Value-at-Risk for Currencies (FX Spot)

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PHP as Quote Currency

- For simplicity, we will assume that the quote currency for all exchange rates is Philippine pesos (PHP).
- Thus, when we say we have a long position on a currency CCY, we are implying that we have a long position on the currency pair $CCYPHP$.
- On the other hand, taking a short position on CCY means shorting the currency pair $CCYPHP$.
- Therefore, all historical exchange rates to be used in calculating VaR must be expressed in $CCYPHP$.

Calculating ΔP

Suppose a FI holds a portfolio consisting solely of currency C_1. Let F_1 be the FI's position on C_1, $y_{1,0}$ be the exchange rate to peso of C_1 today, and y_1 be the exchange rate to peso of C_1 tomorrow. The peso equivalents of the position on C_1 today and tomorrow are $F_1 y_{1,0}$ and $F_1 y_1$, respectively. Hence, the change in the position's peso equivalent today and tomorrow is

$$\begin{aligned} \Delta P &= F_1 y_1 - F_1 y_{1,0} \\ &= F_1(y_1 - y_{1,0}) \end{aligned}$$

Dealing with Long and Short Positions

- When the bank takes a long position on C_1, then F_1 is given a positive sign. Thus, ΔP is positive when $y_1 > y_{1,0}$ and negative when $y_1 < y_{1,0}$. This is consistent with the fact that a long position on a currency gains value when the currency's exchange rate to peso increases but loses value when the exchange rate decreases.
- On the other hand, when the bank takes a short position on C_1, F_1 is assigned a negative sign, Thus, ΔP is positive when when $y_1 < y_{1,0}$ and negative when $y_1 > y_{1,0}$, consistent with the fact that a short position gains value when the exchange rate decreases and loses value when the exchange rate increases.
- Making F_1 a signed quantity ($+$ if long, - if short) ensures the consistency of our interpretation of ΔP:
  - $\Delta P > 0$ implies a gain in the value of the portfolio
  - $\Delta P < 0$ implies a loss in the value of the portfolio

An Illustration

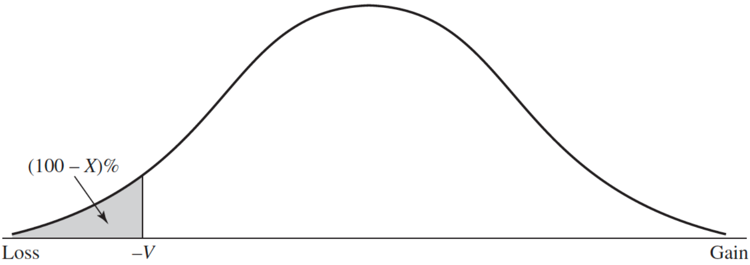

Figure 12.1 Calculation of VaR from the Probability Distribution of the Gain in the Portfolio Value
Losses are negative gains; confidence level is X%; VaR level is V.

Delta-Normal Approach

Single Currency C_1

Recall: Stocks

The one-day 99% VaR, $|V|$, of a portfolio consisting of a long position on N on a stock with current share price S_0 is

$$|V| = NS_0\sigma\Phi^{-1}(0.99)$$

where σ is the daily volatility of stock returns.

Formula for VaR

The one-day 99% VaR, $|V|$, of a portfolio consisting of a position F_1 on currency C_1 is

$$|V| = |F_1|y_{1,0}\sigma_1\Phi^{-1}(0.99)$$

where $y_{1,0}$ is the current exchange rate of C_1 to PHP and σ_1 is the daily volatility of exchange-rate returns.[1]

[1] The formula can be modified for any $100\alpha\%$ VaR.

n-Currency Case

For an n-currency portfolio, we denote the currencies as $C_1, C_2, \ldots, C_n$ each with the current exchange rate to PHP denoted by $y_{1,0}, y_{2,0}, \ldots, y_{n,0}$ respectively.

n-Currency Case

Undiversified VaR

To compute for the undiversified 99% VaR, calculate the VaR for each currency in the portfolio and take the sum

$$|V_u| = \sum_{i=1}^{n} |F_i| y_{i,0} \sigma_i \Phi^{-1}(0.99).$$

n-Currency Case

Diversified VaR

Let $a_i = F_i y_{i,0}, i = 1, 2, \ldots, n$

$\boldsymbol{a}$ be the vector of a_i's

Σ be the covariance matrix

The diversified 99% VaR $|V_d|$ is given by

$$|V_d| = \sigma_P \Phi^{-1}(0.99)$$

where

$$\sigma_P^2 = \boldsymbol{a}^T \boldsymbol{\Sigma} \boldsymbol{a}.$$

N-day VaR and EWMA model

The approaches for calculating N-day VaR and the procedure for computing VaR using Exponentially-Weighted Moving Average (EWMA) model also easily translate to a portfolio of currencies. The returns on the exchange rates are used instead of the returns on stock prices.

Historical Simulation

n-Currency Case

Suppose the historical data consists of the most recent $M + 1$ days of historical exchange rates to PHP for currencies in the portfolio.

Procedure using returns on individual currencies

1. Generate possible 1-day returns based on the previous 1-day returns.
$$R_{1,i,j} = \ln\left(\frac{y_{i,j}}{y_{i,j-1}}\right)$$
2. Compute for the possible change in portfolio value using the generated 1-day return.
$$\Delta P^{(j)} = \sum_{i=1}^{n} F_i y_{i,0} R_{1,i,j}$$
3. The 1-day 99% VaR is the $(0.01M)^{\text{th}}$ smallest value.

BRW Approach

BRW Approach: Single Currency Case

Step 1: Assignment of Weights

1. Calculate the observed one-day continuously compounded returns $R_{1,0}, R_{1,1}, \ldots, R_{1,M-1}$ from the exchange rates of currency C_1 or a portfolio of currencies, where $R_{1,j} = \ln(y_{1,j}/y_{1,j-1})$ is the rate of return observed j days ago.
2. Compute for scenarios of changes in portfolio value $\Delta P^{(j)} = F_1 y_{1,0} R_{1,j}$ for $j = 0, 1, \ldots, M-1$.
3. Let $0 < \lambda_1 < 1$ be constant and let $w_{1,j}$ be the weight assigned to $R_{1,j}$ for all $j = 0, 1, \ldots, M-1$. The weights are assumed to decay exponentially, so we assume that $w_{1,j+1} = \lambda_1 w_{1,j}$. Making sure that the weights add up to 1, it can be shown that

$$w_{1,j} = \left(\frac{1-\lambda_1}{1-\lambda_1^M}\right)\lambda_1^j, \qquad j = 0, 1, \ldots, M-1$$

BRW Approach

Step 2: Constructing the Empirical Distribution

After assigning the weights to each element of the scenario space for ΔP, we then arrange the $\Delta P^{(j)}$'s and w_j's in increasing order of $\Delta P^{(j)}$. We denote the resulting sequence as follows:

$\boldsymbol{\Delta P}$ **Scenario**	$\Delta \widetilde{P}^{(0)}$	$\Delta \widetilde{P}^{(1)}$	$\cdots$	$\Delta \widetilde{P}^{(j)}$	$\cdots$	$\Delta \widetilde{P}^{(M-1)}$
Weights	$w_{(0)}$	$w_{(1)}$	$\cdots$	$w_{(j)}$	$\cdots$	$w_{(M-1)}$

The empirical cumulative distribution at $\Delta \widetilde{P}^{(j)}$, which denoted by ψ_j, is determined by the accumulation of weights

$$\psi_j = \sum_{k=0}^{j} w_{(k)}, \qquad j = 0, 1, \ldots, M-1$$

BRW Approach

Step 3: Calculating the 1-day $100\alpha\%$ VaR

The $100\alpha\%$ value-at-risk is obtained by first identifying successive ordered pairs $(\Delta\tilde{P}^{(k)}, \psi_k)$ and $(\Delta\tilde{P}^{(k+1)}, \psi_{k+1})$ such that

$$\psi_k < 1 - \alpha < \psi_{k+1}. \tag{1}$$

Let V be the linearly interpolated value of ΔP from the ordered pairs $(\Delta\tilde{P}^{(k)}, \psi_k)$ and $(\Delta\tilde{P}^{(k+1)}, \psi_{k+1})$ corresponding to the value $\psi = 1 - \alpha$. **The $100\alpha\%$ value-at-risk is therefore given by $|V|$.**

BRW Approach: **n**-asset Portfolio

Procedure using returns on individual currencies

1. From the historical exchange rates, we can construct M scenarios for the one-day continuously compounded return $R_{1,i}$ on the ith exchange rate.
2. Thus, M scenarios can also be constructed for ΔP_i

$$\Delta P_{i,j} = F_i y_{i,0} R_{1,i,j}.$$

3. The scenario space for ΔP is given by

$$\Delta P^{(j)} = \sum_{i=1}^{n} \Delta P_{i,j} = \sum_{i=1}^{n} F_i y_{i,0} R_{1,i,j}$$

We can then resume the BRW Approach at Step 2 using $\{\Delta P^{(j)}\}$ as the $\{\Delta P\}$ scenarios.

References

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