# State-of-Charge Estimation of Lithium-ion Batteries using Extended Kalman filter and Unscented Kalman filter

Ivan Jokić, *Student Member, IEEE*, Žarko Zečević, *Member, IEEE* and Božo Krstajić, *Member, IEEE*

*Abstract* — **Lithium-ion battery stands for vital segment of the hybrid-electrical vehicles (HEV). Accurate monitoring of battery status, which is the main task of battery management system (BMS), ensures reliable operation and preserve battery performance. Successful management of battery relies on accurate estimation of state-of-charge (SoC) parameter. Proposed methods for SoC estimation tend to consider nonlinear nature of battery system with time and temperature dependent parameters. Kalman-based filters have been widely used for SoC estimation. In this paper, two Kalman-based filters have been used fo SoC estimation, Extended Kalman filter (EKF) and Uscented Kalman filter (UKF). These two methods have been compared and simulation results are presented.**

*Keywords* — **BMS, EKF, Extended Kalman filter, Lithium–ion battery, SoC estimation, UKF, Unscented Kalman filter.**

## I. Introduction

Electric vehicles (EVs) have become widely accepted alternative to commonly used technologies in automotive industry [1]. In that manner, battery system as a most expensive part of EVs have been researched thoroughly. Compared with other types of batteries, Lithium-ion battery can storage considerably amount of energy and has no memory effect [2], [3]. Knowing that battery system is the most expensive part of EVs, reliability of its functioning became an imperative. BMS coordinates with thousands of Li-ion battery cells inside EV with aim to monitor battery vital characteristics and to contribute safe and reliable operation of vehicles [3], [4]. Battery performance can be affected by many factors, such as operating temperature and aging of the battery. State of Charge (SoC) is recognized as a vital parameter of battery. Its precise estimate can extend its lifecycle, increase battery performance and enhance safety and reliability of the system [1], [5]. However, it is not possible to estimate SoC directly. Nonlinearity of the system, temperature and time dependent characteristics of the battery make direct calculation inaccurate. From those reasons, SoC estimation is widely investigated [6].

In this paper two versions of Kalman filter are considered: Extended Kalman filter and Unscented Kalman filter. Their SoC estimates are compared under two simulation tests.

In Section II adopted model for Li-ion battery is discussed. Also some methods for parameter estimation are mentioned. General overview of standard Kalman filter is presented in Section III. Furthermore, EKF and UKF are explained, respectively, together with equations and block diagrams. Conducted simulation tests can be found in Section IV together with comments on their results, while general comment on paper is given in Section V.

Ivan Jokić, Žarko Zečević and Božo Krstajić, Elektrotehnički fakultet Podgorica, Univerzitet Crne Gore, Džordža Vašingtona bb, 81000 Podgorica, Crna Gora (e-mails: ivan.j@ac.me, zarkoz@ac.me, bozok@ac.me)

## II. Baterry Model and SoC estimation

Battery model used in this paper is presented on figure 1 and has been adopted from [1]. Its parameters are estimated using measurements and implementing Least Square approach. Parameter values are given in table 1.

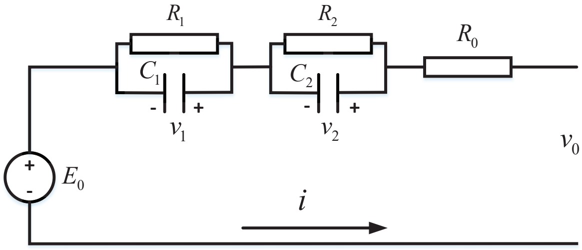


Fig. 1. Equivalent circuit model

### A. SoC estimation

SoC parameter is a state variable in adopted model and is defined as a ratio of available and total battery capacity:

$$s(t) = s(t_0) - \eta \int_{t_0}^{t} i(\tau) d\tau, \tag{1}$$

with $\eta$=1/(3600$C$), where $C$ is battery capacity and its dimension is Ah. Accurate calculating directly the SoC is not possible because of few reasons. Noise presented in measurements of the current would be accumulated in SoC estimates and therefore provide inaccurate estimate. Also, another problem is to calculate initial values of SoC [1].

As it is noted, model of battery has been adopted from [1] and is shown on figure 1. It consists of voltage source $E_0$, called battery *internal open circuit voltage*, while other parameters form circuit, such as $R_0$, $R_1$, $C_1$, $R_2$ and $C_2$ tend to model dynamics of the battery. Voltage on battery terminals is defined as $v_0$. SoC parameter is in nonlinear relation with $E_0$ and this relation can be modeled by polynomial function, that is adopted from [1]:

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$$E_0 = a_1 SoC^7 + a_2 SoC^6 + a_3 SoC^5 + a_4 SoC^4 + a_5 SoC^3 + \\ + a_6 SoC^2 + a_7 SoC + a_8, \tag{2}$$

where coefficients are obtained using fitting curve technique and are:

$$a_1 = 8.4073, a_2 = -19.892, a_3 = 11.497, a_4 = 4.161, \\ a_5 = -4.5533, a_6 = 0.34365, a_7 = 0.64685, a_8 = 3.5016. \tag{3}$$

In order to create a state-space based model of the system, voltage drop $v_1$ over capacitor $C_1$, voltage drop $v_2$ over capacitor $C_2$ and SoC have been adopted for states. Circuit from figure 1 can be modeled with next equation:

$$v_0 = E_0 + v_1 + v_2 + iR_0, \tag{4}$$

where current represents input signal for the model and is used in relations for derivatives of two first states:

$$i = \frac{v_1}{R_1} + C_1 \frac{dv_1}{dt} = \frac{v_2}{R_2} + C_2 \frac{dv_2}{dt}. \tag{5}$$

Using equations (1)-(5), state space model can be constructed:

$$\dot{\mathbf{x}} = \begin{bmatrix} -1/R_1C_1 & 0 & 0 \\ 0 & -1/R_2C_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1/C_1 \\ 1/C_2 \\ -\eta \end{bmatrix} i, \tag{6}$$

with state vector defined as $\mathbf{x} = [v_1 \quad v_2 \quad SoC]^T$. After discretizing, discrete state space model is obtained:

$$\dot{\mathbf{x}}_{\mathbf{k}} = \begin{bmatrix} -\Delta t/R_1C_1 + 1 & 0 & 0 \\ 0 & -\Delta t/R_2C_2 + 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_{\mathbf{k-1}} + \begin{bmatrix} \Delta t/C_1 \\ \Delta t/C_2 \\ -\eta \Delta t \end{bmatrix} i_k, \tag{7}$$

where time interval is denoted by *Δt*, and superscript *k* and *k-1* determines time by defining iteration.

## III. Kalman-based Filters

### *A. Kalman filter*

Kalman filter represents an optimal state estimator of the linear systems, under process and measurement noise [7]. It is named after R.E. Kalman, who has invented this filter [8], [9]. The discrete-time system can be described by equations:

$$\mathbf{x}_{\mathbf{k}} = \mathbf{A}\mathbf{x}_{\mathbf{k-1}} + \mathbf{B}u_{k-1} + w_{k-1} \\ z_k = \mathbf{H}\mathbf{x}_{\mathbf{k}} + v_k, \tag{8}$$

where $\mathbf{x} \in \Re^n$ represents the state of the system, **A** is named as transition matrix, **B** is called input matrix, $\mathbf{x}_{k-1}$ and $u_{k-1}$ are state and input signals from previous iteration, **H** represents measurement matrix, while $w_k$ and $v_k$ are defined as process and measurement noise, respectively. These two random variables are considered as independent. They are also assumed as white and with normal probability distributions [7]:

$$p(w) \sim N(0, \mathbf{Q}) \\ p(v) \sim N(0, \mathbf{P}), \tag{9}$$

where **Q** and **R** represents *process noise* and *measurement noise* covariance matrices. Kalman filter achieves the minimum mean-square state error estimate [1].

The estimation process consists of two steps, as can be seen from figure 2. First step is *time update* for estimating the states, based on the model(8). This estimation is called *a priori* estimation and is denoted with "¯". The input signals in this step are states from previous iteration, input signal sample and process covariance matrix. Second step is *measurement update* and is implemented by feedback. This step uses error in estimating output signal to correct *a priori* estimation of the states and it is named as *a posteriori* estimation, denoted by "^".

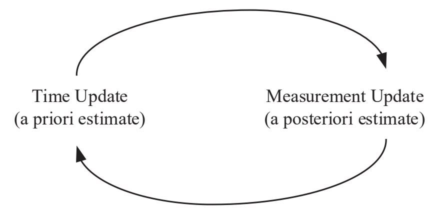


Fig. 2. Discrete Kalman filter cycle

The idea behind Kalman filter is to make an optimum tradeoff between measurements and estimates based on the model, in order to achieve optimal state estimates [1].

### *B. Extended Kalman filter*

The Extended Kalman filter represents a version of Kalman filter for nonlinear systems [10]. Its block diagram is presented on figure 3. EKF linearizes functions around the current mean and covariance [7]. Nonlinear system can be described by state-space model:

$$\mathbf{x}_{\mathbf{k}} = f(\mathbf{x}_{\mathbf{k-1}}, u_k) + w_k \\ y_k = h(\mathbf{x}_{\mathbf{k}}) + v_k, \tag{10}$$

where $f(\mathbf{x}_{k-1}, u_{k-1})$ and $h(\mathbf{x}_k)$ represents nonlinear functions. These nonlinearities are approximated in EKF using linearization through first order Taylor expansion:

$$\mathbf{A}_{\mathbf{k-1}} = \left. \frac{\partial f(\mathbf{x}_{\mathbf{k-1}}, u_{k-1})}{\partial \mathbf{x}_{\mathbf{k-1}}} \right|_{\mathbf{x}_{\mathbf{k-1}} = \hat{\mathbf{x}}_{\mathbf{k-1}}}, \mathbf{C}_{\mathbf{k}} = \left. \frac{\partial h(\mathbf{x}_{\mathbf{k}}, u_k)}{\partial \mathbf{x}_{\mathbf{k}}} \right|_{\mathbf{x}_{\mathbf{k}} = \hat{\mathbf{x}}_{\mathbf{k}}}, \tag{11}$$

where $\mathbf{A}_{\mathbf{k-1}}$ is defined as Jacobian matrix of partial derivatives of $f$ with respect to $\mathbf{x}_{\mathbf{k-1}}$ and $u_{k-1}$, and **C** is defined as Jacobian matrix of partial derivatives of $h$ with respect to $\mathbf{x}_{\mathbf{k}}$ and $u_k$ [7]. A priori and a posteriori error and error covariance are defined as follows:

$$\mathbf{e}_{\mathbf{k}}^{-} = \mathbf{x}_{\mathbf{k}} - \hat{\mathbf{x}}_{\mathbf{k}}^{-}, \mathbf{e}_{\mathbf{k}} = \mathbf{x}_{\mathbf{k}} - \hat{\mathbf{x}}_{\mathbf{k}} \\ \mathbf{P}_{\mathbf{k}}^{-} = E\left[\mathbf{e}_{\mathbf{k}}^{-} \mathbf{e}_{\mathbf{k}}^{-\mathbf{T}}\right], \mathbf{P}_{\mathbf{k}} = E\left[\mathbf{e}_{\mathbf{k}} \mathbf{e}_{\mathbf{k}}^{\mathbf{T}}\right] \tag{12}$$

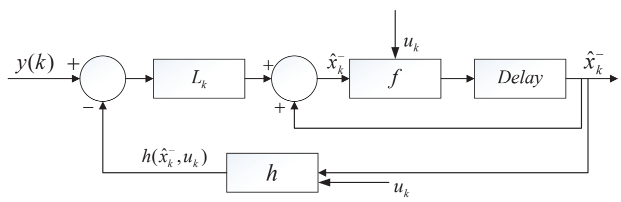


Fig. 3. EKF estimation block diagram

The EKF estimatin process can be presented in five steps with equations (13), repectively:

1. state estimate time update,
2. error covariance time update,
3. Kalman gain matrix,
4. state estimate measurement update,
5. error covariance measurement update [1],

$$
\begin{aligned}
&\hat{\mathbf{x}}_\mathbf{k}^- = f(\hat{\mathbf{x}}_\mathbf{k-1}, u_{k-1}),\\
&\mathbf{P}_\mathbf{k}^- = \mathbf{A}_\mathbf{k-1}\mathbf{P}_\mathbf{k-1}\mathbf{A}_\mathbf{k-1}^\mathbf{T} + \mathbf{Q}(t),\\
&\mathbf{K}_\mathbf{k} = \mathbf{P}_\mathbf{k}^-\mathbf{C}_\mathbf{k}^\mathbf{T}[\mathbf{C}_\mathbf{k}\mathbf{P}_\mathbf{k}^-\mathbf{C}_\mathbf{k}^\mathbf{T} + R(t)]^{-1},\\
&\hat{\mathbf{x}}_\mathbf{k} = \hat{\mathbf{x}}_\mathbf{k}^- + \mathbf{K}_\mathbf{k}[y_k - h(\hat{\mathbf{x}}_\mathbf{k}^-, u_k)],\\
&\mathbf{P}_\mathbf{k} = (\mathbf{I} - \mathbf{K}_\mathbf{k}\mathbf{C}_\mathbf{k}^\mathbf{T})\mathbf{P}_\mathbf{k}^-,
\end{aligned}
\tag{13}
$$

where **I** is unity matrix.

### C. *Unscented Kalman filter*

Unlike EKF, Unscented Kalman filter does not linearize state space equations. It provides sigma points for states instead, by nonlinear unscented transformation (UT), which mean and error covariance are calculated and updated iteratively. These sigma points are then propagated through the nonlinear model functions, by which it is provided a priori estimate of the states and of output signal. The mean and covariance of those variables are then computed based on their statistics [3]. Feedback with output measurement is used to update (*a posterior estimate*) provided estimate and to provide optimal state estimation.

In each iteration *2n+1* sigma points, where *n* is number of states and coefficients $\mathbf{w^c}$, $\mathbf{w^m}$ are calculated by next equations(14)-(15):

$$
\begin{aligned}
&\hat{\mathbf{x}}_\mathbf{k-1}^\mathbf{0} = \hat{\mathbf{x}}_\mathbf{k-1}^+\\
&\hat{\mathbf{x}}_\mathbf{k-1}^\mathbf{i} = \hat{\mathbf{x}}_\mathbf{k-1}^+ + \sqrt{n+\lambda}\left(\sqrt{\mathbf{P}_\mathbf{k-1}}\right), i = 1,2,\ldots,n\\
&\hat{\mathbf{x}}_\mathbf{k-1}^\mathbf{i} = \hat{\mathbf{x}}_\mathbf{k-1}^+ - \sqrt{n+\lambda}\left(\sqrt{\mathbf{P}_\mathbf{k-1}}\right), i = n+1,\ldots,2n
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&w_0^m = \frac{\lambda}{(n+\lambda)}, w_0^c = \frac{\lambda}{(n+\lambda)} + 1 + \beta - \alpha^2\\
&w_i^m = w_i^c = \frac{1}{(2(n+\lambda))}, i = 1,2,\ldots,2n
\end{aligned}
\tag{15}
$$

where $\lambda=\alpha^2(n+k)-n$ is defined as parameter controlling the spread of sigma points around their mean. In this work parameters α and β are set to 1 and 0, respectively. Parameter *k* can take values *0* or *3-n*. Next, $\sqrt{\mathbf{P}_\mathbf{k-1}}$ is decomposed matrix $\mathbf{P}_\mathbf{k-1}$. As it is explained, these sigma points are propagated through state-space equations:

$$
\begin{aligned}
&\hat{\mathbf{x}}_\mathbf{k}^\mathbf{i} = f(\hat{\mathbf{x}}_\mathbf{k-1}^\mathbf{i}, u_k), i = 1,2,\ldots,2n\\
&\hat{\mathbf{x}}_\mathbf{k}^- = \sum_{i=0}^{2n} w_i^m \hat{\mathbf{x}}_\mathbf{k}^\mathbf{i}\\
&\mathbf{P}_\mathbf{k}^- = \sum_{i=0}^{2n} w_i^c (\hat{\mathbf{x}}_\mathbf{k}^\mathbf{i} - \hat{\mathbf{x}}_\mathbf{k}^-)(\hat{\mathbf{x}}_\mathbf{k}^\mathbf{i} - \hat{\mathbf{x}}_\mathbf{k}^-)^T + \mathbf{Q}_\mathbf{k}.
\end{aligned}
\tag{16}
$$

In order to estimate output signal, sigma points of states are propagated through measurement function. Those acquired values are also used for calculating the mean and covariance of the measurement, as well as cross covariance of the state and measurement. It is obtained by next equations(17):

$$
\begin{aligned}
&\hat{y}_k^i = h(\hat{\mathbf{x}}_\mathbf{k}^\mathbf{i}, u_k), i = 0,1,2,\ldots,2n\\
&\hat{y}_k = \sum_{i=0}^{2n} w_i^m \hat{y}_k^i\\
&P_k^h = \sum_{i=0}^{2n} w_i^c (\hat{y}_k^i - \hat{y}_k)(\hat{y}_k^i - \hat{y}_k)^T + R_k\\
&\mathbf{P}_\mathbf{k}^\mathbf{fh} = \sum_{i=0}^{2n} w_i^c (\hat{\mathbf{x}}_\mathbf{k}^\mathbf{i} - \hat{\mathbf{x}}_\mathbf{k}^-)(\hat{y}_k^i - \hat{y}_k)^T,
\end{aligned}
\tag{17}
$$

where $\hat{y}_k^i$ represents measurement estimated from sigma point at $k^{th}$ iteration, $\hat{y}_k$ and $P_k^h$ represent mean and covariance of $\hat{y}_k^i$, while $P_k^{fh}$ is cross covariance of the state $\hat{x}_k^i$ and measurement $\hat{y}_k^i$.

Finally, acquired covariance matrices of measurements and states are used to define Kalman gain(18) and also make an a posteriori estimate of the states(19) and state covariance(20):

$$\mathbf{K}_\mathbf{k} = P_k^{fh}\left(P_k^h\right)^{-1} \tag{18}$$

$$\hat{\mathbf{x}}_\mathbf{k}^+ = \hat{\mathbf{x}}_\mathbf{k}^- + \mathbf{K}_\mathbf{k}(y_k - \hat{y}_k) \tag{19}$$

$$\mathbf{P}_\mathbf{k}^+ = \mathbf{P}_\mathbf{k}^- + \mathbf{K}_\mathbf{k}\mathbf{P}_\mathbf{k}^\mathbf{h}\mathbf{K}_\mathbf{k}^\mathbf{T}. \tag{20}$$

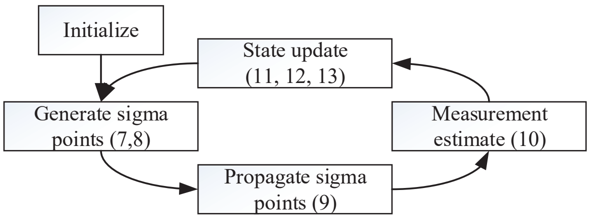


Fig. 4. UKF estimating cycle

## IV. SIMULATION RESULTS

Time step of 1s is used in simulation tests. Model parameters are adopted from[1]. They are estimated by Least Square approach and are given in table 1.

TABLE 1: PAGE LAYOUT DESCRIPTION.

| *Parameters* | *Value* |
|---|---|
| $R_0$ | 0.0013Ω |
| $R_1$ | 0.0042Ω |
| $C_1$ | 17111F |
| $R_2$ | 0.0024Ω |
| $C_2$ | 440.57F |

For EKF, next measurement and process covariance matrices are used:

$$
\begin{aligned}
&\mathbf{Q} = diag\begin{pmatrix}0.1 & 0.1 & 0.01\end{pmatrix}, \mathbf{P} = diag\begin{pmatrix}0.1 & 0.1 & 0.01\end{pmatrix},\\
&R = 1000,
\end{aligned}
\tag{21}
$$

while for UKF they have next values:

$$
\begin{aligned}
&\mathbf{Q} = diag\begin{pmatrix}0.01 & 0.1 & 0\end{pmatrix}, \mathbf{P} = \begin{pmatrix}0.1 & 0.1 & 10^{-6}\end{pmatrix},\\
&R = 15.
\end{aligned}
\tag{22}
$$

### *A. Constant discharge simulation*

First simulation test is battery discharging with constant current of 20A, applied at the input of the model. SoC estimates from both filters are presented on figure 5. On figure 6 SoC estimation error in percentage is shown. It can be seen that EKF estimates more precisely in first part of simulation, while over considerably bigger range of SoC values UKF provides better estimate.

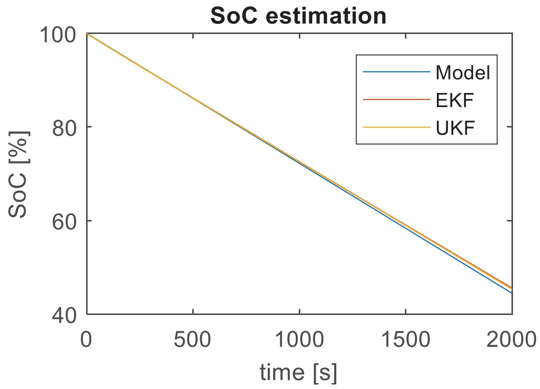


Fig. 5. SoC estimate under constant current discharge

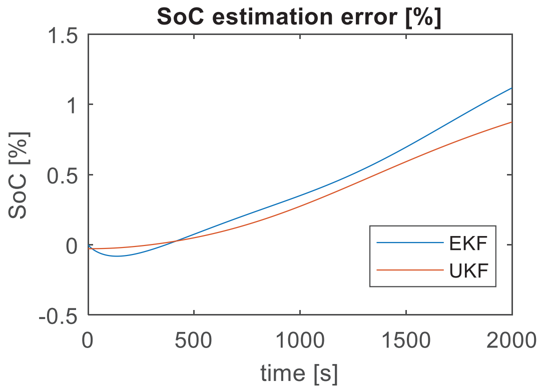


Fig. 6. SoC estimation error under constant current discharge

### *B. Noisy charge/discharge simulation*

Second simulation test is under constant charge/discharge of battery, with noise presented in process. A white Gaussian noise, with covariance of 0.6 is added to constant current of ±20A. Current signal is of square wave, with period of 150s and pulse width of 50%.

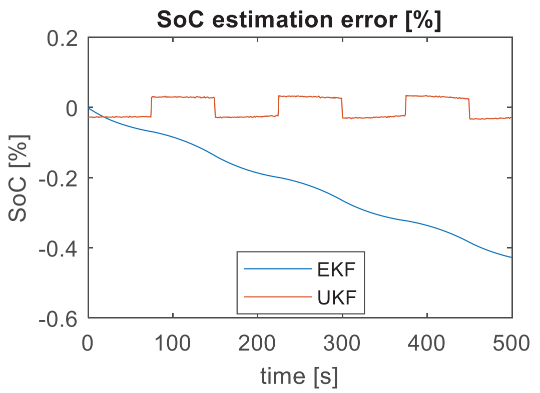


Fig. 7. SoC estimate in percentage under noisy constant charge/discharge

SoC estimation error in percentage is presented on figure 7, while SoC stimate, also in percentage, is presented on figure 8. From both figures, it can be concluded that UKF performs considerably better results under described conditions.

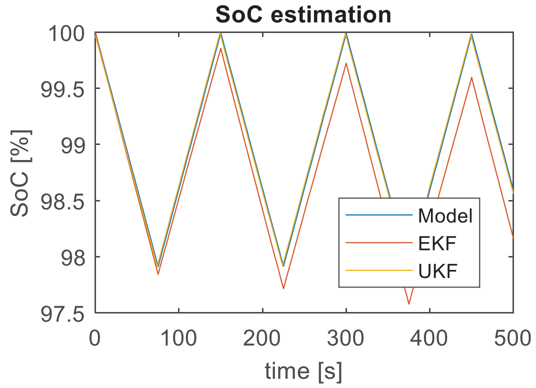


Fig. 8. SoC estimate in percentage under noisy constant charge/discharge

## V. CONCLUSION

Two versions of Kalman filter, Extended Kalman filter and Unscented Kalman filter have been used in this paper for SoC estimation of Lithium-ion battery. Durign battery discharging under constant current it is shown that in most part of simulation UKF reaches better performance. In second simulation test with alternating charging and discharging of battery by constant current and noisy process, UKF has performed considerably better results.

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