Library Imports

```
library(glue)
library(DAAG)

options(repr.plot.width=10, repr.plot.height=5)

set.seed(20)
options(warn=-1)
```

```
Loading required package: lattice
```

Question 7.1

Describe a situation or problem from your job, everyday life, current events, etc., for which exponential smoothing would be appropriate. What data would you need? Would you expect the value of α (the first smoothing parameter) to be closer to 0 or 1, and why?

ANSWER: The first that comes to mind when thinking of exponential smoothing and its uses for short-term forecasting is the stock market. Nowadays, there are numerous firms that try to use machine learning/Al to predict the prices of stocks in order to make lots of profit.

The data I would need is stock price data in minute or maybe even second intervals. Seeing as how volatile stock prices are, I would say the alpha value would be near 1 so a lot of weight would be placed on the true current observation and little weight on the previous observation.

Question 7.2

Using the 20 years of daily high temperature data for Atlanta (July through October) from Question 6.2 (file temps.txt), build and use an exponential smoothing model to help make a judgment of whether the unofficial end of summer has gotten later over the 20 years. (Part of the point of this assignment is for you to think about how you might use exponential smoothing to answer this question. Feel free to combine it with other models if you'd like to. There's certainly more than one reasonable approach.) Note: in R, you can use either HoltWinters (simpler to use) or the smooth package's es function (harder to use, but more general). If you use es, the Holt-Winters model uses model="AAM" in the function call (the first and second constants are used "A"dditively, and the third (seasonality) is used "M"ultiplicatively; the documentation doesn't make that clear).

```
temp_data <- read.table('../data/7.2tempsSummer2018.txt', sep='', header=TRUE)

temp_data$datetime <- as.Date(temp_data$DAY, format = "%d-%b")
```

First, I transformed the data into a time series data frame with a frequency of 123 since there are 123 days in the original data set.

```
temp_ts <- ts(as.vector(temp_data[,c(2:21)]), start=1996, frequency=123)</pre>
```

The transformed data set was then inputted into the Holt Winters function and the smoothed data was plotted as a function of time.

The plot is in the appendix.

```
hw <- HoltWinters(temp_ts)
plot.ts(fitted(hw)[,1], col=3, ylab="Temperature (F)", xlab="Year")</pre>
```

The resulting seasonal factors were transformed into a matrix and used in the CUSUM approach to find the date at which summer ended.

```
m <- matrix(hw$fitted[,4], ncol=123)</pre>
```

Custom CUSUM function that I coded in R is below.

```
# cusum approach takes in the data of interest, threshold (T), critical value (C),
mean (mu),
# The is inc determines whether the change that is being detected is an increase or
decrease.
cusum approach <- function(data, T, C, mu, is inc) {</pre>
   end index <- length(data)</pre>
    if (isTRUE(is inc)) {
        for (i in 1:length(data)) {
            # reset to 0
            # Check if S t is greater than the threshold, if it is then
                break
        for (i in 1:length(data)) {
```

```
# Check if S_t is greater than the threshold, if it is then
# the loop ends
if (S_t > T) {
    end_index <- i
    break
}

return(end_index)
}</pre>
```

I tested a variety of C and T values and this combination of C = 1 and T = 25 seemed to give the most reasonable summer end dates.

```
C <- 1
T <- 20

for (i in 1:nrow(m)) {
    year_data <- m[i,]

    mu <- mean(year_data)
    end_index <- cusum_approach(year_data, T, C, mu, is_inc=FALSE)

    summer_end_date <- temp_data[end_index,1]
    year <- substr(colnames(temp_ts)[i], 2, 5)
    print(glue("Summer ends on {summer_end_date} in the year {year}"))
}</pre>
```

```
Summer ends on 7-Jul in the year 1996
Summer ends on 7-Jul in the year 1997
Summer ends on 7-Jul in the year 1998
Summer ends on 6-Jul in the year 1999
Summer ends on 26-Jul in the year 2000
Summer ends on 13-Jul in the year 2001
Summer ends on 6-Jul in the year 2002
Summer ends on 3-Sep in the year 2003
Summer ends on 26-Jul in the year 2004
Summer ends on 8-Aug in the year 2005
Summer ends on 13-Jul in the year 2006
Summer ends on 13-Jul in the year 2007
Summer ends on 12-Jul in the year 2008
Summer ends on 1-Aug in the year 2009
Summer ends on 6-Jul in the year 2010
Summer ends on 6-Jul in the year 2011
Summer ends on 6-Jul in the year 2012
Summer ends on 27-Aug in the year 2013
Summer ends on 14-Aug in the year 2014
```

For the same C and T values, I ran the original data through the CUSUM approach to compare the summer end dates before and after exponential smoothing.

```
for (i in 2:21) {
    # Initialize the mean (mu)
    mu <- mean(temp_data[,i])

# Get the index at which the threshold is passed.
    end_index <- cusum_approach(temp_data[,i], T, C, mu, is_inc=FALSE)

summer_end_date <- temp_data[end_index,1]
    year <- substr(colnames(temp_data)[i], 2, 5)
    print(glue("Summer ends on {summer_end_date} in the year {year}"))
}</pre>
```

```
Summer ends on 21-Sep in the year 1996
Summer ends on 25-Sep in the year 1997
Summer ends on 30-Sep in the year 1998
Summer ends on 20-Sep in the year 1999
Summer ends on 6-Sep in the year 2000
Summer ends on 25-Sep in the year 2001
Summer ends on 25-Sep in the year 2002
Summer ends on 29-Sep in the year 2003
Summer ends on 19-Sep in the year 2004
Summer ends on 7-Oct in the year 2005
Summer ends on 21-Sep in the year 2006
Summer ends on 17-Sep in the year 2007
Summer ends on 21-Sep in the year 2008
Summer ends on 2-Oct in the year 2009
Summer ends on 28-Sep in the year 2010
Summer ends on 7-Sep in the year 2011
Summer ends on 30-Sep in the year 2012
Summer ends on 16-Aug in the year 2013
Summer ends on 26-Sep in the year 2014
Summer ends on 16-Sep in the year 2015
```

The exponential smoothing provided summer end dates that are much earlier than the end dates estimated on the original data set. This was actually counter intuitive for me. I expected the end dates to actually be later after exponential smoothing because the threshold would be harder to surpass if the data was more stable. However, the opposite was true. My hypothesis as to why this happened is that the volatility in the temperature data causes the S_t value to fluctuate a lot. So, the S_t value can come close to the threshold value but if the next day a sudden drop in temperature happened then S_t could now be very far from the threshold. The exponential smoothing prevents such sudden changes in S_t by smoothing out the temperature data.

Question 8.1

Describe a situation or problem from your job, everyday life, current events, etc., for which a linear regression model would be appropriate. List some (up to 5) predictors that you might use.

ANSWER:

Linear regression is useful when predicting house prices. Especially in the Bay Area where the housing market is so high, its important to be able to accurately predict the housing price so that you don't overpay on an already ridiculously highly priced house.

5 predictors to use: 1. Square footage 2. Distance of bus stop or subway stations 3. Year it was built 4. Number of bedrooms 5. Number of bathrooms

Question 8.2

Using crime data from http://www.statsci.org/data/general/uscrime.txt (file uscrime.txt, description at http://www.statsci.org/data/general/uscrime.html), use regression (a useful R function is lm or glm) to predict the observed crime rate in a city with the following data:

- M = 14.0
- So = 0
- Ed = 10.0
- Po1 = 12.0
- Po2 = 15.5
- LF = 0.640
- M.F = 94.0
- Pop = 150
- NW = 1.1
- U1 = 0.120
- U2 = 3.6
- Wealth = 3200
- Ineq = 20.1
- Prob = 0.04
- Time = 39.0

Show your model (factors used and their coefficients), the software output, and the quality of fit. Note that because there are only 47 data points and 15 predictors, you'll probably notice some overfitting. We'll see ways of dealing with this sort of problem later in the course.

```
crime_data <- read.table('../data/5.1uscrimeSummer2018.txt', sep='', header=TRUE)</pre>
```

```
M <- 14.0

So <- 0

Ed <- 10.0

Po1 <- 12.0

Po2 <- 15.5

LF <- 0.640

M.F <- 94.0

Pop <- 150

NW <- 1.1

U1 <- 0.120

U2 <- 3.6

Wealth <- 3200
```

```
Ineq <- 20.1
Prob <- 0.04
Time <- 39.0

sample_data <- data.frame(M,So,Ed,Po1,Po2,LF,M.F,Pop,NW,U1,U2,Wealth,Ineq,Prob,Time)</pre>
```

Use cross-validation to determine the root mean squared error (RMSE) and the mean absolute error (MAE) for each fold in the cross-validation step.

```
# RMSE function
rmse <- function(resids) {
    sqrt(mean(resids^2))
}
# MAE function
mae <- function(resids) {
    mean(abs(resids))
}</pre>
```

```
# Shuffle data
shuffled df <- crime data[sample(nrow(crime data)), ]</pre>
# 5 fold split
folds <- cut(seq(1, nrow(crime data)), breaks=5, labels=FALSE)</pre>
# For each fold, train and test on Linear Regression algorithm
resids <- c(1:nrow(crime data))</pre>
preds <- c(1:nrow(crime data))</pre>
    print(glue("Iteration #{i}"))
    temp fit <- lm(formula=Crime ~., data=train data)</pre>
    print(summary(temp fit))
    # Perform prediction step on test data since model was fit using the training
data
    preds[test indices] <- predict(temp fit, test data)</pre>
    resids[test_indices] <- test_data$Crime - preds[test_indices]</pre>
overall rmse <- round(rmse(resids), 3)</pre>
print(glue("RMSE: {overall rmse}"))
print(glue("MAE: {overall mae}"))
```

```
Iteration #1
Call:
lm(formula = Crime ~ ., data = train_data)
```

```
Residuals:
           1Q Median
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -6104.9296 2068.7312 -2.951 0.00763 **
            102.2760
                       60.1488 1.700 0.10382
            -114.2915 195.7346 -0.584 0.56550
            174.3027 118.6461 1.469 0.15663
Po1
            -112.9272 131.6819 -0.858 0.40081
LF
            -764.5198 1700.0301 -0.450 0.65753
             15.9682 24.5602 0.650 0.52263
M.F
             -1.2496
                        1.6571 -0.754 0.45919
Pop
                                0.548 0.58921
NW
              4.3656
          -5307.2621 5371.9166 -0.988 0.33442
            148.8000 103.3013 1.440 0.16448
                       0.1493 1.739 0.09666 .
Wealth
              0.2596
             77.4564
                       28.8980 2.680 0.01401 *
Ineq
           -3938.1110 2897.3326 -1.359 0.18850
Prob
                       8.2228 -0.314 0.75625
Time
             -2.5860
Signif. codes: 0 \*** 0.001 \** 0.01 \*' 0.05 \.' 0.1 \ ' 1
Residual standard error: 217.9 on 21 degrees of freedom
Multiple R-squared: 0.7718, Adjusted R-squared: 0.6089
F-statistic: 4.736 on 15 and 21 DF, p-value: 0.0006456
Iteration #2
Call:
lm(formula = Crime ~ ., data = train data)
Residuals:
             1Q Median
                             3Q
-231.958 -83.934 -2.402 64.632 259.599
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -3.485e+03 1.436e+03 -2.427 0.02387 *
          -1.494e+01 1.250e+02 -0.120 0.90589
           2.879e+02 5.360e+01 5.371 2.16e-05 ***
           2.097e+02 8.736e+01
                                2.400 0.02530 *
           -1.139e+02 9.575e+01 -1.189 0.24708
Po2
LF
           -9.398e+02 1.164e+03 -0.807 0.42814
           -1.025e+01 1.726e+01 -0.594 0.55865
M.F
          -1.415e+00 1.026e+00 -1.380 0.18154
Pop
NW
           1.052e+00 6.287e+00 0.167 0.86859
           -7.455e+03 3.994e+03 -1.867 0.07535.
           2.569e+02 7.525e+01 3.414 0.00248 **
Wealth
          -7.612e-02 8.877e-02 -0.858 0.40041
           7.050e+01 1.829e+01 3.854 0.00086 ***
Ineq
          -9.381e+03 2.691e+03 -3.486 0.00209 **
          -1.596e+01 6.722e+00 -2.375 0.02670 *
Time
```

```
Residual standard error: 156 on 22 degrees of freedom
Multiple R-squared: 0.9092, Adjusted R-squared: 0.8473
Iteration #3
Call:
Residuals:
                               Max
-381.60 -108.29 12.58 114.69 521.04
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -6.988e+03 2.101e+03 -3.326 0.00307 **
           6.918e+01 4.982e+01 1.388 0.17892
           -1.415e+01 1.940e+02 -0.073 0.94252
           2.481e+02 7.439e+01 3.335 0.00300 **
           2.203e+02 1.378e+02 1.599 0.12411
Po1
          -1.373e+02 1.447e+02 -0.949 0.35298
          -1.235e+03 1.797e+03 -0.687 0.49898
           3.043e+01 2.462e+01 1.236 0.22951
M.F
Pop
           8.875e+00 8.069e+00 1.100 0.28330
          -6.551e+03 5.050e+03 -1.297 0.20803
           1.143e-02 1.291e-01 0.089 0.93027
Wealth
           5.951e+01 3.137e+01 1.897 0.07099 .
Ineq
          -2.784e+03 2.667e+03 -1.044 0.30789
Prob
          5.304e+00 9.232e+00 0.575 0.57146
Time
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 224.6 on 22 degrees of freedom
Multiple R-squared: 0.763, Adjusted R-squared: 0.6015
F-statistic: 4.723 on 15 and 22 DF, p-value: 0.0005457
Iteration #4
Call:
Residuals:
           1Q Median
                          3Q
                                 Max
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.098e+03 1.884e+03 -2.706 0.0129 *
           6.709e+01 6.386e+01 1.051 0.3048
          -8.101e+01 1.816e+02 -0.446 0.6599
Po1
           2.475e+02 1.417e+02 1.747 0.0946.
          -1.702e+02 1.600e+02 -1.064 0.2989
Po2
```

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

```
-8.118e+02 2.644e+03 -0.307 0.7617
M.F
           2.223e+01 2.878e+01 0.773 0.4480
           1.416e+00 1.836e+00 0.771 0.4487
Pop
           8.377e+00 7.792e+00 1.075 0.2940
          -3.955e+03 4.912e+03 -0.805 0.4294
           1.342e+02 9.980e+01 1.345 0.1923
          4.299e-02 1.281e-01 0.335 0.7404
Wealth
           4.937e+01 3.203e+01 1.541 0.1375
          -4.536e+03 2.597e+03 -1.747 0.0946.
Prob
Time
          -4.215e+00 8.548e+00 -0.493 0.6268
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 218.4 on 22 degrees of freedom
Multiple R-squared: 0.8327, Adjusted R-squared: 0.7187
F-statistic: 7.302 on 15 and 22 DF, p-value: 1.958e-05
Iteration #5
Call:
Residuals:
          1Q Median 3Q
-304.60 -109.24 -38.76 124.87 401.79
Coefficients:
(Intercept) -7536.8930 2240.1291 -3.364 0.00293 **
            72.1891 47.9392 1.506 0.14700
            94.6490 182.1750 0.520 0.60881
            147.4452 73.9349 1.994 0.05927 .
            188.3669 164.1972 1.147 0.26420
Po1
           -110.4119 183.1552 -0.603 0.55308
Po2
          -1139.1182 1740.3542 -0.655 0.51987
LF
M.F
             -0.4180
Pop
NW
          -6183.4096 4799.1947 -1.288 0.21161
           129.5443 94.5294 1.370 0.18503
                       0.1133 1.567 0.13214
Wealth
Ineq
          -5497.2661 2751.7377 -1.998 0.05886 .
Prob
            -1.2663 10.1290 -0.125 0.90170
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 209.9 on 21 degrees of freedom
Multiple R-squared: 0.8489, Adjusted R-squared: 0.741
F-statistic: 7.867 on 15 and 21 DF, p-value: 1.47e-05
RMSE: 289.558
MAE: 221.229
```

I also want to see the residual plot as well for the predicted values.

The plot is in the appendix.

```
xyplot(resids ~ preds,
    xlab = "Fitted Values",
    ylab = "Residuals",
    main = "Residual Diagnostic Plot",
    panel = function(x, y, ...)
    {
        panel.grid(h = -1, v = -1)
        panel.abline(h = 0)
        panel.xyplot(x, y, ...)
    }
}
```

From the summary of the models above, it seems that the features that seemed most useful were Ineq, Ed, M, Po1, U2, Prob, and Time. Using this subset of predictors, I'll build another linear regression model.

```
# Shuffle data
shuffled df <- crime data[sample(nrow(crime data)), ]</pre>
# 5 fold split
folds <- cut(seq(1, nrow(crime data)), breaks=5, labels=FALSE)</pre>
# For each fold, train and test on Linear Regression algorithm
resids <- c(1:nrow(crime data))</pre>
preds <- c(1:nrow(crime data))</pre>
    print(glue("Iteration #{i}"))
    test indices <- which(folds==i, arr.ind=TRUE)</pre>
    test data <- crime data[test indices, ]</pre>
    temp fit <- lm(formula=Crime ~ M + Ed + Po1 + U2 + Ineq + Prob + Time,
data=train data)
    print(summary(temp fit))
    # Perform prediction step on test data since model was fit using the training
    preds[test indices] <- predict(temp fit, test data)</pre>
    resids[test indices] <- test data$Crime - preds[test indices]</pre>
overall rmse <- round(rmse(resids), 3)</pre>
print(glue("RMSE: {overall rmse}"))
print(glue("MAE: {overall mae}"))
```

```
Iteration #1
Call:
```

```
data = train data)
Residuals:
          1Q Median 3Q
-463.41 -113.86     0.58     68.57    592.99
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) -4568.594 1241.738 -3.679 0.000949 ***
                       59.748 2.954 0.006163 **
            176.508
Po1
                      48.031 1.687 0.102287
Ineq
                       17.056 3.418 0.001891 **
Prob
Time
             -1.312
                       6.202 -0.212 0.833895
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 215.3 on 29 degrees of freedom
Multiple R-squared: 0.6924, Adjusted R-squared: 0.6182
Iteration #2
Call:
lm(formula = Crime ~ M + Ed + Po1 + U2 + Ineq + Prob + Time,
   data = train data)
Residuals:
    Min 1Q Median 3Q
Coefficients:
(Intercept) -5213.611 809.713 -6.439 4.12e-07 ***
            201.936
                       13.403 8.275 3.09e-09 ***
Po1
           110.910
Ineq
          -6561.410 2178.158 -3.012 0.005225 **
Prob
           -10.483 5.571 -1.882 0.069617 .
Time
Signif. codes: 0 \*** 0.001 \** 0.01 \*' 0.05 \.' 0.1 \' 1
Residual standard error: 167.6 on 30 degrees of freedom
Multiple R-squared: 0.8571, Adjusted R-squared: 0.8237
F-statistic: 25.7 on 7 and 30 DF, p-value: 5.114e-11
Iteration #3
lm(formula = Crime ~ M + Ed + Po1 + U2 + Ineq + Prob + Time,
   data = train data)
```

```
Min
          10 Median
-393.89 -92.37 -18.49 129.66 565.05
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) -5343.858 1205.693 -4.432 0.000115 ***
            104.572
                      40.033 2.612 0.013922 *
                      57.723 3.715 0.000830 ***
            214.457
Ed
Po1
            80.575
                      53.943 1.494 0.145695
            Inea
Prob
                     6.864 0.746 0.461205
Time
             5.124
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 211.4 on 30 degrees of freedom
Multiple R-squared: 0.7138, Adjusted R-squared: 0.6471
F-statistic: 10.69 on 7 and 30 DF, p-value: 1.117e-06
Iteration #4
Call:
lm(formula = Crime ~ M + Ed + Po1 + U2 + Ineq + Prob + Time,
Residuals:
                               Max
-542.08 -102.08 4.98 135.19 444.37
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -4353.1006 1012.5067 -4.299 0.000167 ***
            82.6074
            158.3024 55.2603 2.865 0.007552 **
           127.6608 15.6173 8.174 4e-09 ***
                      44.3536 1.950 0.060596 .
            86.4822
Ineq
          -3945.8976 2022.6503 -1.951 0.060469 .
             0.1747
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 199.3 on 30 degrees of freedom
Multiple R-squared: 0.8101, Adjusted R-squared: 0.7658
F-statistic: 18.28 on 7 and 30 DF, p-value: 3.182e-09
Iteration #5
Call:
lm(formula = Crime ~ M + Ed + Po1 + U2 + Ineq + Prob + Time,
   data = train data)
Residuals:
          1Q Median 3Q
                               Max
-498.35 -90.91 -10.76 153.29 548.25
```

Residuals:

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -4652.173 1259.826 -3.693 0.000915 ***
                       62.219 2.840 0.008159 **
                       16.805 7.114 7.91e-08 ***
Po1
             82.956
Ineq
             72.333
                        16.831 4.298 0.000177 ***
Prob
Time
                        6.958 -0.536 0.595933
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '' 1
Residual standard error: 218.3 on 29 degrees of freedom
Multiple R-squared: 0.7744,
                             Adjusted R-squared:
F-statistic: 14.22 on 7 and 29 DF, p-value: 7.421e-08
RMSE: 227.23
MAE: 169.714
```

The plot is in the appendix.

```
xyplot(resids ~ preds,
    xlab = "Fitted Values",
    ylab = "Residuals",
    main = "Residual Diagnostic Plot",
    panel = function(x, y, ...)
    {
        panel.grid(h = -1, v = -1)
        panel.abline(h = 0)
        panel.xyplot(x, y, ...)
    }
}
```

The second model with the subset of predictors worked the better than the model built on all the predictors. RMSE reduced from 289.558 to 227.23 and MAE reducted from 221.229 to 169.714.

So, now the final model using all available data will be built using the formula below:

Crime ~ M + Ed + Po1 + U2 + Ineq + Prob + Time

```
final_model <- lm(formula=Crime ~ M + Ed + Po1 + U2 + Ineq + Prob + Time,
data=crime_data)

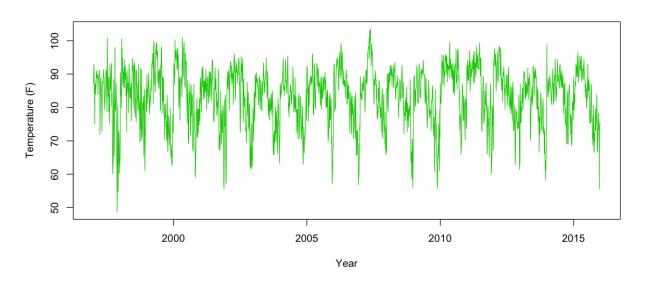
sample_prediction <- round(predict(final_model, sample_data), 3)

print(glue("Prediction for the crime rate for the sample data is
{sample_prediction}"))
print("Final Model")
print(summary(final_model))</pre>
```

```
Prediction for the crime rate for the sample data is 1285.283
[1] "Final Model"
Call:
lm(formula = Crime ~ M + Ed + Po1 + U2 + Ineq + Prob + Time,
   data = crime data)
Residuals:
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
            106.659
                       33.877 3.148 0.003144 **
                       48.288 3.922 0.000345 ***
            189.408
                       13.993 8.269 4.16e-10 ***
Po1
                       41.364 2.145 0.038249 *
            67.728 14.083 4.809 2.28e-05 ***
Ineq
           -4249.756 1880.672 -2.260 0.029502 *
Time
                       5.538 -0.417 0.678810
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 202.8 on 39 degrees of freedom
Multiple R-squared: 0.7669, Adjusted R-squared: 0.7251
F-statistic: 18.33 on 7 and 39 DF, p-value: 1.553e-10
```

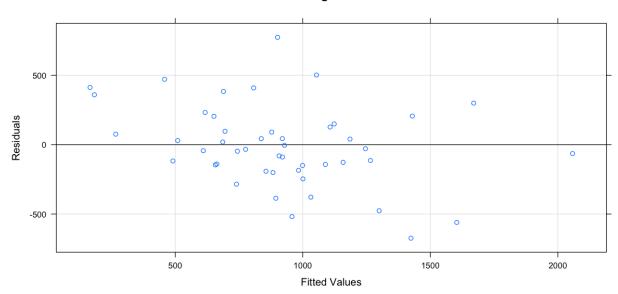
Appendix:

Holt-Winters plot



Linear regression model with every predictor

Residual Diagnostic Plot



Linear regression model with M, Ed, Po1, U2, Ineq, Prob, and Time as the predictors Residual Diagnostic Plot

