Probabilistic Quantitative Precipitation Forecasting Using Bayesian Model Averaging

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Abstract

Bayesian model averaging (BMA) is a statistical way of postprocessing forecast ensembles to create predictive probability density functions (PDFs) for weather quantities. It represents the predictive PDF as a weighted average of PDFs centered on the individual bias-corrected forecasts, where the weights are posterior probabilities of the models generating the forecasts and reflect the forecasts' relative contributions to predictive skill over a training period. It was developed initially for quantities whose PDFs can be approximated by normal distributions, such as temperature and sea-level pressure. BMA does not apply in its original form to precipitation, because the predictive PDF of precipitation is nonnormal in two major ways: it has a positive probability of being equal to zero, and it is skewed. Here we extend BMA to probabilistic quantitative precipitation forecasting. The predictive PDF corresponding to one ensemble member is a mixture of a discrete component at zero and a gamma distribution. Unlike methods that predict the probability of exceeding a threshold, BMA gives a full probability distribution for future precipitation. The method was applied to daily 48-h forecasts of 24-h accumulated precipitation in the US Pacific Northwest in 2003–2004 using the University of Washington mesoscale ensemble. It yielded predictive distributions that were calibrated and sharp. It also gave probability of precipitation (PoP) forecasts that were much better calibrated than those based on consensus voting of the ensemble members.

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1 Introduction

A number of existing methods generate probabilistic precipitation forecasts based on deterministic forecasts. Regression techniques such as model output statistics (MOS) can be used to generate probabilities of exceeding thresholds (Glahn and Lowry 1972; Klein and Glahn 1974; Bermowitz 1975; Charba 1998; Antolik 2000), or to generate quantiles of expected precipitation (Bremnes 2004). Applequist et al. (2002) found that logistic regression can outperform standard regression, and Hamill et al. (2004) found that this can be further refined by using logistic regression on power-transformed forecasts.

These methods, however, do not yield a full predictive probability density function (PDF); rather, they give only probabilities for certain specific events. They also do not make use of all the information available in an ensemble forecast. Ensemble forecasts can give an indication of uncertainty, and a relationship between forecast errors and ensemble spread has been established for several ensemble systems (Buizza et al. 2005). Anderson (1996) suggested using the ensemble member forecasts to partition the real line into a series of bins, assuming each bin to be an equally likely range of possible outcomes, and probabilities uniformly distributed within the inner bins. Hamill and Colucci (1998) noted that this approach is not well-calibrated, with far too many observations appearing at the extreme bins. They proposed an alternative method, fitting gamma distributions with parameters based on corrected ensembles or transformations of the ensemble mean. While they reported good results, it is not obvious how to obtain calibrated probability of precipitation (PoP) forecasts using this approach.

Bayesian model averaging (BMA) was introduced by Raftery et al. (2005) as a statistical postprocessing method for producing probabilistic forecasts from ensembles in the form of predictive PDFs. The BMA predictive PDF of any future weather quantity of interest is a weighted average of PDFs centered on the individual bias-corrected forecasts, where the weights are equal to posterior probabilities of the models generating the forecasts and reflect the forecasts' contributions to overall forecasting skill over a training period. The original development of BMA by Raftery et al. (2005) was for weather quantities whose predictive PDFs are approximately normal, such as temperature and sea-level pressure.

BMA in the form described by Raftery et al. (2005) does not apply directly to precipitation. This is because the predictive distribution of precipitation is far from normal. It is nonnormal in two major ways: it has a positive probability of being equal to zero, and when it is not zero the predictive density is skewed. Here we extend BMA to precipitation by modeling the predictive distribution for a given ensemble member as a mixture of a point mass at zero and a gamma distribution; the BMA PDF is then itself a mixture of such distributions.

In our experiments we show that BMA was calibrated and sharp for the period we considered. This indicates that BMA has the potential to provide both calibrated PoP forecasts, and calibrated and sharp probabilistic quantitative precipitation forecasts (PQPF).

In section 2 we review the BMA technique and describe our extension of it to precipitation. Then in section 3 we give results for daily 48-h forecasts of 24-h accumulated precipitation over the US Pacific Northwest in 2003–2004 based on the 9-member University of Washington mesoscale ensemble (Grimit and Mass 2002; Eckel and Mass 2005). Throughout the paper we use illustrative examples drawn from these data. Finally, in section 4 we discuss possible improvements to the method.

2 Methods

2.1 Bayesian model averaging

Bayesian model averaging (Leamer 1978; Kass and Raftery 1995; Hoeting et al. 1999) was originally developed as a way to combine inferences and predictions from multiple statistical models, and was applied to statistical linear regression and related models in social and health sciences. Raftery et al. (2005) extended BMA to ensembles of dynamical models and showed how it can be used as a statistical postprocessing method for forecast ensembles, yielding calibrated and sharp predictive PDFs of future weather quantities.

In BMA for ensemble forecasting, each ensemble member forecast, f_k, is associated with a conditional PDF, $h_k(y|f_k)$, which can be interpreted as the PDF of the weather quantity y given f_k, conditional on f_k being the best forecast in the ensemble. The BMA predictive PDF is then

$$p(y|f_1, \ldots, f_K) = \sum_{k=1}^{K} w_k \, h_k(y|f_k), \tag{1}$$

where w_k is the posterior probability of forecast k being the best one, and is based on forecast k's relative performance in the training period. The w_k's are probabilities and so they are nonnegative and add up to 1, that is, $\sum_{k=1}^{K} w_k = 1$.

2.2 Discrete-continuous model

For temperature and sea-level pressure, the conditional PDF can be fit reasonably well using a normal distribution centered at a bias-corrected forecast, as shown by Raftery et al. (2005). For precipitation, however, the normal distribution is not appropriate. Figure 1 illustrates the distribution of precipitation accumulation among the verifying observations in our database of ensemble forecasts over the Pacific Northwest in 2003 and 2004, stratified by the accumulation amount predicted by the centroid member (Eckel and Mass 2005) of

the forecast ensemble. These histograms show two important aspects of the distribution of precipitation. First, accumulated precipitation was zero in a large number of cases. Second, for the cases on which the accumulated precipitation was not zero, the distributions were highly skewed. The normal distribution does not fit data of this kind, and to extend BMA to precipitation we must develop a model for the conditional PDF $h_k(y|f_k)$ in (1) that takes account of these facts.

Our model for $h_k(y|f_k)$ is in two parts. The first part specifies PoP as a function of the forecast f_k, given that f_k is the best ensemble member forecast for that day. We follow Hamill et al. (2004) in using logistic regression with a power transformation of the forecast as predictor variable. Hamill et al. (2004) recommended using the fourth root of the forecast as predictor variable, but we found that using the cube root was adequate. All else equal, it seems desirable to use a predictor variable that is as close to the original forecast as possible, and the cube root is closer than the fourth root, and so is preferable if its performance is adequate. We found that this model did not provide the best possible predictions when the forecast was equal to zero, and so we included a second predictor, δ_k, equal to 1 if $f_k = 0$ and equal to 0 otherwise. Our logistic regression model then is

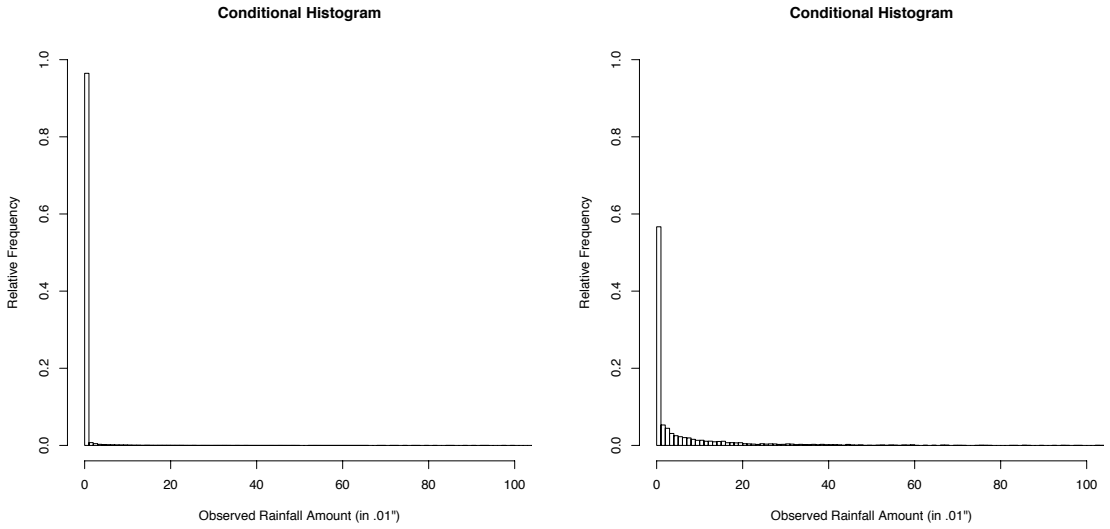
$$\text{logit}\, P(y = 0|f_k) \equiv \log \frac{P(y = 0|f_k)}{P(y > 0|f_k)} = a_0 + a_1 f_k^{1/3} + a_2 \delta_k. \tag{2}$$

The probability $P(y > 0|f_k)$ is the probability of nonzero precipitation given the forecast f_k, if f_k is the best ensemble member forecast for that day.

The second part of our model specifies the PDF of the amount of precipitation given that it is not zero. Previous authors have fit gamma distributions to precipitation amounts (Coe and Stern 1982; Stern and Coe 1984; Wilks 1990; Hamill and Colucci 1998; Wilson et al. 1999), as they can fit skewed data and are quite flexible, and we also took the gamma distribution as our starting point. The gamma distribution with shape parameter α and scale parameter β has the PDF

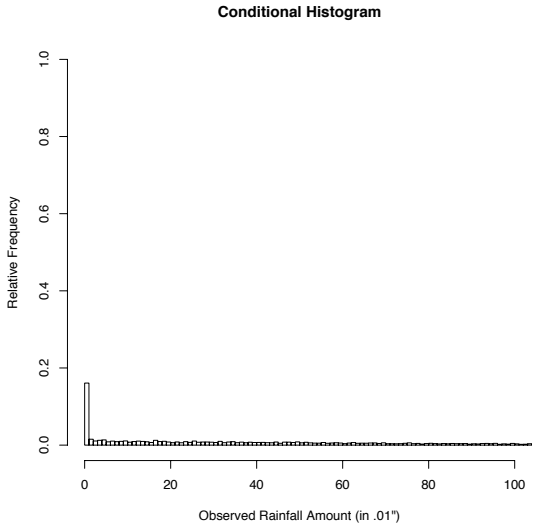
$$g(y) = \frac{1}{\beta^\alpha \Gamma(\alpha)}\, y^{\alpha-1} \exp(-y/\beta)$$

for $y > 0$ and $g(y) = 0$ for $y \leq 0$. The mean of this distribution is $\mu = \alpha\beta$, and its variance is $\sigma^2 = \alpha\beta^2$. We found that fitting gamma distributions to the raw observed accumulation amounts did not give an especially good fit. We found the same issues with high values being fit poorly that Hamill and Colucci (1998) reported. In light of this, rather than fitting the gamma distribution to the observed precipitation amounts themselves, we fit the gamma distribution to powers of the observed values. We found that the best fit was achieved when the gamma distribution was fit to the cube root of the observed precipitation amounts.



(a)

(b)



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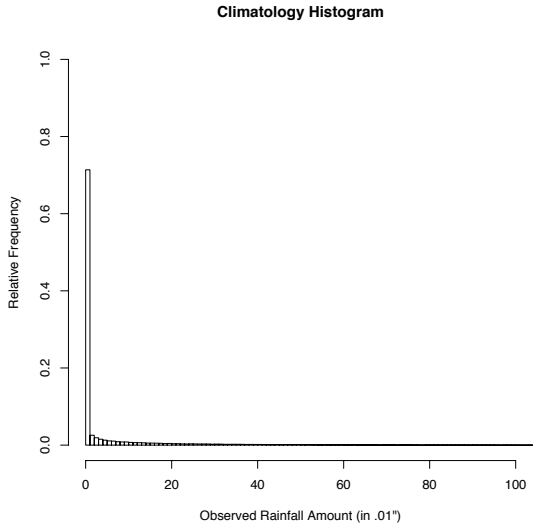


(d)

Figure 1: Histograms of observed precipitation accumulation: (a) for cases on which the centroid member forecast of precipitation was zero, (b) for cases on which it was between 6.4 and 9.6 hundredths of an inch, (c) for cases on which it was greater than 59.4 hundredths of an inch, and (d) for all cases.

It remains to specify how the parameters of the gamma distribution depend on the forecast. We found that the means and variances of the fitted gamma distributions were both approximately linear as functions of the original forecasted accumulation amount. We therefore fit the mean and variance of the gamma distribution as linear functions of the forecasted amount. For the mean, we added an additional predictor variable equal to 1 when the forecast was zero and equal to 0 otherwise, and found that this improved the fit.

Putting these components together, we get the following model for the conditional PDF of precipitation accumulation, given that forecast f_k is best:

$$h_k(y|f_k) = P(y = 0|f_k)\, I[y = 0] + P(y > 0|f_k)\, g_k(y|f_k)\, I[y > 0],$$

where y is the cube root of the precipitation accumulation, $I[y = 0]$ is an indicator function equal to 1 if $y = 0$ and equal to 0 otherwise, and $P(y = 0|f_k)$ is specified in (2). The conditional PDF $g_k(y|f_k)$ of the cube root precipitation amount y given that it is positive is a gamma distribution with PDF

$$g_k(y|f_k) = \frac{1}{\beta_k^{\alpha_k}\Gamma(\alpha_k)}\, y^{\alpha_k - 1} \exp(-y/\beta_k).$$

The parameters of the gamma distribution depend on the original forecast, f_k, through the relationships

$$\mu_k = b_{0k} + b_{1k} f_k + b_{2k}\delta_k$$

and

$$\sigma_k^2 = c_{0k} + c_{1k} f_k, \tag{3}$$

where $\mu_k = \alpha_k\beta_k$ is the mean of the distribution, $\sigma_k^2 = \alpha_k\beta_k^2$ is its variance, and δ_k is equal to 1 if $f_k = 0$ and equal to 0 otherwise.

2.3 BMA for discrete-continuous models

For the variances, we observed that the parameters c_{0k} and c_{1k} in (3) did not vary much from one model to another, and so we restricted the variance parameters to be constant across all ensemble members. This simplifies the model by reducing the number of parameters to be estimated, makes parameter estimation computationally easier, and reduces the risk of overfitting. It is analagous to the assumption of equal variances in Raftery et al. (2005).

Our final BMA model (1) for the predictive PDF of the weather quantity, y, here the cube root of precipitation accumulation, is thus

$$p(y|f_1, \ldots, f_K) = \sum_{k=1}^{K} w_k \Big(P(y = 0|f_k) I[y = 0] + P(y > 0|f_k)\, g_k(y|f_k)\, I[y > 0] \Big), \tag{4}$$

where w_k is the posterior probability of ensemble member k being best, f_k is the original forecast from this member,

$$\text{logit}\, P(y = 0|f_k) = a_{0k} + a_{1k} f_k^{1/3} + a_{2k}\delta_k$$

with δ_k equal to 1 if $f_k = 0$ and equal to 0 otherwise, and

$$g_k(y|f_k) = \frac{1}{\beta_k^{\alpha_k}\Gamma(\alpha_k)}\, y^{\alpha_k - 1} \exp(-y/\beta_k).$$

The parameters $\alpha_k = \mu_k^2/\sigma_k^2$ and $\beta_k = \sigma_k^2/\mu_k$ of the gamma distribution depend on f_k through the relationships

$$\mu_k = b_{0k} + b_{1k} f_k + b_{2k}\delta_k$$

and

$$\sigma_k^2 = c_0 + c_1 f_k,$$

that specify the mean and the variance of the distribution, respectively. While (4) is stated in terms of the cube root of the precipitation amount, it is easy to express the resulting probability statements in terms of the original amounts.

2.4 Parameter estimation

Parameter estimation is based on data from a training period, which we take here to be the 25 days of forecast and observation data preceding initialization, following Raftery et al. (2005). The training period is a sliding window, and the parameters are reestimated for each new initialization period. The required data consist of forecast-observation pairs from a collection of observation sites for each of the ensemble members.

The parameters a_{0k}, a_{1k}, and a_{2k} are member-specific, and they are determined separately for each ensemble member, using the observations and the forecasts from that ensemble member only. They are estimated by logistic regression with precipitation/no precipitation as the dependent variable, and $f_k^{1/3}$ and δ_k as the two predictor variables.

The parameters b_{0k}, b_{1k}, and b_{2k} are also member-specific, and are determined by linear regression with the nonzero precipitation observations as cases, the cube root of the amount of precipitation as the dependent variable, and two predictor variables, the original forecast and the indicator variable for the forecast being equal to zero.

We estimate w_k, $k = 1, \ldots, K$; c_0; and c_1 by the maximum likelihood technique (Fisher 1922) from the training data. The likelihood function is defined as the probability of the training data given the parameters to be estimated, viewed as a function of the parameters. The maximum likelihood estimator is the value of the parameter vector that maximizes the

likelihood function, that is, the value of the parameter vector under which the observed data were most likely to have been observed.

It is convenient to maximize the logarithm of the likelihood function (or log-likelihood function) rather than the likelihood function itself, for reasons of both algebraic simplicity and numerical stability; the same parameter value that maximizes one also maximizes the other. Assuming independence of forecast errors in space and time, the log-likelihood function for the BMA model (4) is

$$\ell(w_1, \ldots, w_K; c_0; c_1) = \sum_{s,t} \log p(y_{st}|f_{1st}, \ldots, f_{Kst}), \tag{5}$$

where the summation is over values of s and t that index observations in the training set by space and time, and $p(y_{st}|f_{1st}, \ldots, f_{Kst})$ is given by (4), with subscripts s and t added to y and f_k. This cannot be maximized analytically, and instead we maximize it numerically using the expectation-maximization, or EM algorithm (Dempster et al. 1977; McLachlan and Krishnan 1997).

The EM algorithm is a method for finding the maximum likelihood estimator when the problem can be recast in terms of unobserved quantities such that, if we knew what they were, the estimation problem would be straightforward. The BMA model (4) is a finite mixture model (McLachlan and Peel 2000). Here we introduce the unobserved quantities z_{kst}, where $z_{kst} = 1$ if ensemble member k is the best forecast for verification site s and time t, and $z_{kst} = 0$ otherwise. For each (s, t), only one of $\{z_{1st}, \ldots, z_{Kst}\}$ is equal to 1; the others are all zero.

The EM algorithm is iterative, and alternates between two steps, the E (or expectation) step, and the M (or maximization) step. It starts with an initial guess for the parameters. In the E step, the z_{kst} are estimated given the current guess for the parameters; the estimates of the z_{kst} are not necessarily integers, even though the true values are 0 or 1. In the M step, the parameters are reestimated given the current values of the z_{kst}.

For the BMA model (4), the E step is

$$\hat{z}_{kst}^{(j+1)} = \frac{w_k^{(j)} p^{(j)}(y_{st}|f_{kst})}{\sum_{l=1}^{K} w_l^{(j)} p^{(j)}(y_{st}|f_{lst})},$$

where the superscript j refers to the jth iteration of the EM algorithm, and thus $w_k^{(j)}$ refers to the estimate of w_k at the jth iteration, and $p^{(j)}(y_{st}|f_{kst})$ is $p(y_{st}|f_{kst})$ as defined in (4), using the estimates of c_0 and c_1 from the jth iteration. The M step then consists of estimating the w_k, c_0, and c_1 using as weights the current estimates of z_{kst}, namely $\hat{z}_{kst}^{(j+1)}$. Thus

$$w_k^{(j+1)} = \frac{1}{n} \sum_{s,t} \hat{z}_{kst}^{(j+1)},$$

where n is the number of cases in the training set, that is, the number of distinct values of (s,t). There are not analytic solutions for the maximum likelihood estimates of the parameters c_0 and c_1, and so they must be estimated numerically by optimizing (5) using the current estimates of the w_k parameters.

The E and M steps are then iterated to convergence, which we defined as changes no greater than some small tolerances in any of the log-likelihood, the parameter values, or the $\hat{z}_{kst}^{(j)}$ in one iteration. The log-likelihood is guaranteed to increase at each EM iteration (Wu 1983), which implies that in general it converges to a local maximum of the likelihood. Convergence to a global maximum cannot be guaranteed, so the solution reached by the algorithm can be sensitive to the starting values. Choosing the starting value for day $t+1$ to be equal to the converged estimate for day t usually leads to a good solution.

2.5 Examples

To illustrate how the method works, we show two examples. Our first example is from station KRNT on 19 May 2003, in Renton, Washington. Table 1 shows the raw ensemble forecasts, the logistic regression PoP results, the BMA results and the verifying observation. All nine ensemble members predicted no rain, but it actually did rain. The BMA predictive PDF is shown in Figure 2(a); the BMA 90th percentile upper bound was slightly above the observed amount, even though the forecast probability of rain was low (below 20%).

Our second example is from station KPWT on 26 January 2003, in Bremerton, Washington. Again, Table 1 shows the raw ensemble forecasts, logistic regression PoP results, BMA results and the observed value. The BMA deterministic forecast, that is, is the median of the BMA predictive PDF, was about 3 hundredths of an inch. The BMA predictive PDF itself is shown in Figure 2(b). The observation is far outside the ensemble range, but it is contained within the BMA upper bound.

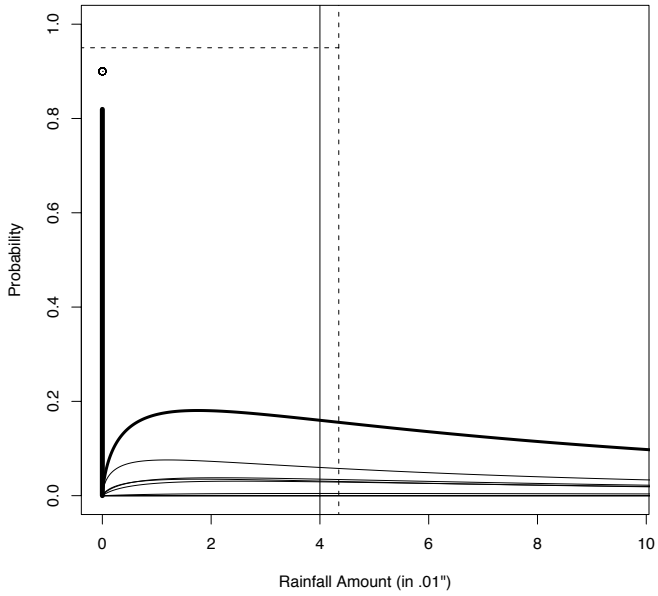
Spatial displays of the BMA PoP forecast, and of the BMA deterministic forecast and 90th percentile upper bound for the precipitation amount, are shown in Figures 3 and 4 for these two dates. Spatial displays of the PoP seem potentially useful in communicating probability forecasts to the general public, and might assist in the use and interpretation of the forecasts (Gigerenzer et al. 2005).

3 Results

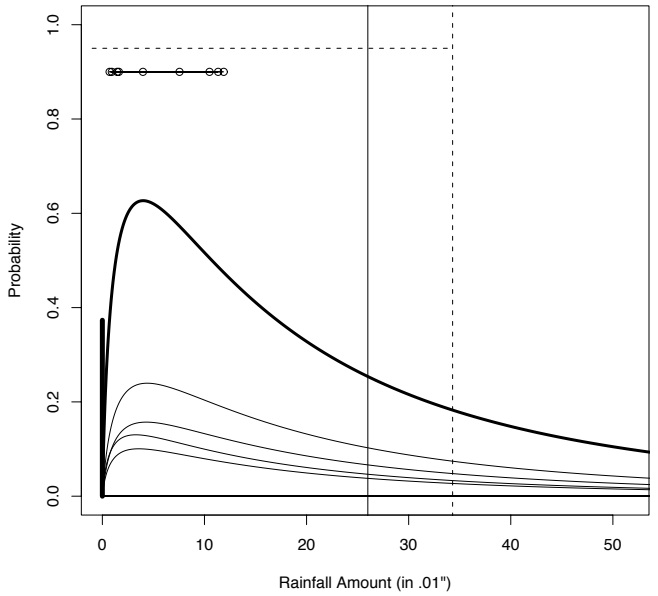
BMA was applied to 48-h forecasts of 24-h precipitation accumulation in the US Pacific Northwest for the 0000 UTC cycle over the two-year period of January 1, 2003 through December 31, 2004, using the 9-member University of Washington mesoscale ensemble (Eckel

Table 1: Raw ensemble, logistic regression PoP and BMA forecasts for two example stations. The quantitative precipitation forecasts and observations are given in hundredths of an inch. Descriptions of the University of Washington ensemble member acronyms can be found in Eckel and Mass (2005).

Ensemble Member	CENT	AVN	CMCG	ETA	GASP	JMA	NGPS	TCWB	UKMO
Station KRNT on 19 May 2003									
BMA Weight	0.00	0.42	0.00	0.22	0.18	0.16	0.00	0.02	0.00
Member PoP	0.20	0.16	0.22	0.18	0.18	0.22	0.23	0.24	0.19
BMA PoP	0.18								
Member Forecast	0	0	0	0	0	0	0	0	0
BMA Forecast	0								
BMA Upper Bound	4								
Observation	4								
Station KPWT on 26 January 2003									
BMA Weight	0.00	0.20	0.33	0.00	0.00	0.00	0.23	0.24	0.00
Member PoP	0.45	0.62	0.75	0.60	0.41	0.40	0.69	0.40	0.71
BMA PoP	0.63								
Member Forecast	2	8	10	4	1	1	11	1	12
BMA Forecast	3								
BMA Upper Bound	34								
Observation	26								



(a)



(b)

Figure 2: BMA-fitted PDFs for (a) station KRNT on 19 May 2003 and (b) station KPWT on 26 January 2003. The thick vertical line at zero represents the BMA estimate of the probability of no precipitation, and the upper solid curve is the BMA PDF of the precipitation amount given that it is nonzero. The lower curves are the components of the BMA PDF. The dashed vertical line represents the 90th percentile upper bound of the BMA PDF, the dashed horizontal line is the respective prediction interval; the dots represent the ensemble member forecasts; and the solid vertical line represents the verifying observation.

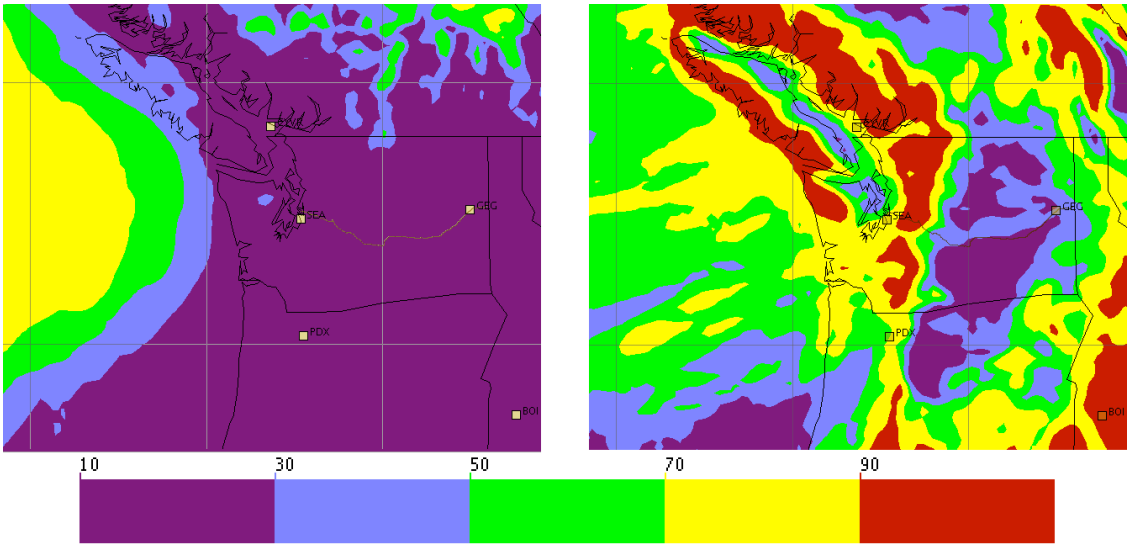
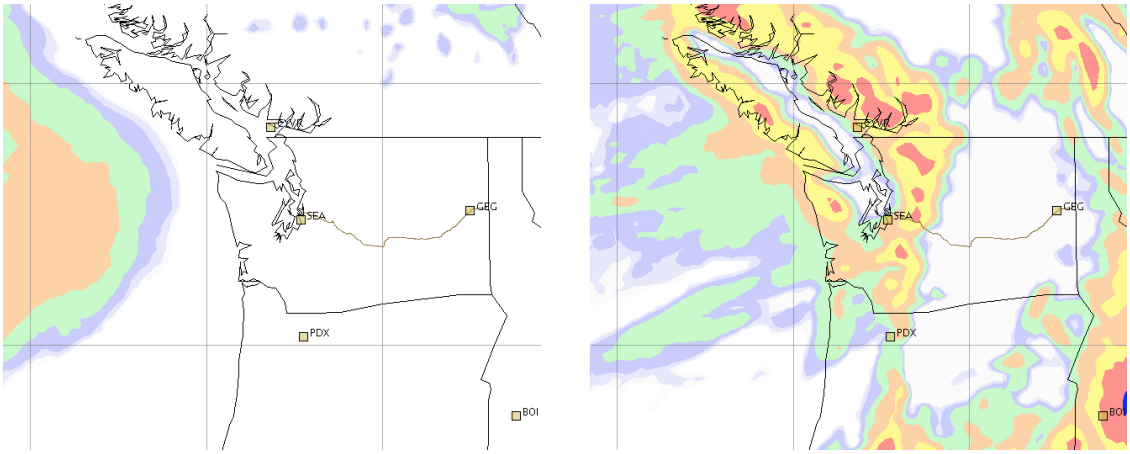


Figure 3: BMA PoP forecast for (left) 19 May 2003 and (right) 26 January 2003.

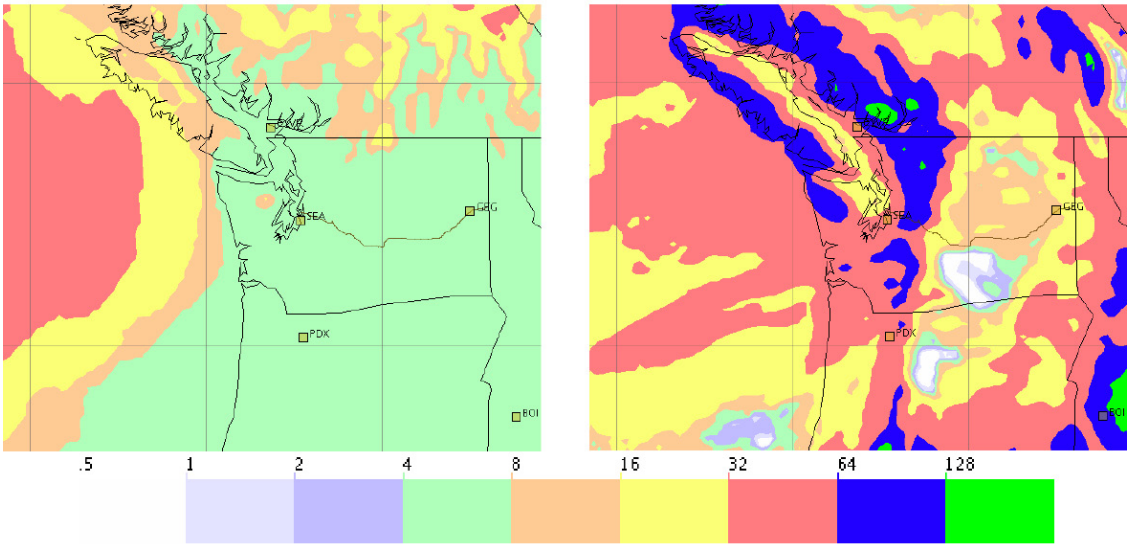
and Mass 2005). The forecasts were produced for observation locations by bilinear interpolation from the forecast grids. The observations were subject to the quality control procedures described by Baars (2005). Following the recommendations of Raftery et al. (2005), a 25-day training period was used.

We begin with a discussion of the PoP forecasts. Figure 5 shows the respective reliability diagram (Wilks 2006, section 7.4.4). As can be seen, BMA produced well-calibrated results, while a consensus vote from the raw ensemble produced uncalibrated results. Table 2 shows that the Brier score (Wilks 2006, p. 284) for the BMA PoP forecasts was better than that for either sample climatology or consensus voting.

In assessing probabilistic forecasts of quantitative precipitation, we follow Gneiting et al. (2005) and aim to maximize the sharpness of the predictive PDFs, subject to calibration. Calibration refers to the statistical consistency between the forecast PDFs and the observations, and in the context of precipitation forecasts was discussed by Krzysztofowicz and Sigrest (1999). To assess calibration, we consider Figure 6, which shows the verification rank histogram for the ensemble forecasts and the probability integral transform (PIT) histogram for the BMA forecast distributions. The verification rank histogram illustrates the lack of calibration in the raw ensemble, similar to results reported by Hamill and Colucci (1998), Eckel and Walters (1998) and Mullen and Buizza (2001) for other ensembles. The PIT histogram is a continuous analogue of the verification rank histogram (Gneiting et al. 2005), and it shows that the BMA forecast distributions were considerably better calibrated than the raw ensemble. For the verification rank histogram, there were incidences where the ob-



(a)



(b)

Figure 4: (a) BMA deterministic forecast and (b) BMA 90th percentile upper bound forecast for (left) 19 May 2003 and (right) 26 January 2003, in hundredths of an inch.

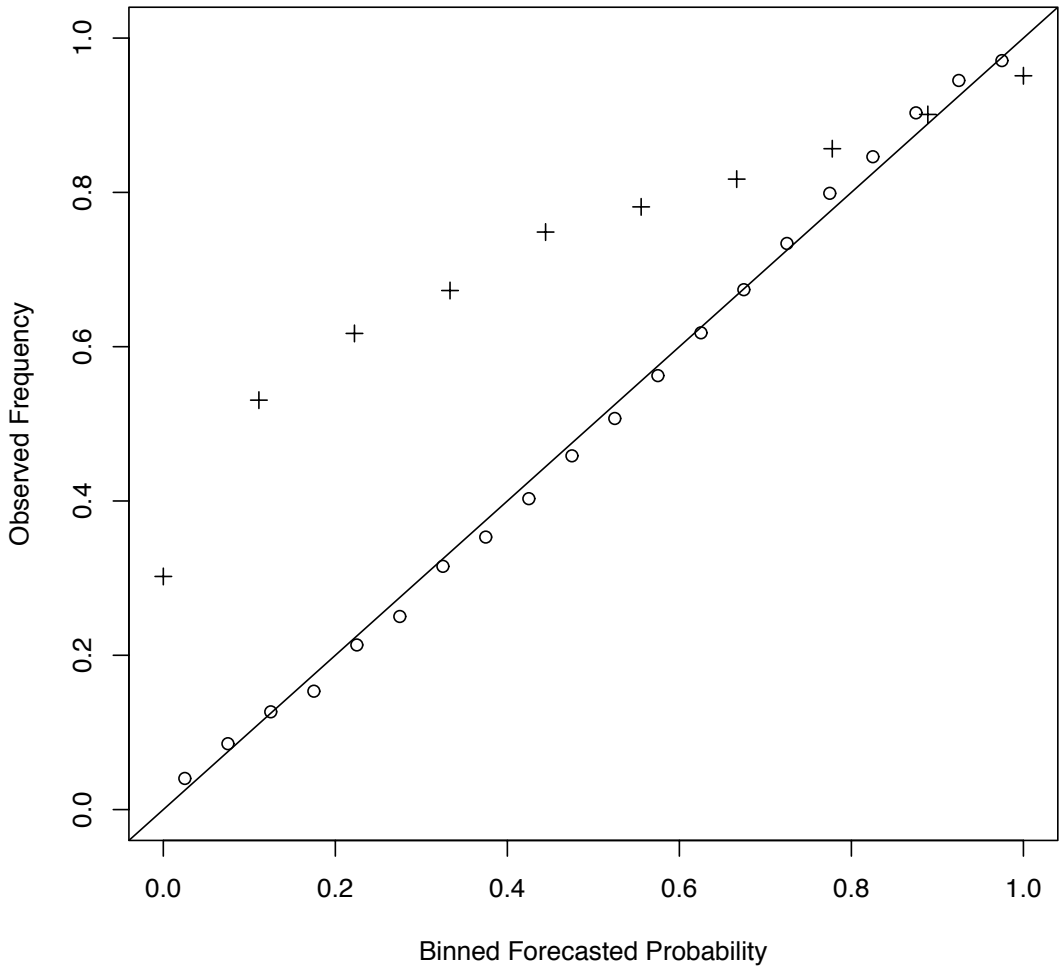


Figure 5: Reliability diagram of binned PoP forecast versus observed relative frequency of precipitation, for consensus voting of the raw ensemble (crosses) and BMA (circles).

served value was zero (no precipitation), and one or more forecasts were also zero. To obtain a rank in these situations, a ranking was randomly chosen between zero and the number of forecasts that equaled zero. To calculate the values for the PIT histogram, each BMA cumulative distribution function was evaluated at its corresponding observation. In the case of an observation of zero, a value was randomly drawn between zero and the probability of no precipitation.

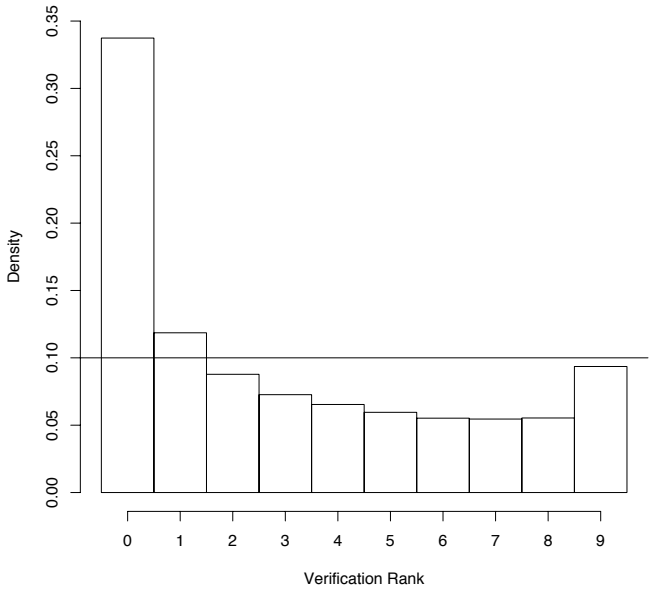
Table 3 shows the empirical coverage of lower 50% and 90% prediction intervals, and the results echo what we saw in the histograms. Sample climatology was perfectly calibrated, as expected, while the raw ensemble was substantially uncalibrated. The BMA intervals were close to being calibrated. The table also shows the average width of the prediction intervals, which characterizes the sharpness of the forecast distributions. The BMA PDFs produced narrower intervals than the raw ensemble forecasts for both intervals considered, and narrower intervals than climatology for 90% intervals.

Scoring rules provide summary measures of predictive performance that address calibration and sharpness simultaneously. A particularly attractive scoring rule for probabilistic forecasts of a scalar variable is the continuous ranked probability score (CRPS), which generalizes the mean absolute error (MAE), and can be directly compared to the latter (Gneiting et al. 2005; Wilks 2006, section 7.5.1). Table 2 shows MAE and CRPS values for sample climatology, raw ensemble forecasts and BMA forecasts, all in units of hundredths of an inch. A deterministic forecast can be created from the BMA forecast by finding the median of the predictive PDF, and the MAE refers to this forecast. Similarly, we show the MAE for the median of the sample climatology and the median of the 9-member forecast ensemble, with the results for BMA being by far the best. We also computed MAE values for deterministic forecasts based on the respective means; these were much higher than the MAE values for the median forecasts, as is generally true when the predictive PDFs are highly skewed. The results for the CRPS were similar, in that the BMA forecast substantially outperformed the others.

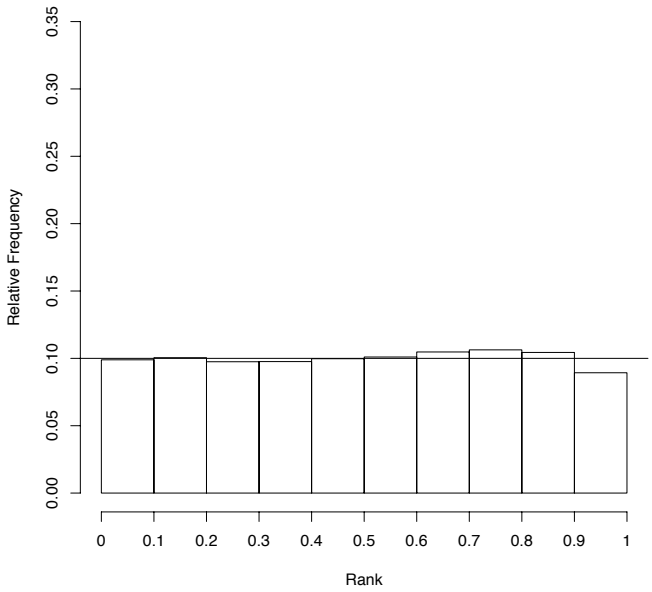
Furthermore, Table 2 shows Brier skill scores (Wilks 2006, p. 285) at various thresholds for ensemble consensus voting, taking PoP to be equal to the proportion of ensemble members that predict precipitation, for BMA forecasts and for logistic regression results based on the cube root of the ensemble mean, all relative to sample climatology.

4 Discussion

We have shown how to apply BMA to precipitation forecasts. This provides a statistical postprocessing method for ensembles that yields a full predictive distribution for quantita-



(a)



(b)

Figure 6: (a) Verification rank histogram for raw ensemble forecasts, and (b) PIT histogram for BMA forecast distributions of precipitation accumulation.

Table 2: Mean absolute error (MAE), continuous ranked probability score (CRPS) and Brier skill score (BSS) for precipitation forecasts. The thresholds, MAE and CRPS values are given in hundredths of an inch, and the MAE refers to the deterministic forecast given by the median of the respective forecast distribution.

Score	MAE	CRPS	BSS	BSS	BSS	BSS	BSS
Threshold			0	5	50	100	200
Sample Climatology	8.7	7.8					
Ensemble Forecast	9.4	7.6	0.08	0.03	−0.01	0.05	0.11
BMA Forecast	7.4	5.1	0.38	0.36	0.28	0.24	0.16
Logistic Regression			0.37	0.37	0.29	0.25	0.13

Table 3: Coverage and average width of lower 50% and 90% prediction intervals for precipitation accumulation, in percent and hundredths of an inch, respectively.

Interval	50%	90%	50%	90%
	Coverage		Width	
Sample Climatology	50.0	90.0	0.0	24.0
Ensemble Forecast	68.5	92.9	11.8	24.2
BMA Forecast	50.5	91.2	3.9	22.2

tive precipitation. The predictive distribution has two components: the probability of zero precipitation, and the PDF for the precipitation accumulation given that it is greater than zero. It thus provides both PoP and PQPF in a unified form. In our experiments with the University of Washington ensemble, the BMA forecast PDFs were better calibrated and sharper than the raw ensemble, which was uncalibrated. The BMA median forecast had lower MAE than the ensemble median, and the BMA forecast PDFs had substantially lower CRPS than the raw ensemble.

BMA PoP forecasts at various thresholds had better Brier skill scores than the ensemble proportion forecasts. Power-transformed logistic regression based on the ensemble mean, as suggested by Hamill et al. (2004), produced Brier skill scores comprable to BMA. BMA offers the added advantage, due to giving a full predictive PDF, of being able to give probabilities of exceeding arbitrary precipitation amounts, rather than having to create a new logistic regression model for each threshold of interest.

Various improvements to the method may be possible. The BMA parameters were estimated using data on observations from the entire Pacific Northwest, and a more local approach, for example partitioning the region into climatologically homogeneous subregions, or fitting BMA locally for each location using only observations within a given radius, might perform better. This latter possibility was suggested by Eric Grimit and Clifford Mass. Our method of estimation assumes independence of forecast errors in space and time. This is unlikely to hold, but it is also unlikely to have much effect on the results, because we are focusing here on the predictive distribution of a single scalar quantity. A calibrated probabilistic forecasting method for temperature and sea-level pressure that does take account of spatial dependence was proposed by Gel et al. (2004), and it would be interesting to extend this to precipitation. Herr and Krzysztofowicz (2005) proposed a generic bivariate probability model for rainfall accumulation in space and gave a critique of the simulation technique of Seo et al. (2000), which generates multiple realizations of downscaled precipitation fields from PQPF.

Our use of a 25-day training period was based on experience with temperature and sea-level pressure, but it may be possible to choose a better training period for the forecasting task at hand, using the kind of experiment reported by Raftery et al. (2005). Hamill et al. (2004) recommended the use of reforecasts from past years computed on the same basis as the current ensemble forecasts. If such reforecasts were available, it seems likely that expanding the training period to include days from the same season in previous years would improve performance. The University of Washington ensemble is frequently updated, however, and as a result prior forecasts were not available to us.

Our experiments were carried out for 24-h precipitation accumulation, and other experi-

ments not reported here suggest that the method also performs well for other accumulation durations, such as 3 hours, 6 hours or 12 hours. This would have to be verified for the particular forecasting task at hand.

Probabilistic forecasts using BMA based on the University of Washington mesoscale ensemble prediction system are currently being produced in real time for temperature and precipitation. They are available online at http://bma.apl.washington.edu. This website provides median forecasts, upper bound forecasts, and forecasts of exceeding thresholds for precipitation accumulation. We apply the BMA technique directly on the model grid, and the website provides the ability to look at the BMA PDF for any grid cell in the forecast domain.

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