

Lessons in this Module

- A. A Customer Analytics Dataset to Illustrate Indicator Variables
- B. Creating and Using Indicator (Dummy) Variables
- C. Interpreting the Coefficients of Indicator Variables
- D. Interaction Term and Interpreting its Coefficient
- E. Another Example of Using Indicator Variables

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Direct Marketing Dataset

- A direct marketing firm has a data set containing information on past customer behavior (actually the amount spent on buying products)
- This is a simulated data which mimics data from a direct marketing company
- We are interested in knowing which customer characteristics can predict AmountSpent (amount spent on buying products)
- To answer questions like this we introduce indicator variable and interaction terms and their interpretation

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The Direct Marketing Dataset

```
Read the file "direct_marketing.csv" into a dataframe called dirmkt dirmkt <- read_csv("direct_marketing.csv", col_types = list(
```

Age = col_factor(c("Old", "Middle", "Young")), # age group category

Gender = col_factor(c("Female", "Male")), # Gender category

OwnHome = col_factor(c("Own", "Rent")), # home owner or renter

Married = col_factor(c("Single", "Married")), # single or married

Location = col_factor(c("Far", "Close")), # near a store or far away

Salary = col_double(), # annual salary

Children = col_integer(), # number of children

History = col_factor(c("High", "Low", "Medium", "None")), # type of customer

Catalogs = col_integer(), # number of catalogs sent to this customer

AmountSpent = col_double())) # \$ amount of purchases made by this customer

The Direct Marketing Dataset...

str(dirmkt) # what happened to the first row of the csv file?

Classes 'tbl_df', 'tbl' and 'data.frame': 1000 obs. of 10 variables:

\$ Age : Factor w/ 3 levels "Old", "Middle", ...: 1 2 3 2 2 3 2 2 2 1 ...

 $\$ Gender $\$: Factor w/ 2 levels "Female", "Male": 1 2 1 2 1 2 1 2 1 2 ...

\$ OwnHome : Factor w/ 2 levels "Own", "Rent": 1 2 2 1 1 1 2 1 1 1 ...

 $\$ Married : Factor w/ 2 levels "Single", "Married": 1 1 1 2 1 2 1 1 2 2 ...

\$ Location : Factor w/ 2 levels "Far", "Close": 1 2 2 2 2 2 2 2 2 1 ...

\$ Salary : num 47500 63600 13500 85600 68400 30400 48100 68400 51900 80700 ...

\$ Children : int 0 0 0 1 0 0 0 0 3 0 ...

\$ History : Factor w/ 4 levels "High", "Low", "Medium", ..: 1 1 2 1 1 2 3 1 2 4 ...

\$ Catalogs : int 6 6 18 18 12 6 12 18 6 18 ...

\$ AmountSpent: num 75.5 131.8 29.6 243.6 130.4 ...

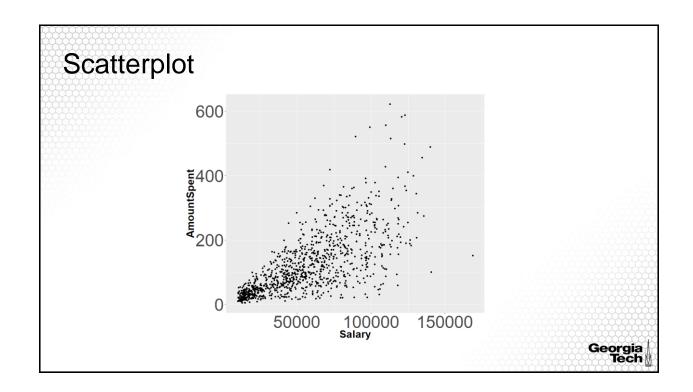
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First 10 Rows of the dirmkt Dataframe

Age	Gender	OwnHome	Married	Location	Salary	Children	History	Catalogs	AmountSpent
Old	Female	Own	Single	Far	47500	0	High	6	75.5
Middle	Male	Rent	Single	Close	63600	0	High	6	131.8
Young	Female	Rent	Single	Close	13500	0	Low	18	29.6
Middle	Male	Own	Married	Close	85600	1	High	18	243.6
Middle	Female	Own	Single	Close	68400	0	High	12	130.4
Young	Male	Own	Married	Close	30400	0	Low	6	49.5
Middle	Female	Rent	Single	Close	48100	0	Medium	12	78.2
Middle	Male	Own	Single	Close	68400	0	High	18	115.5
Middle	Female	Own	Married	Close	51900	3	Low	6	15.8
Old	Male	Own	Married	Far	80700	0	None	18	303.4

Exploring the dirmkt Dataframe

- We would like to understand better the reasons why some individuals spend more than others
- In particular, we would like to investigate whether salary has an influence on AmountSpent
- · So, how do we get started?
- To keep matters clear, we start with AmountSpent, Salary

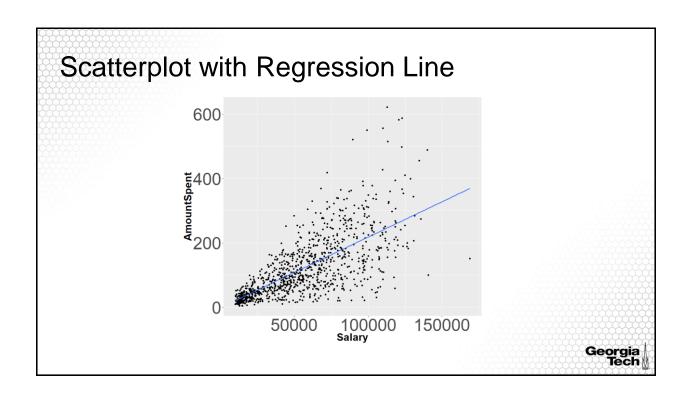


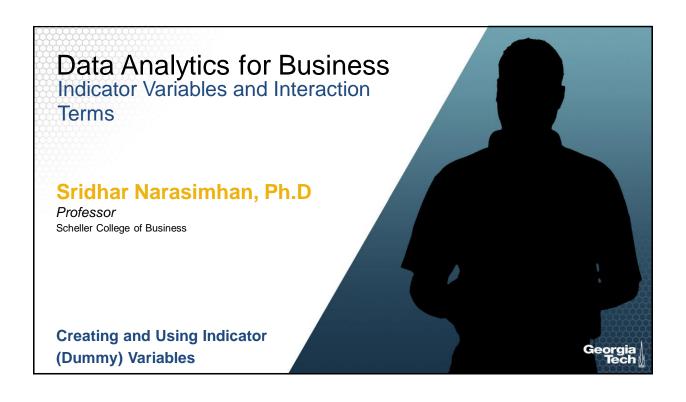
RS: Simple Regression

	Estimate	S.E.	t Value	Pr> t
Intercept	-1.531783	4.537416	-0.338	0.736
Salary	0.002196	0.000071	30.930 ***	<.001

R-squared	Adjusted R-squared	
0.722	0.721	

• AmountSpent = b_0 + b_1 *Salary





What to do about the Categorical Variable

Age?

- Is this categorical variable important?
- Does being Middle-aged or Old potentially have an effect on AmountSpent compared to being Young?
- How can we include the Age variable in a regression model that requires numeric values?

Salary	AmountSpent
47500	75.5
63600	131.8
13500	29.6
85600	243.6
68400	130.4
30400	49.5
48100	78.2
68400	115.5
51900	15.8
80700	303.4
	47500 63600 13500 85600 68400 30400 48100 68400 51900

Doing Regression with Qualitative Predictor Variable

- Consider the variable Age
- We want to investigate the effect of Age on AmountSpent. Note that Age is a qualitative (or categorical) variable with three possible values: Young, Middle, or Old
- · We need to quantify this variable

Age
Old
Middle
Young
Middle
Middle
Young
Middle
Middle
Middle
Old

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Creating Indicator (Dummy) Variables

- Since we have three possible values for Age, we need to create two indicator (or dummy) variables
- The base (or reference) case, with both dummy variables set to 0, is Age = Young. This is the reference group to compare for the other values of the dummy variable. It is up to the modeler to determine which value of the categorical variable is used as the base case
- The two dummy variables that we have created are:

$$\mathsf{AgeMid} = \begin{cases} 1, & \textit{if Age} = \textit{Middle} \\ 0, & \textit{otherwise} \end{cases}$$

$$AgeOld = \begin{cases} 1, & if Age = Old \\ 0, & otherwise \end{cases}$$

Assigning Values (0 or 1) to the New Indicator (Dummy) Variables

$$AgeMid = \begin{cases} 1, & if Age = Middle \\ 0, & otherwise \end{cases}$$

$$AgeOld = \begin{cases} 1, & if Age = Old \\ 0, & otherwise \end{cases}$$

We then run the regression, $AmountSpent = b_0 + b_1*AgeMid + b_2*AgeOld$

90	AgeOld
0	1
1	0
0	0
1	0
1	0
0	0
1	0
1	0
1	0
0	1
	1 0 1 1 0 1 1 1 1 1 1

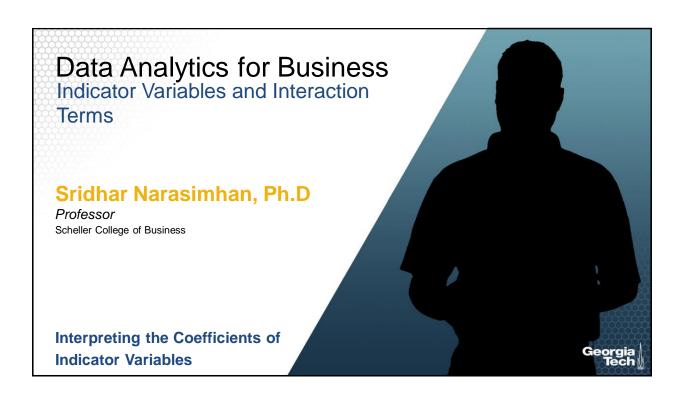
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Quiz

With this Indicator variables coding scheme,

$$\mathsf{AgeMid} = \begin{cases} 1, & \textit{if Age} = \textit{Middle} \\ 0, & \textit{otherwise} \end{cases} \qquad \mathsf{AgeOld} = \begin{cases} 1, & \textit{if Age} = \textit{Old} \\ 0, & \textit{otherwise} \end{cases}$$

- Can a record in the dirmkt dataframe have this value (AgeMid = 0, AgeOld = 0)?
 Answer: YES, because this record is for someone who is Young, i.e., the base case.
- Can a record in the dirmkt dataframe have this value (AgeMid = 1, AgeOld = 1)?
 Answer: NO, every individual has to be in exactly one age category



A Linear Model With Indicator Variables

$$\mathsf{AgeMid} = \begin{cases} 1, & \textit{if Age} = \textit{Middle} \\ 0, & \textit{otherwise} \end{cases} \qquad \mathsf{AgeOld} = \begin{cases} 1, & \textit{if Age} = \textit{Old} \\ 0, & \textit{otherwise} \end{cases}$$

With this Indicator variables coding scheme, We then run the regression, $AmountSpent = b_0 + b_1*AgeMid + b_2*AgeOld$

We then fit it using the data in dirmkt

DR1: 1st Regression with Dummy Variable

	Estimate	S.E.	t Value	Pr> t
Intercept	55.862	5.112	10.93***	<.001
AgeMid	94.307	6.395	14.75***	<.001
AgeOld	87.350	7.919	11.03***	<.001

- AmountSpent = $b_0 + b_1*AgeMid + b_2*AgeOld$
- Which age group's Average AmountSpent is \$55.862? Young, Middle, or Old?
- Correct Answer: With AgeMid = 0 and AgeOld = 0, b₀ captures the average AmountSpent of customers who are Young (base case)

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AmountSpent = b0 + b1*AgeMid + b2*AgeOld

	Estimate	S.E.	t Value	Pr> t
Intercept	55.862	5.112	10.93***	<.001
AgeMid	94.307	6.395	14.75***	<.001
AgeOld	87.350	7.919	11.03***	<.001

- What is the Average AmountSpent for someone who is middle-aged?
- This individual has AgeMid = 1 and AgeOld = 0, so $b_0 + b_1$ captures the average AmountSpent for folks who are middle-aged
- $b_0 + b_1 = $55.862 + $94.307 = 150.169
- So, \$94.307 is the increase in AmountSpent (on average) for middle-aged customers compared to someone who is young

AmountSpent = b0 + b1*AgeMid + b2*AgeOld

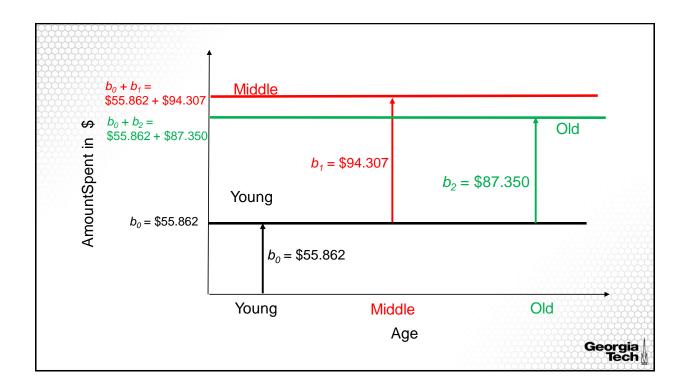
	Estimate	S.E.	t Value	Pr> t
Intercept	55.862	5.112	10.93***	<.001
AgeMid	94.307	6.395	14.75***	<.001
AgeOld	87.350	7.919	11.03***	<.001

- What is the Average AmountSpent for someone who is old?
- This individual has AgeMid = 0 and AgeOld = 1, so $b_0 + b_2$ captures the average AmountSpent for customers who are old
- $b_0 + b_2 = $55.862 + $87.350 = 143.212
- So, \$87.350 is the increase in AmountSpent (on average) for old customers compared to someone who is young

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Graphically

Let's take a look at this graphically...



Important Note

You can directly use a Factor Variable in regression in R instead of creating & using Dummy variables

Im(AmountSpent ~ Age, data=dirmkt)

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	143.213	6.048	23.678	<2e-16 ***
AgeMiddle	6.956	7.165	0.971	0.332
AgeYoung	-87.350	7.919	-11.030	<2e-16 ***

- What is the base case? What is the average AmountSpent for the base case?
 The Base Case is Old with Average AmountSpent = \$143.213
- What is the Average AmountSpent of Young? \$143.213 \$87.350 = \$55.863
- What is the Average AmountSpent of Middle? \$143.213 + \$6.956 = \$150.169
- All three groups have the same answers as our coding scheme where Young was the base case!!!

R's Indicator variable coding

R's indicator variable coding scheme can be found by using:

contrasts(dirmkt\$Age)

	Middle	Young	
Old	0	0	(Old is the base case in this coding scheme)
Middle	1	0	
Young	0	1	

- In this case R uses a different coding scheme for dummy variables
- · I find it more useful to use my own coding scheme!



DR2: 2nd Regression with <u>Salary</u> and Dummy Variables

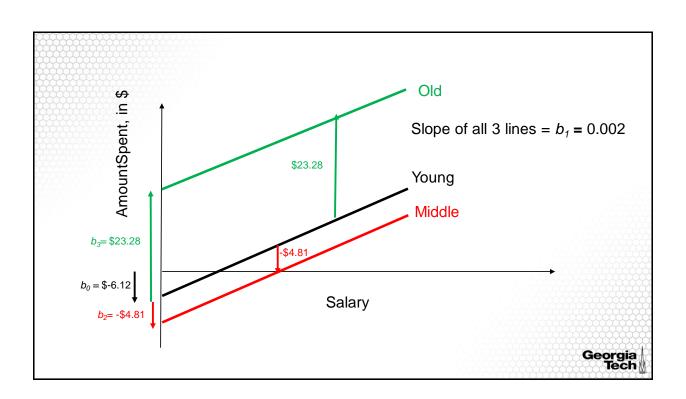
	Estimate	S.E.	t Value	Pr> t
Intercept	-6.12	4.72	-1.30	0.20
Salary	.002	.00009	25	<.001
AgeMid	-4.81	6.39	-0.75	0.45
AgeOld	23.28	6.72	3.46	<.001

- AmountSpent = $b_0 + b_1$ *Salary + b_2 *AgeMid + b_3 *AgeOld
- What is the (average) increase in AmountSpent for a one unit increase in Salary?
- Answer: \$.002



Graphically

Graphically, AmountSpent = b0 + b1*Salary + b2*AgeMid + b3*AgeOld would look like this...

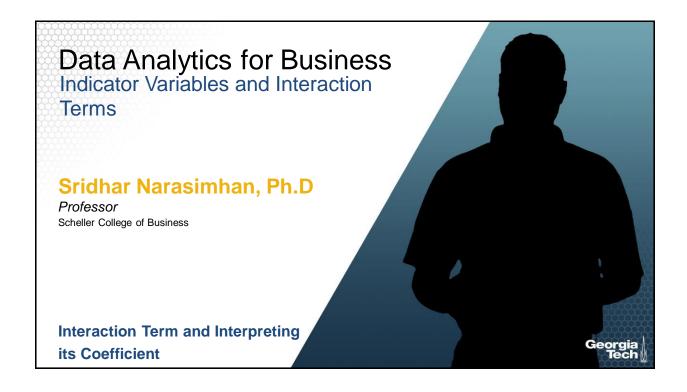


Quiz

	Estimate	S.E.	t Value	Pr> t
Intercept	-6.12	4.72	-1.30	0.20
Salary	.002	.00009	25	<.001
AgeMid	-4.81	6.39	-0.75	0.45
AgeOld	23.28	6.72	3.46	<.001

AmountSpent = b0 + b1*Salary + b2*AgeMid + b3*AgeOld

- · What does this result mean?
- A. Middle-aged customers spend the most
- B. Old customers spend the least
- C. Old customers spend more that young customers
- D. At the same salary level, old customers spend more than young customers What is the current answer?
- D. At the same salary level, old customers spend more than young customers



Next Regression with Dummy Variables

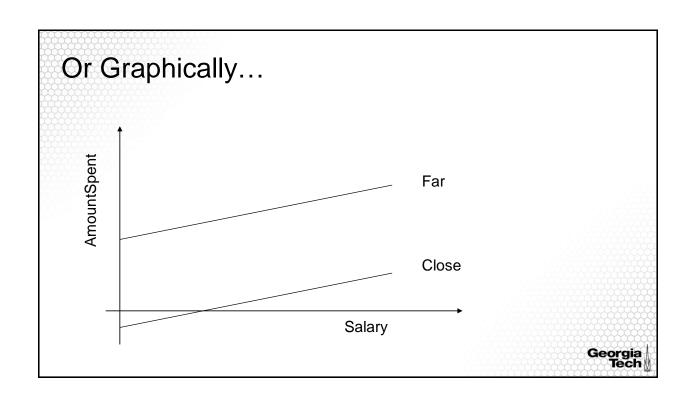
 In the same dataset, Location is a categorical variable with a value equal to "Close" if the customer lives close to a store that sells similar merchandise, and has a value equal to "Far" otherwise

$$Far = \begin{cases} 1, & if \ Location = Far \\ 0, & otherwise \end{cases}$$

- We want to study the impact of Location on AmountSpent
- AmountSpent = $b_0 + b_1$ Salary + b_2 Far

	Estimate	S.E.	t Value	Pr> t
Intercept	-20.480	4.413	-4.64	<.0001
Salary	0.002	0.00007	34.05	<.0001
Far	59.060	4.414	13.38	<.0001

Multiple R-Squared: 0.5672, Adjusted R-squared: 0.5663



	Estimate	S.E.	t Value	Pr> t
Intercept	-20.480	4.413	-4.64	<.0001
Salary	0.002	0.00007	34.05	<.0001
Far	59.060	4.414	13.38	<.0001

 $AmountSpent = b_0 + b_1Salary + b_2Far$

What is the estimated AmountSpent for a customer who lives far?

- A. 59.06
- B. -20.48
- C. -20.48 + 0.002 * Salary
- D. 38.58 + 0.002 * Salary

For this customer since Far =1, the correct answer is

D. because AmountSpent = -20.48 + .002* Salary +59.06 * 1 = 38.58 + .002*Salary

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But...

- AmountSpent = b₀ + b₁Salary + b₂Far
- In the above model, we assume that customers who live far away from a store that sells similar products will spend (at our direct market firm) at the same rate as customers who live close to a store
- · Is this assumption realistic?
- So, can we investigate another scenario that the spending rate may be different? So how should we change the model?

Interaction Term

- Is spending rate higher for customers how live far away?
- To answer this question we need to construct a new variable SalaryFar
- SalaryFar = Salary * Far, is an Interaction Term
- AmountSpent = b_0 + b_1 Salary + b_2 Far + b_3 SalaryFar

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Regression with Dummy Variable and Interaction Term

AmountSpent = $b_0 + b_1$ Salary + b_2 Far + b_3 SalaryFar

	Estimate	S.E.	t Value	Pr> t
Intercept	1.448	4.808	0.30	0.76
Salary	0.002	0.000	24.72	<.0001
Far	-13.460	8.680	-1.55	0.12
SalaryFar	0.001	0.000	9.57	<.0001

Multiple R-Squared: 0.6036, Adjusted R-squared: 0.6024

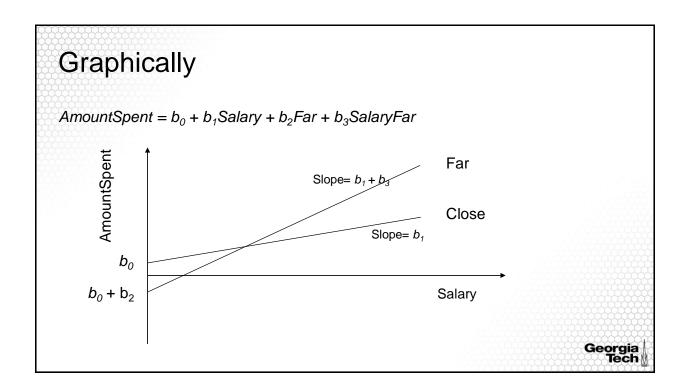
What does this result mean?

Regression with Experience, Dummy Variables, and Interaction Term

	Estimate	S.E.	t Value	Pr> t
Intercept	1.448	4.808	0.30	0.76
Salary	0.002	0.000	24.72	<.0001
Far	-13.460	8.680	-1.55	0.12
SalaryFar	0.001	0.000	9.57	<.0001

 $AmountSpent = b_0 + b_1Salary + b_2Far + b_3SalaryFar$

- How would you interpret b₃ the coefficient of SalaryFar?
- b_3 is the amount to add to b_1 to get the slope for individuals who live far away



Test Your Understanding

AmountSpent = $b_0 + b_1$ Salary + b_2 Far + b_3 SalaryFar

If the salary of a customer who lives close increases by \$10,000, what is the predicted <u>increase</u> in AmountSpent for that customer?

- For this customer (i.e., the baseline case), Far = 0, thus the relevant slope is $b_1 = 0.002$
- Hence, the increase in AmountSpent (on average) for this individual = \$.002*10000 = \$20

If the salary of a customer who lives far away increases by \$10,000, what is the predicted <u>increase</u> in AmountSpent for that customer?

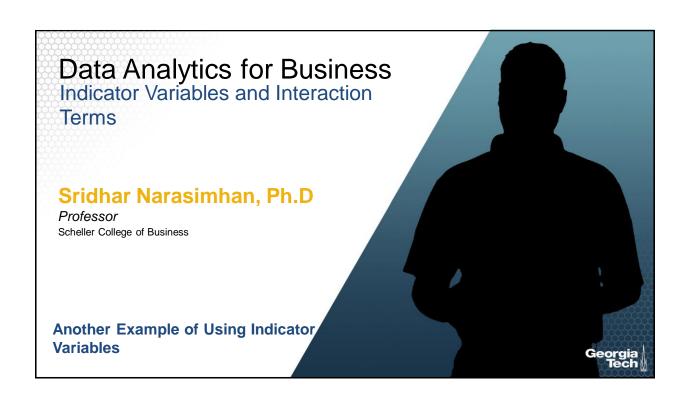
- For this customer, Far = 1, hence the relevant slope is $b_1 + b_3 = 0.002 + 0.001 = 0.003$
- Hence, the increase in AmountSpent (on average) for this individual = \$.003*10000 = \$30

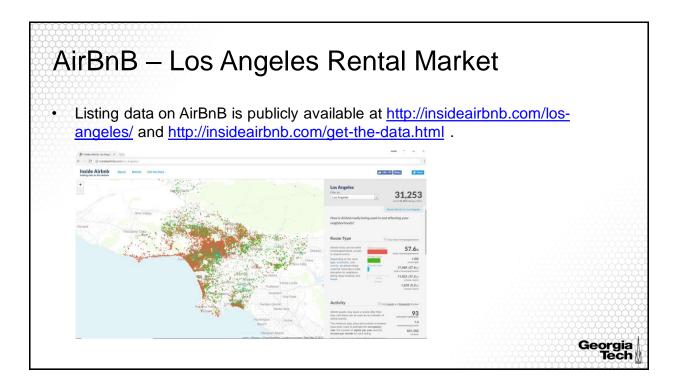


Categorical Variable with M Values

- If a categorical (factor) variable has M possible values, then you will need to construct and use M-1 indicator (dummy) variables
- Be careful when using and interpreting the value of the coefficients of the dummy variable and the value of the coefficients for any interaction terms
- Remember the base case applies to the group where all indicator variables are set to 0
- · All other cases have to be interpreted with reference to the base case







About the Data

- Listing data collected on May 2, 2017
- We discarded listings with price greater than \$1000 and missing values for beds, baths, and rating

\$ Price : num 50 55 150 30 45 80 120 55 50 50 ...

\$ Reviews : int 33 14 22 3 38 42 15 58 19 1 ...

\$ Beds : int 1 1 3 1 1 2 1 2 1 1 ... \$ Baths : num 1 1 1 1 1 1.5 1 2 0 2 ... \$ Capacity : int 2 2 6 1 2 2 2 3 1 2 ...

\$ Monthly_Reviews : num 1.91 1.72 2.12 0.18 7.92 1.89 1.96 2.98 0.53 0.04 ... \$ Room_Type : Factor w/ 3 levels "Shared room",..: 2 2 3 2 2 2 3 2 2 2 ...

\$ Rating : int 93 100 100 93 98 99 99 92 89 NA ...

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Research Questions

If a property owner aims to get a higher price for his or her property, then it is essential to understand the key factors that influence price

- What variables influence listing price?
 - Is there a relationship between capacity and price?
 - Does the type of rental (shared, private or full house) change this relationship?

Data Wrangling

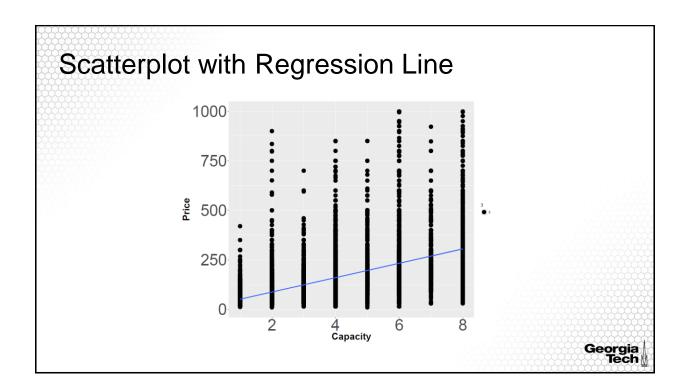
```
la_listing <- la_listing %>%
     mutate(Price = str_replace(Price, "[$]", "")) %>%
                                                              Convert price to numeric and room_type to factor
     mutate(Price = str_replace(Price, "[,]",
     mutate(Price = as.numeric(Price))
     mutate(Room_Type = factor(Room_Type, levels = c("Shared room", "Private room", "Entire home/apt"))) %>%
     mutate(Capacity_Sqr = Capacity * Capacity) %>%
     mutate(Beds_Sqr = Beds * Beds) %>%
                                                              Create squared terms for testing non-linear relations
     mutate(Baths_Sqr = Baths * Baths) %>%
     mutate(In_Reviews = log(1+Reviews)) %>%
     mutate(In_Monthly_Reviews = log(1+Monthly_Reviews))
     mutate(In_Price = log(1+Price)) %>%
     mutate(In_Beds = log(1+Beds)) %>%
                                                              Create log terms for testing non-linear relations
     mutate(In_Baths = log(1+Baths)) %>%
     mutate(In_Capacity = log(1+Capacity)) %>%
     mutate(In_Rating = log(1+Rating)) %>%
     mutate(Shared_ind = ifelse(Room_Type == "Shared room",1,0)) %>%
                                                                            Create dummy variables for room_type
     mutate(House_ind = ifelse(Room_Type == "Entire home/apt",1,0)) %>%
     mutate(Private_ind = ifelse(Room_Type == "Private room", 1,0)) %>%
     mutate(Capacity_x_Shared_ind = Shared_ind * Capacity) %>%
     mutate(Capacity_x_House_ind = House_ind * Capacity) %>%
     mutate(Capacity_x_Private_ind = Private_ind * Capacity) %>%
                                                                             Create interaction terms
     mutate(In_Capacity_x_Shared_ind = Shared_ind * In_Capacity) %>%
     mutate(In_Capacity_x_House_ind = House_ind * In_Capacity) %>%
     mutate(In_Capacity_x_Private_ind = Private_ind * In_Capacity)
                                                                                                                        Georgia
    filter(Price < 1000, !is.na(Beds), !is.na(Baths), !is.na(Price), !is.na(Rating))
                                                                            Filter unwanted data
```

2RS: Simple Regression – How Does Price Vary by Room Capacity?

• Price = $b_0 + b_1$ *Capacity

Ţ.	Estimate	S.E.	t Value	Pr> t
Intercept	15.039	1.141	13.19***	<.001
Capacity	38.272	0.316	114.72***	<.001

R-squared	Adjusted R-squared
0.367	0.367



Creating Indicator (Dummy) variables

We define two dummy variables:

Private_ind =
$$\begin{cases} 1, & if \ Room \ type = "Private \ room" \\ 0, & otherwise \end{cases}$$

$$\label{eq:house_ind} \begin{aligned} \text{House_ind} = \begin{cases} 1, & \textit{if Room type} = "\textit{Entire home/apt"} \\ 0, & \textit{otherwise} \end{cases} \end{aligned}$$

 The base (or reference) case, with both dummy variables set to 0, is Room type = "Shared." This is the reference group to compare for the other values of the dummy variable



2DR1: How Does Price Vary by Room Type?

Price = $b_0 + b_1*Private_ind + b_2*House_ind$ (only dummies)

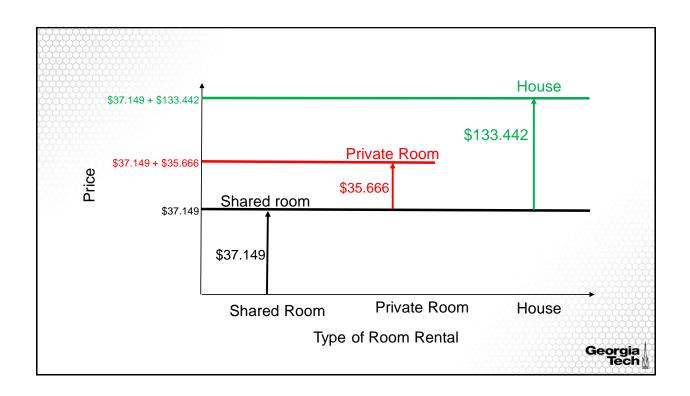
	Estimate	S.E.	t Value	Pr> t
Intercept	37.149	2.954	12.58***	<.001
Private_ind	35.666	3.123	11.42***	<.001
House-ind	133.442	3.058	43.64***	<.001

- Which room type's Average Price is \$37.149? Shared room
- What is the Average Price of a Private Room? \$37.149 + \$35.666
- What is the Average Price of an Entire House? \$37.149 + \$133.442

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Graphically

Let's take a look at this graphically...



2DR2: 2nd Regression with <u>Capacity</u> and Dummy Variables

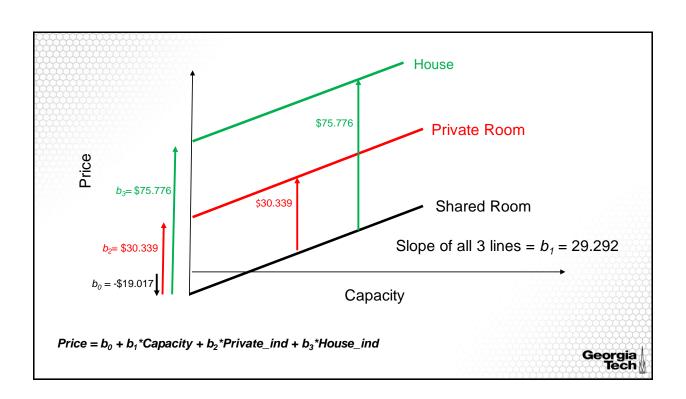
	Estimate	S.E.	t Value	Pr> t
Intercept	-19.017	2.678	-7.101	<.001
Capacity	29.292	0.355	82.605	<.001
Private_ind	30.339	2.739	11.076	<.001
House-ind	75.776	2.771	27.346	<.001

- $Price = b_0 + b_1*Capacity + b_2*Private_ind + b_3*House_ind$
- What is the (average) increase in Price for each additional individual?
 Answer: \$29.292



Graphically

Let's take a look at $Price = b_0 + b_1*Capacity + b_2*Private_ind + b_3*House_ind$ graphically...



Interaction Terms

- Construct two new variables:
- P_Cap = Private-ind*Capacity
- H_Cap = House_ind*Capacity
- P_Cap and H_Cap are the Interaction terms.
- The new regression is: $Price = b_0 + b_1*Capacity + b_2*Private_ind + b_3*House_ind + b_4*P_Cap + b_5*H_Cap$

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DR3: 3rd Regression with Capacity, Dummy Variables, and Interaction Terms

	Estimate	S.E.	t Value	Pr> t
Intercept	35.885	4.111	8.728***	<.001
Capacity	0.659	1.687	0.391	0.695980
Private_ind	20.684	4.672	4.427***	<.001
House_ind	2.293	4.423	0.518	0.604147
P_Cap	7.080	1.947	3.636***	<.001
H_Cap	33.414	1.729	19.323***	<.001

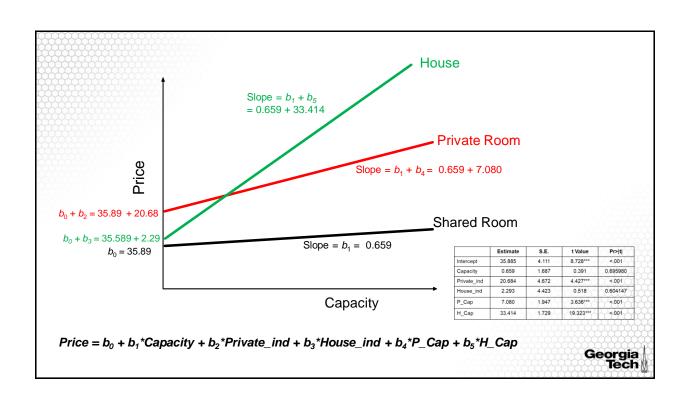
- Price = $b_0 + b_1$ *Capacity + b_2 *Private_ind + b_3 *House_ind + b_4 *P_Cap + b_5 *H_Cap
- How would you interpret b₄ and b₅ the coefficients of P_Cap and H_Cap?
- b_4 is the amount to add to b_1 to get the slope for a Private room
- b₅ is the amount to add to b₁ to get the slope for a House
- Statistically, Capacity (slope) and House_ind (bump in intercept) are not very different from 0



Graphically

Let's take a look at

 $Price = b_0 + b_1*Capacity + b_2*Private_ind + b_3*House_ind + b_4*P_Cap + b_5*H_Cap$ graphically...



Recap of this Module

- A. A Customer Analytics Dataset to Illustrate Indicator Variables
- B. Creating and Using Indicator (Dummy) Variables
- C. Interpreting the Coefficients of Indicator Variables
- D. Interaction Term and Interpreting its Coefficient
- E. Another Example of Using Indicator Variables