

$$|g\rangle \otimes \left(\frac{1}{\sqrt{2}} |1\rangle + \frac{i}{\sqrt{2}} |3\rangle \right) \quad (1)$$

Step 8: Swap

The Hamiltonian for this is:

$$H = \frac{\Omega}{2} (\sigma^+ a + a^\dagger \sigma^-) \quad (2)$$

Our initial state is:

$$c_{g_1}(0) = \frac{1}{\sqrt{2}} \quad (3)$$

$$c_{e_2}(0) = 0 \quad (4)$$

$$c_{g_3}(0) = \frac{i}{\sqrt{2}} \quad (5)$$

Using eqs. (??), (??), (3), (4), and (5), we obtain the following expressions for c_{e_2} and c_{g_3} as functions of time:

$$c_{g_3}(t) = \frac{i}{\sqrt{2}} \cos\left(\frac{\Omega\sqrt{3}}{2}t\right) \quad (6)$$

$$c_{e_2}(t) = -\frac{1}{\sqrt{2}} \sin\left(\frac{\Omega\sqrt{3}}{2}t\right) \quad (7)$$

Since we want the excitation from the cavity to go into the qubit, we set $c_{g_3}(t) = 0$ and obtain:

$$\frac{\Omega\sqrt{3}}{2}t_8 = \frac{\pi}{2} \quad (8)$$

$$\rightarrow t_8 = \frac{\pi}{\Omega\sqrt{3}} \quad (9)$$

$$c_{e_2}(t_8) = -\frac{1}{\sqrt{2}} \sin\left(\frac{\Omega\sqrt{3}}{2} \frac{\pi}{\Omega\sqrt{3}}\right) = -\frac{1}{\sqrt{2}} \quad (10)$$

t_1 is the interaction time for the cavity to exchange excitations with the qubit from $n = 3$ total excitations to $n = 2$ total excitations.

For $n = 2$ excitations,

$$c_{g_2}(0) = 0 \quad (11)$$

$$c_{e_0}(0) = 0 \quad (12)$$

Therefore, there is no change in the second excitation. In other words, for two excitations, there is no interaction between the qubit and the cavity.

For $n = 1$ excitations,

$$c_{g_1}(0) = \frac{1}{\sqrt{2}} \quad (13)$$

$$c_{e_0}(0) = 0 \quad (14)$$

Using eq. (??) and eq. (??),

$$c_{g_1}(t) = \frac{1}{\sqrt{2}} \cos\left(\frac{\Omega}{2}t\right) \quad (15)$$

$$c_{e_0}(t) = \frac{i}{\sqrt{2}} \sin\left(\frac{\Omega}{2}t\right) \quad (16)$$

For time $t = t_8$,

$$c_{g_1}(t_8) = \frac{1}{\sqrt{2}} \cos\left(\frac{\pi}{2\sqrt{3}}\right) \quad (17)$$

$$c_{e_0}(t_8) = \frac{i}{\sqrt{2}} \sin\left(\frac{\pi}{2\sqrt{3}}\right) \quad (18)$$

For $n = 0$ excitations, there cannot be any change in the state of the system because there are no excitations to be shifted around.

Therefore, after time $t = t_8$, we obtain the following state:

$$|\psi(t_8)\rangle = \frac{1}{\sqrt{2}} \cos\left(\frac{\pi}{2\sqrt{3}}\right) |g, 1\rangle + \frac{i}{\sqrt{2}} \sin\left(\frac{\pi}{2\sqrt{3}}\right) |e, 0\rangle - \frac{1}{\sqrt{2}} |e, 2\rangle \quad (19)$$

$$= 0.43571 |g, 1\rangle + i0.556 |e, 0\rangle - 0.7071 |e, 2\rangle \quad (20)$$

Step 7: Pulse

First, rotation.

$$e^{i\Delta_{off}\sigma^+\sigma^-t} |g\rangle = 1 |g\rangle$$

$$e^{i\Delta_{off}\sigma^+\sigma^-t} |e\rangle = e^{i\Delta_{off}t} |e\rangle$$

The state becomes:

$$|\psi\rangle = 0.43571 |g, 1\rangle + i0.556e^{i\Delta_{off}t_7} |e, 0\rangle - 0.7071e^{i\Delta_{off}t_7} |e, 2\rangle$$

Pulse:

$$|e, 1\rangle \rightarrow i |g, 1\rangle$$

$$\Omega_q t_7 = \pi$$

State:

$$|\psi\rangle = i0.43571 |e, 1\rangle - 0.556e^{i\Delta_{off}t_7} |g, 0\rangle - i0.7071e^{i\Delta_{off}t_7} |g, 2\rangle$$

Step 6: Rotation

$$|\psi\rangle = i0.43571e^{i\Delta_{off}t_6} |e, 1\rangle - 0.556e^{i\Delta_{off}t_7} |g, 0\rangle - i0.7071e^{i\Delta_{off}t_7} |g, 2\rangle$$

Step 5: Swap

$$c_{g_2}(t) = -i0.7071 \cos\left(\frac{\Omega\sqrt{n}}{2}t_5\right) e^{i\Delta_{off}t_7} - 0.43571 \sin\left(\frac{\Omega\sqrt{n}}{2}t_5\right) e^{i\Delta_{off}t_6}$$

$$\text{set } c_{g_2}(t_5) = 0$$

$$0.7071 \cos\left(\frac{\Omega\sqrt{n}}{2}t_5\right) e^{i\Delta_{off}t_7} = i0.43571 \sin\left(\frac{\Omega\sqrt{n}}{2}t_5\right) e^{i\Delta_{off}t_6}$$

$$-\frac{i0.7071}{0.43571} e^{i\Delta_{off}(t_7-t_6)} = \tan\left(\frac{\Omega\sqrt{n}}{2}t_5\right)$$

Set Ω to be real and positive:

$$-ie^{i\Delta_{off}(t_7-t_6)} = 1$$

$$e^{i\Delta_{off}(t_7-t_6)} = i = e^{i\frac{\pi}{2}}$$

$$\Delta_{off}(t_7 - t_6) = \frac{\pi}{2}$$

$$t_6 = t_7 - \frac{\pi}{2\Delta_{off}}$$

Solving for t_5 :

$$\tan\left(\frac{\Omega\sqrt{n}}{2}t_5\right) = \frac{0.7071}{0.43571}$$

$$\Omega t_5 = 1.44$$

More n=2 states. Note that c_{g_0} stays constant because there is no state for it to exchange excitations in.

$$c_{e_1}(t_5) = i0.43571 \cos\left(\frac{\Omega\sqrt{2}}{2}t_5\right) e^{i\Delta_{off}t_6} + 0.7071 \sin\left(\frac{\Omega\sqrt{n}}{2}t_5\right) e^{i\Delta_{off}t_7}$$

The final state is:

$$|\psi\rangle = (-0.7969429 - 0.2339386i) |e, 1\rangle + (0.534368071079 + 0.156861063497i) |g, 0\rangle$$

Step 4: Pulse

First, rotation.

$$\begin{aligned} e^{i\Delta_{off}\sigma^+\sigma^-t} |g\rangle &= 1 |g\rangle \\ e^{i\Delta_{off}\sigma^+\sigma^-t} |e\rangle &= e^{i\Delta_{off}t} |e\rangle \end{aligned}$$

The state becomes:

$$|\psi\rangle = (-0.7969429 - 0.2339386i) e^{i\Delta_{off}t_4} |e, 1\rangle - (0.534368071079 + 0.156861063497i) |g, 0\rangle$$

Pulse:

$$|e, 2\rangle \rightarrow i |g, 2\rangle$$

$$\Omega_q t_4 = \pi$$

State:

$$|\psi\rangle = i(-0.7969429 - 0.2339386i) e^{i\Delta_{off}t_4} |g, 1\rangle - i(0.534368071079 + 0.156861063497i) |e, 0\rangle$$

Step 3: Rotation

$$|\psi\rangle = i(-0.7969429 - 0.2339386i) e^{i\Delta_{off}t_4} |g, 1\rangle - i(0.534368071079 + 0.156861063497i) e^{i\Delta_{off}t_3} |e, 0\rangle$$

Step 2: Swap

$$c_{g1}(t_2) = i(-0.7969429 - 0.2339386i) e^{i\Delta_{off}t_4} \cos\left(\frac{\Omega\sqrt{n}}{2}t_2\right) \\ - (0.534368071079 + 0.156861063497i) e^{i\Delta_{off}t_3} \sin\left(\frac{\Omega\sqrt{n}}{2}t_2\right)$$

$$c_{e0}(t_2) = -i(0.534368071079 + 0.156861063497i) e^{i\Delta_{off}t_3} \cos\left(\frac{\Omega\sqrt{n}}{2}t_2\right) \\ - (-0.7969429 - 0.2339386i) e^{i\Delta_{off}t_4} \sin\left(\frac{\Omega\sqrt{n}}{2}t_2\right)$$

$$\text{set } c_{g1}(t_2) = 0$$

$$i \frac{(-0.7969429 - 0.2339386i)}{(0.534368071079 + 0.156861063497i)} e^{i\Delta_{off}t_4} \cos\left(\frac{\Omega\sqrt{n}}{2}t_2\right) = e^{i\Delta_{off}t_3} \sin\left(\frac{\Omega\sqrt{n}}{2}t_2\right)$$

$$(-1.49137460586i) e^{i\Delta_{off}(t_4-t_3)} = \tan\left(\frac{\Omega\sqrt{n}}{2}t_2\right)$$

Set Ω to be real and positive:

$$-ie^{i\Delta_{off}(t_4-t_3)} = 1$$

$$\Delta_{off}(t_4 - t_3) = \frac{\pi}{2}$$

$$t_3 = t_4 - \frac{\pi}{2\Delta_{off}}$$

$$t_3 = 0.00368$$

Solving for Ωt_2 :

$$\tan\left(\frac{\Omega\sqrt{n}}{2}t_2\right) = 1.49137460586$$

$$\Omega t_2 = 1.9602$$

The final state is:

$$|\psi\rangle = (-0.31944698577 - 0.20522836891i) |e, 0\rangle$$

Step 1: Pulse

First, rotation.

$$\begin{aligned} e^{i\Delta_{off}\sigma^+\sigma^-t} |g\rangle &= 1 |g\rangle \\ e^{i\Delta_{off}\sigma^+\sigma^-t} |e\rangle &= e^{i\Delta_{off}t} |e\rangle \end{aligned}$$

The state becomes:

$$|\psi\rangle = (-0.31944698577 - 0.20522836891i) e^{i\Delta_{off}t_1} |e, 0\rangle$$

Pulse:

$$|e, 0\rangle \rightarrow i |g, 0\rangle$$

$$\Omega_q t_1 = \pi$$

$$t_1 = 0.00314$$

State:

$$|\psi\rangle = i (-0.31944698577 - 0.20522836891i) e^{i\Delta_{off}t_1} |g, 0\rangle$$

$$|\psi\rangle = (-0.286895130017 + 0.248709156436j) |g, 0\rangle$$