CS 2102, Fall 2014
Final for sgr7sg
Samantha Rafalowski

Your score on this exam is 79.50 of 100 points.

Your breakdown of points per page is below.

Page	Score	Max
1	0	0
2	6	14
3	16	20
4	7	10
5	16	16
6	13.5	14
7	12	12
8	9	14

A graded scan of each page follows.

CS 2102 Discrete Mathematics
Final Exam
Fall 2014

<u>Time Limit: 75 Minutes</u>

* This is a closed book, closed note exam.

* Write your answers on the exam paper and on page for which it appears, in ink or legible pencil. Any work outside of this will not be graded.

* If your answer cannot be read, understood or is vague it will be marked wrong.

* The numbers in parentheses is the point value for the given problem.

Print Name and Computing ID: Samantha Rafalowski sgr7sg

Pledge (Write Out in Full and Sign):

I have neither given or recieved help on this exam.

Rafalowski

Do not write in this area

Department Course Exam Page Version

(8pts) 1. Prove or disprove the following statement: Any nonzero rational number times any irrational number is irrational.

$1/3 \cdot 3 = 1$ false

0/4

counter example:

2/4

$1/3 \in \neg Q$ $3 \in Q$

$1/3 \cdot 3 = 1$

$1 \in Q$

(6pts) 2. For each of the following statements, (1) write the statement informally without using variables or the symbols $\forall$ or $\exists$, and (2) indicate whether the statement is true or false and briefly justify your answer.

(a) $\forall$ real numbers x, $\exists$ a real number y such that $x < y$.

(b) $\exists$ a real number y such that $\forall$ real numbers x, $x < y$.

2/3

a) For all real numbers x, there is a real number y that is greater than x. 2/3

True, the set of real numbers is infinite & therefore every number will always be smaller than another

b) There exists a real number y that is larger than all other real numbers

False, the set of real numbers is infinite

No variables

Your score on this page is 6/14

(8pts) 3. Find the smallest equivalence relation on $\{1,2,3\}$ that contains $(1,2)$ and $(2,3)$.

$\{x \in X, y \in Y \mid y \geq x\}$

reflexive yes 4/4

antisymmetric 2/4 (3,2)

no unless 2 = 2

transitive

yes

(6pts) 4. Rewrite the following statement formally. Use variables and include both quantifiers $\forall$ and $\exists$ in you answer. Remember ***Q*** is the set of all rational numbers and ***Z*** is the set of all integers.

Every rational number can be written as a ratio of some two integers.

$\{\forall x \in Q(x), \exists a, b \in Z(x) \mid x = \frac{a}{b}\}$ 6/6

(6pts, 3pts each) 5. Write negations for each of the following statements:

(a) For all integers n, if n is prime then n is odd.

(b) Al is absent or Bob is present.

1/3

a) For some integers ~~n~~, if n is prime then n is even.

3/3

b) Al is not absent and Bob is not present.

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Your score on this page is 16/20

6. Consider the following statement: For all integers n, if n^3 is odd then n is odd.

(a. 6pts) Prove the statement either by contradiction or by contraposition. Clearly indicate which method you are using.

(b. 4pts) If you used proof by contradiction in part (a), write what you would "suppose" and what you would "show" to prove the statement by contraposition. If you used proof by contradiction in part (a), write what you would "suppose" and what you would "show" to prove the statement by contradiction.

3/6

4/4

a) Proof by contraposition

n^3 is even then n is

Let $k \in \mathbb{Z}$ (x) | $(2k)^3$ = even

$2^3 \cdot k^3 = 8 \cdot k^3$

8 is even & divisible by 2 so $8 \cdot k^3$ is even

$\sqrt[3]{(2k)^3} = 2k$, all numbers multiplied by 2 are even

∴ if n^3 is even, then n is even

∴ if n^3 is odd, then n is odd

b) proof by contradiction

→ you would suppose

if n^3 is odd then n is even.

→ You would show that when n^3 is odd, n cannot be even (is not divisible by 2) and thus the assumption that n^3 = odd → n = even is false

Your score on this page is 7/10

(8pts, 2 pts for each property) 7. Determine whether the following binary relation is reflexive, symmetric, antisymmetric, or transitive. Give an explanation for your answer.

The relations R on Z where aRb means $a^2 = b^2$.

Z is integers **4/4**

a) Reflexive : yes $a^2 = a^2$ $\boxed{aRa}$ **4/4**

b) symmetric: yes $a^2 = b^2 \rightarrow b^2 = a^2$ $aRb \rightarrow bRa$

c) antisymmetric: NO $a^2 = b^2$, $a = b$ (could be -3 and 3)

d) transitive: yes $a^2 = b^2$, $b^2 = c^2$, $a^2 = c^2$

aRb, $bRc \rightarrow aRc$

(8pts, 4 pts each) 8. Let S be the set of all strings in 0's and 1's, and define a function $g: S \rightarrow Z$ as follows: for each string s in S, $g(s)$= the number of 1's in s minus the number of 0's in s.

$g(s) = \#1s - \#0's$

(a) Is g 1-1? Prove or give a counterexample.

(b) Is g onto? Prove or give a counterexample.

1-1 means every y has an x (exactly one) **4/4**

onto means every x has a y **4/4**

a) NO: counter-example:

$S_1 = (0, 0, 1, 1, 1)$

$S_2 = (1, 1, 1, 0, 0)$

$g(s_1) = 1$, $g(s_2) = 1$ but $s_1 \neq s_2$

* b) yes

$g: S \rightarrow$ $\{x, y \in S, s \in S \mid x - y = g(s)\}$

Thus $\forall s \in S$, $\exists x - y \in g(s)$

Your score on this page is 16/16

(8pts, 4pts each) 9. Prove or disprove the following statements. Assume that the statement applies to all sets.

(a) If $A \cup C = B \cup C$, then $A = B$.

(b) If $A \cap B = A \cup B$, then $A = B$.

4/4

4/4

a) counter-example: $A = \{1,2,3\}$ $B = \{3,4,5,6\}$

$C = \{7,9,3\}$

$A \cup C = \{3\}$, $B \cup C = \{3\}$, but $A \neq B$

b) TRUE

$|A \cup B| = |A| + |B| - \cancel{|A \cap B|}$

$|A \cap B| + |A \cup B| = |A| + |B|$

$2|A \cup B| = |A| + |B|$

so since 2x union = $|A| + |B|$, $|A|$ must equal $|B|$

(6pts) 10. Write the follow two statements in symbolic form and determine whether they are logically equivalent. Include a truth table and a few words explaining how the truth table supports your answer.

a) If Sam bought it at Crown Books, then Sam didn't pay full price.

b) Sam bought it at Crown Books or Sam paid full price.

B = bought @ Crown Books

P = pay full price

5.5/6

a) $B \to \neg P$

b) $B \vee P$

B	P	$\neg P$	$B \to \neg P$	$B \vee P$
T	T	F	F	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	F

These are not logically equivalent. as seen in the truth table, when Sam bought it at Crown Books and paid full price, the values for $B \to \neg P$ and $B \vee P$ are different.

Your score on this page is 13.5/14

Pres Tres ≠ pres Tres
A B ≠ B A so order matters

$perm = \frac{n!}{(n-a)!}$

(12pts, 3pts each) 11. A president and a treasurer are to be chosen from a student club consisting of 10 people. How many different choices of officers are possible if

a. there are no restrictions;

order matters b/c pres & tres are different

3/3

$C = \frac{10!}{8!} = \frac{10 \cdot 9}{1!} = \frac{90}{1!} = \boxed{90}$

b. Person A will serve only if he is president;

3/3

treasurer w/person A: $\frac{9!}{1!\,8!}$ + no pers on A $\frac{9!}{2!\,7!} = \frac{9 \cdot 8}{2} = \frac{72}{2} = 72 + 9 = \boxed{81}$

c. Persons B and C will serve together or not at all;

3/3

B=pres C=tres, B=tres C=pres

$1 + 1 + \frac{8!}{6!} = \frac{8 \cdot 7}{1} = \frac{56}{1} =$

$56 + 2 = \boxed{58}$

d. Persons D and E will not serve together?

3/3

Person D pres or treasurer w/o E $\boxed{8 + 8}$

Person E pres or treasurer w/o D $\boxed{8 + 8}$

neither $\frac{8!}{6!} = 8 \cdot 7 = \boxed{56}$

$56 + 32 = 88$

$\boxed{= 88}$

Your score on this page is 12/12

(10pts) 12. Use the principle of Mathematical Induction to prove that $\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$ for all $n \geq 1$.

base case: $n=1$ $\sum_{i=1}^{1} 1(2) = \frac{1(2)(3)}{3} = \frac{6}{3}$ 4/4 ✓

4/4 base case

1/6

inductive case: $n=k+1$ $\sum_{i=1}^{k+1} i(i+1) = \frac{(k+1)(k+2)(k+3)}{3}$

+1 for inductive step

$\sum_{i=1}^{k+1} (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3} = \frac{(k^2+3k+2)(k+3)}{3}$

incorrect

$= \frac{k^3+3k^2+3k^2+9k+2k+6}{3}$

$= \frac{k(k^2+6k+11)+6}{3}$

never used inductive hypothesis

1/6 for induction

*not proven

$= \frac{k(11+k(k+6))+6}{3}$

since $k=n+1$

$= \frac{(n+1)(11+(n+1)(n+7))+6}{3}$

$\sum_{i=1}^{k+1} i(i+1) = \frac{(n+1)(11+n^2+8n+7)+6}{3}$

$= \frac{(n+1)(n+2)(n+3)}{3}$

(4pts) 13. State your favorite fictional character from any type of media!

Abed Nadir from community!

4/4

Your score on this page is 9/14