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Greedy Kernel PCA Applied to Single-Channel EEG Recordings

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Abstract— **In this work, we propose the correction of univariate, single channel EEGs using a kernel technique. The EEG signal is embedded in its time-delayed coordinates obtaining a multivariate signal. A kernel subspace technique is used for denoising and artefact extraction. The proposed kernel method follows a greedy approach to use a reduced data set to compute a new basis onto which to project the mapped data in feature space. The pre-image of the reconstructed multivariate signal is computed and the embedding is reverted. The resultant signal is the high amplitude artifact which must be subtracted from the original signal to obtain a corrected version of the underlying signal.**

I. INTRODUCTION

Projective subspace techniques can be used favorably to get rid of most of the noise contributions to *multivariate* signals. However, these techniques are not readily available for univariate signals which in signal processing are more the rule than the exception. An example represent biomedical signals like Electroencephalograhic (EEG) recordings. The latter are generally distorted by electrical signals generated by eye movements, eye blinking, muscle activity, head movements, heart beats and line noise. Particularly, eye movements and blinking are major sources of EEG contamination. These ocular movement based signals (EOG) are of larger amplitude than cortical signals (EEG). As they propagate over the scalp, they are recorded in most EEG derivations. Especially the frontal channels often show prominent EOG artifacts which obscure the underlying EEG signals. These artifacts complicate the EEG interpretation, for instance a seizure onset will be difficult to determine in epileptic data analysis. The artifact spectra overlap the EEG spectra and have a higher energy. Thus it is not efficient to apply simple linear filtering to enhance EEG signals. Therefore, it is common practice to exclude, manually or semi-manually, the data segments that contain signals higher than a given threshold. Consequently, ocular artifacts pose a significant problem to the clinicians and neurologists either by the data lost or by masking significant events in the data.

The availability of digitalized EEGs makes possible the application of more sophisticated techniques than simple linear filtering. The primary goal of these correction methods is to remove artifacts without distorting the underlying brain signals of interest. A variety of automatic procedures have been proposed to correct or remove ocular artifacts from EEG recordings. The traditional method is regression analysis which basically consists in the subtraction of the scaled EOG channel (or horizontal and vertical EOG recording channels) from the EEG signal. In [2] a fully experimental set-up based on regression techniques is described. However, this techniques impose the simultaneous recording of EOG channels. It is also reported that the subtraction of the EOG contribution often distorts the results because the EOG channel also suffers from contamination with EEG signals.

More recently independent component analysis(ICA) [13], [8], blind source separation [7] or adaptive filtering techniques [5] have been discussed. The most recent works use independent component analysis: [8] used the INFOMAX algorithm [12], [15] applied the joint approximative diagonalization of eigen-matrices algorithm (JADE), in [7] an approximate joint diagonalization of time-delayed correlation matrices (SOBI) was used while in [13] the fast fixed point algorithm (FastICA) has been applied. In all the works but one [12], the EOG channels are included in the processed data set of signals. Only one of the works [13] considered the possibility to achieve the computation of independent components without the inclusion of EOG recordings. One important issue in ICA methods is the identification of components related with ocular artifacts. The latter need to be eliminated to reconstruct the EEG data without artifacts. All of the works mentioned above are based on a multichannel EEG analysis with EOG channels included. But the latter are not always available. For instance, the EOG electrodes are considered cumbersome for the patient in long term monitoring sessions.

In this work we propose a method that only processes any single EEG channel which suffers from high-amplitude interferences. This proposal is an extension of the strategy followed in [10] but using kernel methods. Every one-dimensional EEG signal is embedded in time-delayed coordinates obtaining a multidimensional signal. In [10] the denoising of multidimensional signals is discussed using local projective subspace techniques. In this work we propose the application of a kernel technique to the multidimensional signal and after reconstruction and reverting the embedding the artifact is isolated. The corrected EEG is obtained subtracting the artifact from the original signal. Naturally, this method can be applied also in parallel to a subset of channels.

II. APPLICATION OF KERNEL-PCA TO UNIVARIATE SIGNAL

Time series analysis techniques often rely on embedding one dimensional sensor signals in the space of their time-delayed coordinates [6]. Embedding can be regarded as

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a mapping that transforms a one-dimensional time series $x = (x[0], x[1], ..., x[N-1])$ to a multidimensional sequence of $K = N - M + 1$ lagged vectors $\mathbf{x}_k = [x[k-1+M-1], \dots, x[k-1]]^T,\ k = 1, \dots, K$. The lagged vectors $\mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_K]$ lie in a space of dimension M. Embedding can be regarded as a non-linear signal manipulation [10], hence a non-linear technique should be appropriate. Kernel Principal Component Analysis (KPCA) relies on a non-linear mapping of given data to a higher dimensional space, called feature space. Without loosing generality, let's assume that the data set is centered and split into two parts yielding the mapped data set $\mathbf{\Phi} = [\phi(\mathbf{x}_1)\phi(\mathbf{x}_2)\dots\phi(\mathbf{x}_r), \phi(\mathbf{x}_{r+1})\dots\phi(\mathbf{x}_K)]$. The denoising is achieved by considering the projections onto directions related to the largest eigenvalues of the covariance matrix. However to avoid an explicit mapping into feature space, all data manipulations are achieved by dot products [9] and the kernel trick is applied. Using this strategy, the kernel matrix is easily computed because its entries depend on the application of a kernel function computed in input space. For instance, using an RBF kernel, the entry (i, j) of the kernel matrix ($\mathbf{K}$) is given by

$$k(i,j) \equiv k(\mathbf{x}_i, \mathbf{x}_j) = exp(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}) \tag{1}$$

In denoising applications, one of the steps of KPCA is to compute the projections of a mapped data set onto a feature subspace. Considering L eigenvectors (columns of $\mathbf{U}$) of a covariance matrix (a correlation matrix if the data is centered) corresponding to the L largest eigenvalues, the projections of the mapped data set $\mathbf{\Phi}$ are

$$\mathbf{Z} = \mathbf{U}^T\mathbf{\Phi} \tag{2}$$

The columns of the matrix $\mathbf{U}$ form the basis in feature space onto which to project the data set. This basis can be written as a linear combination of the mapped data

$$\mathbf{U} = \mathbf{\Phi}_B\mathbf{A} \tag{3}$$

The matrix $\mathbf{A}$ is a matrix of coefficients and either $\mathbf{\Phi}_B = \mathbf{\Phi}$ (KPCA) or $\mathbf{\Phi}_B = \mathbf{\Phi}_R$ (greedy KPCA), representing a subset of the data set only. Note that the column j of $\mathbf{Z}$ depends on the dot products $\mathbf{\Phi}_B^T\phi(\mathbf{x}_j)$. Thus the implementation requires the storage of the matrix of coefficients and of the set of indices of the training subset B in the greedy case. Finally, to recover the noise-reduced signal after denoising in feature space, the non-linear mapping must be reverted, i.e. the pre-image in input space of every signal, denoised and reconstructed in feature space, must be estimated.

Denoising using KPCA thus comprises two steps after the computation of the projections in the feature space: a) the reconstruction in feature space and b) the estimation of the pre-image of the reconstructed point $\hat{\phi}(\mathbf{x}_j) = \mathbf{U}\mathbf{z}_j$, where $\mathbf{z}_j$ represents the projections of a noisy point $\mathbf{x}_j$. These two steps can be joined together by minimizing the Euclidian distance of the image $\phi(\mathbf{p})$ of a yet unknown point $\mathbf{p}$ from $\hat{\phi}(\mathbf{x}_j)$

$$\begin{aligned} \tilde{d}^{(2)} &= \|\phi(\mathbf{p}) - \hat{\phi}(\mathbf{x}_j)\|^2 \\ &= (\phi(\mathbf{p}) - \hat{\phi}(\mathbf{x}_j))^T(\phi(\mathbf{p}) - \hat{\phi}(\mathbf{x}_j)) \end{aligned} \tag{4}$$

The central idea of the fixed-point method consists in computing the unknown pre-image of a reconstructed point in the projected feature subspace by finding a $\mathbf{p}$ which minimizes that distance (4). If an RBF kernel is considered, the iterative procedure is described by the following equation

$$\mathbf{p}_{t+1} = \frac{\mathbf{X}_B(\mathbf{g}\Diamond\mathbf{k}_{p_t})}{\mathbf{g}^T\mathbf{k}_{\mathbf{p_t}}} \tag{5}$$

where $\Diamond$ represents a Hadamard product, $\mathbf{g} = \mathbf{A}\mathbf{z}_j$. The components of the vector $\mathbf{k}_{\mathbf{p_t}} = \mathbf{k}(\mathbf{X}_B, \mathbf{p}_t)$ are given by the dot products between $\phi(\mathbf{p}_t)$ and the images $\mathbf{\Phi}_B$ of the training subset $\mathbf{X}_B$. The algorithm must be initialized and $\mathbf{p}_0 \equiv \mathbf{x}_i$ is a valid choice [10].

The points $\mathbf{p}_k$ then form the columns of $\hat{\mathbf{X}}$, the noise-free multidimensional signal in input space. The one-dimensional signal, $\hat{x}[n]$, is then obtained by reverting the embedding, i.e. by forming the signal with the mean of the values along each descendent diagonal of $\hat{\mathbf{X}}$ [11]. Note that if $\hat{x}[n]$ corresponds to the extracted artifact, then the corrected EEG is computed as $y[n] = x[n] - \hat{x}[n]$.

A. Computing the Basis

The two approaches, KPCA and greedy KPCA, respectively, arise from two distinct strategies to deal with the eigendecomposition of the kernel matrix ($\mathbf{K}$) of the data set. In KPCA the matrix $\mathbf{A}$ is computed using the largest eigenvalues ($\mathbf{D}$ and corresponding eigenvectors ($\mathbf{V}$) of $\mathbf{K}$ [10]. Then the basis vector is

$$\mathbf{U} = \mathbf{\Phi}\mathbf{V}\mathbf{D}^{-1/2} \tag{6}$$

In greedy KPCA a low-rank approximation of the kernel matrix is considered. This leads to the eigendecomposition of matrices with reduced size. Considering the kernel matrix, written in block notation [14],[4]

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_r & \mathbf{K}_{rs} \\ \mathbf{K}_{rs}^T & \mathbf{K}_s \end{bmatrix} \tag{7}$$

This manipulation takes into account that the original training set is divided into two subsets. Where the $\mathbf{K}_r$ is the kernel matrix within subset $\mathbf{\Phi}_R$, $\mathbf{K}_{rs}$ is the kernel matrix between subset $\mathbf{\Phi}_R$ and $\mathbf{\Phi}_S$ and $\mathbf{K}_s$ is the kernel matrix within the subset $\mathbf{\Phi}_S$. The approximation is written using the upper blocks of the original matrix [14], [4]

$$\tilde{\mathbf{K}} = \begin{bmatrix} \mathbf{K}_r \\ \mathbf{K}_{rs}^T \end{bmatrix} \mathbf{K}_r^{-1} \begin{bmatrix} \mathbf{K}_r & \mathbf{K}_{rs} \end{bmatrix} \tag{8}$$

It can be verified that the lower block is approximated by $\mathbf{K}_s \approx \mathbf{K}_{rs}^T\mathbf{K}_r^{-1}\mathbf{K}_{rs}$. There are different suggestions to perform this eigendecomposition [14],[1], one of them is based on an incomplete Cholesky decomposition $\tilde{\mathbf{K}} = \mathbf{C}^T\mathbf{C}$, with $\mathbf{C} = \begin{bmatrix} \mathbf{L} & \mathbf{L}^{-T}\mathbf{K}_{rs} \end{bmatrix}$, where $\mathbf{L}$ is a triangular matrix corresponding to the decomposition of $\mathbf{K}_r = \mathbf{L}^T\mathbf{L}$. The $R \times R$ matrix $\mathbf{Q} = \mathbf{C}\mathbf{C}^T$ and its eigendecomposition $\mathbf{V}_q\mathbf{D}\mathbf{V}_q^T$ in conjunction with $\mathbf{L}$ is used to form the basis

$$\mathbf{U} = \mathbf{\Phi}_R\mathbf{L}^{-1}\mathbf{V}_q \tag{9}$$

Note that the pivoting index within the incomplete Cholesky decomposition [1] leads to the selection of $\mathbf{\Phi}_R$ within the training set.

B. Choosing training and testing subsets

In the last section it was discussed that with an incomplete Cholesky decomposition it is possible to compute the parameters of the model using a training data set divided into two subsets. But, the parameters depend on both subsets. However, there are approaches, where the parameters depend on only one of the subsets [14] which can be chosen randomly. In this application, we consider an hybrid approach which leads to the choice of three subsets of data. We start to split the data into two data sets: the training set with K vectors and the testing set which contains the remaining data to be denoised. In this application, we consider to form the training set by choosing randomly the K vectors. The incomplete Cholesky decomposition is performed using the symmetric pivoting algorithm [1]. The pivoting scheme allows to identify the subset R among the K vectors randomly chosen. The algorithm is iterative and stops when all pivot values are less than a threshold and the absolute difference between consecutive pivots is also very small.

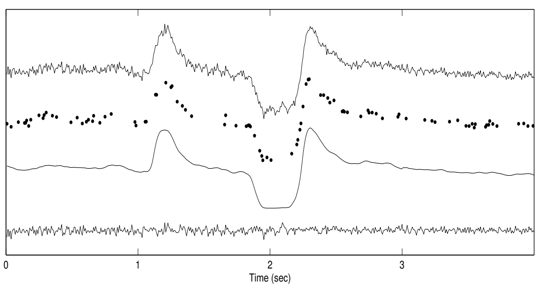

Fig. 1. Illustration of greedy KPCA: The signals in the different steps are: $1strow$ - original EEG, $2ndrow$ - first row of random selected points, $3rdrow$ - extracted EOG signal and $4throw$ - corrected EEG

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TABLE I

PARAMETERS OF THE ALGORITHM

	K	R	L
Segment 1	384	[9 18]	[3 6]
Segment 2	384	[13 15]	4

III. RESULTS

The EEG signals were chosen from a database of epileptic patients recorded on long-term EEG monitoring sessions. The EEG signals were recorded using 19 electrodes placed according to the $10-20$ system (common ground reference at Fz). The signals are filtered, digitalized (sampling rate-$128Hz$) and stored as European Data Format (EDF) files. Monopolar (common Cz reference) brain signals were visualized using EEGLAB [3]. The signals of the set of channels recorded along the monitoring session suffer from distinct forms of distortion. In particular, the high-amplitude interference arising from ocular movements are most visible in frontal channels while electrode artifacts show up in different channels spread over the scalp. We will present results using two data segments (with N=1280 samples) of a patient which suffered from a partial complex seizure from the right temporal focus. The analysis is performed in parallel in different single channels using an embedding dimension of $M = 11$. In the multidimensional signal, the K vectors of the training set are chosen randomly. In figure 1 the first row of the multidimensional signal is plotted according to its time reference (the second signal). The parameters of the model are computed using the greedy approach. The number of patterns in the training subset R is chosen. The number L of eigenvectors ($\mathbf{U}$) to project the data for reconstruction in the feature space is determined according to the eigenspectrum of the matrix $\mathbf{Q}$ which should approximate the eigenvalues of the kernel matrix of the training set. Two segments of $10s$ were chosen to illustrate the method. The table I shows the range of those parameters for both data segments.

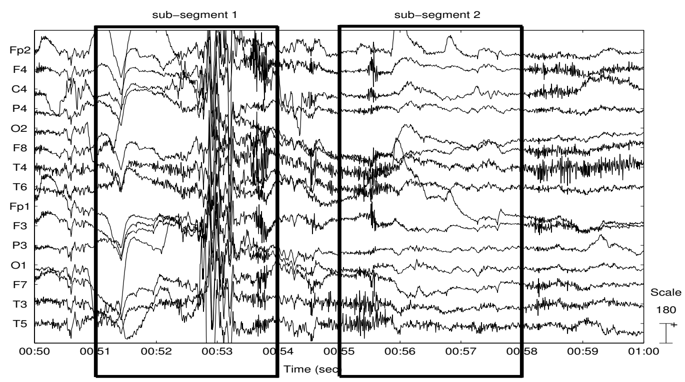
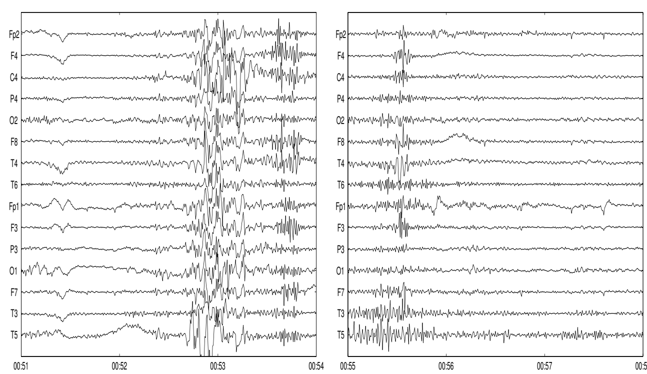


Fig. 2. Original EEG recordings referenced against Cz



(a) sub-segment 1 (b) sub-segment 2

Fig. 3. Corrected sub-segments corresponding to Fig. 2. All channels were processed.

Segment 1 All channels are processed one after the other by the algorithm (see figure 2). The corrected EEG (see fig. 3) mainly possesses the high frequency ($> 10Hz$) contents of the original signal. However, in $T4$ and $T6$ bursts of theta ($3-7Hz$) waves and sharp slow waves can be seen. Also the bursts of spikes are now clearly visible in the frontal channels.

Segment 2 This segment shows prominent eye movement artifacts recorded at the frontal channels (fig. 4). Thus only these frontal channels and channel $T4$, monitoring temporal cortex, were processed. In fig 5 two zoomed subsegments of the corrected EEG are shown. In channel $T4$ a burst of spikes can be seen while in other channels ($F4$ and $F8$) single spikes also occur during the same period. Comparing the corrected recordings of channel $T4$ with the corresponding recording before the seizure, the pronounced burst of spike waves is more clearly seen in the corrected recording. This paroxysmal activity in $T4$ before the seizure onset indicates the possible origin of the the epileptogenic focus.

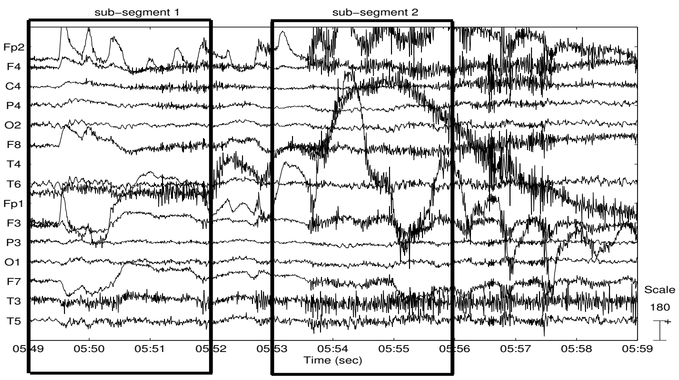
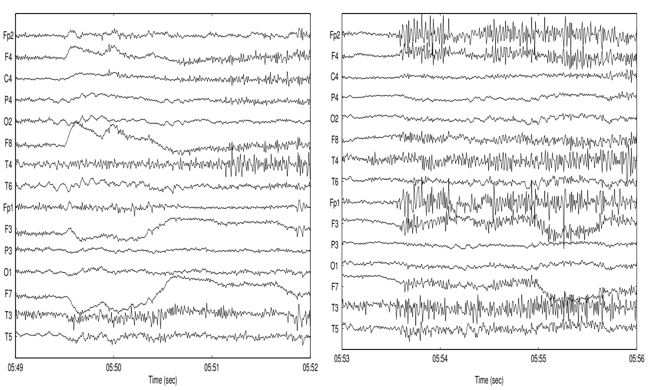


Fig. 4. Original EEG recordings referenced against Cz.



(a) sub-segment 1 (b) sub-segment 2

Fig. 5. Corrected EEG sub-segments corresponding to Fig. 4. Channels processed: Fp2, T4 and Fp1

IV. Conclusions

The proposed method needs the information contained within a single channel only, hence can be applied to each channel separately. Thus only channels which contain such artifacts need to be processed. Our preliminary results show good performance in removing artifacts like eye or head movements. In summary, with the method proposed we can separate EEG signal recordings into two components: artifacts and undistorted EEG. The user can choose to process a subset of channels keeping others unprocessed. This allows a comparison of the outcomes of the algorithm with non-processed channels concerning non-artifact-related signals to corroborate a distortionless correction. Although this is ongoing work, we present a method that is intended to help a visual inspection of the EEG recordings by an experienced clinician, hence might be useful in some critical segment analysis like the onset of ictal seizures. Despite the variety of methods applied, it is not possible to conclude about their performance once they use distinct databases, different measures and goals. The proposed method needs to be evaluated in a more quantitative and systematic approach, not only concerning the spectral distortion in the important frequency ranges of EEG.

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