Banach spaces:

Cauchy sequence u_n in a normed vector space $(E, \|\circ\|_E)$ have the following property:

$$\forall \varepsilon > 0, \ \exists N(\varepsilon) > 0 \text{ such that } n > m > N(\varepsilon)$$

$$\Rightarrow \|u_n - u_m\|_E < \varepsilon .$$

→ for a certain number N, for all other n, m greater than that particular N, the distance between elements u_n, u_m must not be greater than ε. (Thus, N is ε dependent).

Finite element method.

Variational formulation of the method:

Consider a particular problem where u is the unknown function such that u is to be considered as an element of the space of functions V.

The idea is to obtain an approximate function $\tilde{u}$. instead of finding a sequence of points in a mesh $u_1, \dots, u_n$.

In the finite element method, we find the variational problem.

↳ Consider a real valued function v on Ω called the test function.

For the continuous problem

$$-\Delta u = f$$

$$* \quad -\int_{\Omega} \Delta u \cdot v \, d\Omega = \int_{\Omega} f v \, d\Omega \quad \forall v \in \Omega.$$

↳ Energy formulation:

$f \cdot v$: work done

f ↑ force, v ↑ displacement.

$$-\int \Delta u \cdot v \, d\Omega = \int f \cdot v \, d\Omega.$$

According to Green's formula, for Ω bounded in $\mathbb{R}^n$,

$$\boxed{\int_\Omega (\Delta u) v \, d\Omega = -\int \nabla u \cdot \nabla v \, d\Omega + \int_{\partial\Omega} \frac{\partial u}{\partial n} v \, d\Gamma}$$

n is the normal vector to the boundary $\partial\Omega$.
$d\Gamma$ is a surface element.

Example: Dirichlet conditions

$$u\big|_{\partial\Omega} = 0$$

Since u is one of the functions v then

$$v = 0 \text{ on } \partial\Omega.$$

$$\Rightarrow \quad \frac{\partial u}{\partial n} \cdot v = 0$$

Thus

$$\int \nabla u \cdot \nabla v \, d\Omega + \int f \cdot v \, d\Omega = 0.$$

$$\forall v \in V.$$

we need the integrals over Ω to converge.

the Cauchy-Schwartz inequality yields

$$\left| \int_\Omega d\Omega\, f\nu \right| \le \int d\Omega\, |f\nu|$$

$$\le \sqrt{\int d\Omega\, |f|^2 \int d\Omega\, |\nu|^2}.$$

Since f is already an L^2 function, we get that

$$\int d\Omega\, |f|^2 < \infty$$

Thus we need to ~~impose~~

$$\int d\Omega\, |\nu|^2 < \infty$$

in order for $\int d\Omega\, f\nu$ to converge. $\therefore \nu \in L^2$ as well.

$$\frac{\partial c_+}{\partial t} - \nabla^2 c_+ + \nabla(c_+ \nabla \psi) = 0$$

$$\frac{\partial c_-}{\partial t} - \nabla^2 c_- - \nabla(c_- \nabla \psi) = 0.$$

$$\nabla^2 \psi = \kappa_1 c_- - \kappa_1 c_+.$$

$$\Delta x^2 \frac{\left(c_+^{n+1,k} - c_+^{n,k}\right)}{\Delta t} = \left(c_+^{n,k+1} - 2c_+^{n,k} + c^{n,k-1}\right)$$

$$- \left(c_+^{n,k+1} - c_+^{n,k}\right)\left(\psi^{n,k+1} - \psi^{n,k+1}\right)$$

$$- c_+^{n,k}\left(\psi^{n,k+1} - 2\psi^{n,k} + \psi^{n,k}\right)$$

$$-\frac{\sigma x}{2\varepsilon} \qquad \sigma = \frac{-2\varepsilon\kappa}{\beta q} \quad \frac{\tanh\left(\frac{q\beta V_0}{4}\right)}{\tanh\left(\frac{q\beta V_0}{4}\right) - 1}$$

$$\Rightarrow \quad V(x) = \frac{\kappa}{\beta q} \cdot \left(\frac{\tanh\left(\frac{q\beta V_0}{4}\right)}{\tanh\left(\frac{q\beta V_0}{4}\right) - 1}\right) \cdot x$$

$$\psi(x) = \kappa \left(\frac{\tanh\left(\frac{q\beta V_0}{4}\right)}{\tanh\left(\frac{q\beta V_0}{4}\right) - 1}\right) \cdot x.$$

RGE's $\longrightarrow$ Ecuaciones para los couplings.

MSSM.

PSS partial split susy.

Run couplings with RGES.

mass equation in terms of θ_{sol}.

BRpV

Calcula masas de neutrinos según RGES.

$$\frac{\partial c_+}{\partial t} = \nabla^2 c_+ - \nabla c_+ \nabla \psi - c_+ \nabla^2 \psi$$

$$\frac{\partial c_-}{\partial t} = \nabla^2 c_- + \nabla c_- \nabla \psi + c_- \nabla^2 \psi$$

$$\nabla^2 \psi = \kappa (c_- - c_+)$$

We get

$$\frac{\partial c_+}{\partial t} = \nabla^2 c_+ + \nabla c_+ \cdot E + c_+ \nabla \cdot E$$

$$\frac{\partial c_-}{\partial t} = \nabla^2 c_- - \nabla c_- \cdot E - c_- \nabla \cdot E$$

$$\nabla E = \kappa (c_- - c_+)$$

$$\frac{\partial}{\partial t}(c_- - c_+) = \nabla^2 (c_- - c_+) - (\nabla (c_- - c_+)) \cdot E - (c_- - c_+) \nabla \cdot E$$

$$\frac{\partial}{\partial t} \nabla E = \nabla^3 E - (\nabla \cdot E) E - (\nabla \cdot E)^2$$

$$\int_0^x \psi'' \, dx = \kappa \int_0^x (c_- - c_+) \, dx$$

$$\psi'(x) - \psi_0 = \kappa \int_0^x (c_- - c_+) \, dx$$

$$\psi'(x) = \psi_0 + \kappa \int_0^x (c_- - c_+) \, dx .$$

$$\frac{\partial c_+}{\partial t} = \nabla^2 c_+ - \nabla (c_+ \nabla \psi)$$

$$\frac{\partial c_-}{\partial t} = \nabla^2 c_- + \nabla (c_+ \nabla \psi)$$

$$c_+^{n+1,k} - c_+^{n,k} = \frac{\Delta t}{\Delta x^2} \Big[c^{n,k+1} - 2c^{n,k} + c^{n,k-1} - \left(c_+^{n,k+1} - c_+^{n,k}\right) \times \left(\psi^{n,k+1} - \psi^{n,k}\right) + c_+^{n,k}\left(\psi^{n,k+1} - 2\psi^{n,k} + \psi^{n,k-1}\right) \Big]$$

$$c_+^{n+1,k} = c_+^{n,k}\left(1 + \alpha\left(-2 - \psi^{n,k} + \psi^{n,k-1}\right)\right) + \alpha c^{n,k+1}\left(1 - \psi^{n,k+1} + \psi^{n,k}\right) + \alpha c^{n,k-1}$$

$$A\underline{\psi} = \kappa \Delta \underline{c}$$

$$\underline{\psi} = \kappa A^{-1} \Delta \underline{c}$$

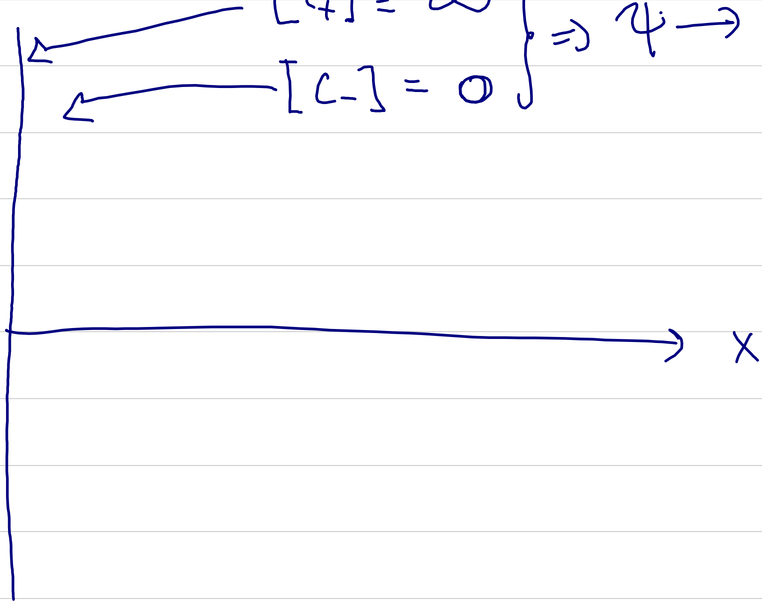
$$\frac{\partial c_+}{\partial t} - \nabla^2 c_+ + \nabla(c_+ \nabla \psi) = \gamma$$

Laplace:

$$\nabla^2 c_+ = \frac{1}{\Delta x^2}\left(c_+^{n+1} - 2c_+^{n} + c_+^{n-1}\right)$$

$K=0$

$$\left.\begin{array}{l}[c_+] = \infty \\ [c_-] = 0\end{array}\right\} \Rightarrow \psi \to \infty$$



$$\left.\frac{\partial c_+}{\partial t} + \nabla \cdot N\right|_{x=0} = r$$

$$\left.c_+^{n+1} - c_+^{n} = r - \nabla^2 c_+ + \nabla (c_+ \nabla \psi)\right|_{x=0}$$

$$c_+^{n+1} = c_+^{n} + r - \nabla^2 c_+ + \nabla (c_+ \nabla \psi)$$

Consider the Laplace - Dirichlet problem:

$$-\Delta u = f$$

$$u(0) = 0.$$

$$\frac{\partial c_+}{\partial t} = \mathcal{D}_+ \nabla^2 c_+ - \mathcal{D}_+ \nabla (c_+ \nabla \psi)$$

$$\frac{\partial c_-}{\partial t} = \mathcal{D}_+ \nabla^2 c_- + \mathcal{D}_- \nabla (c_- \nabla \psi)$$

$$\nabla^2 \psi + \kappa_1 (c_+ - c_-) = 0$$

$$c_\pm = T_\pm(t) \zeta_\pm(x)$$

$$\partial_t \left(T_+(t) \zeta(x)\right) = \mathcal{D}_+ \partial_x^2 \left(T_+(t) \zeta(x)\right) - \mathcal{D}_+ \partial_x \left(T_+(t) \zeta(x) \nabla \psi\right)$$

$$\frac{1}{T_+(t)} \partial_t T_+(t) = \frac{\mathcal{D}_+}{\zeta_+(x)} \partial_x^2 \zeta(x) - \frac{\mathcal{D}_+}{\zeta(x)} \partial_x \left(\zeta(x) \nabla \psi\right)$$