# Notes from 7 May 2018 with SK

May 9, 2018

**Procedure.**

The Hamiltonian for our system is:

$$H = \Delta(t)\sigma^+\sigma^- + \left(\frac{\Omega}{2}\sigma^+ a + \frac{\Omega_q(t)}{2}\sigma^+ + \frac{\Omega_r(t)}{2}a^\dagger\right) + \text{h.c.} \tag{1}$$

| term | meaning |
|---|---|
| $\Delta(t)$ | $= \omega_q(t) - \omega_r$ (detuning) |
| $\sigma^+\sigma^-$ | $\sigma^+\sigma^- = \frac{1}{2}\left(-\sigma^z + 1\right)$ : rotation around $z$ axis of Bloch sphere |
| $\frac{\Omega}{2}\left(\sigma^+ a + \sigma^- a^\dagger\right)$ | JC interaction between cavity and qubit |
| $\Omega$ | Coupling strength between resonator and qubit |
| $\Omega_q(t)$ | Drive on the qubit (complex) |
| $\Omega_r(t)$ | Drive on the cavity. Experimentally tunable parameter (complex) |

Qubit space operators:

$$\sigma^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{2}$$
$$\sigma^x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{3}$$
$$\sigma^y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{4}$$
$$\sigma^+ = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{5}$$
$$\sigma^- = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{6}$$

$$\sigma^x = \sigma^+ + \sigma^- \tag{7}$$
$$\sigma^y = i\left(\sigma^+ - \sigma^-\right) \tag{8}$$

$$\sigma^+\sigma^- = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{9}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{10}$$
$$= \frac{1}{2}\left(-\sigma^z + 1\right) \tag{11}$$

$\sigma^x |g\rangle$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |e\rangle \tag{12}$$

$\sigma^y |g\rangle$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -i \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -i\, |e\rangle \tag{13}$$

$\sigma^z |g\rangle$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = - \begin{bmatrix} 0 \\ 1 \end{bmatrix} = - |g\rangle \tag{14}$$

$\sigma^+ |g\rangle$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |e\rangle \tag{15}$$

$\sigma^- |g\rangle$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 \tag{16}$$

$\sigma^x |e\rangle$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |g\rangle \tag{17}$$

$\sigma^y |e\rangle$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = i \begin{bmatrix} 0 \\ 1 \end{bmatrix} = i\, |g\rangle \tag{18}$$

$\sigma^z |e\rangle$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |e\rangle \tag{19}$$

$\sigma^+ |e\rangle$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \tag{20}$$

$\sigma^- |e\rangle$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |g\rangle \tag{21}$$

Also, note that:

$$e^{ib(\hat{n}\cdot\vec{\sigma})} = I\cos(b) + i(\hat{n}\cdot\vec{\sigma})\sin(b) \tag{22}$$

We use the following constants:

$$\Delta_{off} = 2\pi \cdot (-463)\,\mathrm{MHz}$$

$$\Omega_q = 1000\,\mathrm{MHz}$$

Our initial and target states are:

$$|g\rangle\,|0\rangle \rightarrow |g\rangle \otimes \sum_n c_n\,|n\rangle \tag{23}$$

In order to understand how to get to the final state, we will will work backwards by seeing what steps would be required to go from the final state to the initial state. These steps can be summarized as follows:

1. Place the cavity and the qubit on resonance with each other so that $\Delta = 0$. Let them interact until the cavity transfers the photon excitation into the qubit.
2. Put the cavity and the qubit off resonance and apply a pi-pulse so that the qubit goes from the excited state to the ground state.
3. Rotate the state around the $z$-axis so that the phase changes to allow $\Omega$ to be real.
4. Repeat steps 1-3 until all the excitations have been eliminated and the state $|g\rangle\,|0\rangle$ has been reached.
5. Time reverse everything

Note that the Hamiltonian is being solved in reverse time (so the times are negative). Therefore, the Schrödinger Equation looks like:

$$-i\hbar\frac{\partial}{\partial t}\,|\psi(t)\rangle = \hat{H}\,|\psi(t)\rangle \tag{24}$$

Where the dash $(-)$ is a negative sign. Therefore, we can just stick the dash (negative sign) (or whatever you want to call it) (maybe a minus sign for example) into the $i$'s when we are solving this. $|\cdot\rangle$ is a ket.
The $\hat{H}$ does not refer to the first letter of Prof. Tureci's first name. And the $\psi$ is not pounds per square inch, since we are assuming absolute zero, so we never really think about pressure. Except the pressure to make better jokes than these. In order to understand the theory, we start with the target state and work backwards. Take the following target state as an example:

$$|g\rangle \otimes \left(\frac{1}{\sqrt{2}}\,|1\rangle + \frac{i}{\sqrt{2}}\,|3\rangle\right) \tag{25}$$

Where the $i$ represents a rotation by $\pi/2$ since $e^{i\pi/2} = i$.

$$c_{g_n}(t) = c_{g_n}(0)\cos\left(\frac{\Omega\sqrt{n}}{2}t\right) + ic_{e_{n-1}}(0)\sin\left(\frac{\Omega\sqrt{n}}{2}t\right) \tag{26}$$

$$c_{e_{n-1}}(t) = c_{e_{n-1}}(0)\cos\left(\frac{\Omega\sqrt{n}}{2}t\right) + ic_{g_n}(0)\sin\left(\frac{\Omega\sqrt{n}}{2}t\right) \tag{27}$$

Then we use eq. (26) eq. (27) to determine the times for which the system needs to be on-resonance and off-resonance. These equations come from solving the Schrödinger Equation and using the rotating wave approximation. (We solved this in the first p-set.) The Hamiltonian for this is:

$$H = \frac{\Omega}{2}\left(\sigma^+ a + a^\dagger \sigma^-\right) \tag{28}$$

Our initial state is:

$$c_{g_1}(0) = \frac{1}{\sqrt{2}} \tag{29}$$

$$c_{e_2}(0) = 0 \tag{30}$$

$$c_{g_3}(0) = -\frac{i}{\sqrt{2}} \tag{31}$$

Using eqs. (26), (27), (29), (30), and (31), we obtain the following expressions for $c_{e2}$ and $c_{g3}$ as functions of time:

$$c_{g_3}(t) = -\frac{i}{\sqrt{2}}\cos\left(\frac{\Omega\sqrt{3}}{2}t\right) \tag{32}$$

$$c_{e_2}(t) = \frac{1}{\sqrt{2}}\sin\left(\frac{\Omega\sqrt{3}}{2}t\right) \tag{33}$$

Since we want the excitation from the cavity to go into the qubit, we set $c_{g_3}(t) = 0$ and obtain:

$$\frac{\Omega\sqrt{3}}{2}t_1 = \frac{\pi}{2} \tag{34}$$

$$\rightarrow t_1 = \frac{\pi}{\Omega\sqrt{3}} \tag{35}$$

$$c_{e_2}(t_1) = \frac{1}{\sqrt{2}}\sin\left(\frac{\Omega\sqrt{3}}{2}\frac{\pi}{\Omega\sqrt{3}}\right) = \frac{1}{\sqrt{2}} \tag{36}$$

$t_1$ is the interaction time for the cavity to exchange excitations with the qubit from $n = 3$ total excitations to $n = 2$ total excitations.
For $n = 2$ excitations,

$$c_{g_2}(0) = 0 \tag{37}$$

$$c_{e_0}(0) = 0 \tag{38}$$

Therefore, there is no change in the second excitation. In other words, for two excitations, there is no interaction between the qubit and the cavity.
For $n = 1$ excitations,

$$c_{g_1}(0) = \frac{1}{\sqrt{2}} \tag{39}$$

$$c_{e_0}(0) = 0 \tag{40}$$

Using eq. (26) and eq. (27),

$$c_{g_1}(t) = \frac{1}{\sqrt{2}}\cos\left(\frac{\Omega}{2}t\right) \tag{41}$$

$$c_{e_0}(t) = \frac{i}{\sqrt{2}}\sin\left(\frac{\Omega}{2}t\right) \tag{42}$$

For time $t = t_1$,

$$c_{g_1}(t_1) = \frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{2\sqrt{3}}\right) \tag{43}$$

$$c_{e_0}(t_1) = \frac{i}{\sqrt{2}}\sin\left(\frac{\pi}{2\sqrt{3}}\right) \tag{44}$$

For $n = 0$ excitations, there cannot be any change in the state of the system because there are no excitations to be shifted around.
Therefore, after time $t = t_1$, we obtain the following state:

$$|\psi(t_1)\rangle = \frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{2\sqrt{3}}\right)|g,1\rangle + \frac{i}{\sqrt{2}}\sin\left(\frac{\pi}{2\sqrt{3}}\right)|e,0\rangle + \frac{1}{\sqrt{2}}|e,2\rangle \tag{45}$$

Then, we use the (reverse) qubit pulse to go from the excited state to the ground state. This is like a rotation around the $x$ axis of the Bloch sphere ($e^{i\Delta_{off}t}$) where $\Delta_{off}$ determines the extent of rotation.
The relevant Hamiltonian is:

$$H = \frac{\Omega_q}{2}\sigma^+ + \frac{\Omega_q}{2}\sigma^- \tag{46}$$

We look at the time evolution of the state so that we can determine when the qubit will have gone into the ground state. Then, we change the frequency of the cubit in negligible time in order to bring the cubit and resonator out of resonance. However, we first have to apply the phase rotation, since we are going in reverse time. We use the following time evolution operator:

$$e^{i\Delta_{off}\sigma^+\sigma^- t} = e^{\frac{i}{2}\Delta_{off}(-\sigma^z+1)t} \tag{47}$$

Disregard the $\Delta_{off}\frac{i}{2}t$ in the exponential because it doesn't change the state.

$$e^{-\frac{i}{2}\Delta_{off}\sigma^z t_r} = \cos\left(\frac{\Delta_{off}}{2}t_r\right) - i\sigma^z \sin\left(\frac{\Delta_{off}}{2}t_r\right) \tag{48}$$

Applying this phase rotation:

$$\begin{aligned}
|\psi(t_2)\rangle_1 = & \frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right) + i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|g,1\rangle \\
& + \frac{i}{\sqrt{2}}\sin\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right) - i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|e,0\rangle \\
& + \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right) - i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|e,2\rangle
\end{aligned} \tag{49}$$

Now we apply the pulse:

$$e^{iHt} = e^{i\left(\frac{\Omega_q}{2}\sigma^+ + \frac{\Omega_q^*}{2}\sigma^-\right)t} \tag{50}$$

$$= e^{i\left[\frac{\mathrm{Re}\Omega_q\left(\sigma^+ + \sigma^-\right)}{2} - i\frac{\mathrm{Im}\Omega_q\left(\sigma^+ - \sigma^-\right)}{2}\right]t} \tag{51}$$

$$= e^{i\frac{|\Omega_q|}{2}t\left[-\frac{\mathrm{Re}\Omega_q}{|\Omega_q|}\sigma^x + \frac{\mathrm{Im}\Omega_q}{|\Omega_q|}\sigma^y\right]} \tag{52}$$

$$= e^{i\frac{|\Omega_q|}{2}\hat{n}\cdot\vec{\sigma}t} \tag{53}$$

where:

$$\hat{n} = -\frac{\mathrm{Re}\Omega_q}{|\Omega_q|}\hat{x} + \frac{\mathrm{Im}\Omega_q}{|\Omega_q|}\hat{y} \tag{54}$$

Expanding eq. (53):

$$\mathbb{I}\cos\left(\frac{|\Omega_q|}{2}t\right) - i\left[\frac{\mathrm{Re}\Omega_q}{|\Omega_q|}\sigma^x - \frac{\mathrm{Im}\Omega_q}{|\Omega_q|}\sigma^y\right]\sin\left(\frac{|\Omega_q|}{2}t\right) \tag{55}$$

Applying eq. (55) to the state (49), term by term:

$$
\begin{aligned}
|\psi(t_2)\rangle =& \cos\left(\frac{|\Omega_q|}{2}t_2\right)\frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)+i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|g,1\rangle \\
&- i\frac{\mathrm{Re}\Omega_q}{|\Omega_q|}\sin\left(\frac{|\Omega_q|}{2}t_2\right)\frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)+i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|e,1\rangle \\
&+ \frac{\mathrm{Im}\Omega_q}{|\Omega_q|}\sin\left(\frac{|\Omega_q|}{2}t_2\right)\frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)+i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|e,1\rangle \\
\\
&+ \cos\left(\frac{|\Omega_q|}{2}t_2\right)\frac{i}{\sqrt{2}}\sin\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)-i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|e,0\rangle \\
&- i\frac{\mathrm{Re}\Omega_q}{|\Omega_q|}\sin\left(\frac{|\Omega_q|}{2}t_2\right)\frac{i}{\sqrt{2}}\sin\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)-i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|g,0\rangle \\
&- \frac{\mathrm{Im}\Omega_q}{|\Omega_q|}\sin\left(\frac{|\Omega_q|}{2}t_2\right)\frac{i}{\sqrt{2}}\sin\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)-i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|g,0\rangle \\
\\
&+ \cos\left(\frac{|\Omega_q|}{2}t_2\right)\frac{1}{\sqrt{2}}\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)-i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|e,2\rangle \\
&- i\frac{\mathrm{Re}\Omega_q}{|\Omega_q|}\sin\left(\frac{|\Omega_q|}{2}t_2\right)\frac{1}{\sqrt{2}}\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)-i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|g,2\rangle \\
&- \frac{\mathrm{Im}\Omega_q}{|\Omega_q|}\sin\left(\frac{|\Omega_q|}{2}t_2\right)\frac{1}{\sqrt{2}}\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)-i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|g,2\rangle
\end{aligned} \tag{56}
$$

The time $t_2$ must be set so that the $|e,2\rangle$ term is zero.

$$\frac{|\Omega_q|}{2}t_2 = \frac{\pi}{2} \tag{57}$$

$$\rightarrow t_2 = \frac{\pi}{|\Omega_q|} \tag{58}$$

Since we can control the drive on the qubit, we can control the phase as well, so let's set that equal to zero. That means that this is true: $\frac{\mathrm{Im}\Omega_q}{|\Omega_q|} = 0$ and $\frac{\mathrm{Re}\Omega_q}{|\Omega_q|} = 1$. Therefore, eq. (56) becomes:

$$
\begin{aligned}
|\psi(t_2)\rangle =& \frac{1}{\sqrt{2}}\cos\left(\frac{\pi}{2\sqrt{3}}\right)\left[-i\cos\left(\frac{\Delta_{off}}{2}t_2\right)+\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|e,1\rangle \\
&+ \frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{2\sqrt{3}}\right)\left[\cos\left(\frac{\Delta_{off}}{2}t_2\right)-i\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|g,0\rangle \\
&- \frac{1}{\sqrt{2}}\left[i\cos\left(\frac{\Delta_{off}}{2}t_2\right)+\sin\left(\frac{\Delta_{off}}{2}t_2\right)\right]|g,2\rangle
\end{aligned} \tag{59}
$$

$$
\begin{aligned}
&= (0.43128 - 0.06199i)\,|e,1\rangle \\
&+ (-0.07924 + 0.55125i)\,|g,0\rangle \\
&+ (-0.69991 + 0.10061i)\,|g,2\rangle
\end{aligned} \tag{60}
$$

Now, we rotate. Apply the following:

$$
\begin{aligned}
e^{i\Delta_{off}\sigma^+\sigma^- t} &= e^{\frac{i}{2}(\Delta_{off}(-\sigma^z+1))t} && (61)\\
&= e^{-i\frac{\Delta_{off}\sigma^z}{2}t} && (62)\\
&= \cos\left(\frac{\Delta_{off}}{2}t\right) - i\sigma^z \sin\left(\frac{\Delta_{off}}{2}t\right) && (63)
\end{aligned}
$$

We can do the step from eq. (61) to eq. (62) because the constant does not change the state.

$$
\begin{aligned}
|\psi(t_3)\rangle = \cos&\left(\frac{\Delta_{off}}{2}t_3\right)(0.43128 - 0.06199i)\,|e,1\rangle \\
- i\sin&\left(\frac{\Delta_{off}}{2}t_3\right)(0.43128 - 0.06199i)\,|e,1\rangle \\
\\
+ \cos&\left(\frac{\Delta_{off}}{2}t_3\right)(-0.07924 + 0.55125i)\,|g,0\rangle \\
- i\sin&\left(\frac{\Delta_{off}}{2}t_3\right)(-0.07924 + 0.55125i)\,|g,0\rangle \qquad (64)\\
\\
+ \cos&\left(\frac{\Delta_{off}}{2}t_3\right)(-0.69991 + 0.10061i)\,|g,2\rangle \\
+ i\sin&\left(\frac{\Delta_{off}}{2}t_3\right)(-0.69991 + 0.10061i)\,|g,2\rangle
\end{aligned}
$$

$$
\begin{aligned}
|\psi(t_3)\rangle &= e^{-i\frac{\Delta_{off}}{2}t_3}(0.43128 - 0.06199i)\,|e,1\rangle \\
&+ e^{-i\frac{\Delta_{off}}{2}t_3}(-0.07924 + 0.55125i)\,|g,0\rangle \qquad (65)\\
&+ e^{i\frac{\Delta_{off}}{2}t_3}(-0.69991 + 0.10061i)\,|g,2\rangle
\end{aligned}
$$

We leave $t_3$ as a free parameter while we do the swap. The initial states are:

$$
\begin{aligned}
c_{g_0}(0) &= e^{-i\frac{\Delta_{off}}{2}t_3}(-0.07924 + 0.55125i) && (66)\\
c_{e_1}(0) &= e^{-i\frac{\Delta_{off}}{2}t_3}(0.43128 - 0.06199i) && (67)\\
c_{g_2}(0) &= e^{i\frac{\Delta_{off}}{2}t_3}(-0.69991 + 0.10061i) && (68)
\end{aligned}
$$

Using eqs. (26), (27), (66), (67), and (68), we obtain the following expressions for $c_{g_0}(t)$, $c_{e_1}(t)$, and $c_{g_2}(t)$.

$$
\begin{aligned}
c_{g_0}(t) &= e^{-i\frac{\Delta_{off}}{2}t_3}(-0.07924 + 0.55125i)\cos\left(\frac{\Omega\sqrt{n}}{2}t\right) \\
c_{g_2}(t) &= e^{i\frac{\Delta_{off}}{2}t_3}(-0.69991 + 0.10061i)\cos\left(\frac{\Omega\sqrt{n}}{2}t\right) \\
&+ e^{-i\frac{\Delta_{off}}{2}t_3}(0.43128i + 0.06199)\sin\left(\frac{\Omega\sqrt{n}}{2}t\right) \qquad (69)\\
c_{e_1}(t) &= e^{-i\frac{\Delta_{off}}{2}t_3}(0.43128 - 0.06199i)\cos\left(\frac{\Omega\sqrt{n}}{2}t\right) \\
&+ e^{i\frac{\Delta_{off}}{2}t_3}(-0.69991i - 0.10061)\sin\left(\frac{\Omega\sqrt{n}}{2}t\right)
\end{aligned}
$$

The excitation goes from the cavity to the cubit, so we set $c_{g_2}(t) = 0$:

$$e^{i\frac{\Delta_{off}}{2}t_3}(-0.69991 + 0.10061i)\cos\left(\frac{\Omega\sqrt{n}}{2}t\right) = -e^{-i\frac{\Delta_{off}}{2}t_3}(0.43128i + 0.06199)\sin\left(\frac{\Omega\sqrt{n}}{2}t\right) \tag{70}$$

$$\frac{(0.69991 - 0.10061i)}{(0.43128i + 0.06199)}e^{i\Delta_{off}t_3} = \tan\left(\frac{\Omega\sqrt{n}}{2}t_4\right) \tag{71}$$

$$\frac{(0.69991 - 0.10061i)(-0.43128i + 0.06199)}{(0.43128i + 0.06199)(-0.43128i + 0.06199)} = -0.457 - 1.557i \tag{72}$$

$$(-0.457 - 1.557i)\,e^{i\Delta_{off}t_3} = \tan\left(\frac{\Omega\sqrt{n}}{2}t_4\right) \tag{73}$$

$$1.62287e^{i0.2714}e^{i\Delta_{off}t_3} = \tan\left(\frac{\Omega\sqrt{n}}{2}t_4\right) \tag{74}$$

Set $n = 2$ because we are in the second photon state:

$$t_4\Omega = \arctan(1.62287)\cdot\sqrt{2} = 1.44\text{s} \tag{75}$$

Set the imaginary part equal to zero:

$$\begin{aligned} t_3 &= \frac{-0.2714}{\Delta_{off}} && (76)\\ &= 9.33018\times 10^{-5} && (77)\end{aligned}$$

Now, the states are:

$$\begin{aligned}
c_{g_0}(t) &= \left(\cos\left(\frac{-0.2714}{2}\right) + i\sin\left(\frac{0.2714}{2}\right)\right)(-0.07924 + 0.55125i)\\
c_{g_2}(t) &= \left(\cos\left(\frac{-0.2714}{2}\right) - i\sin\left(\frac{0.2714}{2}\right)\right)(-0.69991 + 0.10061i)\cos\left(\frac{1.44\sqrt{2}}{2}\right)\\
&+ \left(\cos\left(\frac{-0.2714}{2}\right) + i\sin\left(\frac{0.2714}{2}\right)\right)(0.43128i + 0.06199)\sin\left(\frac{1.44\sqrt{2}}{2}\right)\\
c_{e_1}(t) &= \left(\cos\left(\frac{-0.2714}{2}\right) + i\sin\left(\frac{0.2714}{2}\right)\right)(0.43128 - 0.06199i)\cos\left(\frac{1.44\sqrt{1}}{2}\right)\\
&+ \left(\cos\left(\frac{-0.2714}{2}\right) - i\sin\left(\frac{0.2714}{2}\right)\right)(-0.69991i - 0.10061)\sin\left(\frac{1.44\sqrt{1}}{2}\right)
\end{aligned} \tag{78}$$

**Wigner Function**

The Wigner function is a transformation into phase space.

**Experimentally:** We transform the system by applying the displacement operator on the system. They coherently displace the resonator state by driving the system with the $\Omega_r$ given by: $-\alpha = \frac{1}{2}\int\Omega_r(t)dt$, and use the operator $D(-\alpha) = D^\dagger(\alpha) = \exp\left(\alpha^* a - \alpha a^\dagger\right)$.
The parity operator detects the probability of the function being even or odd because it returns either positive or negative eigenvalues, respectively. We know the parity from the applying the displacement operator to the system. Then, we can use the distribution:

$$P_e(\tau) \approx \frac{1}{2}\left(1 - P_g \sum_{n=0}^{\infty} P_n \cos\left(\sqrt{n}\Omega\tau\right)\right)$$

The $P_n$ can be found from the $P_e$ by doing a fit of the $P_e$ to this distribution. The $P_e$ is found by...

do we know the parity by applying a certain displacement?
We have to find $P_e$ somehow?

**Theoretically:** Use Qutip to do Qutip.Wigner