Homework_Week2_GanapathyRaamanBalaji

May 29, 2019

1 Homework - Week2 - Ganapathy Raaman Balaji

1.1 Problem 4.1

```
In a recent performance analysis of a fleet of mining trucks, I used GPS data
(latitude and longitude) recorded by the machine to cluster the turck operating in different fi
I used this data to summarize truck operation and performance in different mine sites.
The predictors I used were
GPS coordinates,
Truck speed,
engine RPM,
operation hours and
aftertreatment (emission) performance values.
```

[ ]:

```

```

1.2 Problem 4.2

[20]:

```r
# install.packages("dplyr", repos='http://cran.us.r-project.org')
# install.packages("tidyverse", repos='http://cran.us.r-project.org')
# install.packages("cluster", repos='http://cran.us.r-project.org')
# install.packages("fpc", repos='http://cran.us.r-project.org')
# install.packages("factoextra", repos='http://cran.us.r-project.org')
```

[9]:

```r
oldw <- getOption("warn")
options(warn = -1)
library(dplyr)
library(tidyverse)
library(cluster)
library(fpc)
library(factoextra)
require(gridExtra)
```

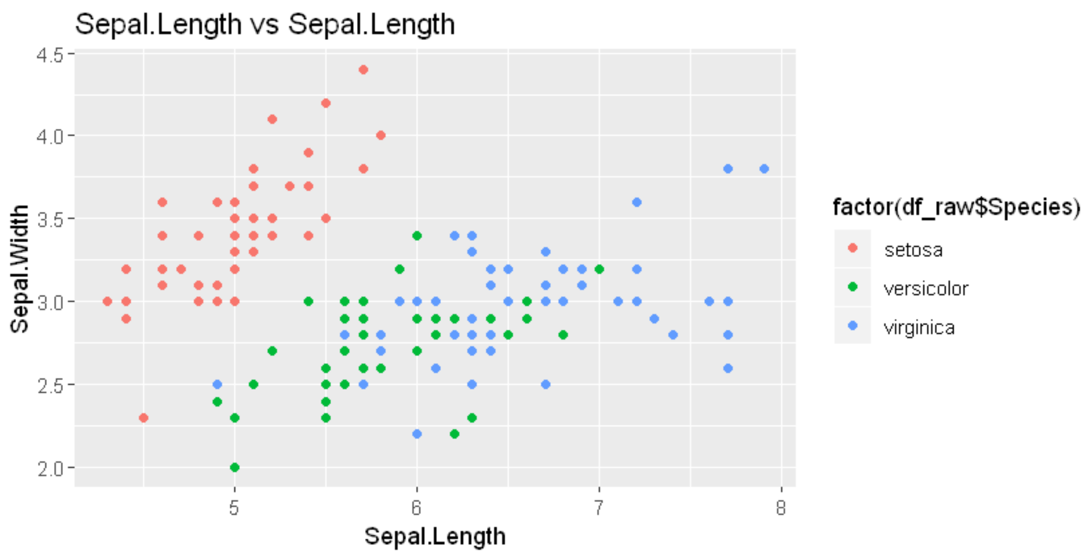
```
Welcome! Related Books: `Practical Guide To Cluster Analysis in R` at
https://goo.gl/13EFCZ
Loading required package: gridExtra
```

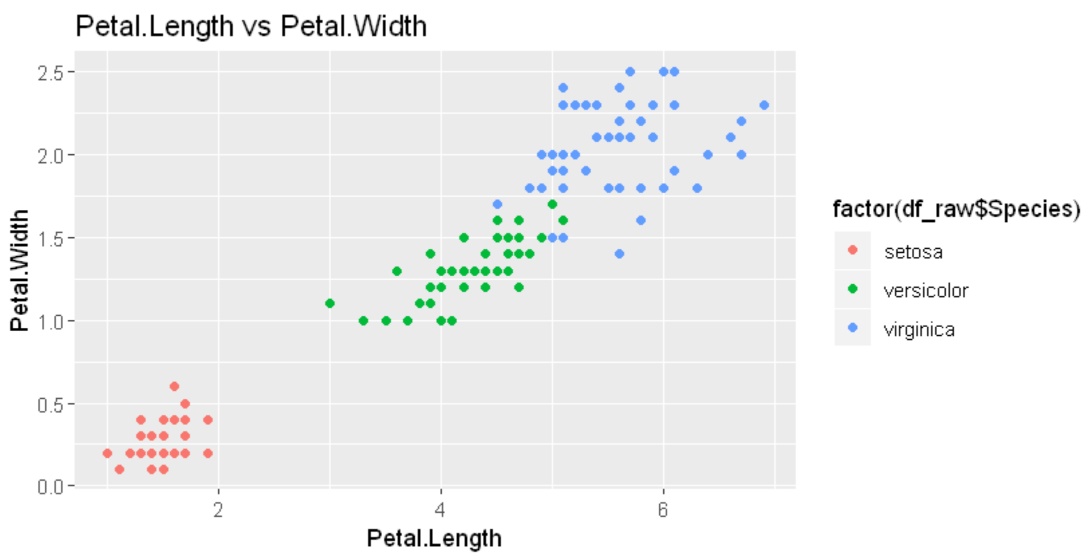
```
Attaching package: 'gridExtra'

The following object is masked from 'package:dplyr':

    combine
```

[10]:
```
# Read the iris.txt to a dataframe using read.table function.
# Writing the first four columns (containing the predictors) to a separate␣
 ↪dataframe.

df_raw <- read.table("iris.txt", header = TRUE)
df <- df_raw[,1:4]
head(df)
```

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
5.1	3.5	1.4	0.2
4.9	3.0	1.4	0.2
4.7	3.2	1.3	0.2
4.6	3.1	1.5	0.2
5.0	3.6	1.4	0.2
5.4	3.9	1.7	0.4

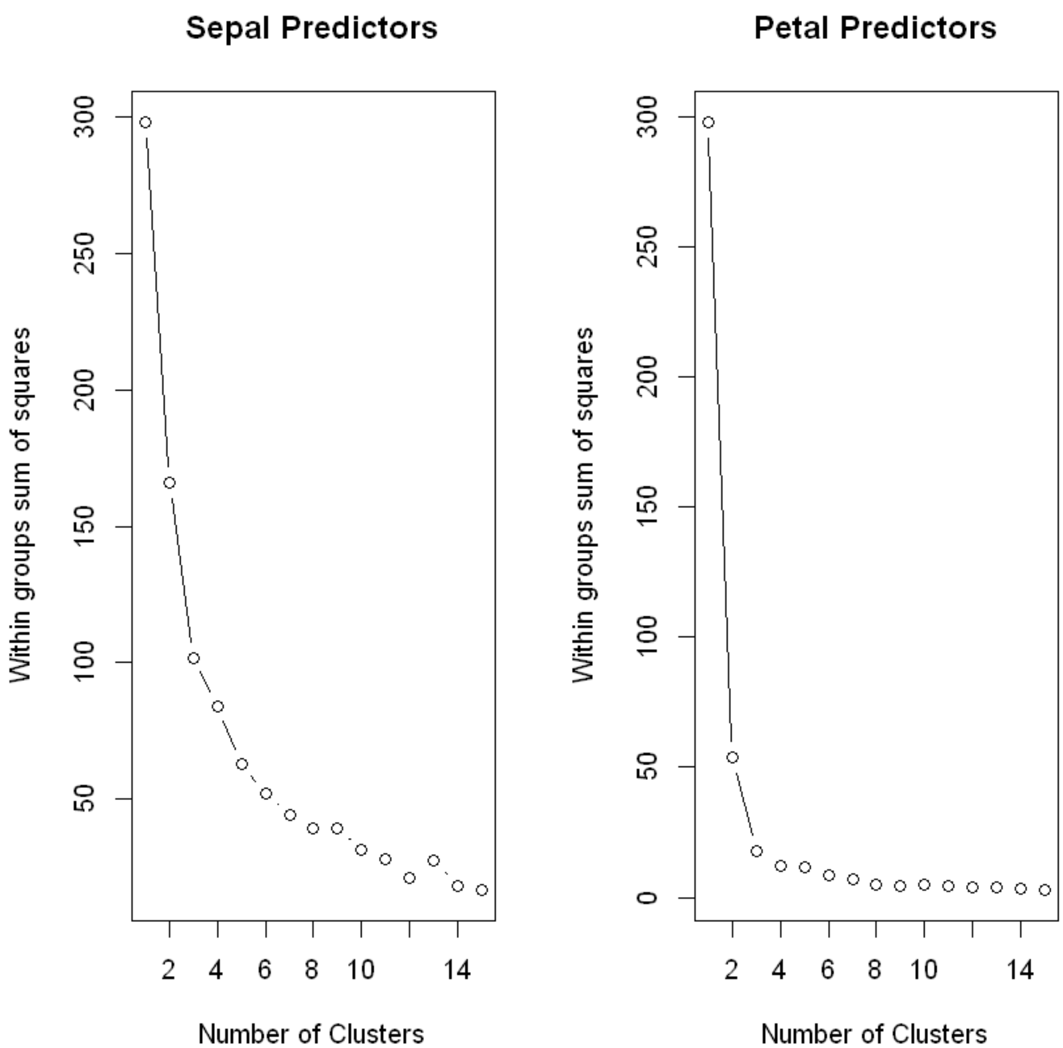
[11]:
```
# plotting the petal and sepal features separately to view the different␣
 ↪features. This helps understand the distincr feature
# that will help cluster the data.

library(ggplot2)
plot1 <- ggplot(df, aes(x = df[,1], y = df[,2]))+geom_point(aes(color = factor
                                                    ␣
 ↪(df_raw$Species)))+labs(x="Sepal.Length", y = "Sepal.Width",
                                                                          ␣
 ↪title="Sepal.Length vs Sepal.Length")
plot2 <- ggplot(df, aes(x = df[,3], y = df[,4]))+geom_point(aes(color = factor
                                                    ␣
 ↪(df_raw$Species)))+labs(x="Petal.Length", y = "Petal.Width",
                                                                          ␣
 ↪title="Petal.Length vs Petal.Width")
grid.arrange(plot1, plot2, ncol=1)

# It is clear the petal features can get a better clustering of the iris␣
 ↪dataset
```





[12]:
```
# For the next step, I am going to try to determine the number of clusters␣
↪using the elbow method. From the previous
# method, I determined that I am going to be using petal features as the input␣
↪for kmeans method. I am using the elbow
# method on both the petal and sepal features.

# FROM THE ELBOW METHOD, I am going to be using 3 clusters as input to kmeans␣
↪function.

sepals_df = df[,1:2]
petals_df = df[,3:4]

par(mfrow=c(1,2))
```

```
sepals_df <- scale(sepals_df)
wss <- (nrow(sepals_df)-1)*sum(apply(sepals_df,2,var))
for (i in 2:15) wss[i] <- sum(kmeans(sepals_df, centers=i)$withinss)
plot1 <- plot(1:15, wss, type="b", xlab="Number of Clusters", ylab="Within␣
 ↪groups sum of squares", main="Sepal Predictors")

petals_df <- scale(petals_df)
wss <- (nrow(petals_df)-1)*sum(apply(petals_df,2,var))
for (i in 2:15) wss[i] <- sum(kmeans(petals_df, centers=i)$withinss)
plot2 <- plot(1:15, wss, type="b", xlab="Number of Clusters", ylab="Within␣
 ↪groups sum of squares", main="Petal Predictors")
```



The plot above represents the variance within the clusters. It decreases as k increases, but it can be seen a bend (or "elbow") at k = 3 for the petal predictor. This bend indicates that additional clusters beyond the third have little value. In the next section, we'll classify the observations into 3 clusters.

[14]:
```
# kmeans(x, centers, iter.max = 10, nstart = 1, algorithm = c("Hartigan-Wong",
↪"Lloyd", "Forgy", "MacQueen"), trace=FALSE)

set.seed(123)
model <- kmeans(petals_df, 3, nstart = 25, iter.max = 10)
```

[15]:
```
cluster_centroids <- aggregate(petals_df,by=list(model$cluster),FUN=mean)
cluster_centroids
```

Group.1	Petal.Length	Petal.Width
1	-1.3006301	-1.2507035
2	1.0245672	1.1242119
3	0.3048515	0.1648655

[16]:
```
# Plotting the clusters and showing the location of the centroid in the cluster
fviz_cluster(object = model, data = df, geom = "point", choose.vars = c("Petal.
↪Length", "Petal.Width"),
             stand = FALSE, ellipse.type = "norm") + theme_bw()
```

1.3 Question 5.1

[17]:
```
install.packages("outliers", repos = 'http://cran.us.r-project.org')
```

```
package 'outliers' successfully unpacked and MD5 sums checked

The downloaded binary packages are in
        C:\Users\bgraa\AppData\Local\Temp\Rtmp4cz1Wh\downloaded_packages
```

[18]:
```
library(outliers)
crime_df <- read.table("uscrime.txt", header = TRUE)
head(crime_df, n=3)
```

M	So	Ed	Po1	Po2	LF	M.F	Pop	NW	U1	U2	Wealth	Ineq	Prob	Time
15.1	1	9.1	5.8	5.6	0.510	95.0	33	30.1	0.108	4.1	3940	26.1	0.084602	26.2011
14.3	0	11.3	10.3	9.5	0.583	101.2	13	10.2	0.096	3.6	5570	19.4	0.029599	25.2999
14.2	1	8.9	4.5	4.4	0.533	96.9	18	21.9	0.094	3.3	3180	25.0	0.083401	24.3006

[19]:
```
crime <- crime_df$Crime
grubbs.test(crime, type = 11, opposite = FALSE, two.sided = FALSE)
```

```
Grubbs test for two opposite outliers

data:  crime
G = 4.26880, U = 0.78103, p-value = 1
alternative hypothesis: 342 and 1993 are outliers
```

[ ]:

1.4 Problem 6.1

Describe a situation or problem from your job, everyday life, current events, etc., for which a Change Detection model would be appropriate. Applying the CUSUM technique, how would you choose the critical value and the threshold?

As a Performance analytics engineer at CAT, I bin the engine and machine performance metrics to view as a 1D histogram or heat maps. To correlate these histograms, I often find time weighted values of key performance indicators. Depending on the importance of the metric, I vary the time window from minutes to days. After tabulating the time weighted values, I compare the values to the threshold to detect failures.

For example, I recently performed fatigue analysis where I had to calculate remaining life of a truck component based on stress-strain values. I chose my critical value based on varying the elastic and plastic constants of the material of component. The threshold is a million cycles (General rule of thumb when looking at cyclic fatigue life of a material). I identified trucks and instances where the component lasted over the million cycles threshold to summarize optimum performance.

[ ]:

1.5 Problem 6.2 (a)

In this problem, I varied the values of C from 0 through 3, keeping the threshold at 75 degrees Farenheit. I calculated average temperatures of each year. For different values of C, I used CUSUM approach, based on the following equation:

S(t) = max{0, S(t-1)+(mu - x(t) - C)}

to identify the day in each year when temperature (in Farenheit) decreased to unofficially end summer.

From my solution, I plotted the unofficial end of summer day per year for each value of C. October 8 seemed to be the average of all years when summer unofficially ended.

Summer unofficially ended earliest in the year 2000 across all values of C. (for C=0, the minimum day of end of summer was September 17).

The plot corroborates this data.

1.6 Problem 6.2 (b)

Using CUSUM approach for C=0,1,2,3, the values of temperature seems to rise above threshold of 3 degrees in the year 2011 (and onwards) for C = 0. For C=1, 2012 and 2013 seem to be hotter than the previous years by 3 degrees, but gets cooler from 2014. So, for C=0. Atalanta seems to get warmer from 2011 (the day is September 19 for C=0 - calculated from the previous part of the problem).

The answer seems to complement the average temperature trend plotted in the chart.

[ ]:

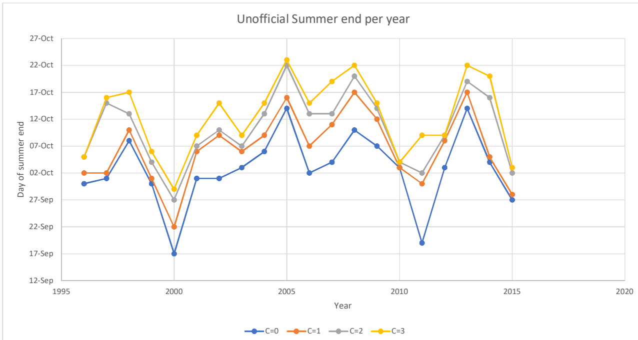
C	Threshold
3	75

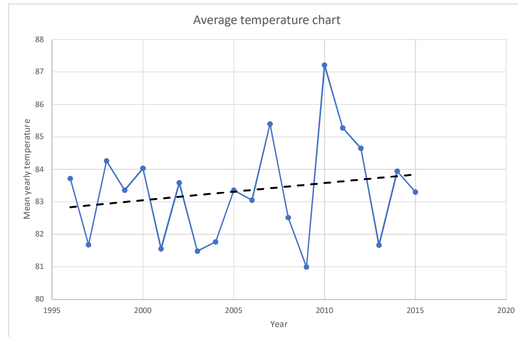
mu values	83.71544715	81.67479675	84.2601626	83.35772358	84.03252033	81.55284553	83.58537	81.47967	81.76422764	83.3577236	83.04878	85.39837	82.5122	80.99187	87.21138	85.27642	84.65041	81.66667	83.94309	83.30081
DAY	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
01-Jul	98	86	91	84	89	84	90	73	82	91	93	95	85	95	87	92	105	82	90	85
02-Jul	97	90	88	82	91	87	90	81	81	89	93	85	87	90	84	94	93	85	93	87
03-Jul	97	93	91	87	93	87	87	87	86	86	93	82	91	89	83	95	99	76	87	79
04-Jul	90	91	91	88	95	84	89	86	88	86	91	86	90	91	85	92	98	77	84	85
05-Jul	89	84	91	90	96	86	93	80	90	89	90	88	88	80	88	90	100	83	86	84
06-Jul	93	84	89	91	96	87	93	84	90	82	81	87	82	87	89	90	98	83	87	84
07-Jul	93	75	93	82	96	87	89	87	89	76	80	82	88	86	94	94	93	79	89	90
08-Jul	91	87	95	86	91	89	89	90	87	88	82	82	90	82	97	94	95	88	90	90
09-Jul	93	84	95	87	96	91	90	89	88	89	84	89	89	84	96	91	97	88	90	91
10-Jul	93	87	91	87	99	87	91	84	89	78	84	86	87	84	90	92	95	87	87	93
11-Jul	90	84	91	82	96	90	84	84	90	83	90	85	89	86	93	95	90	80	85	92
12-Jul	91	88	86	77	93	90	77	86	89	86	91	87	93	90	90	95	84	87	90	93
13-Jul	93	86	88	73	91	86	82	87	91	84	91	86	85	84	91	97	90	78	89	92
14-Jul	93	90	87	81	93	82	88	84	91	87	91	84	88	89	91	90	90	85	90	90
15-Jul	82	91	91	81	93	82	91	86	84	84	91	81	89	89	94	80	90	86	86	89
16-Jul	91	91	87	86	93	84	93	88	84	85	91	86	89	90	89	85	92	87	83	88
17-Jul	96	89	90	82	91	87	93	88	84	89	93	89	88	88	87	87	93	91	86	93
18-Jul	95	89	91	87	97	88	93	88	87	90	93	89	90	82	83	89	93	87	82	92
19-Jul	96	89	95	88	100	90	93	88	84	89	96	88	91	80	90	94	91	90	85	91
20-Jul	99	90	91	90	99	87	91	88	88	89	93	86	94	82	91	91	84	86	76	93
21-Jul	91	89	91	90	93	84	95	89	89	90	93	86	95	86	94	92	90	87	82	93
22-Jul	95	84	89	91	96	87	91	86	89	91	91	79	92	84	95	94	95	85	83	92
23-Jul	91	87	91	93	87	90	89	81	93	91	86	82	87	87	97	92	97	84	88	88
24-Jul	93	88	91	93	82	84	87	82	95	90	87	87	88	88	94	92	97	86	87	91
25-Jul	84	89	86	91	75	82	84	84	89	92	88	87	89	90	95	90	98	89	88	90
26-Jul	84	89	88	93	82	88	86	87	87	94	93	87	87	92	95	94	98	86	89	91
27-Jul	82	91	80	93	88	90	89	87	84	92	95	90	90	90	93	94	97	82	92	92
28-Jul	79	91	88	93	91	84	91	89	89	90	96	89	93	89	90	90	97	86	90	94
29-Jul	90	89	89	93	89	89	91	88	87	83	91	87	92	85	94	93	94	86	82	93
30-Jul	91	88	90	97	87	89	88	84	89	78	91	92	90	82	95	96	96	90	84	94
31-Jul	87	72	86	99	86	87	90	88	90	84	94	90	88	85	95	96	88	80	85	93
01-Aug	86	80	86	96	86	84	93	84	91	82	95	92	89	89	96	91	94	87	81	89
02-Aug	90	84	82	93	81	84	91	84	90	86	95	92	92	83	84	96	99	89	84	94
03-Aug	84	88	84	88	84	84	91	84	91	88	97	94	91	90	92	97	94	88	88	94
04-Aug	91	89	86	89	88	86	91	82	91	91	98	97	91	92	95	85	87	90	90	97
05-Aug	93	88	90	91	91	88	93	84	90	88	96	96	92	92	93	96	90	88	89	95
06-Aug	88	84	89	93	91	84	97	82	84	86	89	98	94	89	93	93	86	88	92	88
07-Aug	91	84	89	93	91	86	87	84	81	80	97	98	90	91	91	93	84	86	95	88
08-Aug	84	80	86	93	91	88	87	84	82	82	96	100	86	92	93	94	92	83	90	92
09-Aug	90	73	82	91	96	87	86	86	84	85	95	103	85	93	94	91	88	89	89	93
10-Aug	89	80	87	90	95	88	88	87	75	83	96	103	85	93	94	95	87	90	86	94
11-Aug	88	86	88	96	89	86	89	84	82	87	88	100	88	95	95	94	85	90	83	91
12-Aug	86	88	84	98	89	86	91	81	80	88	84	90	81	86	95	95	88	90	88	90
13-Aug	84	88	86	97	89	81	91	87	77	86	81	100	81	90	96	95	91	89	84	89
14-Aug	86	87	80	98	89	87	89	89	82	90	87	99	84	90	89	94	88	83	85	90
15-Aug	89	88	82	93	94	84	88	90	82	92	86	102	87	90	90	88	85	73	87	90
16-Aug	90	91	86	93	97	90	90	86	84	89	89	101	86	88	90	90	91	67	88	90
17-Aug	91	91	84	96	99	91	91	89	86	90	86	101	85	87	91	92	87	66	89	89
18-Aug	91	89	87	98	101	91	93	90	86	90	88	97	86	88	93	94	87	77	89	88
19-Aug	90	89	90	98	101	87	91	90	89	89	88	95	90	90	92	96	84	82	86	89
20-Aug	89	88	79	89	97	86	93	87	88	92	93	96	90	88	93	93	84	84	89	88
21-Aug	90	82	84	91	87	88	93	88	82	94	91	99	85	88	93	94	88	84	92	89
22-Aug	91	79	87	91	86	90	91	88	84	93	88	104	82	85	94	98	84	88	93	92
23-Aug	91	81	87	90	88	88	95	90	84	87	87	98	78	81	93	92	88	90	93	87
24-Aug	91	82	88	80	92	93	93	89	87	85	83	95	83	86	90	93	86	84	88	89
25-Aug	84	84	90	82	92	90	91	88	82	84	85	94	78	87	89	95	85	82	84	84
26-Aug	88	87	91	89	90	91	88	89	86	84	88	92	83	90	90	99	90	82	86	86
27-Aug	84	90	89	88	90	91	84	90	88	86	88	88	80	83	89	95	90	86	88	85
28-Aug	86	90	90	90	92	81	82	91	90	86	90	88	86	75	87	95	80	90	91	83
29-Aug	88	91	93	91	92	86	82	89	87	85	90	89	89	86	84	93	86	92	92	81
30-Aug	84	91	93	91	88	81	78	88	88	85	88	89	89	79	85	90	80	87	88	74
31-Aug	82	88	91	84	87	82	77	89	87	85	80	86	88	79	89	92	89	90	89	84
01-Sep	80	88	87	88	79	80	84	88	82	85	85	84	81	71	90	95	91	90	90	87
02-Sep	73	91	84	91	81	75	84	86	80	88	86	83	85	78	91	96	89	84	90	90
03-Sep	87	93	77	84	82	73	89	87	81	87	85	88	83	79	92	95	85	90	92	89
04-Sep	84	81	90	93	87	81	95	87	82	85	88	91	85	83	84	80	77	89	82	92
05-Sep	87	81	91	96	81	90	93	84	84	81	83	89	88	83	85	78	85	89	89	87
06-Sep	89	82	89	96	66	88	91	73	81	81	85	85	87	85	90	75	85	88	91	85
07-Sep	89	86	90	91	66	87	88	75	86	83	80	86	89	84	91	69	92	88	90	85
08-Sep	89	88	89	91	75	86	87	81	73	85	83	88	90	87	93	73	88	91	84	84
09-Sep	91	84	79	77	80	86	91	82	84	86	83	89	88	84	92	81	83	90	84	87
10-Sep	84	80	78	87	82	89	95	79	84	84	85	89	87	80	94	84	84	89	86	85
11-Sep	86	82	81	87	84	87	95	80	84	84	84	89	83	75	96	86	83	89	90	86
12-Sep	88	86	84	87	86	84	90	81	81	86	82	86	87	81	89	87	81	90	92	78
13-Sep	78	87	89	86	87	84	75	84	79	88	70	85	86	80	86	89	81	87	86	75
14-Sep	79	87	87	87	86	86	78	82	79	88	80	81	88	82	91	92	83	82	78	77
15-Sep	86	88	87	89	80	77	91	82	73	91	82	82	79	79	91	86	87	84	80	80
16-Sep	82	88	88	81	75	77	88	81	75	88	83	76	80	82	89	72	86	89	86	79
17-Sep	82	90	87	81	73	81	86	81	80	86	85	78	69	73	95	79	83	79	86	83
18-Sep	78	88	82	82	73	81	81	81	79	88	85	79	82	80	93	77	79	78	85	83
19-Sep	79	91	80	79	84	82	80	84	78	90	79	82	81	74	92	77	81	84	84	87
20-Sep	79	95	82	68	87	84	86	87	73	90	73	81	79	81	96	82	79	86	83	89
21-Sep	78	89	82	79	77	86	84	82	75	90	75	78	75	79	95	86	85	73	87	77
22-Sep	81	70	88	72	73	87	77	75	80	86	82	86	84	84	92	80	87	82	82	76
23-Sep	84	80	84	75	81	88	82	81	84	87	86	83	82	83	91	83	81	82	77	81
24-Sep	84	82	81	78	84	69	73	80	82	88	84	89	78	85	88	82	78	71	78	74
25-Sep	87	66	82	81	82	66	69	82	81	85	75	87	82	87	93	88	82	67	77	67
26-Sep	84	70	84	82	68	72	75	82	79	77	78	84	80	85	76	86	86	78	74	71
27-Sep	79	64	87	78	71	75	75	82	72	86	79	85	77	80	81	84	88	79	78	71
28-Sep	75	68	80	80	75	78	79	73	78	85	81	85	86	83	76	79	86	77	74	75
29-Sep	72	77	75	77	73	71	73	66	78	85	70	81	86	72	79	84	84	76	71	77
30-Sep	64	86	75	71	75	71	79	71	80	82	75	79	86	74	76	78	72	77	84	85
01-Oct	66	75	86	73	77	75	82	72	82	83	83	80	74	76	79	65	75	82	86	71
02-Oct	72	73	78	75	79	80	84	68	82	85	81	82	74	75	78	68	72	82	85	66
03-Oct	84	75	77	84	82	81	84	66	80	83	82	77	80	76	68	75	74	82	78	66
04-Oct	70	78	82	71	81	80	82	77	81	85	84	80	83	74	67	80	82	85	65	70
05-Oct	66	81	82	73	82	79	87	78	80	81	86	81	83	62	70	83	82	84	71	73
06-Oct	64	82	73	71	73	70	86	75	75	72	76	82	82	71	73	81	83	84	78	76
07-Oct	60	82	82	73	66	68	80	73	75	72	72	83	82	79	81	79	68	74	82	81
08-Oct	78	82	69	73	55	79	71	73	73	73	72	83	72	80	82	78	63	72	86	82
09-Oct	70	80	72	72	55	66	66	73	71	70	79	81	75	85	85	72	70	76	86	81
10-Oct	72	82	73	72	64	73	70	73	71	77	80	81	77	74	86	68	73	80	86	71
11-Oct	69	82	78	73	71	75	78	66	77	82	80	67	78	77	86	65	75	79	86	73
12-Oct	69	79	78	70	73	78	84	78	73	74	71	72	77	66	80	73	79	81	85	76
13-Oct	73	80	78	64	75	78	79	78	64	77	62	74	77	73	80	74	75	82	85	81
14-Oct	79	68	75	75	75	75	68	78	63	78	69	78	80	66	73	77	77	77	75	78
15-Oct	81	63	79	73	77	75	57	69	62	79	70	78	81	61	78	80	77	68	69	81
16-Oct	80	57	78	77	80	62	66	72	71	76	59	76	83	61	76	84	74	74	70	77
17-Oct	82	66	77	80	80	60	64	68	75	75	71	82	69	51	80	85	75	72	80	70
18-Oct	66	64	78	71	80	64	68	70	73	81	77	77	67	55	78	80	74	73	76	66
19-Oct	63	69	82	66	73	71	71	75	68	83	76	76	65	61	82	67	73	63	73	64
20-Oct	68	70	75	60	73	75	73	78	71	83	69	75	66	68	77	59	71	70	73	71
21-Oct	79	70	73	64	75	79	71	84	73	80	69	78	72	71	80	63	76	72	77	76
22-Oct	81	62	63	73	79	80	64	78	73	67	70	72	68	74	78	68	79	69	70	79
23-Oct	69	63	63	57	75	81	59	78	70	70	53	81	62	72	76	70	78	63	72	81
24-Oct	73	62	72	59	75	79	68	73	73	56	56	59	54	69	81	73	79	66	74	76
25-Oct	73	75	75	64	78	73	60	73	78	54	55	61	67	65	76	76	80	56	77	71
26-Oct	75	71	79	69	75	64	68	68	79	61	62	68	70	65	85	77	80	61	84	67
27-Oct	75	57	79	75	78	51	69	64	81	63	66	67	59	60	76	79	70	69	84	56
28-Oct	81	55	79	73	80	55	75	57	78	62	63	70	50	71	74	74	56	64	77	78
29-Oct	82	64	78	72	75	63	75	70	75	64	72	62	59	75	68	59	56	75	73	70
30-Oct	82	66	82	75	77	72	68	77	78	69	73	67	65	66	71	61	56	78	68	70

31-Oct	81	60	79	75	78	71	60	75	82	70	68	71	67	69	75	65	65	74	63	62

DAY	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
01-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.211382	0	0	0	0	0
03-Jul	0	0	0	0	0	0	0	0	0	0	0	0.398374	0	0	1.422764	0	0	2.666667	0	1.300813
04-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.634146	0	0	4.333333	0	0
05-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.42E-14	0	0
06-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
07-Jul	0	3.674796748	0	0	0	0	0	0	0	4.35772358	0.04878	0.398374	0	0	0	0	0	0	0	0
08-Jul	0	0	0	0	0	0	0	0	0	0	0	0.796748	0	0	0	0	0	0	0	0
09-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10-Jul	0	0	0	0	0	0	0	0	0	2.35772358	0	0	0	0	0	0	0	0	0	0
11-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12-Jul	0	0	0	3.357723577	0	0	3.585366	0	0	0	0	0	0	0	0	0	0	0	0	0
13-Jul	0	0	0	10.71544715	0	0	2.170732	0	0	0	0	0	0	0	0	0	0	0.666667	0	0
14-Jul	0	0	0	10.07317073	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15-Jul	0	0	0	9.430894309	0	0	0	0	0	0	0	1.398374	0	0	0	2.276423	0	0	0	0
16-Jul	0	0	0	3.788617886	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17-Jul	0	0	0	2.146341463	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.211382	0	0	0	0	0
19-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.943089	0
21-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.886179	0
22-Jul	0	0	0	0	0	0	0	0	0	0	0	3.398374	0	0	0	0	0	0	1.829268	0
23-Jul	0	0	0	0	0	0	0	0	0	0	0	3.796748	0	0	0	0	0	0	0	0
24-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25-Jul	0	0	0	0	6.032520325	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26-Jul	0	0	0	0	5.06504065	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27-Jul	0	0	1.260162602	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28-Jul	1.715447154	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29-Jul	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30-Jul	0	0	0	0	0	0	0	0	0	2.35772358	0	0	0	0	0	0	0	0	0	0
31-Jul	0	6.674796748	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
01-Aug	0	5.349593496	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02-Aug	0	0.024390244	0	0	0.032520325	0	0	0	0	0	0	0	0	0	0.211382	0	0	0	0	0
03-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
05-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
06-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
07-Aug	0	0	0	0	0	0	0	0	0	0.35772358	0	0	0	0	0	0	0	0	0	0
08-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
09-Aug	0	5.674796748	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10-Aug	0	4.349593496	0	0	0	0	0	0	3.764227642	0	0	0	0	0	0	0	0	0	0	0
11-Aug	0	0	0	0	0	0	0	0	0.528455285	0	0	0	0	0	0	0	0	0	0	0
12-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13-Aug	0	0	0	0	0	0	0	0	1.764227642	0	0	0	0	0	0	0	0	0	0	0
14-Aug	0	0	1.260162602	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15-Aug	0	0	0.520325203	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5.666667	0	0
16-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17.33333	0	0
17-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30	0	0
18-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	31.66667	0	0
19-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	28.33333	0	0
20-Aug	0	0	2.260162602	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23	0	0
21-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17.66667	0	0
22-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8.333333	0	0
23-Aug	0	0	0	0	0	0	0	0	0	0	0	0	1.512195	0	0	0	0	0	0	0
24-Aug	0	0	0	0.357723577	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25-Aug	0	0	0	0	0	0	0	0	0	0	0	0	1.512195	0	0	0	0	0	0	0
26-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	2.99187	0	0	1.650407	0	0	0
29-Aug	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.211382	0	0	0	0	0
30-Aug	0	0	0	0	0	0	2.585366	0	0	0	0	0	0	0	0	0	1.650407	0	0	6.300813
31-Aug	0	0	0	0	0	0	6.170732	0	0	0	0.04878	0	0	0	0	0	0	0	0	2.601626
01-Sep	0.715447154	0	0	0	2.032520325	0	2.756098	0	0	0	0	0	0	6.99187	0	0	0	0	0	0
02-Sep	8.430894309	0	0	0	2.06504065	3.552845528	0	0	0	0	0	0	0	6.98374	0	0	0	0	0	0
03-Sep	2.146341463	0	4.260162602	0	1.097560976	9.105691057	0	0	0	0	0	0	0	5.97561	0	0	0	0	0	0
04-Sep	0	0	0	0	0	6.658536585	0	0	0	0	0	0	0	0.96748	0.211382	2.276423	4.650407	0	0	0
05-Sep	0	0	0	0	0.032520325	0	0	0	0	0	0	0	0	0	0	6.552846	1.300813	0	0	0
06-Sep	0	0	0	0	15.06504065	0	0	5.479675	0	0	0	0	0	0	0	13.82927	0	0	0	0
07-Sep	0	0	0	0	30.09756098	0	0	8.95935	0	0	0.04878	0	0	0	0	27.10569	0	0	0	0
08-Sep	0	0	0	0	36.1300813	0	0	6.439024	5.764227642	0	0	0	0	0	0	36.38211	0	0	0	0
09-Sep	0	0	2.260162602	3.357723577	37.16260163	0	0	2.918699	0.528455285	0	0	0	0	0	0	37.65854	0	0	0	0
10-Sep	0	0	5.520325203	0	36.19512195	0	0	2.398374	0	0	0	0	0	0	0	35.93496	0	0	0	0
11-Sep	0	0	5.780487805	0	33.22764228	0	0	0.878049	0	0	0	0	0	2.99187	0	32.21138	0	0	0	0
12-Sep	0	0	3.040650407	0	28.2601626	0	0	0	0	0	0	0	0	0	0	27.4878	0.650407	0	0	2.300813
13-Sep	2.715447154	0	0	0	22.29268293	0	5.585366	0	0	0	10.04878	0	0	0	0	20.76423	1.300813	0	0	7.601626
14-Sep	4.430894309	0	0	0	17.32520325	0	8.170732	0	0	0	10.09756	1.398374	0	0	0	11.04065	0	0	2.943089	10.90244
15-Sep	0	0	0	0	18.35772358	1.552845528	0	0	5.764227642	0	8.146341	1.796748	0.512195	0	0	7.317073	0	0	3.886179	11.20325
16-Sep	0	0	0	0	24.3902439	3.105691057	0	0	9.528455285	0	5.195122	8.195122	0.02439	0	0	17.5935	0	0	0	12.50407
17-Sep	0	0	0	0	32.42276423	0.658536585	0	0	8.292682927	0	0.243902	12.5935	10.53659	4.99187	0	20.86992	0	0	0	9.804878
18-Sep	2.715447154	0	0	0	40.45528455	0	0	0	8.056910569	0	0	15.99187	8.04878	2.98374	0	26.14634	2.650407	0.666667	0	7.105691
19-Sep	4.430894309	0	1.260162602	1.357723577	37.48780488	0	0.585366	0	8.821138211	0	1.04878	16.39024	6.560976	6.97561	0	31.42276	3.300813	0	0	0.406504
20-Sep	6.146341463	0	0.520325203	13.71544715	31.5203252	0	0	0	14.58536585	0	8.097561	17.78862	7.073171	3.96748	0	31.69919	5.95122	0	0	0
21-Sep	8.861788618	0	0	15.07317073	35.55284553	0	0	0	18.3495935	0	13.14634	22.18699	11.58537	2.95935	0	27.97561	2.601626	5.666667	0	3.300813
22-Sep	8.577235772	8.674796748	0	23.43089431	43.58536585	0	3.585366	3.479675	17.11382114	0	11.19512	18.58537	7.097561	0	0	30.25203	0	2.333333	0	7.601626
23-Sep	5.292682927	7.349593496	0	28.78861789	43.61788618	0	2.170732	0.95935	11.87804878	0	5.243902	17.98374	4.609756	0	0	29.52846	0.650407	0	3.943089	6.902439
24-Sep	2.008130081	4.024390244	0.260162602	31.14634146	40.6504065	9.552845528	9.756098	0	8.642276423	0	1.292683	11.38211	6.121951	0	0	29.80488	4.300813	7.666667	6.886179	13.20325
25-Sep	0	16.69918699	0	30.50406504	39.68292683	22.10569106	21.34146	0	6.406504065	0	6.341463	6.780488	3.634146	0	0	24.0813	3.95122	19.33333	10.82927	26.50407
26-Sep	0	25.37398374	0	28.86178862	52.71544715	28.65853659	26.92683	0	6.170731707	3.35772358	8.390244	5.178862	3.146341	0	8.211382	20.35772	0	20	17.77236	35.80488
27-Sep	1.715447154	40.04878049	0	31.2195122	62.74796748	32.21138211	32.5122	0	12.93495935	0	9.439024	2.577236	5.658537	0	11.42276	18.63415	0	19.66667	20.71545	45.10569
28-Sep	7.430894309	50.72357724	1.260162602	31.57723577	68.7804878	32.76422764	34.09756	5.479675	13.69918699	0	8.487805	0	0	0	19.63415	21.91057	0	21.33333	27.65854	50.4065
29-Sep	16.14634146	52.39837398	7.520325203	34.93495935	76.81300813	40.31707317	41.68293	17.95935	14.46341463	0	18.53659	1.398374	0	5.99187	24.84553	20.18699	0	24	37.60163	53.70732
30-Sep	32.86178862	45.07317073	13.7804878	44.29268293	82.84552846	47.8699187	43.26829	25.43902	13.22764228	0	23.58537	4.796748	0	9.98374	33.05691	24.46341	9.650407	25.66667	34.54472	49.00813
01-Oct	47.57723577	48.74796748	9.040650407	51.6504065	86.87804878	51.42276423	41.85366	31.9187	9.991869919	0	20.63415	7.195122	5.512195	11.97561	38.26829	41.73984	16.30081	22.33333	29.4878	58.30894
02-Oct	56.29268293	54.42276423	12.30081301	57.00813008	88.91056911	49.97560976	38.43902	42.39837	6.756097561	0	19.68293	7.593496	11.02439	14.96748	44.47967	56.01626	25.95122	19	25.43089	72.60976
03-Oct	53.00813008	58.09756098	16.56097561	53.36585366	87.94308943	47.52845528	35.02439	54.87805	5.520325203	0	17.73171	12.99187	10.53659	16.95935	60.69106	63.29268	33.60163	15.66667	28.37398	86.91057
04-Oct	63.72357724	58.77235772	15.82113821	62.72357724	87.97560976	46.08130081	33.60976	56.35772	3.284552846	0	13.78049	15.39024	7.04878	20.95122	77.90244	65.56911	33.25203	9.333333	44.31707	97.21138
05-Oct	78.43902439	56.44715447	15.08130081	70.08130081	87.00813008	45.63414634	27.19512	56.8374	2.048780488	0	7.829268	16.78862	3.560976	36.94309	92.11382	64.84553	32.90244	4	54.26016	104.5122
06-Oct	95.15447154	53.12195122	23.34146341	79.43902439	95.04065041	54.18699187	21.78049	60.31707	5.81300813	8.35772358	11.87805	17.18699	1.073171	43.93496	103.3252	66.12195	31.55285	0	57.20325	108.813
07-Oct	115.8699187	49.79674797	22.60162602	86.79674797	110.0731707	64.7398374	22.36585	65.79675	9.577235772	16.7154472	19.92683	16.58537	0	42.92683	106.5366	69.39837	45.20325	4.666667	56.14634	108.1138
08-Oct	118.5853659	46.47154472	34.86178862	94.15447154	136.1056911	64.29268293	31.95122	71.27642	15.34146341	24.0731707	27.97561	15.98374	7.512195	40.9187	108.748	73.6748	63.85366	11.33333	51.08943	106.4146
09-Oct	129.300813	45.14634146	44.12195122	102.5121951	162.1382114	76.84552846	46.53659	76.7561	23.10569106	34.4308943	29.02439	17.38211	12.02439	33.91057	107.9593	83.95122	75.50407	14	46.03252	105.7154
10-Oct	138.0162602	41.82113821	52.38211382	110.8699187	179.1707317	82.39837398	57.12195	82.23577	30.8699187	37.7886179	29.07317	18.78049	14.53659	37.90244	106.1707	98.22764	84.15447	12.66667	40.97561	115.0163
11-Oct	149.7317073	38.49593496	55.64227642	118.2276423	189.203252	85.95121951	59.70732	94.71545	32.63414634	36.1463415	29.12195	34.17886	16.04878	38.89431	104.3821	115.5041	90.80488	12.33333	35.9187	122.3171
12-Oct	161.4471545	38.17073171	58.90243902	128.5853659	197.2357724	86.50406504	56.29268	95.19512	38.39837398	42.504065	38.17073	44.57724	18.56098	50.88618	108.5935	124.7805	93.45528	10	31.86179	126.6179
13-Oct	169.1626016	36.84552846	62.16260163	144.9430894	203.2682927	87.05691057	57.87805	95.6748	53.16260163	45.8617886	56.21951	52.97561	21.07317	55.87805	112.8049	133.0569	100.1057	6.666667	27.80488	125.9187
14-Oct	170.8780488	47.5203252	68.42276423	150.300813	209.300813	90.6097561	70.46341	96.15447	68.92682927	48.2195122	67.26829	57.37398	20.58537	67.86992	124.0163	138.3333	104.7561	8.333333	33.74797	128.2195
15-Oct	170.5934959	63.19512195	70.68292683	157.6585366	213.3333333	94.16260163	94.04878	105.6341	85.69105691	49.5772358	77.31707	61.77236	19.09756	84.86179	130.2276	140.6098	109.4065	19	45.69106	127.5203
16-Oct	171.3089431	84.8699187	73.94308943	161.0162602	214.3658537	110.7154472	108.6341	112.1138	93.45528455	53.9349593	98.36585	68.17073	15.60976	101.8537	138.439	138.8862	117.0569	23.66667	56.63415	130.8211
17-Oct	170.0243902	97.54471545	78.20325203	161.3739837	215.398374	129.2682927	125.2195	122.5935	97.2195122	59.2926829	107.4146	68.56911	26.12195	128.8455	142.6504	136.1626	123.7073	30.33333	57.57724	141.122
18-Oct	184.7398374	112.2195122	81.46341463	170.7317073	216.4308943	143.8211382	137.8049	131.0732	102.9837398	58.6504065	110.4634	73.96748	38.63415	151.8374	148.8618	138.439	131.3577	36	62.52033	155.4228
19-Oct	202.4552846	121.8943089	80.72357724	185.0894309	224.4634146	151.3739837	147.3902	134.5528	113.7479675	56.0081301	114.5122	80.36585	53.14634	168.8293	151.0732	153.7154	140.0081	51.66667	70.46341	171.7236
20-Oct	215.1707317	130.5691057	86.98373984	205.4471545	232.495935	154.9268293	154.9756	135.0325	121.5121951	53.3658537	125.561	87.76423	66.65854	178.8211	158.2846	176.9919	150.6585	60.33333	78.4065	181.0244
21-Oct	216.8861789	139.2439024	95.24390244	221.804878	238.5284553	154.4796748	164.561	129.5122	127.2764228	53.7235772	136.6098	92.1626	74.17073	185.813	162.4959	196.2683	156.3089	67	82.34959	185.3252
22-Oct	216.601626	155.9186992	113.504065	229.1626016	240.5609756	153.0325203	181.1463	129.9919	133.0406504	67.0813008	146.6585	102.561	85.68293	189.8049	168.7073	210.5447	158.9593	76.66667	93.29268	186.626
23-Oct	228.3170732	171.5934959	131.7642276	252.5203252	246.5934959	150.5853659	202.7317	130.4715	141.804878	77.4390244	173.7073	103.9593	103.1951	195.7967	176.9187	222.8211	162.6098	92.33333	102.2358	185.9268
24-Oct	236.0325203	188.2682927	141.0243902	273.8780488	252.6260163	150.1382114	215.3171	135.9512	147.5691057	101.796748	197.7561	127.3577	128.7073	204.7886	180.1301	232.0976	165.2602	105	109.1789	190.2276
25-Oct	243.7479675	191.9430894	147.2845528	290.2357724	255.6585366	155.6910569	235.9024	141.4309	148.3333333	128.154472	222.8049	148.7561	141.2195	217.7805	188.3415	238.374	166.9106	127.6667	113.122	199.5285
26-Oct	249.4634146	199.6178862	149.5447154	301.5934959	261.6910569	170.2439024	248.4878	151.9106	148.097561	147.512195	240.8537	163.1545	150.7317	230.7724	187.5528	243.6504	168.561	145.3333	110.065	212.8293
27-Oct	255.1788618	221.2926829	151.804878	306.9512195	264.7235772	197.796748	260.0732	166.3902	145.8617886	164.869919	254.9024	178.5528	171.2439	248.7642	195.7642	246.9268	180.2114	155	107.0081	237.1301
28-Oct	254.8943089	244.9674797	154.0650407	314.3089431	265.7560976	221.3495935	265.6585	187.8699	146.6260163	183.227642	271.9512	190.9512	200.7561	255.7561	205.9756	255.2033	205.8618	169.6667	110.9512	239.4309
29-Oct	253.6097561	259.6422764	157.3252033	322.6666667	271.7886179	236.902439	271.2439	196.3496	150.3902439	199.585366	280	211.3496	221.2683	258.748	222.187	278.4797	231.5122	173.3333	118.8943	249.7317
30-Oct	252.3252033	273.3170732	156.5853659	328.0243902	275.8211382	243.4552846	283.8293	197.8293	151.1544715	210.943089	287.0488	226.748	235.7805	270.7398	235.3984	299.7561	257.1626	174	131.8374	260.0325
31-Oct	252.0406504	290.9918699	158.8455285	333.3821138	278.8536585	251.0081301	304.4146	201.3089	147.9186992	221.300813	299.0976	238.1463	248.2927	279.7317	244.6098	317.0325	273.813	178.6667	149.7805	278.3333

The first day of temperature drop for each year is (unofficial summer ends)																						
	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	
0	30-Sep	01-Oct	08-Oct	30-Sep	17-Sep	01-Oct	01-Oct	03-Oct	06-Oct	14-Oct	02-Oct	04-Oct	10-Oct	07-Oct	03-Oct	19-Sep	03-Oct	14-Oct	04-Oct	27-Sep	
1	02-Oct	02-Oct	10-Oct	01-Oct	22-Sep	06-Oct	09-Oct	06-Oct	09-Oct	16-Oct	07-Oct	11-Oct	17-Oct	12-Oct	03-Oct	30-Sep	08-Oct	17-Oct	05-Oct	28-Sep	
2	05-Oct	15-Oct	13-Oct	04-Oct	27-Sep	07-Oct	10-Oct	07-Oct	13-Oct	22-Oct	13-Oct	13-Oct	20-Oct	14-Oct	04-Oct	02-Oct	09-Oct	19-Oct	16-Oct	02-Oct	
3	05-Oct	16-Oct	17-Oct	06-Oct	29-Sep	09-Oct	15-Oct	09-Oct	15-Oct	23-Oct	15-Oct	19-Oct	22-Oct	15-Oct	04-Oct	09-Oct	09-Oct	22-Oct	20-Oct	03-Oct	
AVERAGE	03-Oct	08-Oct	12-Oct	02-Oct	23-Sep	05-Oct	08-Oct	06-Oct	10-Oct	18-Oct	09-Oct	11-Oct	17-Oct	12-Oct	03-Oct	30-Sep	07-Oct	18-Oct	11-Oct	30-Sep	08-Oct





	mu	83.33902439				
		C ---->	0	1	2	3
	xt	xt-mu	xt - mu - C	xt - mu - C	xt - mu - C	xt - mu - C
1996	83.715	0.376	0.376	-0.624	-1.624	-2.624
1997	81.675	-1.664	-1.664	-2.664	-3.664	-4.664
1998	84.260	0.921	0.921	-0.079	-1.079	-2.079
1999	83.358	0.019	0.019	-0.981	-1.981	-2.981
2000	84.033	0.693	0.693	-0.307	-1.307	-2.307
2001	81.553	-1.786	-1.786	-2.786	-3.786	-4.786
2002	83.585	0.246	0.246	-0.754	-1.754	-2.754
2003	81.480	-1.859	-1.859	-2.859	-3.859	-4.859
2004	81.764	-1.575	-1.575	-2.575	-3.575	-4.575
2005	83.358	0.019	0.019	-0.981	-1.981	-2.981
2006	83.049	-0.290	-0.290	-1.290	-2.290	-3.290
2007	85.398	2.059	2.059	1.059	0.059	-0.941
2008	82.512	-0.827	-0.827	-1.827	-2.827	-3.827
2009	80.992	-2.347	-2.347	-3.347	-4.347	-5.347
2010	87.211	3.872	3.872	2.872	1.872	0.872
2011	85.276	1.937	1.937	0.937	-0.063	-1.063
2012	84.650	1.311	1.311	0.311	-0.689	-1.689
2013	81.667	-1.672	-1.672	-2.672	-3.672	-4.672
2014	83.943	0.604	0.604	-0.396	-1.396	-2.396
2015	83.301	-0.038	-0.038	-1.038	-2.038	-3.038

	Threshold	3		
C ---->	0	1	2	3
	St	St	St	St
	0	0	0	0
	0.376	0.000	0.000	0.000
	0.000	0.000	0.000	0.000
	0.921	0.000	0.000	0.000
	0.940	0.000	0.000	0.000
	1.633	0.000	0.000	0.000
	0.000	0.000	0.000	0.000
	0.246	0.000	0.000	0.000
	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000
	0.019	0.000	0.000	0.000
	0.000	0.000	0.000	0.000
	2.059	1.059	0.059	0.000
	1.233	0.000	0.000	0.000
	0.000	0.000	0.000	0.000
	3.872	2.872	1.872	0.872
	5.810	3.810	1.810	0.000
	7.121	4.121	1.121	0.000
	5.449	1.449	0.000	0.000
	6.053	1.053	0.000	0.000

PROBLEM 6.2 (b)