

Homework3 - Ganapathy Raaman Balaji

Question 7.1

Describe a situation or problem from your job, everyday life, current events, etc., for which exponential smoothing would be appropriate. What data would you need? Would you expect the value of α (the first smoothing parameter) to be closer to 0 or 1, and why?

As an Analytics Engineer, I look at time series data every day. Most of the data has some sort of seasonality and trend. Currently I am working on a project where I am summarizing the operation performance of a drill engine. The seasonality is almost always additive in most of the data I have looked at in the past 3 years.

The data collection frequency varies from 1Hz to as much as 100Hz. Even for the same recorded data, sometimes the Control Module records data at a different sample rate. I have built exponential smoothing models to look at the engine loading trend with respect to time. Along with aftertreatment and sometimes machine channels, the model helps understand the engine performance.

To start with a baseline model, I tend to use the previous value to estimate the current value of a calculated channel. I end up using $\alpha = 1$.

In []:

Question 7.2

Using the 20 years of daily high temperature data for Atlanta (July through October) from Question 6.2 (file temps.txt), build and use an exponential smoothing model to help make a judgment of whether the unofficial end of summer has gotten later over the 20 years. (Part of the point of this assignment is for you to think about how you might use exponential smoothing to answer this question. Feel free to combine it with other models if you'd like to. There's certainly more than one reasonable approach.)

Note: in R, you can use either HoltWinters (simpler to use) or the smooth package's es function (harder to use, but more general). If you use es, the Holt-Winters model uses model="AAM" in the function call (the first and second constants are used "A"dditively, and the third (seasonality) is used "M"ultiplicatively; the documentation doesn't make that clear).

```
In [1]: # install.packages("forecast", repos='http://cran.us.r-project.org')
# install.packages("xlsx", repos='http://cran.us.r-project.org')

suppressWarnings(suppressMessages(library("TTR")))
suppressWarnings(suppressMessages(library("forecast")))
```

```
In [6]: # install.packages("xlsx", repos='http://cran.us.r-project.org')
suppressWarnings(suppressMessages(require(xlsx)))
```

Firstly, read the text file, flatten the data frame to a single vector (time series data of frequency = 1 day).

```
In [7]: temperature <- read.table("temps.txt", header = T, sep = '\t')
temp_vector <- as.vector(unlist(temperature[,2:21], recursive = TRUE, use.names = TRUE))

# There are 123 days of data. We want frequency of 1 day. So, deltat = 1/123
temp_ts <- ts(temp_vector, start=c(1996,1), end = c(2015,123), deltat = 1/123)
```

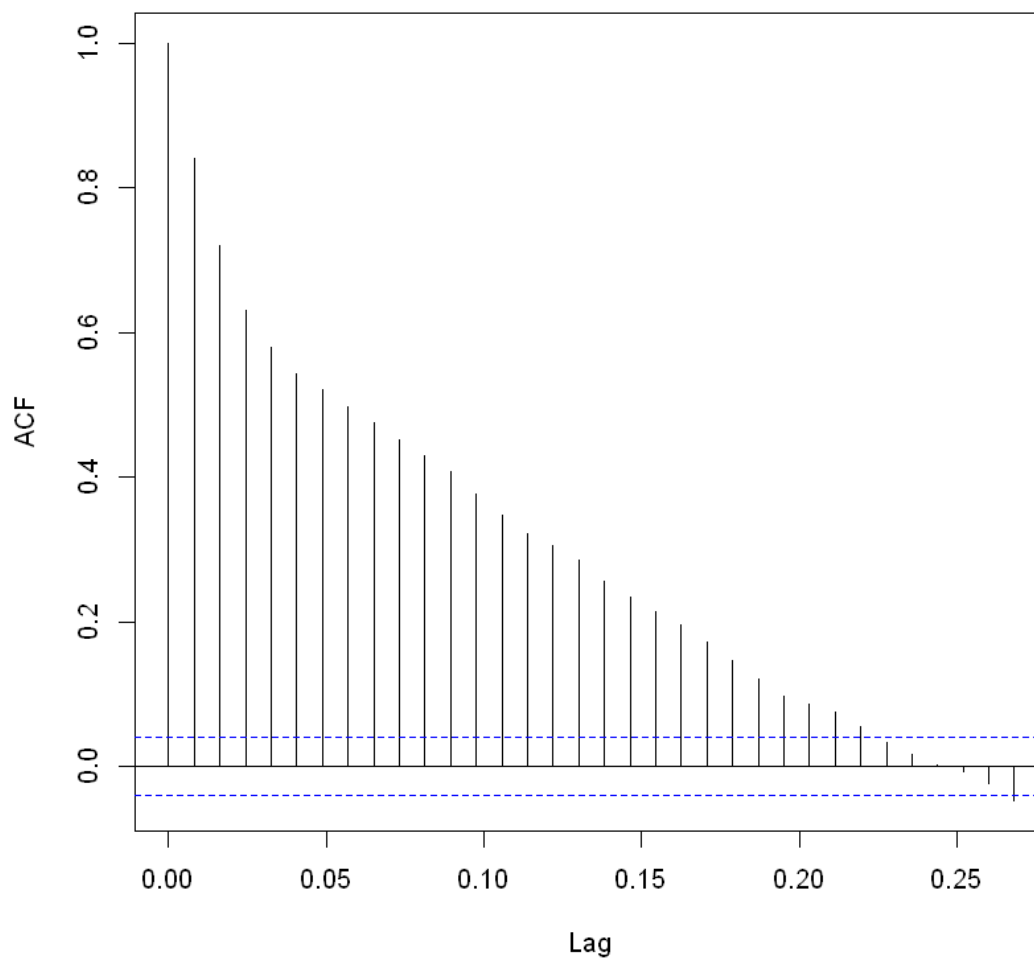
```
In [8]: str(temp_ts)
```

```
Time-Series [1:2460] from 1996 to 2016: 98 97 97 90 89 93 93 91 93 93 ...
```

```
In [9]: # Auto Correlation plot showing that only one value lying outside the 95% limits and  
# the Ljung box test has a p-value < 2.2e-16
```

```
acf(temp_ts, lag.max = NULL, type = "correlation", plot = TRUE)
```

Series temp_ts

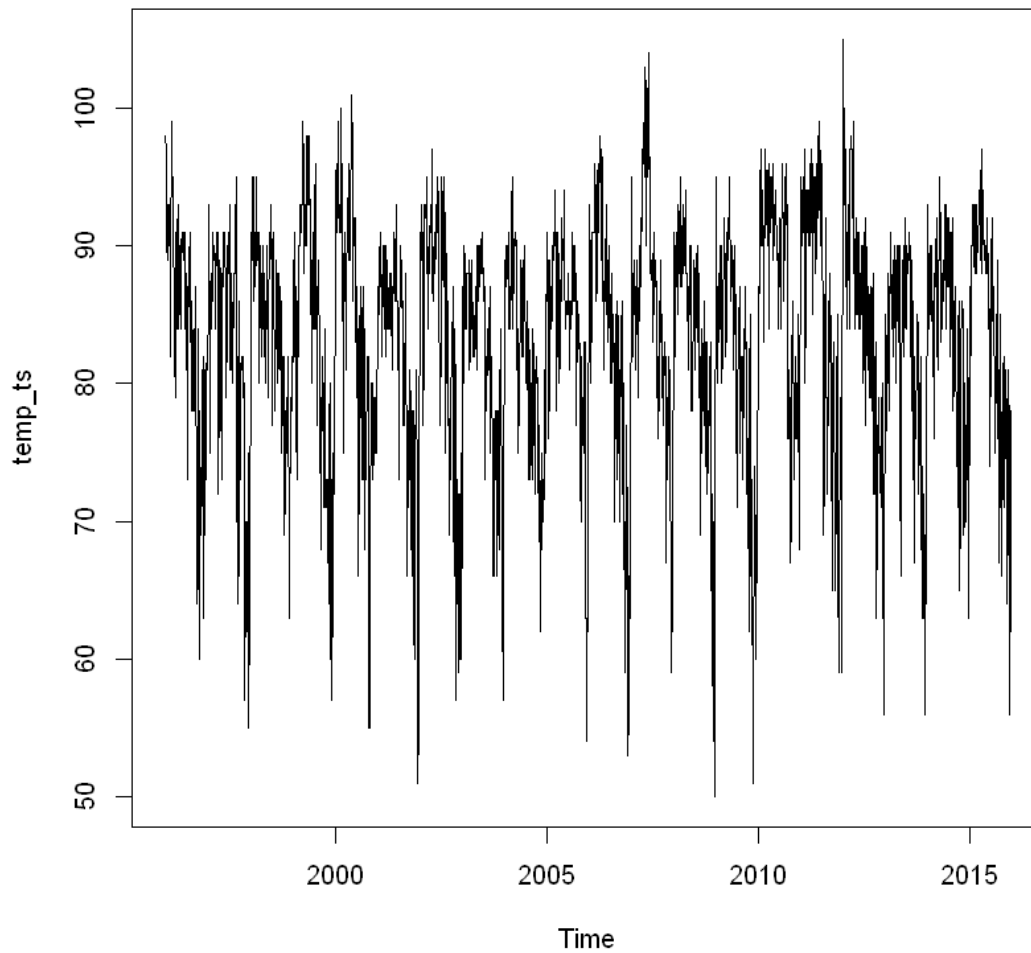


```
In [10]: Box.test(temp_ts, lag = 10, type = "Ljung-Box", fitdf = 0)
```

Box-Ljung test

data: temp_ts
X-squared = 8350.9, df = 10, p-value < 2.2e-16

```
In [11]: # Plotting the time series data  
plot.ts(temp_ts)
```



Now I am going to decompose the time series data, separating it into its constituent components, which are trend component and an irregular component, and if it is a seasonal time series, a seasonal component.

```
In [12]: temp_components <- decompose(temp_ts)
temp_components$seasonal
autoplot(temp_components)
```

Time Series:

Start = c(1996, 1)

End = c(2015, 123)

Frequency = 123

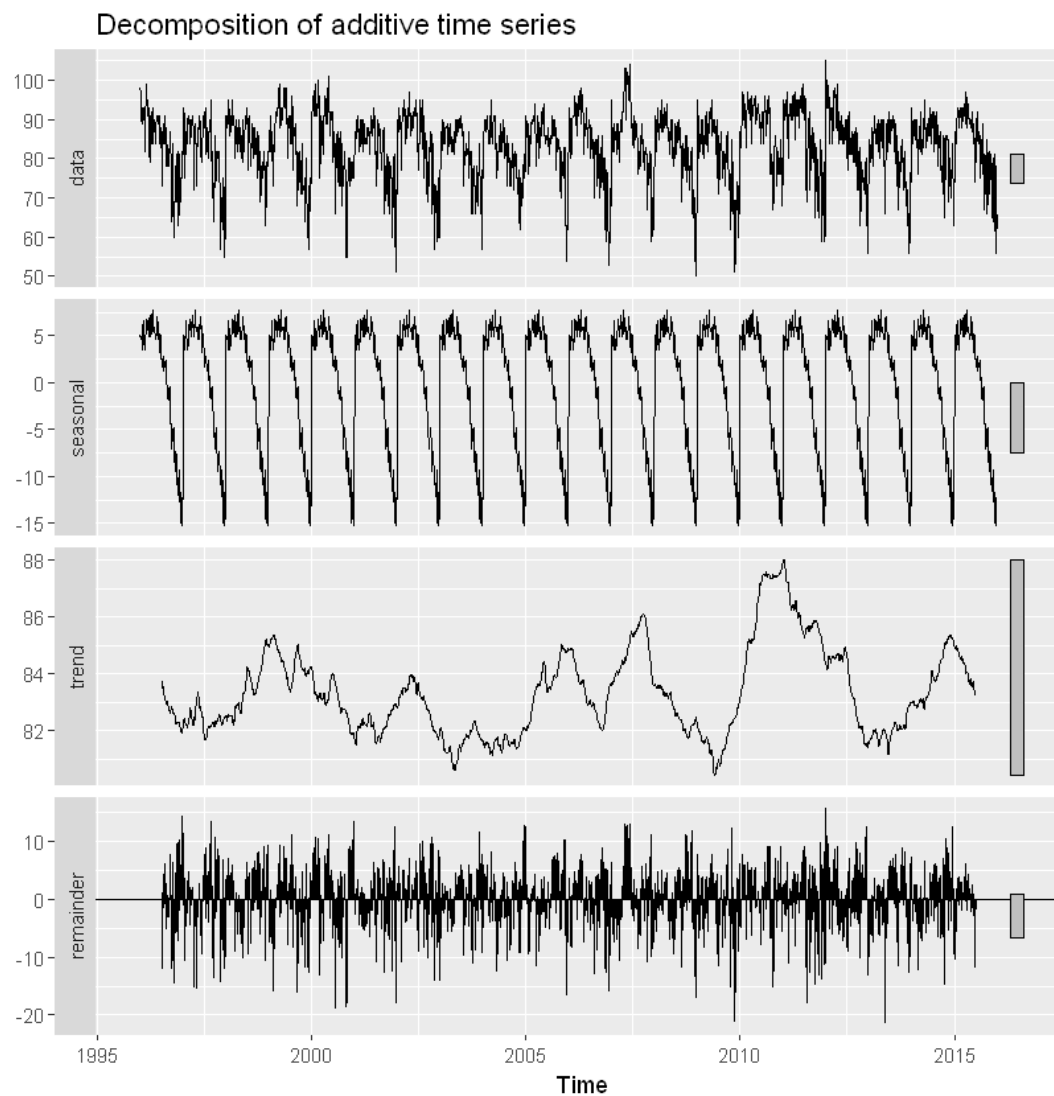
[1]	5.0322211	4.5555416	4.6008989	4.9158326	4.8597778	4.2281989
[7]	3.4404368	5.7053063	6.5495511	4.9723153	4.4982032	4.6560979
[13]	3.5551137	4.6090290	3.9256743	4.6124522	5.5084727	5.6133080
[19]	6.7690632	5.5551137	6.5508348	5.8144205	5.6060337	5.5546858
[25]	4.8747543	6.4622597	6.6257171	7.2080876	5.8922982	6.1533166
[31]	4.8811728	5.3527175	5.2500217	6.5208818	6.9419345	7.7284128
[37]	6.1443307	5.5037659	5.6599490	5.6026105	5.9188279	5.9171163
[43]	4.6509631	4.9633294	5.0690205	4.5953362	5.3860936	5.6017547
[49]	6.7596495	6.9697479	5.9684642	5.7065900	5.9706037	4.9128373
[55]	4.3326063	3.2808305	5.1263589	4.4502785	3.8726148	4.9830128
[61]	2.5670949	2.5606764	1.8221227	1.6685070	2.4130513	2.1520329
[67]	2.8383829	1.2632866	1.4224650	2.0018403	1.6342751	1.9500645
[73]	2.2123666	2.3694056	0.2119387	0.6869066	0.5786482	-0.8937523
[79]	-1.5240476	-1.6806586	-1.1522033	-0.6233201	-1.4662812	-1.8334186
[85]	-0.4637138	-2.5681212	-3.3601623	-4.3631575	-4.3674365	-4.4791182
[91]	-7.0067177	-6.7448435	-5.9053056	-5.8013261	-5.0135641	-5.0704746
[97]	-4.8625157	-6.9686347	-8.0212662	-8.9147194	-9.4444584	-7.9720578
[103]	-7.3426184	-7.5544284	-8.1877190	-9.3477532	-10.8757806	-10.9288400
[109]	-10.0867348	-11.1385106	-11.9267006	-12.0841674	-9.3468974	-11.3464695
[115]	-14.2942658	-14.8715016	-14.0811721	-11.7127510	-13.7645268	-15.2386389
[121]	-14.0268289	-12.2343599	-12.3879757	5.0322211	4.5555416	4.6008989
[127]	4.9158326	4.8597778	4.2281989	3.4404368	5.7053063	6.5495511
[133]	4.9723153	4.4982032	4.6560979	3.5551137	4.6090290	3.9256743
[139]	4.6124522	5.5084727	5.6133080	6.7690632	5.5551137	6.5508348
[145]	5.8144205	5.6060337	5.5546858	4.8747543	6.4622597	6.6257171
[151]	7.2080876	5.8922982	6.1533166	4.8811728	5.3527175	5.2500217
[157]	6.5208818	6.9419345	7.7284128	6.1443307	5.5037659	5.6599490
[163]	5.6026105	5.9188279	5.9171163	4.6509631	4.9633294	5.0690205
[169]	4.5953362	5.3860936	5.6017547	6.7596495	6.9697479	5.9684642
[175]	5.7065900	5.9706037	4.9128373	4.3326063	3.2808305	5.1263589
[181]	4.4502785	3.8726148	4.9830128	2.5670949	2.5606764	1.8221227
[187]	1.6685070	2.4130513	2.1520329	2.8383829	1.2632866	1.4224650
[193]	2.0018403	1.6342751	1.9500645	2.2123666	2.3694056	0.2119387
[199]	0.6869066	0.5786482	-0.8937523	-1.5240476	-1.6806586	-1.1522033
[205]	-0.6233201	-1.4662812	-1.8334186	-0.4637138	-2.5681212	-3.3601623
[211]	-4.3631575	-4.3674365	-4.4791182	-7.0067177	-6.7448435	-5.9053056
[217]	-5.8013261	-5.0135641	-5.0704746	-4.8625157	-6.9686347	-8.0212662
[223]	-8.9147194	-9.4444584	-7.9720578	-7.3426184	-7.5544284	-8.1877190
[229]	-9.3477532	-10.8757806	-10.9288400	-10.0867348	-11.1385106	-11.9267006
[235]	-12.0841674	-9.3468974	-11.3464695	-14.2942658	-14.8715016	-14.0811721
[241]	-11.7127510	-13.7645268	-15.2386389	-14.0268289	-12.2343599	-12.3879757
[247]	5.0322211	4.5555416	4.6008989	4.9158326	4.8597778	4.2281989
[253]	3.4404368	5.7053063	6.5495511	4.9723153	4.4982032	4.6560979
[259]	3.5551137	4.6090290	3.9256743	4.6124522	5.5084727	5.6133080
[265]	6.7690632	5.5551137	6.5508348	5.8144205	5.6060337	5.5546858
[271]	4.8747543	6.4622597	6.6257171	7.2080876	5.8922982	6.1533166
[277]	4.8811728	5.3527175	5.2500217	6.5208818	6.9419345	7.7284128
[283]	6.1443307	5.5037659	5.6599490	5.6026105	5.9188279	5.9171163
[289]	4.6509631	4.9633294	5.0690205	4.5953362	5.3860936	5.6017547
[295]	6.7596495	6.9697479	5.9684642	5.7065900	5.9706037	4.9128373
[301]	4.3326063	3.2808305	5.1263589	4.4502785	3.8726148	4.9830128
[307]	2.5670949	2.5606764	1.8221227	1.6685070	2.4130513	2.1520329
[313]	2.8383829	1.2632866	1.4224650	2.0018403	1.6342751	1.9500645
[319]	2.2123666	2.3694056	0.2119387	0.6869066	0.5786482	-0.8937523
[325]	-1.5240476	-1.6806586	-1.1522033	-0.6233201	-1.4662812	-1.8334186
[331]	-0.4637138	-2.5681212	-3.3601623	-4.3631575	-4.3674365	-4.4791182
[337]	-7.0067177	-6.7448435	-5.9053056	-5.8013261	-5.0135641	-5.0704746
[343]	-4.8625157	-6.9686347	-8.0212662	-8.9147194	-9.4444584	-7.9720578
[349]	-7.3426184	-7.5544284	-8.1877190	-9.3477532	-10.8757806	-10.9288400
[355]	-10.0867348	-11.1385106	-11.9267006	-12.0841674	-9.3468974	-11.3464695
[361]	-14.2942658	-14.8715016	-14.0811721	-11.7127510	-13.7645268	-15.2386389
[367]	-14.0268289	-12.2343599	-12.3879757	5.0322211	4.5555416	4.6008989
[373]	4.9158326	4.8597778	4.2281989	3.4404368	5.7053063	6.5495511
[379]	4.9723153	4.4982032	4.6560979	3.5551137	4.6090290	3.9256743
[385]	4.6124522	5.5084727	5.6133080	6.7690632	5.5551137	6.5508348
[391]	5.8144205	5.6060337	5.5546858	4.8747543	6.4622597	6.6257171
[397]	7.2080876	5.8922982	6.1533166	4.8811728	5.3527175	5.2500217
[403]	6.5208818	6.9419345	7.7284128	6.1443307	5.5037659	5.6599490
[409]	5.6026105	5.9188279	5.9171163	4.6509631	4.9633294	5.0690205
[415]	4.5953362	5.3860936	5.6017547	6.7596495	6.9697479	5.9684642
[421]	5.7065900	5.9706037	4.9128373	4.3326063	3.2808305	5.1263589
[427]	4.4502785	3.8726148	4.9830128	2.5670949	2.5606764	1.8221227
[433]	1.6685070	2.4130513	2.1520329	2.8383829	1.2632866	1.4224650
[439]	2.0018403	1.6342751	1.9500645	2.2123666	2.3694056	0.2119387
[445]	0.6869066	0.5786482	-0.8937523	-1.5240476	-1.6806586	-1.1522033
[451]	-0.6233201	-1.4662812	-1.8334186	-0.4637138	-2.5681212	-3.3601623
[457]	-4.3631575	-4.3674365	-4.4791182	-7.0067177	-6.7448435	-5.9053056
[463]	-5.8013261	-5.0135641	-5.0704746	-4.8625157	-6.9686347	-8.0212662
[469]	-8.9147194	-9.4444584	-7.9720578	-7.3426184	-7.5544284	-8.1877190
[475]	-9.3477532	-10.8757806	-10.9288400	-10.0867348	-11.1385106	-11.9267006
[481]	-12.0841674	-9.3468974	-11.3464695	-14.2942658	-14.8715016	-14.0811721
[487]	-11.7127510	-13.7645268	-15.2386389	-14.0268289	-12.2343599	-12.3879757

[493]	5.0322211	4.5555416	4.6008989	4.9158326	4.8597778	4.2281989
[499]	3.4404368	5.7053063	6.5495511	4.9723153	4.4982032	4.6560979
[505]	3.5551137	4.6090290	3.9256743	4.6124522	5.5084727	5.6133080
[511]	6.7690632	5.5551137	6.5508348	5.8144205	5.6060337	5.5546858
[517]	4.8747543	6.4622597	6.6257171	7.2080876	5.8922982	6.1533166
[523]	4.8811728	5.3527175	5.2500217	6.5208818	6.9419345	7.7284128
[529]	6.1443307	5.5037659	5.6599490	5.6026105	5.9188279	5.9171163
[535]	4.6509631	4.9633294	5.0690205	4.5953362	5.3860936	5.6017547
[541]	6.7596495	6.9697479	5.9684642	5.7065900	5.9706037	4.9128373
[547]	4.3326063	3.2808305	5.1263589	4.4502785	3.8726148	4.9830128
[553]	2.5670949	2.5606764	1.8221227	1.6685070	2.4130513	2.1520329
[559]	2.8383829	1.2632866	1.4224650	2.0018403	1.6342751	1.9500645
[565]	2.2123666	2.3694056	0.2119387	0.6869066	0.5786482	-0.8937523
[571]	-1.5240476	-1.6806586	-1.1522033	-0.6233201	-1.4662812	-1.8334186
[577]	-0.4637138	-2.5681212	-3.3601623	-4.3631575	-4.3674365	-4.4791182
[583]	-7.0067177	-6.7448435	-5.9053056	-5.8013261	-5.0135641	-5.0704746
[589]	-4.8625157	-6.9686347	-8.0212662	-8.9147194	-9.4444584	-7.9720578
[595]	-7.3426184	-7.5544284	-8.1877190	-9.3477532	-10.8757806	-10.9288400
[601]	-10.0867348	-11.1385106	-11.9267006	-12.0841674	-9.3468974	-11.3464695
[607]	-14.2942658	-14.8715016	-14.0811721	-11.7127510	-13.7645268	-15.2386389
[613]	-14.0268289	-12.2343599	-12.3879757	5.0322211	4.5555416	4.6008989
[619]	4.9158326	4.8597778	4.2281989	3.4404368	5.7053063	6.5495511
[625]	4.9723153	4.4982032	4.6560979	3.5551137	4.6090290	3.9256743
[631]	4.6124522	5.5084727	5.6133080	6.7690632	5.5551137	6.5508348
[637]	5.8144205	5.6060337	5.5546858	4.8747543	6.4622597	6.6257171
[643]	7.2080876	5.8922982	6.1533166	4.8811728	5.3527175	5.2500217
[649]	6.5208818	6.9419345	7.7284128	6.1443307	5.5037659	5.6599490
[655]	5.6026105	5.9188279	5.9171163	4.6509631	4.9633294	5.0690205
[661]	4.5953362	5.3860936	5.6017547	6.7596495	6.9697479	5.9684642
[667]	5.7065900	5.9706037	4.9128373	4.3326063	3.2808305	5.1263589
[673]	4.4502785	3.8726148	4.9830128	2.5670949	2.5606764	1.8221227
[679]	1.6685070	2.4130513	2.1520329	2.8383829	1.2632866	1.4224650
[685]	2.0018403	1.6342751	1.9500645	2.2123666	2.3694056	0.2119387
[691]	0.6869066	0.5786482	-0.8937523	-1.5240476	-1.6806586	-1.1522033
[697]	-0.6233201	-1.4662812	-1.8334186	-0.4637138	-2.5681212	-3.3601623
[703]	-4.3631575	-4.3674365	-4.4791182	-7.0067177	-6.7448435	-5.9053056
[709]	-5.8013261	-5.0135641	-5.0704746	-4.8625157	-6.9686347	-8.0212662
[715]	-8.9147194	-9.4444584	-7.9720578	-7.3426184	-7.5544284	-8.1877190
[721]	-9.3477532	-10.8757806	-10.9288400	-10.0867348	-11.1385106	-11.9267006
[727]	-12.0841674	-9.3468974	-11.3464695	-14.2942658	-14.8715016	-14.0811721
[733]	-11.7127510	-13.7645268	-15.2386389	-14.0268289	-12.2343599	-12.3879757
[739]	5.0322211	4.5555416	4.6008989	4.9158326	4.8597778	4.2281989
[745]	3.4404368	5.7053063	6.5495511	4.9723153	4.4982032	4.6560979
[751]	3.5551137	4.6090290	3.9256743	4.6124522	5.5084727	5.6133080
[757]	6.7690632	5.5551137	6.5508348	5.8144205	5.6060337	5.5546858
[763]	4.8747543	6.4622597	6.6257171	7.2080876	5.8922982	6.1533166
[769]	4.8811728	5.3527175	5.2500217	6.5208818	6.9419345	7.7284128
[775]	6.1443307	5.5037659	5.6599490	5.6026105	5.9188279	5.9171163
[781]	4.6509631	4.9633294	5.0690205	4.5953362	5.3860936	5.6017547
[787]	6.7596495	6.9697479	5.9684642	5.7065900	5.9706037	4.9128373
[793]	4.3326063	3.2808305	5.1263589	4.4502785	3.8726148	4.9830128
[799]	2.5670949	2.5606764	1.8221227	1.6685070	2.4130513	2.1520329
[805]	2.8383829	1.2632866	1.4224650	2.0018403	1.6342751	1.9500645
[811]	2.2123666	2.3694056	0.2119387	0.6869066	0.5786482	-0.8937523
[817]	-1.5240476	-1.6806586	-1.1522033	-0.6233201	-1.4662812	-1.8334186
[823]	-0.4637138	-2.5681212	-3.3601623	-4.3631575	-4.3674365	-4.4791182
[829]	-7.0067177	-6.7448435	-5.9053056	-5.8013261	-5.0135641	-5.0704746
[835]	-4.8625157	-6.9686347	-8.0212662	-8.9147194	-9.4444584	-7.9720578
[841]	-7.3426184	-7.5544284	-8.1877190	-9.3477532	-10.8757806	-10.9288400
[847]	-10.0867348	-11.1385106	-11.9267006	-12.0841674	-9.3468974	-11.3464695
[853]	-14.2942658	-14.8715016	-14.0811721	-11.7127510	-13.7645268	-15.2386389
[859]	-14.0268289	-12.2343599	-12.3879757	5.0322211	4.5555416	4.6008989
[865]	4.9158326	4.8597778	4.2281989	3.4404368	5.7053063	6.5495511
[871]	4.9723153	4.4982032	4.6560979	3.5551137	4.6090290	3.9256743
[877]	4.6124522	5.5084727	5.6133080	6.7690632	5.5551137	6.5508348
[883]	5.8144205	5.6060337	5.5546858	4.8747543	6.4622597	6.6257171
[889]	7.2080876	5.8922982	6.1533166	4.8811728	5.3527175	5.2500217
[895]	6.5208818	6.9419345	7.7284128	6.1443307	5.5037659	5.6599490
[901]	5.6026105	5.9188279	5.9171163	4.6509631	4.9633294	5.0690205
[907]	4.5953362	5.3860936	5.6017547	6.7596495	6.9697479	5.9684642
[913]	5.7065900	5.9706037	4.9128373	4.3326063	3.2808305	5.1263589
[919]	4.4502785	3.8726148	4.9830128	2.5670949	2.5606764	1.8221227
[925]	1.6685070	2.4130513	2.1520329	2.8383829	1.2632866	1.4224650
[931]	2.0018403	1.6342751	1.9500645	2.2123666	2.3694056	0.2119387
[937]	0.6869066	0.5786482	-0.8937523	-1.5240476	-1.6806586	-1.1522033
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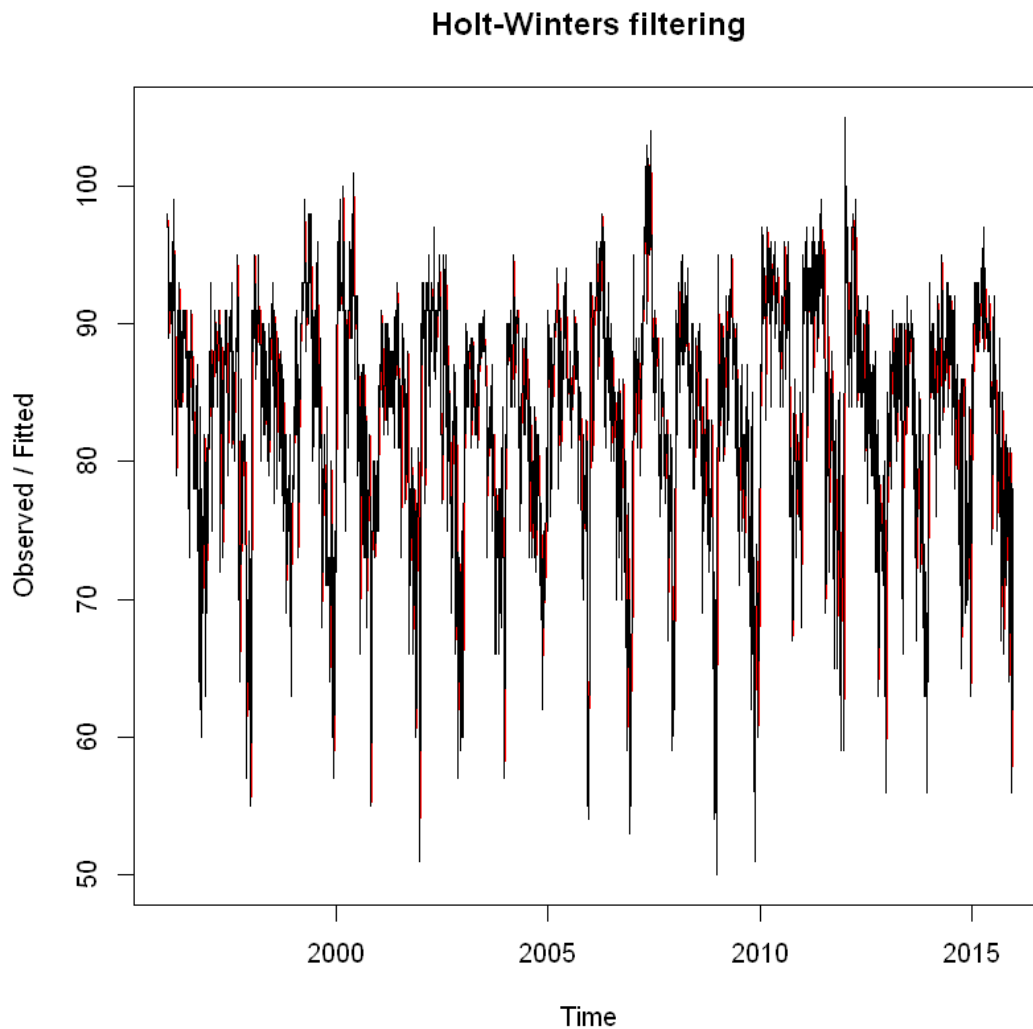
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[2011]	4.6509631	4.9633294	5.0690205	4.5953362	5.3860936	5.6017547
[2017]	6.7596495	6.9697479	5.9684642	5.7065900	5.9706037	4.9128373
[2023]	4.3326063	3.2808305	5.1263589	4.4502785	3.8726148	4.9830128
[2029]	2.5670949	2.5606764	1.8221227	1.6685070	2.4130513	2.1520329
[2035]	2.8383829	1.2632866	1.4224650	2.0018403	1.6342751	1.9500645
[2041]	2.2123666	2.3694056	0.2119387	0.6869066	0.5786482	-0.8937523
[2047]	-1.5240476	-1.6806586	-1.1522033	-0.6233201	-1.4662812	-1.8334186
[2053]	-0.4637138	-2.5681212	-3.3601623	-4.3631575	-4.3674365	-4.4791182

[2059]	-7.0067177	-6.7448435	-5.9053056	-5.8013261	-5.0135641	-5.0704746
[2065]	-4.8625157	-6.9686347	-8.0212662	-8.9147194	-9.4444584	-7.9720578
[2071]	-7.3426184	-7.5544284	-8.1877190	-9.3477532	-10.8757806	-10.9288400
[2077]	-10.0867348	-11.1385106	-11.9267006	-12.0841674	-9.3468974	-11.3464695
[2083]	-14.2942658	-14.8715016	-14.0811721	-11.7127510	-13.7645268	-15.2386389
[2089]	-14.0268289	-12.2343599	-12.3879757	5.0322211	4.5555416	4.6008989
[2095]	4.9158326	4.8597778	4.2281989	3.4404368	5.7053063	6.5495511
[2101]	4.9723153	4.4982032	4.6560979	3.5551137	4.6090290	3.9256743
[2107]	4.6124522	5.5084727	5.6133080	6.7690632	5.5551137	6.5508348
[2113]	5.8144205	5.6060337	5.5546858	4.8747543	6.4622597	6.6257171
[2119]	7.2080876	5.8922982	6.1533166	4.8811728	5.3527175	5.2500217
[2125]	6.5208818	6.9419345	7.7284128	6.1443307	5.5037659	5.6599490
[2131]	5.6026105	5.9188279	5.9171163	4.6509631	4.9633294	5.0690205
[2137]	4.5953362	5.3860936	5.6017547	6.7596495	6.9697479	5.9684642
[2143]	5.7065900	5.9706037	4.9128373	4.3326063	3.2808305	5.1263589
[2149]	4.4502785	3.8726148	4.9830128	2.5670949	2.5606764	1.8221227
[2155]	1.6685070	2.4130513	2.1520329	2.8383829	1.2632866	1.4224650
[2161]	2.0018403	1.6342751	1.9500645	2.2123666	2.3694056	0.2119387
[2167]	0.6869066	0.5786482	-0.8937523	-1.5240476	-1.6806586	-1.1522033
[2173]	-0.6233201	-1.4662812	-1.8334186	-0.4637138	-2.5681212	-3.3601623
[2179]	-4.3631575	-4.3674365	-4.4791182	-7.0067177	-6.7448435	-5.9053056
[2185]	-5.8013261	-5.0135641	-5.0704746	-4.8625157	-6.9686347	-8.0212662
[2191]	-8.9147194	-9.4444584	-7.9720578	-7.3426184	-7.5544284	-8.1877190
[2197]	-9.3477532	-10.8757806	-10.9288400	-10.0867348	-11.1385106	-11.9267006
[2203]	-12.0841674	-9.3468974	-11.3464695	-14.2942658	-14.8715016	-14.0811721
[2209]	-11.7127510	-13.7645268	-15.2386389	-14.0268289	-12.2343599	-12.3879757
[2215]	5.0322211	4.5555416	4.6008989	4.9158326	4.8597778	4.2281989
[2221]	3.4404368	5.7053063	6.5495511	4.9723153	4.4982032	4.6560979
[2227]	3.5551137	4.6090290	3.9256743	4.6124522	5.5084727	5.6133080
[2233]	6.7690632	5.5551137	6.5508348	5.8144205	5.6060337	5.5546858
[2239]	4.8747543	6.4622597	6.6257171	7.2080876	5.8922982	6.1533166
[2245]	4.8811728	5.3527175	5.2500217	6.5208818	6.9419345	7.7284128
[2251]	6.1443307	5.5037659	5.6599490	5.6026105	5.9188279	5.9171163
[2257]	4.6509631	4.9633294	5.0690205	4.5953362	5.3860936	5.6017547
[2263]	6.7596495	6.9697479	5.9684642	5.7065900	5.9706037	4.9128373
[2269]	4.3326063	3.2808305	5.1263589	4.4502785	3.8726148	4.9830128
[2275]	2.5670949	2.5606764	1.8221227	1.6685070	2.4130513	2.1520329
[2281]	2.8383829	1.2632866	1.4224650	2.0018403	1.6342751	1.9500645
[2287]	2.2123666	2.3694056	0.2119387	0.6869066	0.5786482	-0.8937523
[2293]	-1.5240476	-1.6806586	-1.1522033	-0.6233201	-1.4662812	-1.8334186
[2299]	-0.4637138	-2.5681212	-3.3601623	-4.3631575	-4.3674365	-4.4791182
[2305]	-7.0067177	-6.7448435	-5.9053056	-5.8013261	-5.0135641	-5.0704746
[2311]	-4.8625157	-6.9686347	-8.0212662	-8.9147194	-9.4444584	-7.9720578
[2317]	-7.3426184	-7.5544284	-8.1877190	-9.3477532	-10.8757806	-10.9288400
[2323]	-10.0867348	-11.1385106	-11.9267006	-12.0841674	-9.3468974	-11.3464695
[2329]	-14.2942658	-14.8715016	-14.0811721	-11.7127510	-13.7645268	-15.2386389
[2335]	-14.0268289	-12.2343599	-12.3879757	5.0322211	4.5555416	4.6008989
[2341]	4.9158326	4.8597778	4.2281989	3.4404368	5.7053063	6.5495511
[2347]	4.9723153	4.4982032	4.6560979	3.5551137	4.6090290	3.9256743
[2353]	4.6124522	5.5084727	5.6133080	6.7690632	5.5551137	6.5508348
[2359]	5.8144205	5.6060337	5.5546858	4.8747543	6.4622597	6.6257171
[2365]	7.2080876	5.8922982	6.1533166	4.8811728	5.3527175	5.2500217
[2371]	6.5208818	6.9419345	7.7284128	6.1443307	5.5037659	5.6599490
[2377]	5.6026105	5.9188279	5.9171163	4.6509631	4.9633294	5.0690205
[2383]	4.5953362	5.3860936	5.6017547	6.7596495	6.9697479	5.9684642
[2389]	5.7065900	5.9706037	4.9128373	4.3326063	3.2808305	5.1263589
[2395]	4.4502785	3.8726148	4.9830128	2.5670949	2.5606764	1.8221227
[2401]	1.6685070	2.4130513	2.1520329	2.8383829	1.2632866	1.4224650
[2407]	2.0018403	1.6342751	1.9500645	2.2123666	2.3694056	0.2119387
[2413]	0.6869066	0.5786482	-0.8937523	-1.5240476	-1.6806586	-1.1522033
[2419]	-0.6233201	-1.4662812	-1.8334186	-0.4637138	-2.5681212	-3.3601623
[2425]	-4.3631575	-4.3674365	-4.4791182	-7.0067177	-6.7448435	-5.9053056
[2431]	-5.8013261	-5.0135641	-5.0704746	-4.8625157	-6.9686347	-8.0212662
[2437]	-8.9147194	-9.4444584	-7.9720578	-7.3426184	-7.5544284	-8.1877190
[2443]	-9.3477532	-10.8757806	-10.9288400	-10.0867348	-11.1385106	-11.9267006
[2449]	-12.0841674	-9.3468974	-11.3464695	-14.2942658	-14.8715016	-14.0811721
[2455]	-11.7127510	-13.7645268	-15.2386389	-14.0268289	-12.2343599	-12.3879757



Now, I am going to perform single exponential smoothing. Here, I'm using a model with no trend and seasonality. I am going to let R determine the value of α .

```
In [13]: temp_single_es <- HoltWinters(temp_ts, beta = FALSE, gamma = FALSE)
plot(temp_single_es)
```



```
In [14]: temp_single_es
temp_single_es$SSE
```

Holt-Winters exponential smoothing without trend and without seasonal component.

Call:

```
HoltWinters(x = temp_ts, beta = FALSE, gamma = FALSE)
```

Smoothing parameters:

```
alpha: 0.8388021
beta : FALSE
gamma: FALSE
```

Coefficients:

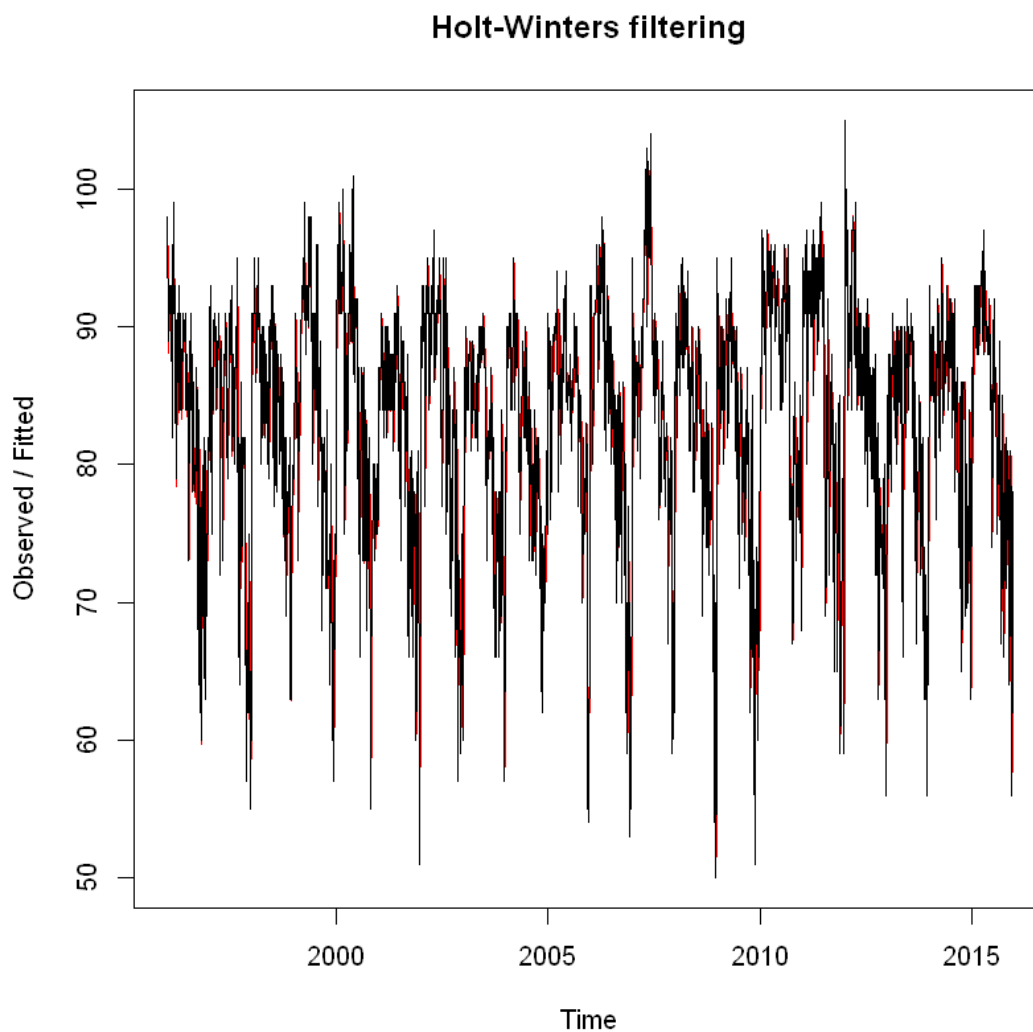
```
[,1]
a 63.30952
```

```
56198.0955314733
```

The estimated value of alpha is 0.8388021. This is high value indicating that the estimate of the current value of the level is based mostly upon very recent observations in the time series. The value of the SSE for the in-sample forecast errors is 56198.0955314733.

I am going to perform double exponential smoothing (gamma = FALSE). I am going to let R determine the value of alpha.

```
In [15]: temp_double_es <- HoltWinters(temp_ts, gamma = FALSE)
plot(temp_double_es)
```



```
In [16]: temp_double_es
temp_double_es$SSE
```

Holt-Winters exponential smoothing with trend and without seasonal component.

Call:

```
HoltWinters(x = temp_ts, gamma = FALSE)
```

Smoothing parameters:

```
alpha: 0.8445729
beta : 0.003720884
gamma: FALSE
```

Coefficients:

```
[,1]
a 63.2530022
b -0.0729933
```

```
56572.5375681139
```

The estimated value of alpha is 0.8388021. Beta is 0.0037. This means that the trend value from the recent observations has relatively very little weight when forecasting for future values. The value of the sum-of-squared-errors for the in-sample forecast errors is 56572.537568114.

Next, I am going to check if the data can be described using an additive model. I am going to use Holt-Winters triple exponential smoothing to estimate the level (alpha), slope (beta) and seasonal (gamma) components.

```
In [17]: temp_add_hw <- HoltWinters(temp_ts)
temp_add_hw
temp_add_hw$SSE
```

Holt-Winters exponential smoothing with trend and additive seasonal component.

Call:

HoltWinters(x = temp_ts)

Smoothing parameters:

alpha: 0.6610618

beta : 0

gamma: 0.6248076

Coefficients:

```
[,1]
a      71.477236414
b      -0.004362918
s1     18.590169842
s2     17.803098732
s3     12.204442890
s4     13.233948865
s5     12.957258705
s6     11.525341233
s7     10.854441534
s8     10.199632666
s9      8.694767348
s10    5.983076192
s11    3.123493477
s12    4.698228193
s13    2.730023168
s14    2.995935818
s15    1.714600919
s16    2.486701224
s17    6.382595268
s18    5.081837636
s19    7.571432660
s20    6.165047647
s21    9.560458487
s22    9.700133847
s23    8.808383245
s24    8.505505527
s25    7.406809208
s26    6.839204571
s27    6.368261304
s28    6.382080380
s29    4.552058253
s30    6.877476437
s31    4.823330209
s32    4.931885957
s33    7.109879628
s34    6.178469084
s35    4.886891317
s36    3.890547248
s37    2.148316257
s38    2.524866001
s39    3.008098232
s40    3.041663870
s41    2.251741386
s42    0.101091985
s43   -0.123337548
s44   -1.445675315
s45   -1.802768181
s46   -2.192036338
s47   -0.180954242
s48    1.538987281
s49    5.075394760
s50    6.740978049
s51    7.737089782
s52    8.579515859
s53    8.408834158
s54    4.704976718
s55    1.827215229
s56   -1.275747384
s57    1.389899699
s58    1.376842871
s59    0.509553410
s60    1.886439429
s61   -0.806454923
s62    5.221873550
s63    5.383073482
s64    4.265584552
s65    3.841481452
s66   -0.231239928
s67    0.542761270
s68    0.780131779
s69    1.096690727
s70    0.690525998
s71    2.301303414
s72    2.965913580
```

s73 4.393732595
s74 2.744547070
s75 1.035278911
s76 1.170709479
s77 2.796838283
s78 2.000312540
s79 0.007337449
s80 -1.203916069
s81 0.352397232
s82 0.675108103
s83 -3.169643942
s84 -1.913321175
s85 -1.647780450
s86 -5.281261301
s87 -5.126493027
s88 -2.637666754
s89 -2.342133004
s90 -3.281910970
s91 -4.242033198
s92 -2.596010530
s93 -7.821281290
s94 -8.814741200
s95 -8.996689798
s96 -7.835655534
s97 -5.749139155
s98 -5.196182693
s99 -8.623793296
s100 -11.809355220
s101 -13.129428554
s102 -16.095143067
s103 -15.125436350
s104 -13.963606549
s105 -12.953304848
s106 -16.097179844
s107 -15.489223470
s108 -13.680122300
s109 -11.921434142
s110 -12.035411347
s111 -12.837047727
s112 -9.095808127
s113 -5.433029341
s114 -6.800835107
s115 -8.413639598
s116 -10.912409484
s117 -13.553826535
s118 -10.652543677
s119 -12.627298331
s120 -9.906981556
s121 -12.668519900
s122 -9.805502547
s123 -7.775306633

66244.2504058464

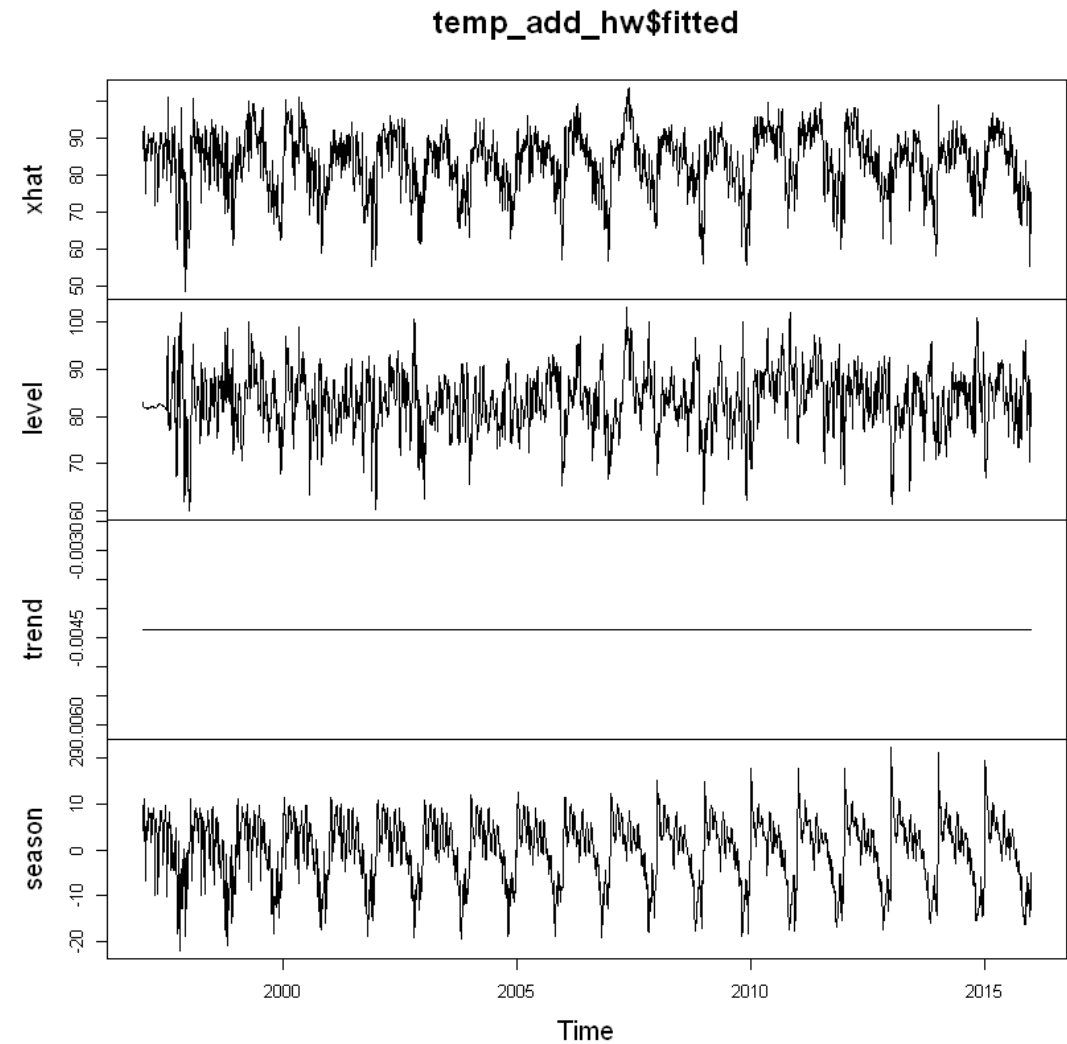
The value of beta is zero, suggesting no trend from recent observations on forecasting future values. The level parameter is 0.6610618, and the seasonal smoothing parameter, gamma is 0.6248076. SSE is 66244.2504058466.

The forecasts made by HoltWinters function are stored in a named element of this list variable called fitted.

```
In [18]: temp_add_hw$fitted  
plot(temp_add_hw$fitted)
```


xhat	level	trend	season
87.17619	82.87739	-0.004362918	4.303159
90.32925	82.09550	-0.004362918	8.238119
92.96089	81.87348	-0.004362918	11.091777
90.93360	81.89497	-0.004362918	9.042997
83.99752	81.93450	-0.004362918	2.067387
84.04358	81.93177	-0.004362918	2.116168
75.06732	81.89860	-0.004362918	-6.826922
87.04284	81.84974	-0.004362918	5.197468
84.01829	81.81705	-0.004362918	2.205599
87.05875	81.80060	-0.004362918	5.262509
84.04807	81.75740	-0.004362918	2.295029
88.04445	81.72126	-0.004362918	6.327550
86.02696	81.68752	-0.004362918	4.343810
89.93161	81.66533	-0.004362918	8.270639
90.90741	81.70618	-0.004362918	9.205599
90.94800	81.76302	-0.004362918	9.189338
88.92923	81.79304	-0.004362918	7.140558
88.90661	81.83546	-0.004362918	7.075517
88.88268	81.89283	-0.004362918	6.994216
89.85831	81.96602	-0.004362918	7.896655
88.81753	82.05532	-0.004362918	6.766574
83.84436	82.17158	-0.004362918	1.677143
87.03232	82.27010	-0.004362918	4.766574
88.03911	82.24438	-0.004362918	5.799094
89.02515	82.21416	-0.004362918	6.815355
89.17489	82.19317	-0.004362918	6.986086
91.16874	82.07319	-0.004362918	9.099907
91.17478	81.95728	-0.004362918	9.221859
89.11179	81.83738	-0.004362918	7.278769
87.99288	81.75912	-0.004362918	6.238119
...
72.64256	80.05496	-0.004362918	-7.408039
66.83529	75.65946	-0.004362918	-8.819799
66.52752	75.10291	-0.004362918	-8.571026
71.27533	77.39407	-0.004362918	-6.114374
72.61174	78.52982	-0.004362918	-5.913719
69.75599	80.76531	-0.004362918	-11.004950
74.87035	88.19393	-0.004362918	-13.319209
79.43815	92.90270	-0.004362918	-13.460183
79.66666	93.93081	-0.004362918	-14.259795
73.08558	88.19726	-0.004362918	-15.107313
73.67625	88.13632	-0.004362918	-14.455711
75.55797	89.66810	-0.004362918	-14.105770
76.93396	93.26126	-0.004362918	-16.322937
77.78777	93.96161	-0.004362918	-16.169481
83.84604	96.08073	-0.004362918	-12.230328
82.21082	91.55071	-0.004362918	-9.335535
72.89454	83.47425	-0.004362918	-10.575347
66.62710	78.91217	-0.004362918	-12.280703
67.28402	77.17113	-0.004362918	-9.882746
73.69847	79.62326	-0.004362918	-5.920428
73.08186	81.14035	-0.004362918	-8.054127
75.45623	85.04824	-0.004362918	-9.587651
78.27330	88.70865	-0.004362918	-10.430990
74.35348	87.20150	-0.004362918	-12.843656

xhat	level	trend	season
76.29092	84.98028	-0.004362918	-8.684994
68.94343	78.83404	-0.004362918	-9.886253
55.62316	70.27328	-0.004362918	-14.645752
73.03021	85.06139	-0.004362918	-12.026808
74.11555	83.05386	-0.004362918	-8.933947
75.38342	80.32887	-0.004362918	-4.941084



I am going to check if the data can be described using an multiplicative model. I am going to use Holt-Winters triple exponential smoothing to estimate the level (alpha), slope (beta) and seasonal (gamma) components.

```
In [19]: temp_mul_hw <- HoltWinters(temp_ts, seasonal = "multiplicative")
temp_mul_hw
temp_mul_hw$SSE
```

Holt-Winters exponential smoothing with trend and multiplicative seasonal component.

Call:

```
HoltWinters(x = temp_ts, seasonal = "multiplicative")
```

Smoothing parameters:

alpha: 0.615003

beta : 0

gamma: 0.5495256

Coefficients:

```
      [,1]  
a    73.679517064  
b   -0.004362918  
s1    1.239022317  
s2    1.234344062  
s3    1.159509551  
s4    1.175247483  
s5    1.171344196  
s6    1.151038408  
s7    1.139383104  
s8    1.130484528  
s9    1.110487514  
s10   1.076242879  
s11   1.041044609  
s12   1.058139281  
s13   1.032496529  
s14   1.036257448  
s15   1.019348815  
s16   1.026754142  
s17   1.071170378  
s18   1.054819556  
s19   1.084397734  
s20   1.064605879  
s21   1.109827336  
s22   1.112670130  
s23   1.103970506  
s24   1.102771209  
s25   1.091264692  
s26   1.084518342  
s27   1.077914660  
s28   1.077696145  
s29   1.053788854  
s30   1.079454300  
s31   1.053481186  
s32   1.054023885  
s33   1.078221405  
s34   1.070145761  
s35   1.054891375  
s36   1.044587771  
s37   1.023285461  
s38   1.025836722  
s39   1.031075732  
s40   1.031419152  
s41   1.021827552  
s42   0.998177248  
s43   0.996049257  
s44   0.981570825  
s45   0.976510542  
s46   0.967977608  
s47   0.985788411  
s48   1.004748195  
s49   1.050965934  
s50   1.072515008  
s51   1.086532279  
s52   1.098357400  
s53   1.097158461  
s54   1.054827180  
s55   1.022866587  
s56   0.987259326  
s57   1.016923524  
s58   1.016604903  
s59   1.004320951  
s60   1.019102781  
s61   0.983848662  
s62   1.055888360  
s63   1.056122844  
s64   1.043478958  
s65   1.039475693  
s66   0.991019224  
s67   1.001437488  
s68   1.002221759  
s69   1.003949213  
s70   0.999566344  
s71   1.018636837  
s72   1.026490773
```

```

s73 1.042507768
s74 1.022500795
s75 1.002503740
s76 1.004560984
s77 1.025536556
s78 1.015357769
s79 0.992176558
s80 0.979377825
s81 0.998058079
s82 1.002553395
s83 0.955429116
s84 0.970970220
s85 0.975543504
s86 0.931515830
s87 0.926764603
s88 0.958565273
s89 0.963250387
s90 0.951644060
s91 0.937362688
s92 0.954257999
s93 0.892485444
s94 0.879537700
s95 0.879946892
s96 0.890633648
s97 0.917134959
s98 0.925991769
s99 0.884247686
s100 0.846648167
s101 0.833696369
s102 0.800001437
s103 0.807934782
s104 0.819343668
s105 0.828571029
s106 0.795608740
s107 0.796609993
s108 0.815503509
s109 0.830111282
s110 0.829086181
s111 0.818367239
s112 0.863958784
s113 0.912057203
s114 0.898308248
s115 0.878723779
s116 0.848971946
s117 0.813891909
s118 0.846821392
s119 0.819121827
s120 0.851036184
s121 0.820416491
s122 0.851581233
s123 0.874038407

68904.5693317479

```

Again, the value of beta is zero, suggesting no trend from recent observations on forecasting future values. The level parameter is 0.615003, and the seasonal smoothing parameter, gamma is 0.5495256. SSE is 68904.5693317479.

I am going to write the fitted values to a csv file to perform CUSUM approach to detect unofficial end of summer.

```
In [20]: df_temp1 <- matrix(temp_mul_hw$fitted[,4], nrow = 123)
```

```
In [27]: write.csv(df_temp1, file = 'smoothed_temperature.csv')
```

Next, I am going to try to predict the temperatures for July 1 through Oct 31 for 2016 and 2017 using the Holt-Winters multiplicative model. To do this, I am using the predict() function that inputs the HW object, prediction interval, number of predictions, and confidence level.

```
In [28]: predicted_temp <- predict(temp_mul_hw, n.ahead = 123*2, prediction.interval = FALSE, level = 0.95)
new_df <- t(as.data.frame(matrix(round(predicted_temp), ncol = 123, byrow = T)))
names(new_df) <- c('X2016', 'X2017')
temperature_data <- cbind(temperature, new_df)
names(temperature_data) <- c('DAY', 'X1996', 'X1997', 'X1998', 'X1999', 'X2000', 'X2001', 'X2002', 'X2003', 'X2004',
'X2005', 'X2006', 'X2007', 'X2008', 'X2009', 'X2010', 'X2011', 'X2012', 'X2013', 'X2014',
'X2015', 'X2016', 'X2017')
```

```
In [29]: head(temperature_data)
```

	DAY	X1996	X1997	X1998	X1999	X2000	X2001	X2002	X2003	X2004	...	X2008	X2009	X2010	X2011	X2012	X2013	X2014	X2015	X2016	X2017
V1	1-Jul	98	86	91	84	89	84	90	73	82	...	85	95	87	92	105	82	90	85	91	91
V2	2-Jul	97	90	88	82	91	87	90	81	81	...	87	90	84	94	93	85	93	87	91	90
V3	3-Jul	97	93	91	87	93	87	87	87	86	...	91	89	83	95	99	76	87	79	85	85
V4	4-Jul	90	91	91	88	95	84	89	86	88	...	90	91	85	92	98	77	84	85	87	86
V5	5-Jul	89	84	91	90	96	86	93	80	90	...	88	80	88	90	100	83	86	84	86	86
V6	6-Jul	93	84	89	91	96	87	93	84	90	...	82	87	89	90	98	83	87	84	85	84

In this problem, I have used Holt Winters approach to exponentially smoothe data, and use the smoothed data to predict unofficial end of summer using CUSUM approach. In addition, I have also used the smoothed data to predict temperatures of each day (July1 to Oct 31) for the next two years.

```
In [ ]:
```

C	Threshold
0	0.01

mu ---->	1.000	0.998	0.998	0.998	0.997	0.996	0.996	0.996	0.996	0.995	0.995	0.995	0.994	0.994	0.994	0.993	0.993	0.994	0.993
v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12	v13	v14	v15	v16	v17	v18	v19	
1	1.0527	1.0495	1.1206	1.1033	1.1184	1.1082	1.1409	1.1406	1.1254	1.1221	1.1614	1.1981	1.1989	1.2430	1.2438	1.3002	1.2906	1.2545	
2	1.1007	1.0997	1.1080	1.0983	1.1102	1.1162	1.1268	1.1541	1.1422	1.1319	1.1445	1.1347	1.1534	1.1654	1.1729	1.1907	1.1920	1.2192	1.2288
3	1.1354	1.1354	1.1391	1.1428	1.1432	1.1385	1.1297	1.1561	1.1657	1.1480	1.1495	1.1358	1.1533	1.1552	1.1573	1.1698	1.1899	1.1723	1.1690
4	1.1103	1.1105	1.1171	1.1258	1.1345	1.1261	1.1308	1.1377	1.1506	1.1470	1.1425	1.1502	1.1512	1.1578	1.1638	1.1593	1.1666	1.1680	1.1590
5	1.0252	1.0252	1.0447	1.0673	1.0847	1.0972	1.1151	1.1039	1.1208	1.1337	1.1322	1.1427	1.1392	1.1129	1.1324	1.1320	1.1452	1.1682	1.1704
6	1.0258	1.0257	1.0282	1.0423	1.0540	1.0675	1.0802	1.0943	1.1027	1.0922	1.0758	1.0885	1.0822	1.1031	1.1151	1.1186	1.1216	1.1350	1.1455
7	0.9166	0.9164	0.9470	0.9476	0.9704	0.9918	1.0025	1.0303	1.0439	1.0353	1.0383	1.0378	1.0611	1.0706	1.0939	1.1096	1.1003	1.0983	1.1150
8	1.0635	1.0634	1.0479	1.0362	1.0140	1.0179	1.0165	1.0313	1.0313	1.0621	1.0631	1.0576	1.0673	1.0589	1.0769	1.0863	1.0926	1.1180	1.1229
9	1.0270	1.0269	1.0286	1.0307	1.0349	1.0368	1.0347	1.0341	1.0360	1.0490	1.0575	1.0747	1.0723	1.0734	1.0749	1.0716	1.0817	1.0934	1.1004
10	1.0644	1.0642	1.0470	1.0439	1.0495	1.0371	1.0387	1.0248	1.0299	1.0072	1.0189	1.0258	1.0301	1.0392	1.0327	1.0426	1.0478	1.0566	1.0593
11	1.0281	1.0279	1.0290	1.0187	1.0194	1.0283	1.0132	1.0133	1.0200	1.0267	1.0419	1.0371	1.0414	1.0477	1.0506	1.0577	1.0443	1.0298	1.0313
12	1.0775	1.0773	1.0539	1.0302	1.0211	1.0241	1.0000	1.0078	1.0092	1.0215	1.0307	1.0363	1.0476	1.0588	1.0502	1.0530	1.0333	1.0482	1.0575
13	1.0532	1.0531	1.0543	1.0328	1.0239	1.0146	1.0207	1.0218	1.0242	1.0206	1.0243	1.0253	1.0123	1.0090	1.0184	1.0305	1.0426	1.0221	1.0286
14	1.1012	1.1014	1.0887	1.0965	1.0840	1.0558	1.0651	1.0479	1.0432	1.0455	1.0417	1.0331	1.0340	1.0407	1.0376	1.0216	1.0280	1.0418	1.0426
15	1.1125	1.1128	1.1160	1.1132	1.1047	1.0888	1.0931	1.0856	1.0569	1.0470	1.0452	1.0330	1.0358	1.0380	1.0445	1.0102	1.0150	1.0254	1.0193
16	1.1123	1.1125	1.1032	1.1187	1.1142	1.1116	1.1135	1.1117	1.0944	1.0850	1.0761	1.0782	1.0696	1.0657	1.0499	1.0485	1.0471	1.0468	1.0319
17	1.0872	1.0874	1.0969	1.0930	1.0917	1.1035	1.1059	1.1068	1.1011	1.1067	1.1037	1.1066	1.0944	1.0823	1.0672	1.0682	1.0659	1.0719	1.0686
18	1.0864	1.0866	1.0930	1.1059	1.1175	1.1193	1.1169	1.1149	1.1193	1.1202	1.1162	1.1147	1.1135	1.0877	1.0697	1.0747	1.0724	1.0631	1.0530
19	1.0853	1.0856	1.0981	1.1046	1.1164	1.1226	1.1210	1.1194	1.1118	1.1111	1.1189	1.1151	1.1172	1.1003	1.1093	1.1155	1.1006	1.0988	1.0954
20	1.0962	1.0966	1.0891	1.0986	1.1020	1.0995	1.0987	1.1027	1.1141	1.1134	1.1084	1.1038	1.1145	1.1143	1.1174	1.1104	1.0867	1.0784	1.0556
21	1.0823	1.0828	1.0829	1.0878	1.0769	1.0732	1.0881	1.0944	1.1031	1.1077	1.1070	1.1055	1.1117	1.1234	1.1304	1.1275	1.1308	1.1207	1.1203
22	1.0204	1.0208	1.0288	1.0440	1.0560	1.0667	1.0640	1.0637	1.0733	1.0837	1.0835	1.0683	1.0711	1.0785	1.0927	1.1043	1.1229	1.1151	1.1190
23	1.0580	1.0579	1.0581	1.0626	1.0409	1.0555	1.0519	1.0409	1.0588	1.0658	1.0567	1.0648	1.0540	1.0681	1.0803	1.0824	1.0985	1.0979	1.1164
24	1.0706	1.0705	1.0679	1.0675	1.0443	1.0339	1.0321	1.0347	1.0474	1.0500	1.0524	1.0694	1.0668	1.0721	1.0688	1.0720	1.0802	1.0892	1.0954
25	1.0829	1.0829	1.0661	1.0615	1.0333	1.0263	1.0196	1.0284	1.0203	1.0316	1.0388	1.0480	1.0540	1.0637	1.0666	1.0633	1.0706	1.0837	1.0898
26	1.0852	1.0847	1.0831	1.0826	1.0872	1.0889	1.0779	1.0768	1.0577	1.0589	1.0683	1.0656	1.0590	1.0666	1.0671	1.0755	1.0757	1.0719	1.0792
27	1.1111	1.1106	1.0817	1.0818	1.0997	1.1030	1.1036	1.0977	1.0773	1.0686	1.0750	1.0802	1.0823	1.0752	1.0687	1.0716	1.0702	1.0591	1.0719
28	1.1128	1.1122	1.1228	1.1140	1.1220	1.1019	1.1076	1.1098	1.1127	1.0963	1.0953	1.0904	1.0966	1.0882	1.0753	1.0651	1.0659	1.0732	1.0702
29	1.0891	1.0887	1.1001	1.1015	1.1220	1.1119	1.1120	1.1093	1.1052	1.0812	1.0718	1.0697	1.0739	1.0650	1.0747	1.0783	1.0686	1.0709	1.0489
30	1.0763	1.0763	1.0857	1.0989	1.0943	1.0997	1.0945	1.0864	1.0948	1.0731	1.0712	1.0831	1.0768	1.0645	1.0707	1.0801	1.0808	1.0894	1.0822
31	0.8787	0.8791	0.9105	0.9514	0.9762	0.9967	1.0200	1.0420	1.0567	1.0728	1.0795	1.0773	1.0710	1.0756	1.0756	1.0782	1.0588	1.0401	1.0502
32	0.9754	0.9758	0.9673	0.9647	0.9713	0.9728	0.9929	0.9969	1.0137	1.0233	1.0381	1.0507	1.0563	1.0713	1.0746	1.0636	1.0740	1.0824	1.0661
33	1.0241	1.0243	0.9997	0.9849	0.9706	0.9713	0.9747	0.9800	0.9875	1.0063	1.0154	1.0248	1.0394	1.0332	1.0130	1.0328	1.0542	1.0665	1.0718
34	1.0738	1.0737	1.0582	1.0266	1.0196	1.0092	1.0024	0.9986	0.9999	1.0093	1.0166	1.0245	1.0277	1.0453	1.0548	1.0559	1.0476	1.0511	1.0666
35	1.0852	1.0853	1.0822	1.0667	1.0658	1.0568	1.0437	1.0278	1.0220	1.0282	1.0292	1.0364	1.0352	1.0451	1.0558	1.0261	1.0123	1.0257	1.0415
36	1.0714	1.0718	1.0837	1.0824	1.0864	1.0838	1.0777	1.0695	1.0556	1.0433	1.0359	1.0350	1.0372	1.0406	1.0409	1.0591	1.0540	1.0453	1.0448
37	1.0209	1.0214	1.0337	1.0482	1.0568	1.0516	1.0653	1.0593	1.0405	1.0338	1.0163	1.0245	1.0322	1.0273	1.0302	1.0324	1.0261	1.0286	1.0391
38	1.0187	1.0194	1.0245	1.0319	1.0385	1.0455	1.0278	1.0385	1.0276	1.0119	1.0281	1.0287	1.0213	1.0265	1.0231	1.0254	1.0194	1.0168	1.0300
39	0.9698	0.9701	0.9751	0.9879	0.9994	1.0148	1.0145	1.0213	1.0234	1.0236	1.0251	1.0303	1.0171	1.0223	1.0266	1.0291	1.0456	1.0310	1.0213
40	0.8841	0.8844	0.8945	0.9110	0.9404	0.9556	0.9652	0.9825	0.9963	1.0095	1.0108	1.0220	1.0161	1.0207	1.0250	1.0192	1.0179	1.0331	1.0264
41	0.9679	0.9683	0.9660	0.9547	0.9547	0.9598	0.9673	0.9757	0.9588	0.9666	0.9780	0.9889	0.9936	1.0000	1.0059	1.0168	1.0144	1.0232	1.0153
42	1.0392	1.0396	1.0255	1.0219	0.9927	0.9816	0.9822	0.9750	0.9869	0.9937	0.9739	0.9728	0.9850	0.9937	0.9995	1.0027	0.9998	1.0062	0.9992
43	1.0623	1.0627	1.0415	1.0412	1.0252	1.0137	1.0118	0.9947	0.9899	0.9943	0.9764	0.9534	0.9453	0.9368	0.9504	0.9638	0.9777	0.9847	0.9987
44	1.0616	1.0619	1.0591	1.0529	1.0440	1.0220	1.0195	1.0265	1.0093	1.0020	0.9855	0.9963	0.9837	0.9814	0.9794	0.9784	0.9876	0.9859	0.9812
45	1.0507	1.0505	1.0357	1.0405	1.0397	1.0477	1.0362	1.0404	1.0445	1.0440	1.0334	1.0282	1.0176	0.9924	0.9870	0.9817	0.9677	0.9722	
46	1.0646	1.0642	1.0610	1.0456	1.0562	1.0480	1.0409	1.0440	1.0448	1.0494	1.0457	1.0478	1.0505	1.0415	1.0286	1.0040	0.9909	0.9559	0.9648
47	1.1034	1.1027	1.1042	1.0885	1.0907	1.0959	1.0879	1.0683	1.0689	1.0580	1.0624	1.0575	1.0539	1.0446	1.0389	1.0319	1.0355	0.9917	0.9899
48	1.1051	1.1045	1.0992	1.1016	1.1044	1.1061	1.1034	1.1003	1.0992	1.0909	1.0776	1.0724	1.0652	1.0565	1.0542	1.0531	1.0399	1.0178	1.0140
49	1.0810	1.0808	1.0917	1.0989	1.1052	1.1057	1.1098	1.1099	1.1075	1.1025	1.1001	1.0838	1.0812	1.0770	1.0767	1.0764	1.0661	1.0862	1.0695
50	1.0804	1.0804	1.0924	1.0950	1.0982	1.0894	1.0894	1.0937	1.1041	1.1003	1.0999	1.0886	1.0970	1.0973	1.0903	1.0921	1.0765	1.0861	1.0794
51	1.0680	1.0681	1.0455	1.0348	1.0395	1.0458	1.0596	1.0598	1.0682	1.0816	1.0979	1.0964	1.0980	1.0927	1.0934	1.0861	1.0813	1.0932	1.0953
52	0.9961	0.9960	1.0151	1.0222	1.0029	1.0176	1.0285	1.0376	1.0298	1.0472	1.0555	1.0705	1.0636	1.0687	1.0740	1.0778	1.0883	1.0914	1.1002
53	0.9615	0.9610	0.9822	0.9916	0.9882	1.0014	1.0043	1.0127	1.0201	1.0264	1.0266	1.0486	1.0426	1.0413	1.0513	1.0666	1.0624	1.0793	1.0876
54	0.9864	0.9861	0.9889	0.9888	0.9929	0.9923	1.0042	1.0122	1.0152	1.0042	1.0065	1.0062	1.0021	0.9998	1.0098	1.0105	1.0303	1.0479	1.0575
55	0.9996	0.9993	1.0000	0.9723	0.9864	0.9990	0.9976	0.9997	1.										

[illegible]

04-Sep	0.1118	0.0813	0.1044	0.0854	0.0662	0.0739	0.0335	0.0198	0.0127	0.0015	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
05-Sep	0.1005	0.0988	0.0965	0.0680	0.0451	0.0436	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0105	0.0246
06-Sep	0.0526	0.0698	0.0650	0.0307	0.0262	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0035	0.0060
07-Sep	0.0000	0.0059	0.0126	0.0000	0.0322	0.0000	0.0000	0.0000	0.0204	0.0164	0.0117	0.0047	0.0110	0.0042	0.0000	0.0000	0.0000	0.0000
08-Sep	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0110	0.0000	0.0000	0.0036	0.0111	0.0000	0.0000	0.0000	0.0068	0.0000	0.0000
09-Sep	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10-Sep	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11-Sep	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12-Sep	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13-Sep	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14-Sep	0.0548	0.0252	0.0000	0.0000	0.0000	0.0000	0.0012	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15-Sep	0.0972	0.0579	0.0237	0.0109	0.0015	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16-Sep	0.0557	0.0274	0.0033	0.0000	0.0000	0.0158	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17-Sep	0.0626	0.0272	0.0000	0.0111	0.0248	0.0413	0.0163	0.0120	0.0066	0.0068	0.0000	0.0064	0.0049	0.0000	0.0000	0.0339	0.0205	0.0000
18-Sep	0.0689	0.0251	0.0000	0.0150	0.0411	0.0487	0.0283	0.0234	0.0027	0.0096	0.0000	0.0000	0.0320	0.0425	0.0203	0.0443	0.0381	0.0346
19-Sep	0.1230	0.0697	0.0471	0.0512	0.0751	0.0750	0.0643	0.0543	0.0264	0.0260	0.0093	0.0030	0.0102	0.0179	0.0060	0.0387	0.0491	0.0567
20-Sep	0.1644	0.1022	0.0881	0.0966	0.0897	0.0877	0.0859	0.0697	0.0448	0.0353	0.0344	0.0147	0.0132	0.0309	0.0173	0.0474	0.0560	0.0534
21-Sep	0.2049	0.1297	0.1149	0.1592	0.1305	0.1169	0.1000	0.0758	0.0681	0.0554	0.0736	0.0483	0.0439	0.0412	0.0187	0.0364	0.0537	0.0444
22-Sep	0.2574	0.1867	0.1654	0.1886	0.1836	0.1578	0.1371	0.1173	0.1032	0.0860	0.1039	0.0860	0.0899	0.0814	0.0515	0.0481	0.0488	0.0729
23-Sep	0.2724	0.2622	0.2210	0.2534	0.2604	0.2223	0.2120	0.2036	0.1657	0.1497	0.1434	0.1067	0.0942	0.0793	0.0624	0.0703	0.0578	0.0680
24-Sep	0.2505	0.2440	0.2255	0.2638	0.2657	0.2354	0.2270	0.2206	0.1782	0.1707	0.1542	0.1237	0.1104	0.0930	0.0801	0.0828	0.0821	0.0841
25-Sep	0.2281	0.2167	0.2171	0.2518	0.2485	0.2793	0.2884	0.2757	0.2284	0.2137	0.1934	0.1453	0.1409	0.1146	0.1084	0.1090	0.1176	0.1438
26-Sep	0.1697	0.2129	0.2129	0.2371	0.2372	0.3010	0.3334	0.3169	0.2731	0.2633	0.2630	0.2064	0.1872	0.1495	0.1319	0.1179	0.1234	0.1784
27-Sep	0.1483	0.1914	0.1897	0.2108	0.2560	0.3077	0.3348	0.3264	0.2961	0.3131	0.3127	0.2640	0.2446	0.2047	0.2192	0.1910	0.1732	0.2004
28-Sep	0.1885	0.2359	0.2132	0.2336	0.2767	0.3145	0.3390	0.3333	0.3284	0.3312	0.3348	0.2927	0.2871	0.2618	0.2760	0.2530	0.2241	0.2384
29-Sep	0.2784	0.3046	0.2856	0.2904	0.3147	0.3353	0.3449	0.3646	0.3481	0.3501	0.3498	0.3117	0.2907	0.2719	0.3096	0.3059	0.2792	0.2930
30-Sep	0.4047	0.3925	0.3832	0.3839	0.3987	0.4212	0.4269	0.4602	0.4265	0.4158	0.4332	0.3917	0.3528	0.3524	0.3770	0.3612	0.3394	0.3546
01-Oct	0.6278	0.5662	0.5431	0.5437	0.5369	0.5484	0.5296	0.5501	0.5063	0.4970	0.5046	0.4692	0.4238	0.4232	0.4523	0.4443	0.4485	0.4520
02-Oct	0.8254	0.7729	0.7158	0.7085	0.6879	0.6823	0.6460	0.6564	0.6007	0.5862	0.5692	0.5350	0.5194	0.5084	0.5289	0.5593	0.5591	0.5464
03-Oct	0.9496	0.9215	0.8812	0.8673	0.8379	0.8142	0.7668	0.7833	0.7188	0.6926	0.6675	0.6221	0.6133	0.6012	0.6191	0.6528	0.6633	0.6459
04-Oct	0.9271	0.9377	0.9386	0.9233	0.9056	0.8896	0.8497	0.8828	0.8254	0.8012	0.7702	0.7313	0.7059	0.6909	0.7352	0.7464	0.7556	0.7392
05-Oct	1.0755	1.0582	1.0414	1.0479	1.0225	1.0010	0.9605	0.9659	0.9124	0.8882	0.8547	0.8149	0.7813	0.7739	0.8330	0.8269	0.8188	0.8021
06-Oct	1.2723	1.2281	1.1949	1.1939	1.1588	1.1328	1.0767	1.0685	1.0146	0.9960	0.9528	0.9073	0.8685	0.8960	0.9445	0.9220	0.9043	0.8852
07-Oct	1.4931	1.4312	1.4046	1.3938	1.3635	1.3418	1.2664	1.2456	1.1876	1.1767	1.1365	1.0699	1.0171	1.0210	1.0573	1.0324	1.0055	0.9824
08-Oct	1.7623	1.6832	1.6311	1.6083	1.5934	1.5721	1.4966	1.4674	1.3988	1.3824	1.3471	1.2653	1.2004	1.1711	1.1796	1.1559	1.1600	1.1473
09-Oct	1.8116	1.7770	1.7848	1.7728	1.8085	1.7655	1.7182	1.6878	1.6197	1.5965	1.5612	1.4729	1.4225	1.3716	1.3582	1.3253	1.3449	1.3346
10-Oct	1.9581	1.9234	1.9358	1.9308	1.9888	1.9758	1.9479	1.9155	1.8506	1.8291	1.7751	1.6882	1.6350	1.5679	1.5412	1.5172	1.5203	1.5038
11-Oct	2.0787	2.0439	2.0610	2.0641	2.1085	2.1025	2.0907	2.0771	2.0281	2.0022	1.9514	1.8732	1.8206	1.7769	1.7407	1.7258	1.7125	1.6818
12-Oct	2.2353	2.1918	2.1932	2.1950	2.2159	2.2096	2.1876	2.2118	2.1602	2.1304	2.0907	2.0594	2.0044	1.9634	1.9283	1.9270	1.9043	1.8706
13-Oct	2.3916	2.3504	2.3436	2.3483	2.3509	2.3311	2.2884	2.2956	2.2645	2.2584	2.2452	2.2165	2.1689	2.1607	2.1359	2.1161	2.0836	2.0472
14-Oct	2.4992	2.4662	2.4653	2.4939	2.4862	2.4606	2.4193	2.4142	2.4083	2.3955	2.4159	2.3799	2.3337	2.3206	2.3082	2.2859	2.2627	2.2234
15-Oct	2.5339	2.5338	2.5689	2.5809	2.5816	2.5693	2.5614	2.5495	2.5527	2.5344	2.5484	2.5081	2.4628	2.4742	2.4846	2.4532	2.4295	2.4029
16-Oct	2.5442	2.6072	2.6257	2.6464	2.6495	2.6481	2.6956	2.7055	2.7107	2.6848	2.6913	2.6470	2.5974	2.6317	2.6385	2.6012	2.5813	2.5858
17-Oct	2.5666	2.6644	2.6859	2.6983	2.6974	2.7433	2.7813	2.8028	2.7959	2.7902	2.8363	2.7947	2.7370	2.7792	2.7914	2.7429	2.7330	2.7350
18-Oct	2.5647	2.6499	2.6912	2.7038	2.7112	2.7910	2.8414	2.8847	2.8658	2.8738	2.9052	2.8665	2.8593	2.9421	2.9441	2.8889	2.8794	2.8867
19-Oct	2.7583	2.8074	2.8203	2.8301	2.8164	2.8775	2.9125	2.9564	2.9403	2.9373	2.9514	2.9314	2.9523	3.0437	3.0599	3.0210	3.0170	3.0259
20-Oct	2.9884	3.0046	2.9911	3.0037	2.9884	3.0117	3.0234	3.0469	3.0414	3.0238	3.0292	3.0121	3.0464	3.1213	3.1386	3.1495	3.1508	3.1885
21-Oct	3.1573	3.1701	3.1690	3.1974	3.1772	3.1738	3.1661	3.1689	3.1529	3.1273	3.1440	3.1227	3.1528	3.1990	3.2289	3.2821	3.2896	3.3188
22-Oct	3.1916	3.2331	3.2662	3.3068	3.2980	3.2886	3.2882	3.2740	3.2523	3.2327	3.2534	3.2242	3.2381	3.2695	3.2984	3.3603	3.3689	3.4024
23-Oct	3.2008	3.2845	3.3643	3.3844	3.3766	3.3709	3.4006	3.3961	3.3686	3.3809	3.3916	3.3679	3.3773	3.3851	3.4091	3.4512	3.4501	3.4932
24-Oct	3.3557	3.4216	3.5011	3.5510	3.5347	3.5116	3.5550	3.5455	3.5176	3.5271	3.5854	3.5329	3.5523	3.5488	3.5657	3.5859	3.5746	3.6293
25-Oct	3.4608	3.5323	3.5925	3.6579	3.6510	3.6342	3.6604	3.6724	3.6414	3.6970	3.7595	3.7563	3.7899	3.7765	3.7680	3.7640	3.7379	3.7810
26-Oct	3.5650	3.6024	3.6557	3.7171	3.7163	3.7286	3.7779	3.7959	3.7522	3.8365	3.9104	3.9271	3.9406	3.9533	3.9569	3.9394	3.9060	3.9728
27-Oct	3.6447	3.6839	3.7221	3.7662	3.7778	3.8288	3.8619	3.9036	3.8559	3.9298	3.9955	4.0141	4.0292	4.0669	4.0686	4.0597	4.0353	4.1049
28-Oct	3.7244	3.8068	3.8311	3.8447	3.8470	3.9573	3.9757	4.0318	3.9737	4.0344	4.0830	4.1045	4.1526	4.2073	4.2230	4.2036	4.2019	4.2440
29-Oct	3.7309	3.8557	3.8905	3.9079	3.9046	4.0266	4.0351	4.1329	4.0838	4.1436	4.1933	4.2030	4.2946	4.3219	4.3539	4.3470	4.3957	4.4351
30-Oct	3.7256	3.8368	3.8937	3.9277	3.9448	4.0473	4.0621	4.1424	4.1260	4.1936	4.2301	4.2762	4.3588	4.3813	4.4494	4.4994	4.5666	4.5828
31-Oct	3.7207	3.8190	3.8735	3.9100	3.9377	4.0141	4.0647	4.1238	4.1184	4.1836	4.2248	4.2787	4.3526	4.4162	4.4957	4.5743	4.6668	4.6819

Question 8.1

Describe a situation or problem from your job, everyday life, current events, etc., for which a linear regression model would be appropriate. List some (up to 5) predictors that you might use.

I have not worked on regression analysis, although I would love to. But one application where I think I might be able to apply this in real life is, soccer. I play soccer manager game with my friends. We often hire virtual scouts to scout for potential talents in the game. Thus far, I have bought players who have had the best goals to matches ratio or tackles to matches ratio.

But I think that based on the stats that the scout provides such as (other than the mainstream stats such as age, matches played, goals scored etc) minutes played, number of successful passes completed, number and recurrence of injuries, nation (I can assign numerical metrics to this categorical variable), build up plays leading to goal, or the number of commanding saves when it comes to a goalkeeper, I might be able to build a mathematical regression model, and even remove insignificant predictors to build a successful model, buy the successful players and maybe even top my league.

In []:

Question 8.2

Using crime data from <http://www.statsci.org/data/general/uscrime.txt> (file uscrime.txt, description at <http://www.statsci.org/data/general/uscrime.html>), use regression (a useful R function is `lm` or `glm`) to predict the observed crime rate in a city with the following data:

```
M = 14.0
So = 0
Ed = 10.0
Po1 = 12.0
Po2 = 15.5
LF = 0.640
M.F = 94.0
Pop = 150
NW = 1.1
U1 = 0.120
U2 = 3.6
Wealth = 3200
Ineq = 20.1
Prob = 0.04
Time = 39.0
```

Show your model (factors used and their coefficients), the software output, and the quality of fit.

Note that because there are only 47 data points and 15 predictors, you'll probably notice some overfitting. We'll see ways of dealing with this sort of problem later in the course.

```
In [24]: install.packages("Amelia", repos='http://cran.us.r-project.org')
```

```
Installing package into 'C:/Users/balajg/Documents/R/win-library/3.5'
(as 'lib' is unspecified)
```

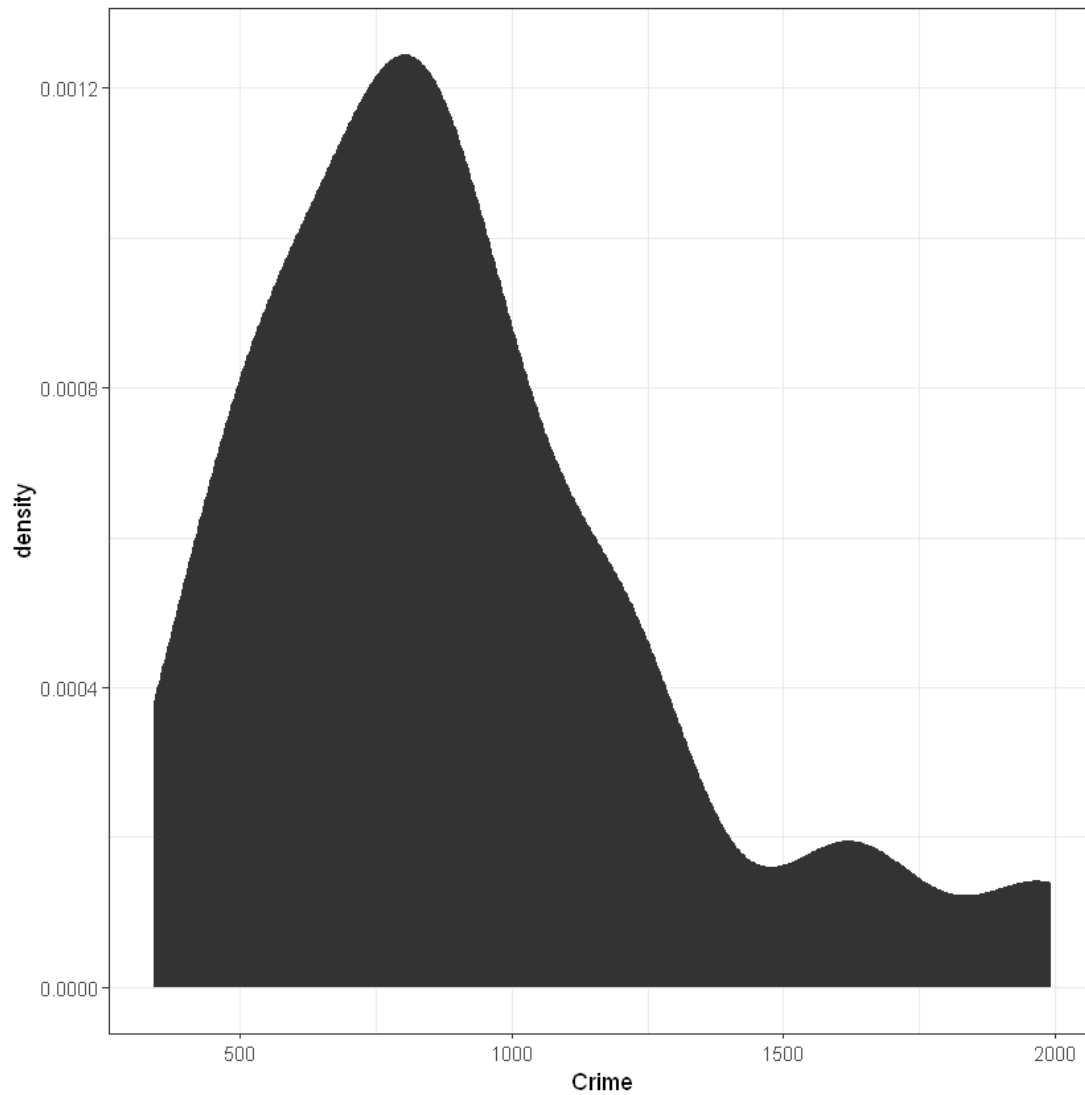
```
package 'Amelia' successfully unpacked and MD5 sums checked
```

```
The downloaded binary packages are in
C:\Users\balajg\AppData\Local\Temp\RtmpSE1k3R\downloaded_packages
```

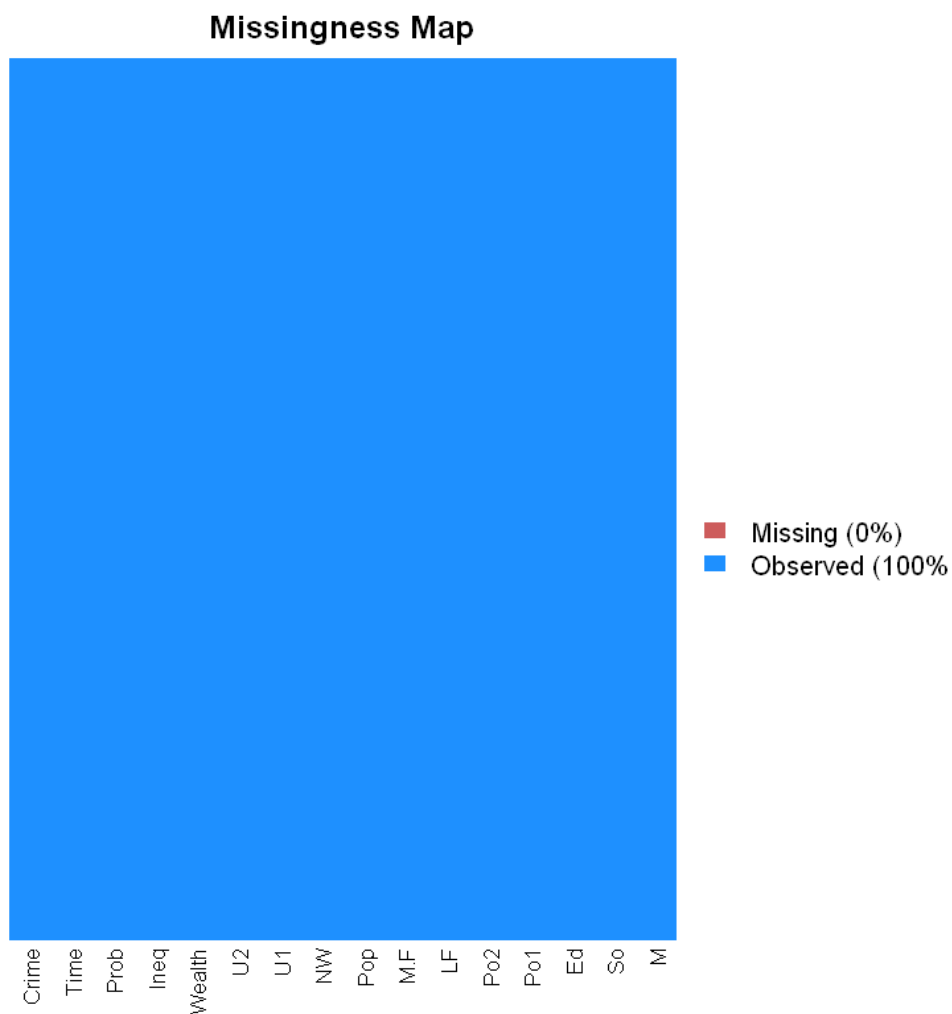
```
In [2]: uscrimes <- read.table("uscrime.txt", header=TRUE, sep="\t")
head(uscrimes)
```

	M	So	Ed	Po1	Po2	LF	M.F	Pop	NW	U1	U2	Wealth	Ineq	Prob	Time	Crime
15.1	1	9.1	5.8	5.6	0.510	95.0	33	30.1	0.108	4.1	3940	26.1	0.084602	26.2011	791	
14.3	0	11.3	10.3	9.5	0.583	101.2	13	10.2	0.096	3.6	5570	19.4	0.029599	25.2999	1635	
14.2	1	8.9	4.5	4.4	0.533	96.9	18	21.9	0.094	3.3	3180	25.0	0.083401	24.3006	578	
13.6	0	12.1	14.9	14.1	0.577	99.4	157	8.0	0.102	3.9	6730	16.7	0.015801	29.9012	1969	
14.1	0	12.1	10.9	10.1	0.591	98.5	18	3.0	0.091	2.0	5780	17.4	0.041399	21.2998	1234	
12.1	0	11.0	11.8	11.5	0.547	96.4	25	4.4	0.084	2.9	6890	12.6	0.034201	20.9995	682	

```
In [17]: # Graphical summary of our response variable  
library(ggplot2)  
ggplot(uscrimes, aes(Crime)) + stat_density() + theme_bw()
```

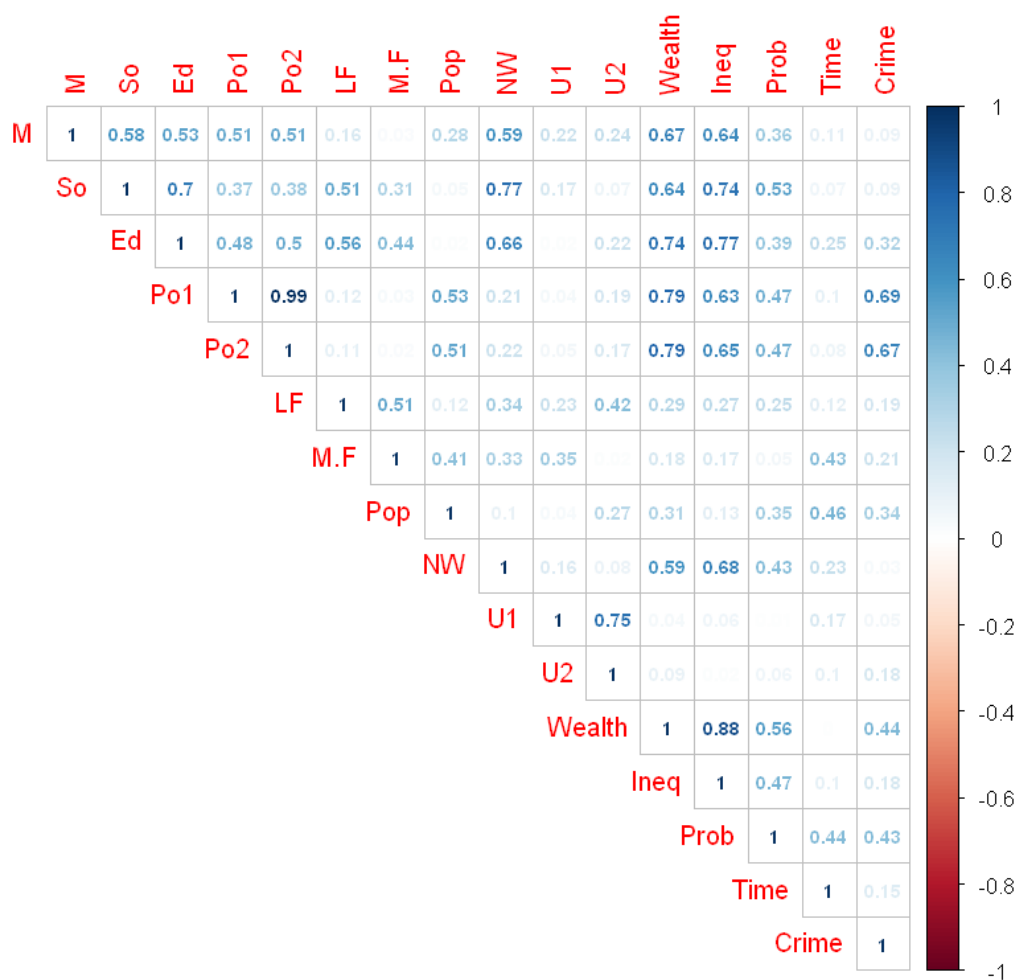


```
In [28]: # In this step, I am going to check for missing data. Using mismap function in Amelia package which checks for missing data.  
library(Amelia)  
mismatch(uscrimes, legend = TRUE, col = c("indianred", "dodgerblue"), y.at=1, y.labels='')
```

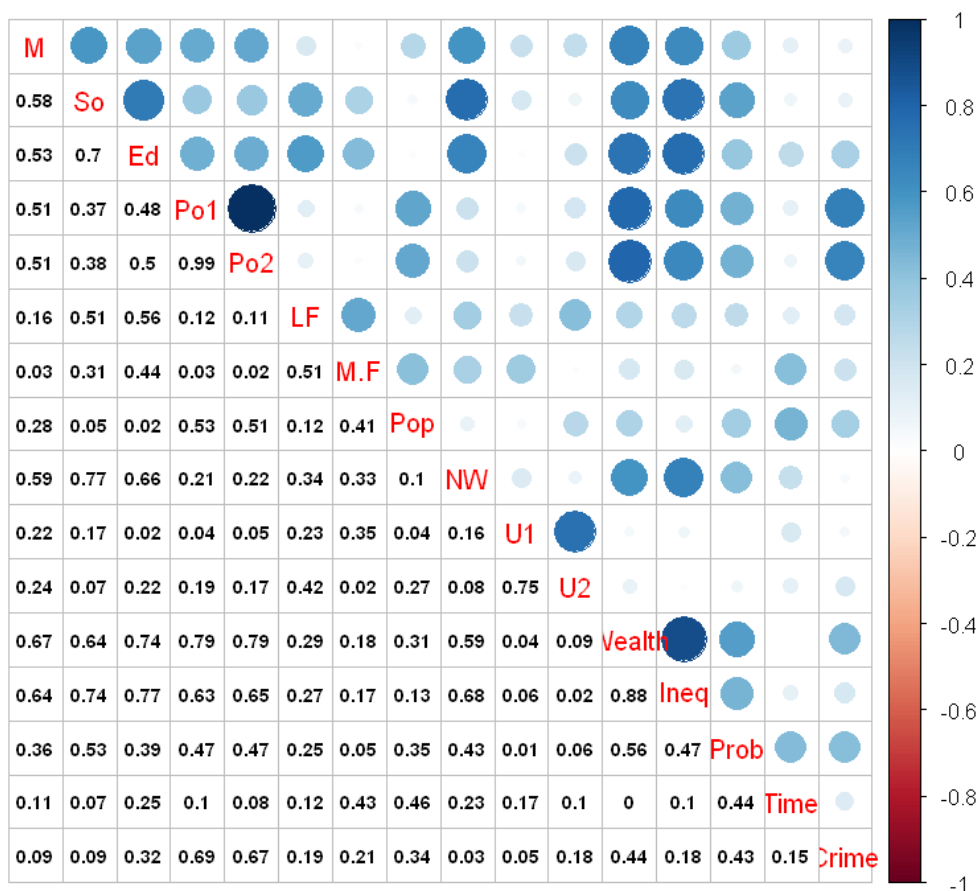


The figure shows no missing data. Next I am going to check for the correlation of variables. I'm going to build a correlation plot to show the dependence of the response (Crime) on the different input variables.

```
In [43]: # suppressWarnings(suppressMessages(install.packages("corrplot", repos='http://cran.us.r-project.org')))
suppressWarnings(suppressMessages(library(corrplot)))
corrplot(abs(cor(uscrimes)), method = "number", type = "upper", number.cex = .7)
```



```
In [44]: # Another way of representing corrpplot, mix of number and circle.
corrplot.mixed(abs(cor(uscrimes)), lower = "number", upper = "circle", lower.col = "black", number.cex = .7)
```



From the corrpplot, it can be inferred that Crime response is more dependent on Po1, Po2, Pop, Wealth, Prob, and Ed than on the rest of input data. I am picking all inputs with more than 30% correlation to Crime response.

Next, I am going to create a test dataframe that I will use to predict the regression model. The test dataframe inputs the following predictors:

```
M = 14.0
So = 0
Ed = 10.0
Po1 = 12.0
Po2 = 15.5
LF = 0.640
M.F = 94.0
Pop = 150
NW = 1.1
U1 = 0.120
U2 = 3.6
Wealth = 3200
Ineq = 20.1
Prob = 0.04
Time = 39.0
```

I am going to fit the regression model with all the predictors. Then I will test the quality of this model by predicting the Crime rate for the baseline dataframe.

```
In [60]: baseline_df <- data.frame(M = 14.0,
                                   So = 0,
                                   Ed = 10.0,
                                   Po1 = 12.0,
                                   Po2 = 15.5,
                                   LF = 0.640,
                                   M.F = 94.0,
                                   Pop = 150,
                                   NW = 1.1,
                                   U1 = 0.120,
                                   U2 = 3.6,
                                   Wealth = 3200,
                                   Ineq = 20.1,
                                   Prob = 0.040,
                                   Time = 39.0
                                   )
```

```
In [61]: base_model <- lm(Crime~., data=uscrimes)
summary(base_model)
```

```
Call:
lm(formula = Crime ~ ., data = uscrimes)

Residuals:
    Min       1Q   Median       3Q      Max
-395.74  -98.09   -6.69  112.99  512.67

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.984e+03  1.628e+03  -3.675 0.000893 ***
M              8.783e+01  4.171e+01   2.106 0.043443 *
So            -3.803e+00  1.488e+02  -0.026 0.979765
Ed             1.883e+02  6.209e+01   3.033 0.004861 **
Po1            1.928e+02  1.061e+02   1.817 0.078892 .
Po2           -1.094e+02  1.175e+02  -0.931 0.358830
LF            -6.638e+02  1.470e+03  -0.452 0.654654
M.F            1.741e+01  2.035e+01   0.855 0.398995
Pop           -7.330e-01  1.290e+00  -0.568 0.573845
NW             4.204e+00  6.481e+00   0.649 0.521279
U1            -5.827e+03  4.210e+03  -1.384 0.176238
U2             1.678e+02  8.234e+01   2.038 0.050161 .
Wealth        9.617e-02  1.037e-01   0.928 0.360754
Ineq          7.067e+01  2.272e+01   3.111 0.003983 **
Prob         -4.855e+03  2.272e+03  -2.137 0.040627 *
Time         -3.479e+00  7.165e+00  -0.486 0.630708
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 209.1 on 31 degrees of freedom
Multiple R-squared:  0.8031,    Adjusted R-squared:  0.7078
F-statistic: 8.429 on 15 and 31 DF,  p-value: 3.539e-07
```

```
In [63]: base_model_predict <- predict(base_model, baseline_df)
base_model_predict
```

```
1: 155.434896887449
```

The baseline regression model with all predictors has predicted a crime rate of 155.434896887449.

Next, I am going to use only those predictors with more than 30% correlation to Crime rate response. Thus my model is going to include only Po1, Po2, Pop, Wealth, Prob, and Ed (as determined from corrplot).

```
In [64]: mod_df <- data.frame(Ed = 10.0,
                               Po1 = 12.0,
                               Po2 = 15.5,
                               Pop = 150,
                               Wealth = 3200,
                               Prob = 0.040
                               )
```

```
In [65]: new_model <- lm(Crime~ + Ed + Po1 + Po2 + Pop + Wealth + Prob, data=uscrimes)
summary(new_model)
```

Call:

```
lm(formula = Crime ~ +Ed + Po1 + Po2 + Pop + Wealth + Prob, data = uscrimes)
```

Residuals:

Min	1Q	Median	3Q	Max
-597.05	-133.34	23.56	152.63	578.04

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	532.1121	493.0890	1.079	0.2870
Ed	64.4792	57.6568	1.118	0.2701
Po1	277.2720	121.6071	2.280	0.0280 *
Po2	-163.4147	129.6253	-1.261	0.2147
Pop	-0.9084	1.3658	-0.665	0.5098
Wealth	-0.2155	0.0932	-2.313	0.0260 *
Prob	-3996.2829	2180.1075	-1.833	0.0742 .

 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 271.8 on 40 degrees of freedom

Multiple R-squared: 0.5705, Adjusted R-squared: 0.506

F-statistic: 8.854 on 6 and 40 DF, p-value: 3.692e-06

```
In [66]: new_model_predict <- predict(new_model, mod_df)
new_model_predict
```

1: 985.407406170355

In the next steps, I am going to split the data (80-20) to training and test sets. I am going to create a regression model using training set, and make predictions and test the quality of the model on test sets. I am using caret package for its cross validation functions.

Reference: <http://www.sthda.com/english/articles/38-regression-model-validation/157-cross-validation-essentials-in-r/> (<http://www.sthda.com/english/articles/38-regression-model-validation/157-cross-validation-essentials-in-r/>)

```
In [93]: suppressWarnings(suppressMessages(library(tidyverse)))
suppressWarnings(suppressMessages(library(caret)))
```

```
In [87]: # Split the data into training and test set
set.seed(123)
training.samples <- uscrimes$Crime %>%
  createDataPartition(p = 0.8, list = FALSE)
train.data <- uscrimes[training.samples, ]
test.data <- uscrimes[-training.samples, ]

# Build the model
model <- lm(Crime~ + Ed + Po1 + Po2 + Pop + Wealth + Prob, data = train.data)

# Make predictions and compute the R2, RMSE and MAE
predictions <- model %>% predict(test.data)
data.frame( R2 = R2(predictions, test.data$Crime),
            RMSE = RMSE(predictions, test.data$Crime),
            MAE = MAE(predictions, test.data$Crime))
```

R2	RMSE	MAE
0.6226139	210.5413	162.1704

Cross Validation Methods

First, Leave one out cross validation - LOOCV


```
In [90]: # Define training control
train.control <- trainControl(method = "LOOCV")
# Train the model
model <- train(Crime~ + Ed + Po1 + Po2 + Pop + Wealth + Prob, data = uscrimes, method = "lm", trControl = train.control)
# Summarize the results
print(model)
```

Linear Regression

47 samples
6 predictor

No pre-processing
Resampling: Leave-One-Out Cross-Validation
Summary of sample sizes: 46, 46, 46, 46, 46, 46, ...
Resampling results:

RMSE	Rsquared	MAE
312.7607	0.3615977	231.8999

Tuning parameter 'intercept' was held constant at a value of TRUE

Next, K-fold cross-validation

```
In [91]: # Define training control
set.seed(123)
train.control <- trainControl(method = "cv", number = 10)
# Train the model
model <- train(Crime~ + Ed + Po1 + Po2 + Pop + Wealth + Prob, data = uscrimes, method = "lm", trControl = train.control)
# Summarize the results
print(model)
```

Linear Regression

47 samples
6 predictor

No pre-processing
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 43, 42, 42, 41, 43, 41, ...
Resampling results:

RMSE	Rsquared	MAE
289.7675	0.5865918	234.0705

Tuning parameter 'intercept' was held constant at a value of TRUE

Lastly, Repeated K-fold cross-validation

The process of splitting the data into k-folds can be repeated a number of times, this is called repeated k-fold cross validation. The final model error is taken as the mean error from the number of repeats. I am performing 10-fold cross validation with 3 repeats.

```
In [92]: # Define training control
set.seed(123)
train.control <- trainControl(method = "repeatedcv", number = 10, repeats = 3)
# Train the model
model <- train(Crime~ + Ed + Po1 + Po2 + Pop + Wealth + Prob, data = uscrimes, method = "lm", trControl = train.control)
# Summarize the results
print(model)
```

Linear Regression

47 samples
6 predictor

No pre-processing
Resampling: Cross-Validated (10 fold, repeated 3 times)
Summary of sample sizes: 43, 42, 42, 41, 43, 41, ...
Resampling results:

RMSE	Rsquared	MAE
295.8304	0.5290194	233.3643

Tuning parameter 'intercept' was held constant at a value of TRUE

The repeated k-fold cv approach gives an R-Squared value of 0.5290194.