Synthesizing arbitrary quantum states in a superconducting resonator

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1 Meaning

Fock states are attractive for quantum computing due to individual addressability, easy manipulation, and superior scalability [1]. Fock states always have a specified number of excitations. They contrast with coherent states, which are still quantum, but closest to classical theory since they have a [Poisson] probability distribution. Coherent states are easier to generate than Fock states since they require a classical driving force. Hofheinz and friends were able to generate an arbitrary superposition of Fock states by creating a particular qubit state and then having the qubit interact with the resonator for a controlled time [2]. To evaluate their results, they analyzed the resonator state using Wigner tomography and compared this to the theoretical prediction.

2 Jay-C Hamiltonian

What is remarkable about this paper is that they show that the Jaynes-Cummings Hamiltonian accurately (not taking into account a slight phase shift) can fully describe the distribution of Fock states.

Their Hamiltonian was in the resonator rotating frame so that the resonator had zero frequency:

$$\frac{H}{\hbar} = \Delta(t)\sigma^+\sigma^- + \left(\frac{\Omega}{2}\sigma^+ a + \frac{\Omega_q(t)}{2}\sigma^+ + \frac{\Omega_r(t)}{2}a^\dagger\right) + \text{h.c.}$$

$\Delta(t) = \omega_q(t) - \omega_r$ is the qubit-resonator detuning
$\sigma^+\sigma^-$ is the occupational number of the cubit
Ω_R and Ω_q are the strength of the oscillations of the resonator and cubit, respectively
$\frac{\Omega_R}{2}\left(a + a^\dagger\right)$ is the driving term of the resonator
$\frac{\Omega_q}{2}\left(\sigma^+ + \sigma^-\right)$ is the driving term of the cubit

3 Absence of democracy.

In order to *control the states*, they tuned the qubit frequency between state 1: on resonance with cavity ($\Delta_{on} = 0$) and state 2: off resonance ($|\Delta_{off}| \gg \Omega$). When the cubit and the cavity are on resonance, the coupling allows the cubit and resonator to exchange their respective ground/excited and photon number states. This occurs at the Rabi "swap" frequency, $\Omega\sqrt{n}$, where Ω is the coupling between the cavity and the qubit. Off resonance, the system oscillates at a frequency of $\sqrt{n\Omega^2 + \Delta^2}$, where the cavity and the qubit are detuned from each other.

The Rabi swap frequency is determined by Figure 1. Using the Rabi swap frequency, they pump photons one at a time into the cavity when the cubit is on resonance with the cavity. Applying the microwave π-pulse, followed by a controlled time (corresponding to the Rabi swap frequency), the cubit goes from $|g\rangle$ to $|e\rangle$. The combination of two Josephson Junctions allows control over the cubit frequency.

Instead of calibrating each microwave signal for each sequence step, they calibrated the experimental system independent of each state preparation.

4 $\odot$ state. Expect ψ. Pay $|g\rangle \otimes \sum_{n=0}^{N} c_n |n\rangle$.

The Target state of the coupled system is an arbitrary superposition of Fock states:

$$|\psi\rangle = |g\rangle \otimes \sum_{n=0}^{N} c_n |n\rangle$$

with a complex amplitude c_n for the nth Fock state. These states are generated by exchanging the number state for an excited state and subsequently decreasing from the cubit excited state to the ground state by producing a classical microwave signal [3]. The following interaction Hamiltonian governs this transaction as well as any retail store employee:

$$H_I(t) = [r(t) + g(t)a]\,\sigma^+ + \left[r^*(t) + g^*(t)a^\dagger\right]\sigma^- \tag{1}$$

This Hamiltonian does not *discount* a phase shift, so our friends at UCSB correct the relative phases of $|g, n\rangle$ and $|e, n-1\rangle$ by adjusting the time over which the qubit and resonator are detuned.

5 Last on the shopping list: c_n

By bringing the qubit and the resonator into resonance and measuring the probability of the cubit in the excited state, Hofheinz and friends determined the n-photon probabilities. In order to find the c_n's for the Target state, they shopped around for the Wigner tomography, which they found in aisle [4]. Generally:

$$W(\alpha) = \frac{2}{\pi}\mathrm{Tr}\left(D(-\alpha)\rho D(\alpha)\Pi\right)$$

For a pure state:

$$W(\alpha) = \frac{2}{\pi}\langle\psi|\, D^\dagger(-\alpha)\Pi D(-\alpha)\,|\psi\rangle$$

Where $D(\alpha)$ is the displacement operator and Π is the parity operator

6 What we don't know might hurt us:

1. How the Rabi swap frequency was determined.
2. The time evolution of the interaction Hamiltonian given by eq. (1) reduces the cubit to the ground state [3].
3. How the calibration was done to determine Rabi swap frequencies and phase accumulation.
4. E. P. Wigner's cray cray theories

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