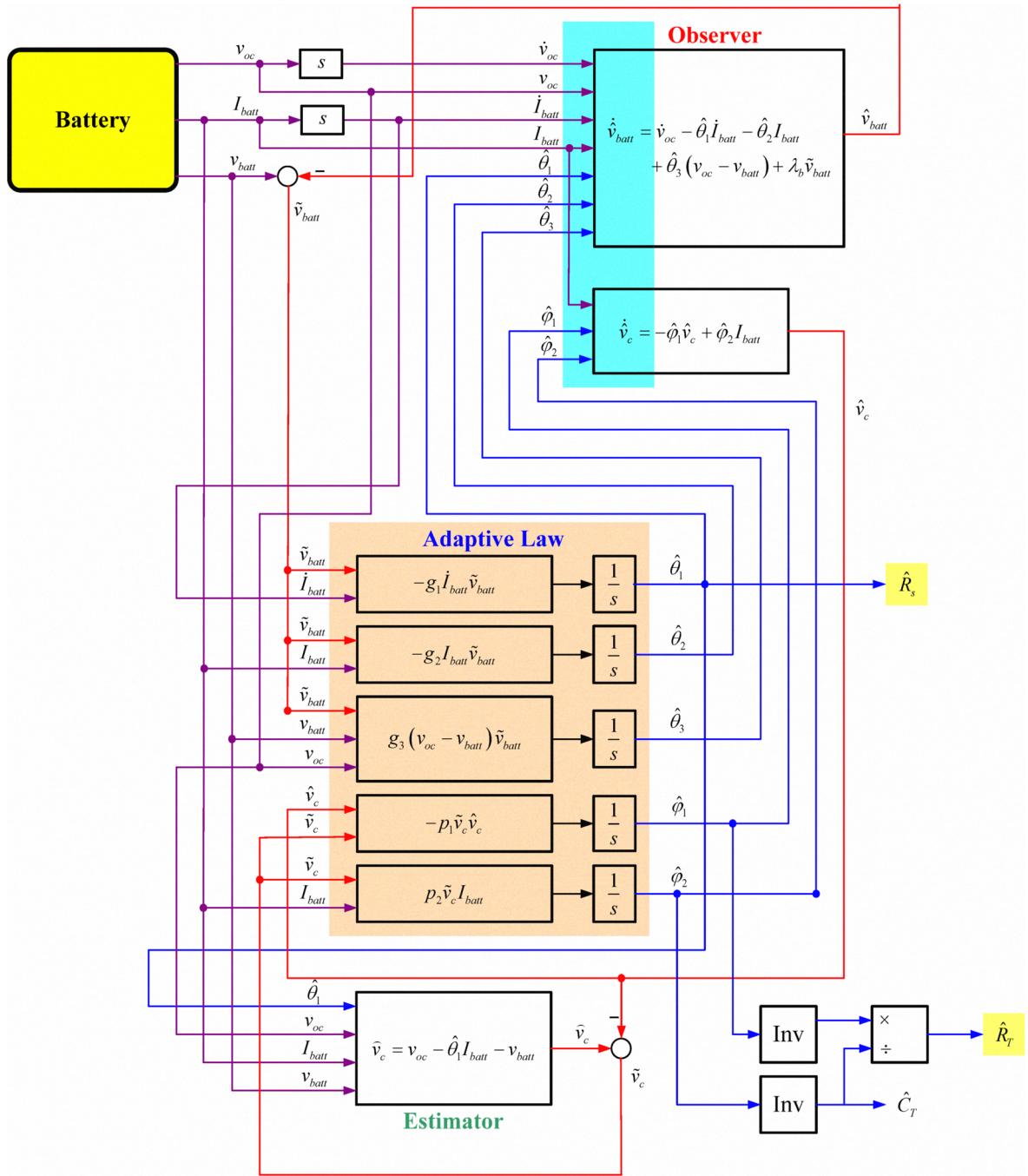


Figure 2. Scheme of adaptive observer and estimator.

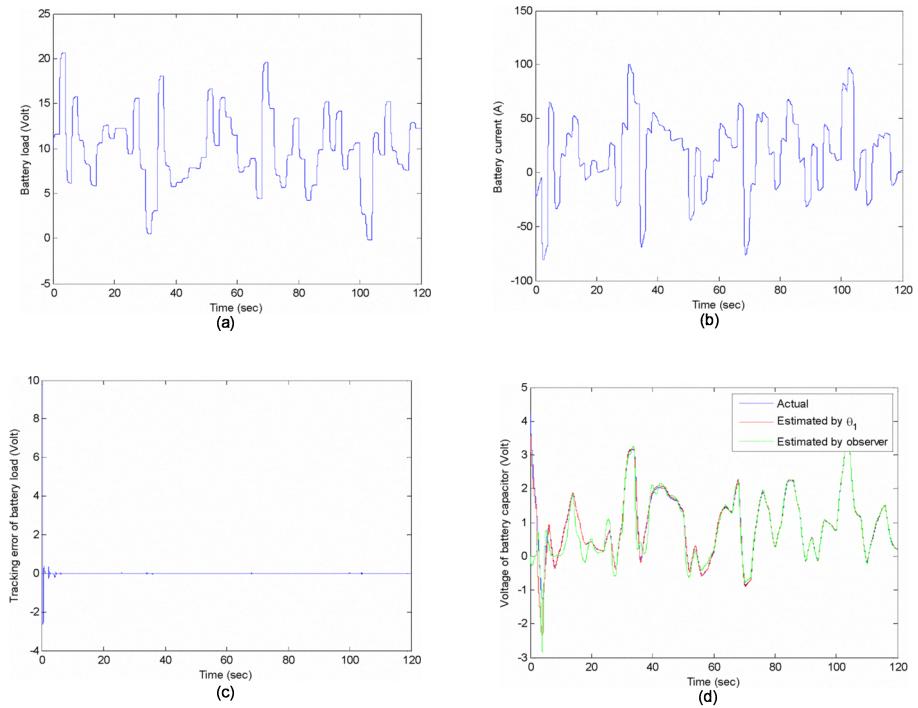


Figure 3. Signals in the adaptive observer system: (a) Load voltage, (b) Battery current, (c) Estimated load voltage error, and (d) Estimated capacitor voltage.

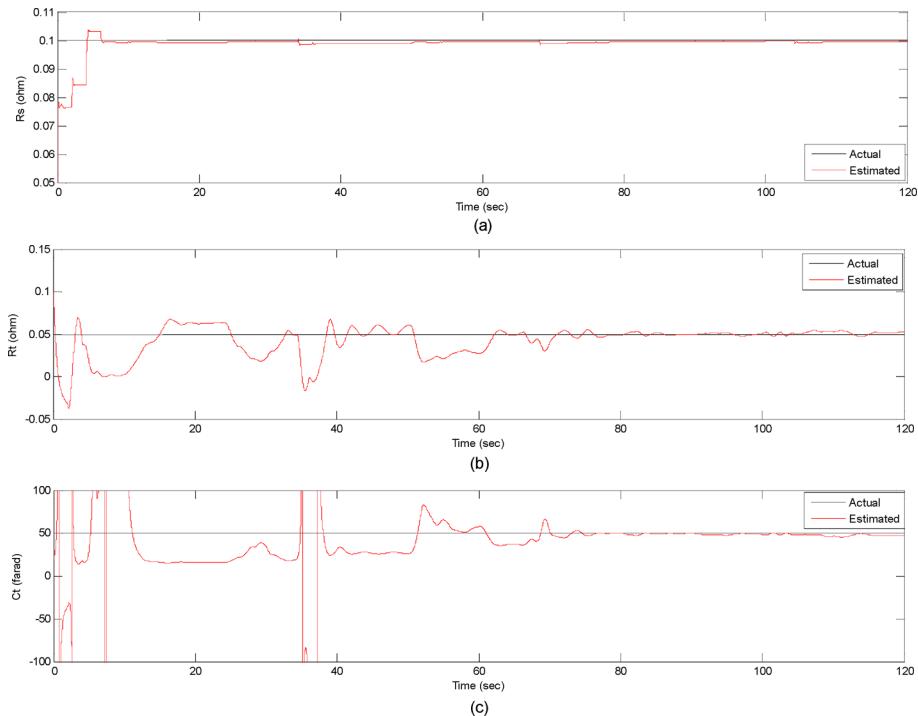


Figure 4. Parameter Estimation: (a)  $R_s$ , (b)  $R_T$ , and (c)  $C_T$ .