



Uncertainty in Software Models

Lola Burgueño, Paula Muñoz, Antonio Vallecillo
Universidad de Málaga, Spain

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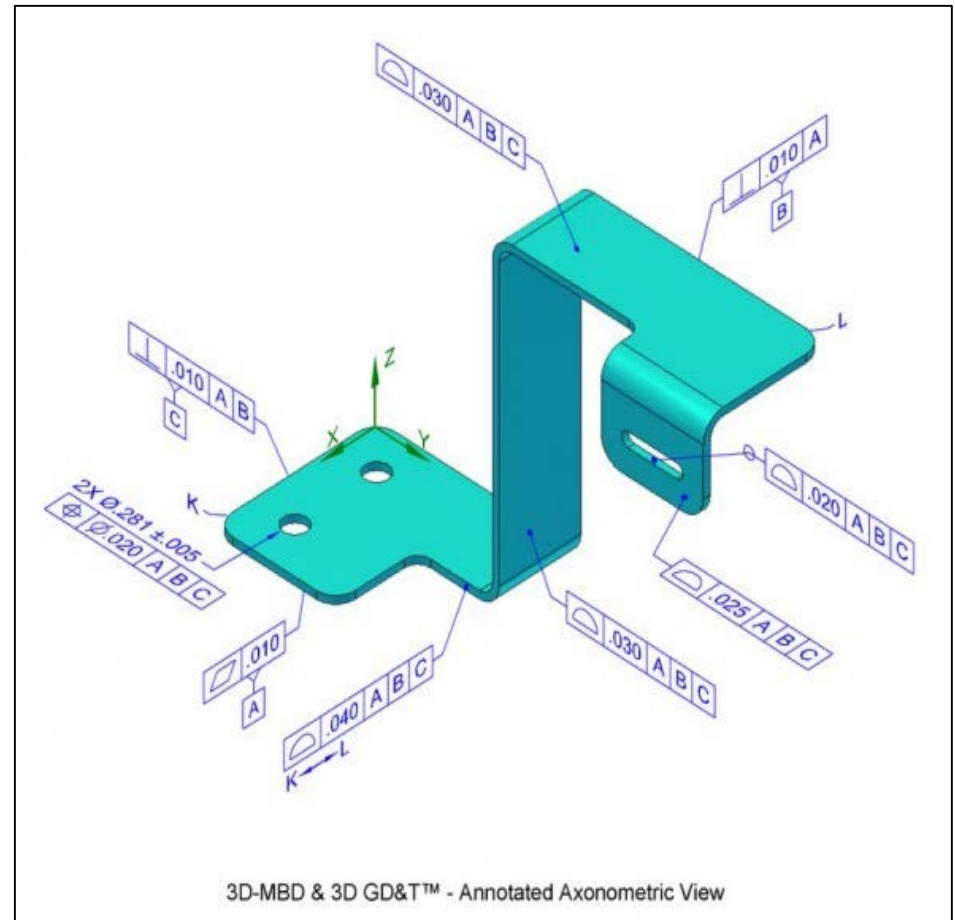
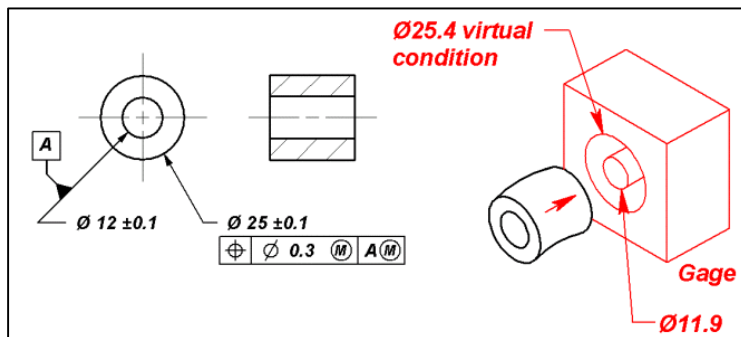
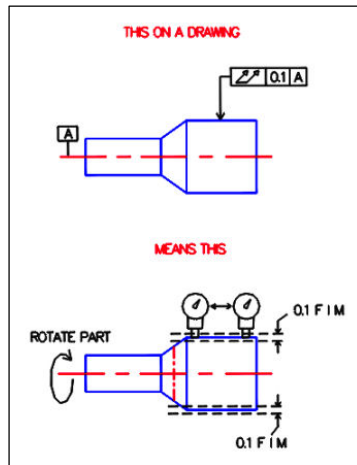
Uncertainty

Uncertainty: Quality or state that involves imperfect and/or unknown information

- It applies to: predictions of future events, estimations, physical measurements, or properties of a system, its elements or its environment
- due to:
 - Underspecification of the problem or solution domains
 - Lack of knowledge of the system, its environment, or its underlying physics
 - Lack of precision in measurements
 - Imperfect, incorrect, or missing information
 - Numerical approximations
 - Values and parameters indeterminacy
 - Different interpretations of the same evidences by separate parties

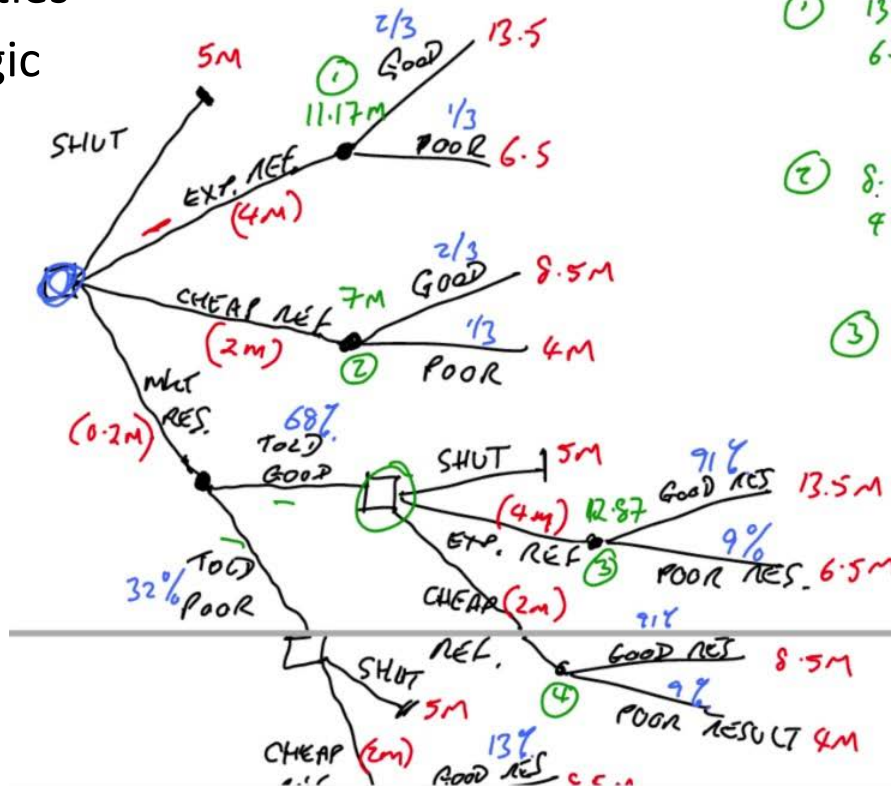
“There is nothing certain, but the uncertain” (*proverb*)

Uncertainty in mechanical systems



Many different formalisms and theories to quantify uncertainty

- Bayesian Belief Networks (BBN)
- Monte Carlo simulations
- Decision theory/trees
- Probabilities
- Fuzzy Logic
- ...



① $13.5 \times \frac{2}{3} = 9$
 $6.5 \times \frac{1}{3} = \frac{2.17}{11.17}$

(2) $8.5 \times 2/3 = 5.67$
 $9 \times 1/3 = 1.33$
7.00

③ $13.5 \times 0.91 = 12.29$
 $6.5 \times 0.09 = 0.58$
12.87

④ $8.5 \times 0.91 = 7.7$
 $9 \times 0.09 =$



A classification of uncertainty (according to its nature)

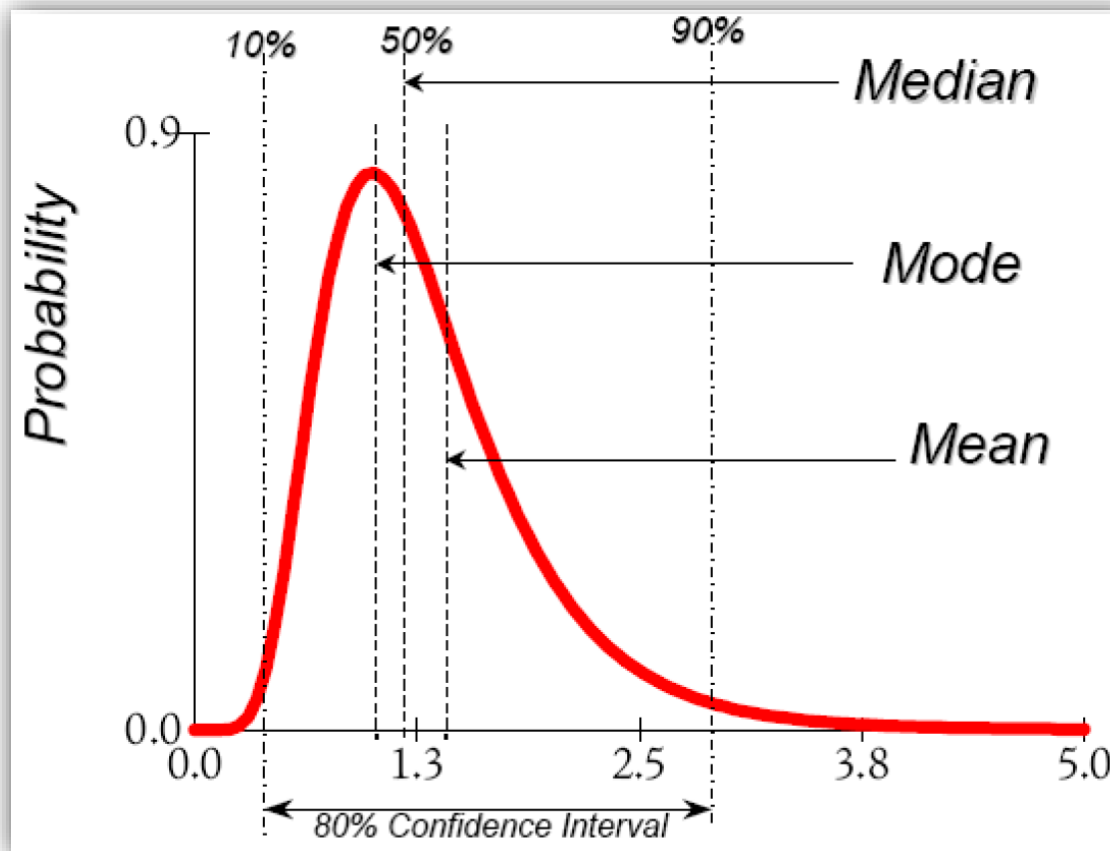
- **Aleatory** Uncertainty – A kind of uncertainty that refers to the *inherent* uncertainty due to the *probabilistic variability or randomness* of a phenomenon
 - Examples: measuring the speed of a car, or the duration of a software development process
 - This type of uncertainty is **irreducible**, in that there will always be variability in the underlying variables.

- **Epistemic** Uncertainty – A kind of uncertainty that refers to *the lack of knowledge* we may have about the system (modeled or real).
 - Examples: Ambiguous or imprecise requirements about the expected system functionality, its envisioned operating environment, etc.
 - This type of uncertainty is **reducible**, in that additional information or knowledge may reduce it.

Types of uncertainty (according to their sources)

- **Measurement uncertainty:** A kind of *aleatory* uncertainty that refers to a set of possible states or outcomes of a measurement, where probabilities are assigned to each possible state or outcome
- **Occurrence uncertainty:** a kind of *epistemic* uncertainty that refers to the degree of belief that we have on the actual existence of an entity, i.e., the real entity that a model element represents
- **Belief uncertainty:** A kind of *epistemic* uncertainty in which a *belief agent* is uncertain about any of the *statements* made about the system or its environment.
- **Design uncertainty:** A kind of *epistemic* uncertainty that refers to a set of possible design decisions or options, where probabilities are assigned to each decision or option
- **Environment uncertainty:** lack of certainty about the surroundings, boundaries and usages of a system and of its elements
- **Spatiotemporal uncertainty:** lack of certainty about the geographical or physical location of a system, its elements or its environment, or about the time properties expressed in a statement about the system or its environment

Measurement Uncertainty



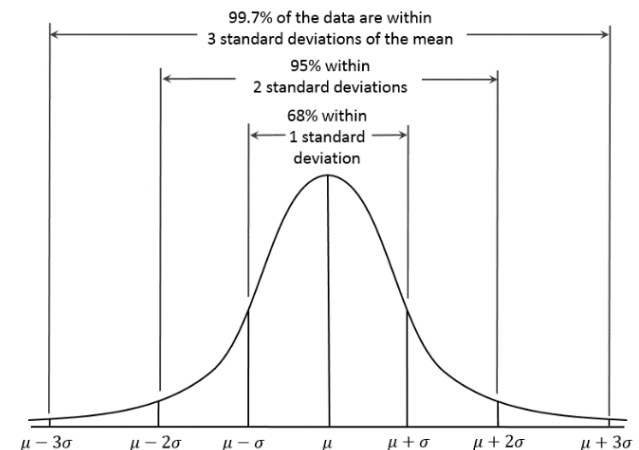
Measurement uncertainty

- **Measurement uncertainty**: A kind of *aleatory* uncertainty that refers to a set of possible states or outcomes of a measurement
- Normally expressed by a parameter, associated with the result of a measurement x , that characterizes the dispersion of the values that could reasonably be attributed to the measurand: the standard deviation u of the possible variation of the values of x
- Representation: $x \pm u$ or (x, u)

- **Examples:**

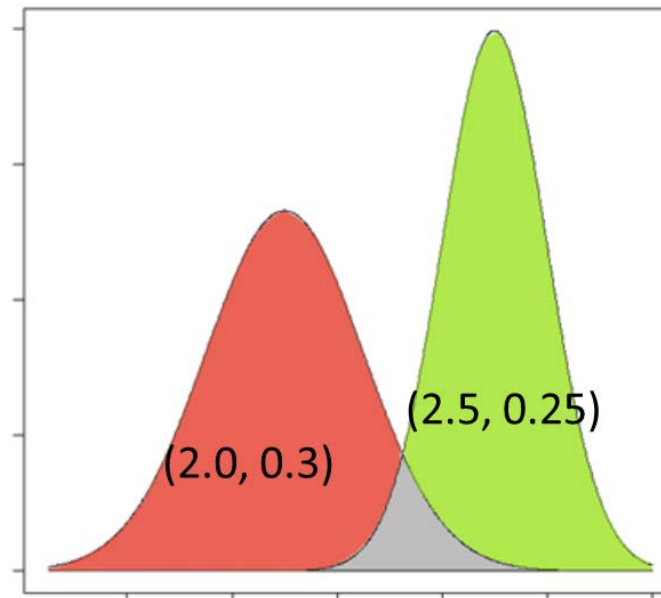
- Normal distribution: (x, σ) with mean x , and standard deviation σ
- Interval $[a, b]$: Uniform distribution is assumed

$$(x, u) \text{ with } x = \frac{a+b}{2}, u = \frac{(b-a)}{2\sqrt{3}}$$



Some problems with Measurement Uncertainty

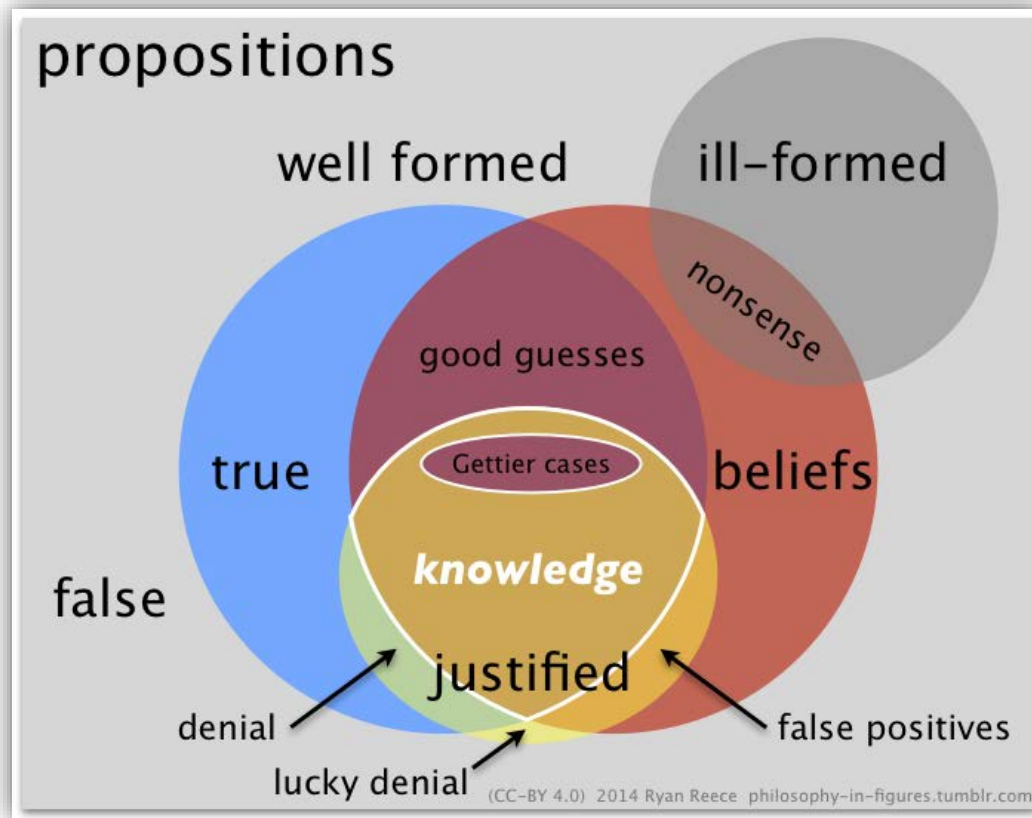
- Computations with uncertain values must respect the *propagation of uncertainty* (uncertainty analysis)
 - In general, this is a complex problem, which cannot be manually managed
- *Comparison of uncertain values* is no longer a Boolean property!
 - How to compare 17.7 ± 0.2 with 17.8 ± 0.2 ?
- Other primitive datatypes are also affected by uncertainty
 - Strings (OCR)
 - Enumerations
 - Collections



UBoolean

- UBooleans are pairs (b, c)
 - where $b:\text{Boolean}$ and $c:\text{Real}$, $c \in [0, 1]$
 - c represents the confidence that the actual value of the value is indeed b
 - Canonical form: (true, c)
 - Equivalence relation: $(b, c) = (\text{not } b, 1 - c)$
- Constants
 - $\text{UBoolean}(\text{true}, 0.999)$, $\text{UBoolean}(\text{false}, 0.001)$
- Operations
 - Redefined basic operations: and, or, not
 - Redefined secondary operations: implies, equivalent, xor
 - Conversion operations: $\text{toBoolean}()$ and $\text{toBooleanC}(c:\text{Real})$

Belief Uncertainty



Belief uncertainty

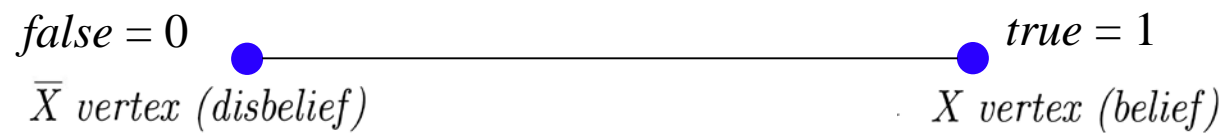
- **Belief uncertainty:** A kind of *epistemic* uncertainty in which the modeler, or any other *belief agent*, is uncertain about any of the *statements* made about the system or its environment.
 - By nature, it is always *subjective*
 - *It may not always be possible to determine whether or not a belief statement is valid.*
 - A belief statement may not necessarily correspond to objective reality.
 - This means that it could be completely false, or only partially true, or completely true.
 - The validity of a statement may only be *meaningfully defined within a given context or purpose.*
 - Thus, the statement that “the Earth can be represented as a perfect sphere” may be perfectly valid for some purposes but invalid or only partly valid for others.
- **Belief agent:** An entity (human, institution, even a machine) that holds one or more beliefs
- **Belief statement:** *Statement* qualified by a *degree of belief*
- **Degree of belief:** Confidence assigned to a statement by a belief agent. Normally expressed by quantitative or qualitative methods (e.g., a grade or a probability “*credence*”)

Classical Boolean Logic

$false = 0$ ●
 \bar{X} vertex (disbelief)

● $true = 1$
 X vertex (belief)

Probabilistic Logic



Kleene (three-valued) Logic

Uncertainty



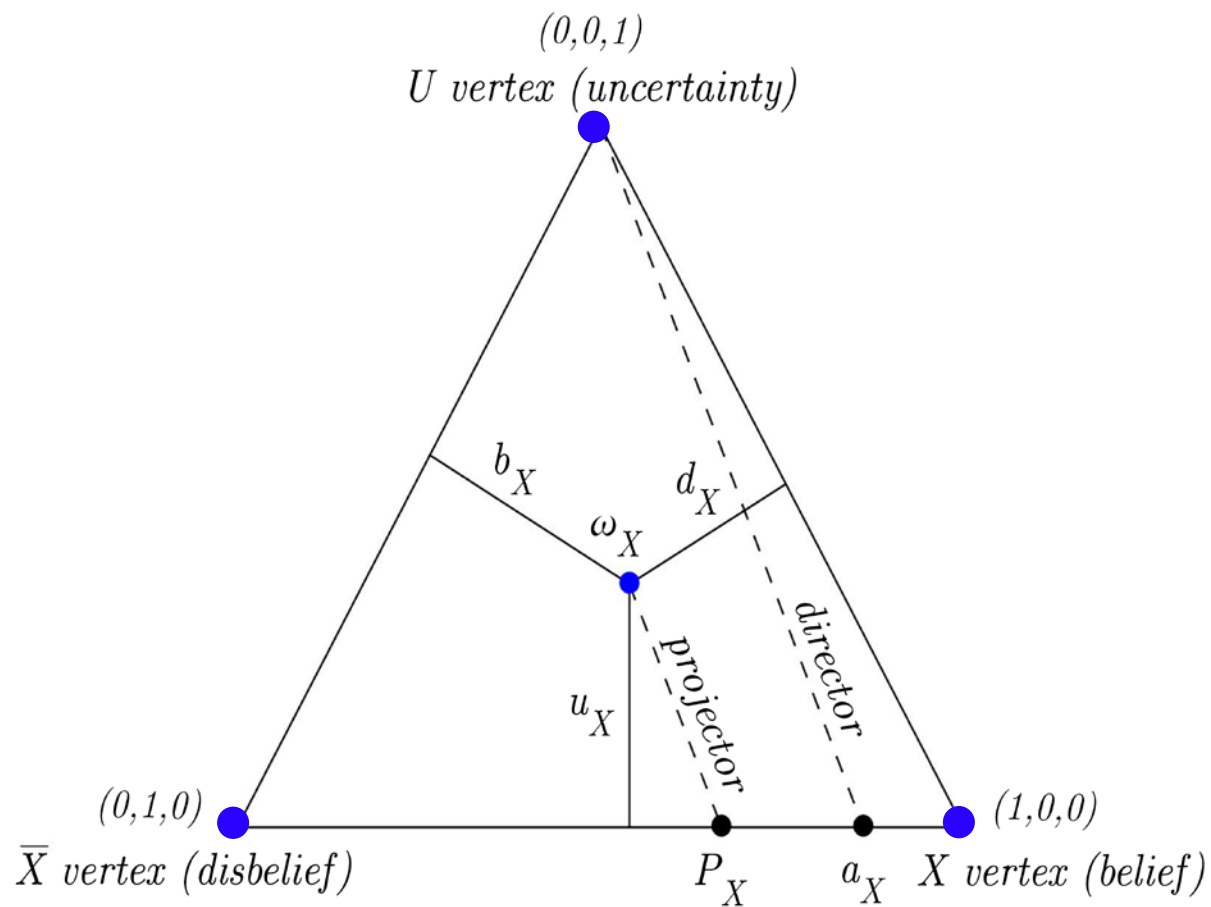
\bar{X} vertex (*disbelief*)



X vertex (*belief*)



Subjective Logic



Subjective logic in UML/OCL

- $SBoolean(b, d, u, a)$
 - b represents the degree of **belief** that the agent has about the statement
 - d represents the degree of **disbelief**
 - u represents the **uncertainty** that the agent expressing the opinion has about the statement, i.e., the degree of trust
 - a is the (objective) **prior probability** assigned to the statement, based on previous evidence (also called “**base rate**”).

- $b + d + u = 1$

- Boolean values lifted to SBoolean:
 - $true \equiv SBoolean(1, 0, 0, 1)$
 - $false \equiv SBoolean(0, 1, 0, 0)$
- UBoolean values lifted to SBoolean:
 - $Uboolean(true, c) = SBoolean(c, 1-c, 0, c)$
- Projection from SBoolean to UBoolean:
 - $SBoolean(b, d, u, a).projection() = UBoolean(true, b + u*a)$