Project 1

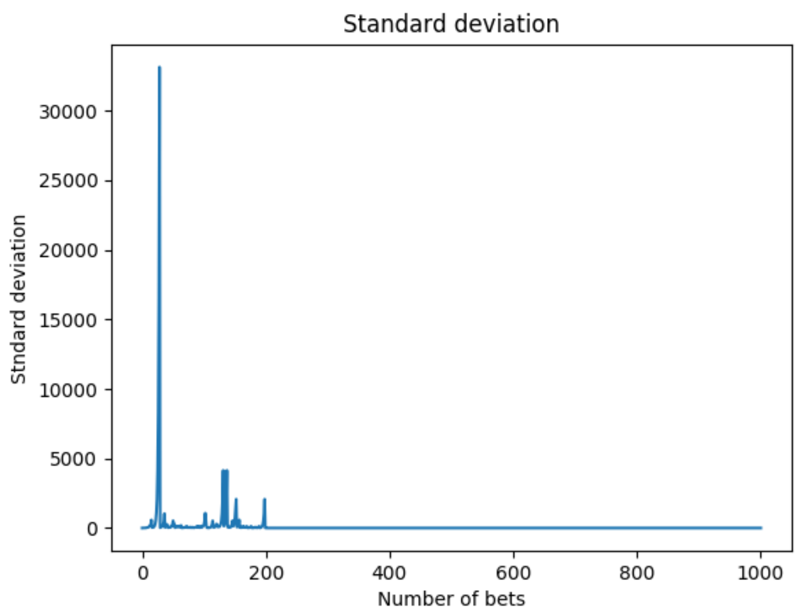
$$p = \text{probability of winning} = 0.53$$
$$q = \text{probability of losing} = 0.47$$

1. The probability of winning \$80 within 1000 sequential bets is 1.0 as estimated from the 1000 simulations.

$$\begin{aligned}
P(X) &= \text{Probability of winning X amount in 1000 bets} \\
P(80) &= 1 - (P(0) + P(1) + \ldots + P(79)) \\
P(n) &= p^n q^{1000-n} \binom{1000}{n} \\
&= \left(\frac{p}{q}\right)^n q^{1000} \binom{1000}{n} \\
P(80) &= 1 - q^{1000} \sum_{n=0}^{n=79} \left(\frac{p}{q}\right)^n \binom{1000}{n} \\
&= 1 - q^{1000} \left(1 + \frac{p}{q}\right)^{79} \\
&\approx 1.0
\end{aligned}$$

2. The expected value of winnings after 1000 sequential bets is \$80.0 as estimated from the 1000 simulations.
   Since the probability of winning \$80 in 1000 sequential bets is almost 1.0, the winnings after any 1000 sequential bets will be \$80. Averaging this also gives \$80. This is also observed from the experiment.

3. The standard deviation fluctuates in initial around 200 rounds and then converges to zero as the number of rounds increase.
   This can be explained from the above reasons. As the probability of winning approaches to 1 in 200 bets, the standard deviation of winning also converges since there is no effect in the winnings after \$80.



4. The probability of winning \$80 within 1000 sequential bets is 0.66 as estimated from the 1000 simulations. On losing 9 bets, one loses all the money.

$$
\begin{aligned}
P(X) &= \text{Probability of winning X amount without going bankrupt.} \\
P(1) &= p + qp + q^2p + \ldots + q^8p \\
&= p\frac{(1-q^9)}{(1-q)} \\
&= 1 - q^9 \\
P(80) &= \underbrace{P(1) * P(1) * \ldots * P(1)}_{80 \text{ times}} \\
&= \left(1-q^9\right)^{80} \\
&\approx 0.76
\end{aligned}
$$

5. There are only two possible scenarios one can end up with in experiment 2.
   Case 1 - The person wins \$80 and stops betting after that. Case 2- The person goes bankrupt in one of the games. This would be a case if the person does not win \$80. It is so because then the person can survive atmost 9*79 = 711 bets without going bankrupt after which it loses every bet, leading to bankruptcy. In this case, the winnings would be -\$256.

Theoretically Expected winnings

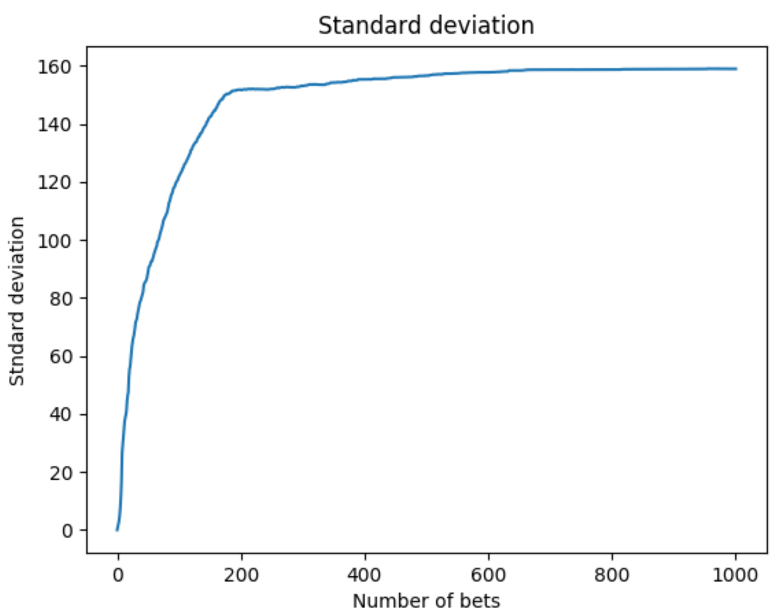
$$\begin{aligned} X &= \text{Winnings after 1000 bets} \\ P(X) &= \text{Probability of winning X in 1000 bets} \\ E(X) &= P(80) * 80 + (1 - P(80)) * (-256) \\ &= 0.76 * 80 - 256 * 0.24 \\ &= -0.64 \end{aligned}$$

Expected winnings from simulations
We can actually calculate the expected winnings after 1000 bets from the above observed probability of winning.

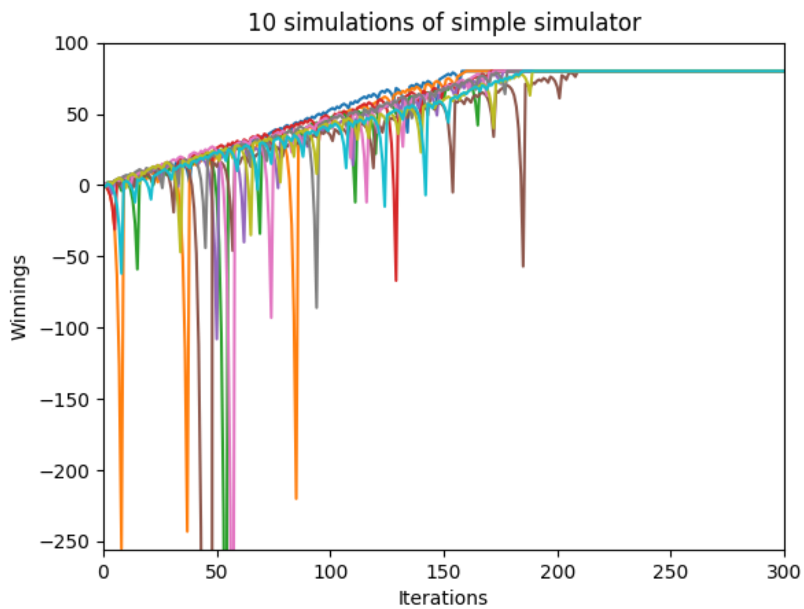
$$\begin{aligned} P(X) &= \text{Probability of winning X in 1000 bets} \\ P(80) &= 0.66 \\ E(X) &= P(80) * 80 + (1 - P(80)) * (-256) \\ &= 0.66 * 80 - 256 * 0.34 \\ &= -34.24 \end{aligned}$$

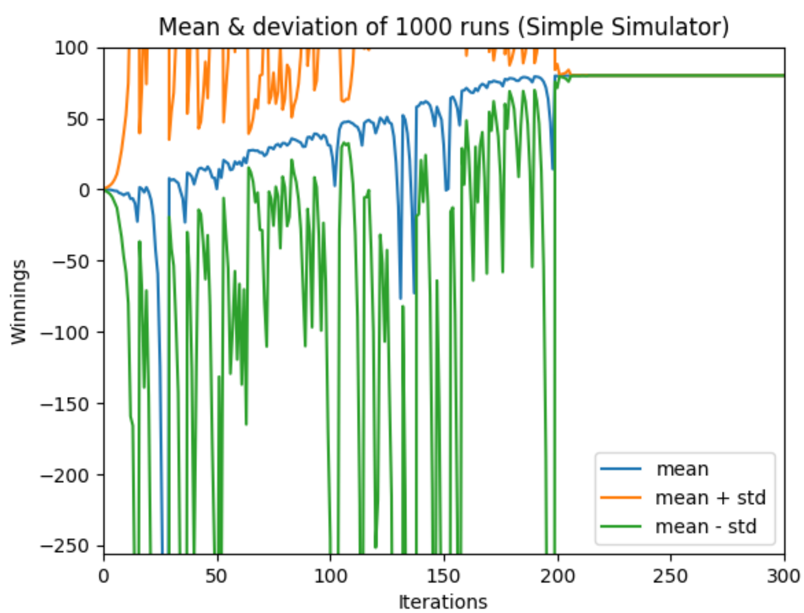
The calculated expected value of winnings after 1000 sequential bets is very close to the observed value $-34.0007 from 1000 simulations.

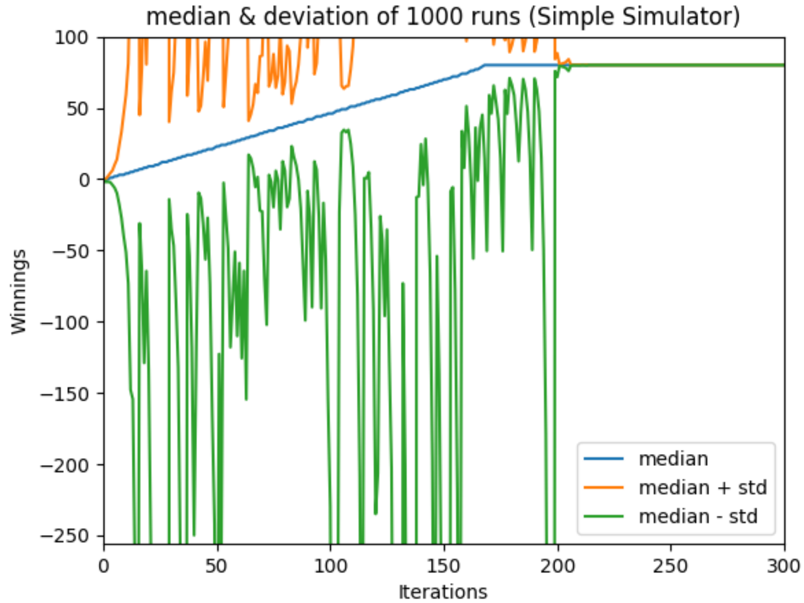
6. The standard deviation for realistic experiment keeps on increasing since the expected value saturates at either 80 or -256.

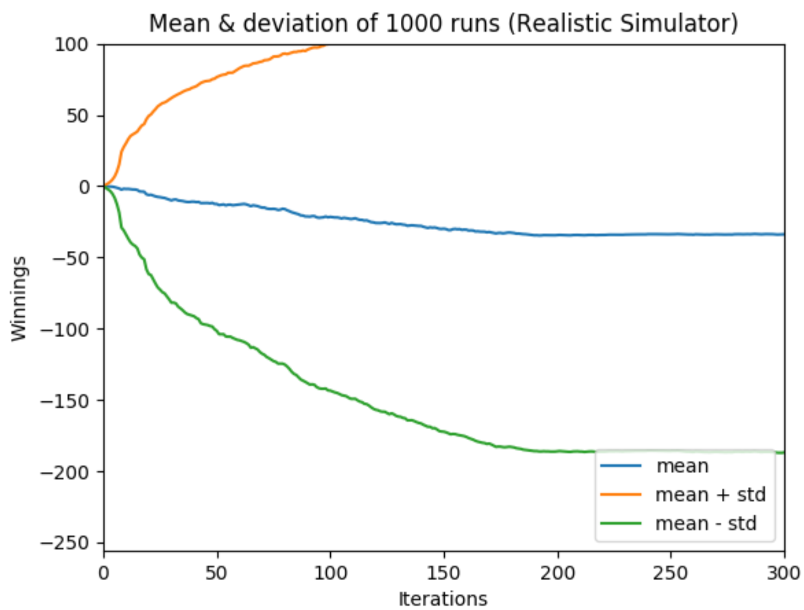


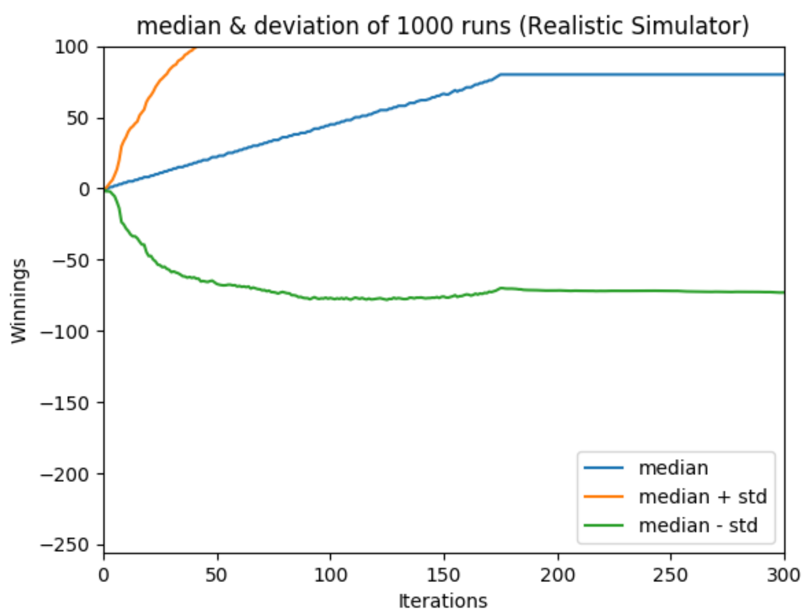
7. Plots asked in the report.






median & deviation of 1000 runs (Simple Simulator)
Winnings
Iterations
median
median + std
median - std
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300


Mean & deviation of 1000 runs (Realistic Simulator)
Winnings
Iterations
mean
mean + std
mean - std
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300


median & deviation of 1000 runs (Realistic Simulator)
Winnings
Iterations
median
median + std
median - std
100
50
0
−50
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−200
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