Homework 1

ISyE 6420
Fall 2019

Due September 8, 2019, 11:55pm. HW1 is not time limited except the due date. Late submissions will not be accepted.

Use of all available electronic and printed resources is allowed except direct communication that violates Georgia Tech Academic Integrity Rules.

1. Circuit.

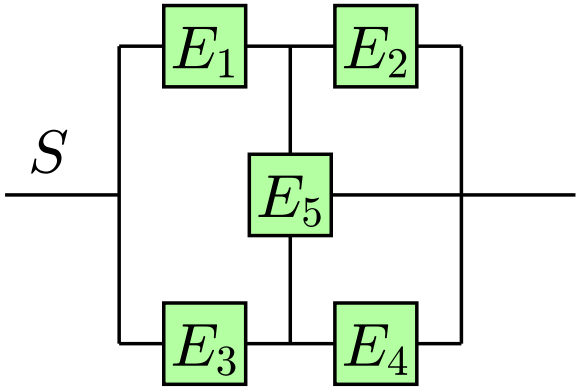

Figure 1: Components $E_1, \ldots, E_5$ at operational at time t with probabilities $e^{-t}, e^{-2t}, e^{-t/2}, e^{-t/3}$ and e^{-t}, respectively.

The system S consists of five independent elements E_i, $i = 1, \ldots, 5$, connected as in Figure 1. Probability that the element E_i is operational at the end of time interval $[0, t]$ is given as

$$p_i(t) = e^{-\lambda_i t}, \ t \geq 0,$$

for $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 1/2, \lambda_4 = 1/3$, and $\lambda_5 = 1$.

(a) Find the probability that the system S will be operational at time t. Plot this probability as a function of time t. What is this probability for $t = 1/2$.

(b) Find the probability that component E_5 was operational at time $t = 1/2$, if the system was operational at that time.

Hint: If you consider (b), it is conditional probability, more precisely, a posterior probability of the hypothesis $H_1 : E_5$ operational at time t, given that system S is operational at t. Thus,

solve part (a) as a total probability with H_1 and $H_2 = H_1^c$ as hypotheses. Under the two hypotheses the system simplifies as in Figure 2 and it is easy to find $P(S|H_1)$ and $P(S|H_2)$. Then (b) is just a Bayes formula. The results for arbitrary t will be messy – do not simplify. For plotting in part (a) take some reasonable interval for time t, say $[0, 4]$.

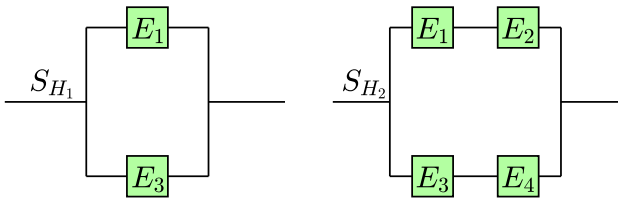

Figure 2: Left: System under hypothesis $H_1 : E_5$ operational; Right: System under hypothesis $H_2 : E_5$ not operational.

2. Two Batches. There are two batches of the same product. In one batch all products are conforming. The other batch contains 20% non-conforming products. A batch is selected at random and one randomly selected product from that batch is inspected. The inspected product was found conforming and was returned back to its batch.

What is the probability that the second product, randomly selected from the same batch, is found non-conforming?

Hint. This problem uses both Bayes' rule and Total Probability. The two hypotheses concern the type of batch. For the first draw the hypotheses are equally likely (the batch is selected at random), but for the second draw, the probabilities of hypotheses are updated by the information on the result of the first draw via Bayes rule. Updated probabilities of hypotheses are then used in the Total Probability Formula for the second draw.

3. Classifier. In a machine learning classification procedure the items are classified as 1 or 0. Based on a training sample of size 120 in which there are 65 1's and 55 0's, the classifier predicts 70 1's and 50 0's. Out of 70 items predicted by the classifier as 1, 52 are correctly classified.

From the population of items where the proportion of 0-labels is 99% (and 1-labels 1%), an item is selected at random. What is the probability that the item is of label 1, if the classifier says it was.

Hint. Think about the following interpretation. If **1** is a specific disease present, **0** no disease present, and the classifier is a medical test for the disease, then you are asked to

find a positive predictive value of a test for a subject coming from population where the prevalence of the disease is 1%.