$$|g\rangle \otimes \left(\frac{1}{\sqrt{2}} |1\rangle + \frac{i}{\sqrt{2}} |3\rangle \right) \tag{1}$$

Step 8: Swap

The Hamiltonian for this is:

$$H = \frac{\Omega}{2} \left(\sigma^{+} a + a^{\dagger} \sigma^{-} \right) \tag{2}$$

Our initial state is:

$$c_{g_1}(0) = \frac{1}{\sqrt{2}} \tag{3}$$

$$c_{e_2}(0) = 0 \tag{4}$$

$$c_{g_3}(0) = \frac{i}{\sqrt{2}} \tag{5}$$

Using eqs. (**??**), (**??**), (3), (4), and (5), we obtain the following expressions for c_{e2} and c_{g3} as functions of time:

$$c_{g_3}(t) = \frac{i}{\sqrt{2}} \cos \left(\frac{\Omega\sqrt{3}}{2} t \right) \tag{6}$$

$$c_{e_2}(t) = -\frac{1}{\sqrt{2}} \sin \left(\frac{\Omega\sqrt{3}}{2} t \right) \tag{7}$$

Since we want the excitation from the cavity to go into the qubit, we set $c_{g_3}(t) = 0$ and obtain:

$$\frac{\Omega\sqrt{3}}{2} t_8 = \frac{\pi}{2} \tag{8}$$

$$\rightarrow t_8 = \frac{\pi}{\Omega\sqrt{3}} \tag{9}$$

$$c_{e_2}(t_8) = -\frac{1}{\sqrt{2}} \sin \left(\frac{\Omega\sqrt{3}}{2} \frac{\pi}{\Omega\sqrt{3}} \right) = -\frac{1}{\sqrt{2}} \tag{10}$$

t_1 is the interaction time for the cavity to exchange excitations with the qubit from $n = 3$ total excitations to $n = 2$ total excitations.

For $n = 2$ excitations,

$$c_{g_2}(0) = 0 \tag{11}$$

$$c_{e_0}(0) = 0 \tag{12}$$

Therefore, there is no change in the second excitation. In other words, for two excitations, there is no interaction between the qubit and the cavity.

For $n = 1$ excitations,

$$c_{g_1}(0) = \frac{1}{\sqrt{2}} \tag{13}$$

$$c_{e_0}(0) = 0 \tag{14}$$

Using eq. (**??**) and eq. (**??**),

$$c_{g_1}(t) = \frac{1}{\sqrt{2}} \cos\left(\frac{\Omega}{2} t\right) \tag{15}$$

$$c_{e_0}(t) = \frac{i}{\sqrt{2}} \sin\left(\frac{\Omega}{2} t\right) \tag{16}$$

For time $t = t_8$,

$$c_{g_1}(t_8) = \frac{1}{\sqrt{2}} \cos\left(\frac{\pi}{2\sqrt{3}}\right) \tag{17}$$

$$c_{e_0}(t_8) = \frac{i}{\sqrt{2}} \sin\left(\frac{\pi}{2\sqrt{3}}\right) \tag{18}$$

For $n = 0$ excitations, there cannot be any change in the state of the system because there are no excitations to be shifted around.

Therefore, after time $t = t_8$, we obtain the following state:

$$|\psi(t_8)\rangle = \frac{1}{\sqrt{2}} \cos\left(\frac{\pi}{2\sqrt{3}}\right) |g, 1\rangle + \frac{i}{\sqrt{2}} \sin\left(\frac{\pi}{2\sqrt{3}}\right) |e, 0\rangle - \frac{1}{\sqrt{2}} |e, 2\rangle \tag{19}$$

$$= 0.43571 |g, 1\rangle + i0.556 |e, 0\rangle - 0.7071 |e, 2\rangle \tag{20}$$

Step 7: Pulse

First, rotation.

$$e^{i\Delta_{off}\sigma^+\sigma^- t} |g\rangle = 1 |g\rangle$$

$$e^{i\Delta_{off}\sigma^+\sigma^- t} |e\rangle = e^{i\Delta_{off} t} |e\rangle$$

The state becomes:

$$|\psi\rangle = 0.43571 |g, 1\rangle + i0.556 e^{i\Delta_{off} t_7} |e, 0\rangle - 0.7071 e^{i\Delta_{off} t_7} |e, 2\rangle$$

Pulse:

$$|e,2\rangle \rightarrow i\,|g,2\rangle$$

$$\Omega_q t_7 = \pi$$

State:

$$|\psi\rangle = i0.43571\,|e,1\rangle - 0.556e^{i\Delta_{off}t_7}\,|g,0\rangle - i0.7071e^{i\Delta_{off}t_7}\,|g,2\rangle$$

Step 6: Rotation

$$|\psi\rangle = i0.43571e^{i\Delta_{off}t_6}\,|e,1\rangle - 0.556e^{i\Delta_{off}t_7}\,|g,0\rangle - i0.7071e^{i\Delta_{off}t_7}\,|g,2\rangle$$

Step 5: Swap

$$c_{g_2}(t) = -i0.7071\cos\left(\frac{\Omega\sqrt{n}}{2}t_5\right)e^{i\Delta_{off}t_7} - 0.43571\sin\left(\frac{\Omega\sqrt{n}}{2}t_5\right)e^{i\Delta_{off}t_6}$$

set $c_{g_2}(t_5) = 0$

$$0.7071\cos\left(\frac{\Omega\sqrt{n}}{2}t_5\right)e^{i\Delta_{off}t_7} = i0.43571\sin\left(\frac{\Omega\sqrt{n}}{2}t_5\right)e^{i\Delta_{off}t_6}$$

$$-\frac{i0.7071}{0.43571}e^{i\Delta_{off}(t_7-t_6)} = \tan\left(\frac{\Omega\sqrt{n}}{2}t_5\right)$$

Set Ω to be real and positive:

$$-ie^{i\Delta_{off}(t_7-t_6)} = 1$$

$$e^{i\Delta_{off}(t_7-t_6)} = i = e^{i\frac{\pi}{2}}$$

$$\Delta_{off}(t_7 - t_6) = \frac{\pi}{2}$$

$$t_6 = t_7 - \frac{\pi}{2\Delta_{off}}$$

Solving for t_5:

$$\tan\left(\frac{\Omega\sqrt{n}}{2}t_5\right) = \frac{0.7071}{0.43571}$$

$$\Omega t_5 = 1.44$$

More n=2 states. Note that c_{g_0} stays constant because there is no state for it to exchange excitations in.

$$c_{e_1}(t_5) = i0.43571 \cos\left(\frac{\Omega\sqrt{2}}{2}t_5\right) e^{i\Delta_{off}t_6} + 0.7071 \sin\left(\frac{\Omega\sqrt{n}}{2}t_5\right) e^{i\Delta_{off}t_7}$$

The final state is: