Interest Rate Swaps

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Interest Rate Swaps

Definition

An **interest rate swap (IRS)** is a contractual agreement between two parties under which each party agrees to make periodic payments to the other for a prespecified time period based on a notional amount of principal.

Similar to FRAs, there is no exchange of principal in an IRS.

Interest Rate Swaps

Definition

In a **plain vanilla interest rate swap/fixed-for-floating interest rate swap**, fixed cash flows computed using fixed interest rate are exchanged for floating cash flows computed using a floating interest rate.

- The fixed cash flows constitute the **fixed leg**.
- The floating cash flows constitute the **floating leg**.
- The fixed interest rate that makes the value of an IRS 0 at inception is called the **swap rate**. This is sometimes quoted as a **swap spread**, which is the difference between the swap rate and the yield of a government bond with the same maturity.
- The dates on which the floating rates are determined are called **reset dates/fixing dates**.

Notes on the Fixed and Floating Legs

- The number of payments in the floating leg may not be the same as the number of payments in the fixed leg.
- The dates on which fixed cash flows are made may not be the same as the dates on which floating cash flows are made.

IRS Valuation

General Idea

Let $V_{\text{fixed}}(t)$ and $V_{\text{float}}(t)$ be the total present value at time t of the fixed and floating legs, respectively.

- For the **fixed-rate payer**, the value of the IRS at time t is

$$V_{\text{swap-fixed}}(t) = V_{\text{float}}(t) - V_{\text{fixed}}(t)$$

- For the **floating-rate payer**, the value of the IRS at time t is

$$V_{\text{swap-float}}(t) = V_{\text{fixed}}(t) - V_{\text{float}}(t)$$

IRS Valuation

Valuation Procedure: Preliminaries

- Let L be the notional for both legs.
- The fixed leg consists of N fixed payments at times $S_1, S_2, ..., S_N$, where $S_i < S_{i+1}\ \forall i$.
- The floating leg consists of n floating payments at times $T_1, T_2, ..., T_n$, where $T_i < T_{i+1}\ \forall i$.
- The floating leg payments are based on the floating interest rates determined at the reset dates $T_0, T_1, ..., T_{n-1}$.
- The last payment dates of both legs are the same: $S_N = T_n$.

Note

In the succeeding discussion, we assume that rates are simple.

IRS Valuation

Valuation Procedure: V_{fixed}

- Let R_{swap} be the fixed rate of the swap. At $t = S_{i+1}$, the fixed-rate payment is

$$L \cdot R_{\text{swap}} \cdot (S_{i+1} - S_i).$$

- The value of the fixed leg at time S_0 is

$$\begin{aligned} V_{\text{fixed}}(S_0) &= \sum_{i=0}^{N-1} L \cdot R_{\text{swap}} \cdot (S_{i+1} - S_i) \cdot DF(S_0, S_{i+1}) \\ &= L \cdot R_{\text{swap}} \cdot \sum_{i=0}^{N-1} (S_{i+1} - S_i) \cdot DF(S_0, S_{i+1}) \end{aligned}$$

IRS Valuation

Valuation Procedure: V_{fixed}

- At time $t < S_0$ (Note: $S_0 = T_0$),

$$\begin{aligned} V_{\text{fixed}}(t) &= L \cdot R_{\text{swap}} \cdot \sum_{i=0}^{N-1} (S_{i+1} - S_i) \cdot DF(S_0, S_{i+1}) DF(t, S_0) \\ &= L \cdot R_{\text{swap}} \cdot \sum_{i=0}^{N-1} (S_{i+1} - S_i) \cdot DF(t, S_{i+1}) \end{aligned}$$

IRS Valuation

Valuation Procedure: V_{float}

- At $t = T_0$ (the first reset date), we can assume that the floating rate between the ith and $(i+1)$st reset dates is the forward rate

$$R_F(T_0, T_i, T_{i+1}) = \left(\frac{DF(T_0, T_i)}{DF(T_0, T_{i+1})} - 1 \right) \frac{1}{T_{i+1} - T_i},$$

- The value of the floating leg at time T_0 is

$$V_{\text{float}}(T_0) = \sum_{i=0}^{n-1} L \cdot R_F(T_0, T_i, T_{i+1}) \cdot (T_{i+1} - T_i) \cdot DF(T_0, T_{i+1})$$

Let us simplify $V_{\text{float}}(T_0)$ using $R_F(T_0, T_i, T_{i+1})$.

IRS Valuation

Valuation Procedure: V_{float}

- Using $R_F(T_0, T_i, T_{i+1})$, we have

$$V_{\text{float}}(T_0) = L\left[1 - DF(T_0, T_n)\right].$$

- At time $t < T_0$,

$$\begin{aligned} V_{\text{float}}(t) &= L\left[1 - DF(T_0, T_n)\right] DF(t, T_0) \\ &= L\left[DF(t, T_0) - DF(t, T_n)\right] \end{aligned}$$

IRS Valuation

Valuation Procedure: Values to the Parties (Recall)

- For the **fixed-rate payer**, the value of the IRS at time t is

$$V_{\text{swap-fixed}}(t) = V_{\text{float}}(t) - V_{\text{fixed}}(t)$$

- For the **floating-rate payer**, the value of the IRS at time t is

$$V_{\text{swap-float}}(t) = V_{\text{fixed}}(t) - V_{\text{float}}(t)$$

IRS Valuation

Valuation Procedure: Values to the Parties (Recall)

The swap rate at time t, denoted by $s(t)$, is the fixed rate R_{swap} that makes the value of the swap equal to 0:

$$V_{\text{fixed}}(t) = V_{\text{float}}(t)$$

$$L \cdot s(t) \cdot \sum_{i=0}^{N-1} (S_{i+1} - S_i) \cdot DF(t, S_{i+1}) = L\left[DF(t, T_0) - DF(t, T_n)\right]$$

$$s(t) = \frac{DF(t, T_0) - DF(t, T_n)}{\sum_{i=1}^{N-1} (S_{i+1} - S_i) \cdot DF(t, S_{i+1})}$$

IRS Valuation Using Bonds

An Exchange of Bonds

To see an IRS as an exchange of a fixed-rate bond and a floating-rate bond, the notional L must be added to the fixed and floating legs at maturity $S_N = T_n$.

IRS Valuation Using Bonds

An Exchange of Bonds

- The price of the fixed-rate bond at time t is given by

$$
\begin{aligned}
B_{\text{fixed-bond}}(t) &= V_{\text{fixed}}(t) + L \cdot DF(t, S_N) \\
&= L \cdot R_{\text{swap}} \cdot \sum_{i=0}^{N-1} (S_{i+1} - S_i) \cdot DF(t, S_{i+1}) \\
&\quad + L \cdot DF(t, S_N)
\end{aligned}
$$

- The price of the floating-rate bond at time t is given by

$$
\begin{aligned}
B_{\text{float-bond}}(t) &= V_{\text{float}}(t) + L \cdot DF(t, T_n) \\
&= L\left[DF(t, T_0) - DF(t, T_n)\right] + L \cdot DF(t, T_n) \\
&= L \cdot DF(t, T_0)
\end{aligned}
$$

IRS Valuation Using Bonds

An Exchange of Bonds

- Since $S_N = T_n$,

$$B_{\text{float-bond}}(t) - B_{\text{fixed-bond}}(t) = V_{\text{float}}(t) - V_{\text{fixed}}(t)$$

  and

$$B_{\text{fixed-bond}}(t) - B_{\text{float-bond}}(t) = V_{\text{fixed}}(t) - V_{\text{float}}(t).$$

Examples

Example 1

A 2-year IRS is initiated on June 25, 2014 for a semi-annual exchange of fixed rate and floating rate payments beginning 6 months from the initiation date. The continuously compounded interest rates with day count convention Actual/365 are as follows:

tenor	6 mo	1 yr	18 mo	2 yr
rate	2.5%	3.25%	4.1%	5%

Determine the price of the swap (the swap rate) at initiation.

Example 2

Today is June 25, 2015. Consider a \$10 million 1-year fixed-for-floating swap with quarterly payments where the floating rate is 3-month LIBOR. Suppose the swap rate is 6% and that it is now 1 month into the swap. The LIBOR rates (simple, Actual/360) are currently at 5%, 5.5%, 6%, and 6.5% at the 2-, 5-, 8-, and 11-month maturities, respectively. The first payment on the floating leg was fixed 1 month ago, based on the (then) current 3-month forward rate of 5.5%. Find the value of the swap now. If we were to fix the swap rate now, what would it be?

Exercises

Answer items 3 and 4 in the uploaded IRS handout.