7: Time Series

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Objectives

1. Discuss the purpose and application of time series analysis for environmental data
2. Explore the components of times series: trend, seasonal, random
3. Learn how to perform stationarity test

lesson on Time Series Analysis

Main questions

1. Has there been an increasing or decreasing *trend* in the response variable over time?
2. Can we *forecast* conditions in the future?

Learning Goals

- Introduction to Time Series Analysis (TSA) + What is TSA> + Examples + TSA Components (trend, cycle, seasonal, random)
- Autocorrelation Function (ACF)
- Partial Autocorrelation Function (PACF)
- Trend and Seasonal Component
- Stationary Tests

Introduction to Time Series Analysis

Meaning and Definitions What is a Time Series? * A set of observations on a variable collected over time * Discrete and continuous time series + Example: stock prices, interest rate, retail sales, electric power consumption, etc. * Mathematical representation: a time series is defined by the values $Y_1, Y_2, ...$ of a variable Y at times $t_1, t_2, ...$ Thus, $Y = F(t)$

What is Time Series Analysis (TSA)? * In TSA, we analyze the past behavior of a variable in order to predict its future behavior * Causes of variation of Time Series Data + Seasons, holidays, etc. + Natural calamities: earthquakes, epidemic, flood, drought, etc. + Political movements or changes, war, etc.

Importance of TSA

- Very popular tool for business forecasting
- Basis for understanding past behavior
- Can forecast future activities/planning for future operations

Components of TSA

- **Time frame**: short, medium, long-term + How far can we predict?
- **Trend** + General tendenc to grow or decline over a long period + Easy to detect + May be linear or non-linear

- **Cycle** + An up and down repetitive movement + Repeats itself over a long period of time + Example: business cycle (prosperity, decline, depression, recovery)
- **Seasonal Variation** + An up and down repetitive movement occuring periodically (short duration) + Factors that cause seasonal variation: climate and weather condition, or custom traditions and habits
- **Random Variations** + Erratic movements that are not predictable because they don't follow a pattern + Examples: strike, fire, war, flood, earthquake, etc.

TSA Terms **Stationary Data** - a time series variable exhibiting no significant upward or downward trend over time **Nonstationary Data** - a time series variable exhibiting a significant upward or downward trend over time **Seasonal Data** - a time series variable exhibiting a repeating pattern at regular intervals over time

Meaning of Autocorrelation Function (ACF)

What is correlation? From statistics, covariance and correlation measure *joint variability* of two variables (i.e., how the two variables are related). It is a measure of linear dependence between two variables.

In Time Series Analysis, we talk about **autocorrelation**, which is a measure of dependence between two adjacent values of the same variables (same variable at different times). The prefix *auto* is to convey the notion of self-correlation, that is, correlation between variables from the same time series.

In the context of a single variable, Y_t is the original series and Y_s is a lagged version of the series. *Autocovariance and autocorrelation function five information about the dependence structure of a time series.* If the covariance is significant, it means that the observation at t_1 is highly dependent on the observation at t_1.

Stationary Processes

The basic idea of stationarity is that the probability laws that govern behavior of the process do not change over time (i.e., the distribution (mean, stdev, autocorrelation) of points in earlier time is equal to the distribution of points at a later time).

Partial Autocorrelation Function

The ACF of a stationary process Y_t at lag h $\rho_{t,t-h} = Corr(Y_t, Y_{t-h})$ measures the linear dependency among the process variables Y_t and Y_{t-h}. But the dependence structure among the *intermediate variables* $Y_t, Y_{t-1}, Y_{t-2}, ...Y_{t-h+2}, Y_{t-h+1}, Y_{t-h}$ also plays an important role on the value of the ACF.

The **partial autocorrelation function** attempts to *remove* the influence of intermediate variables. You would have only the direct correlation between Y_t and T_{t-h}. This correlation relationship is called the *partial autocorrelation function (PACF).*

The ACF and PACF measure the temporal dependency of a stochastic process. You will always build the ACF and PACF before fitting a model to a stochastic process because they give us information about the *autoregressive component* of the series. When ACFs have both increasing (positive) and decreasing (negative) trends we can see seasonality in the dataset.

Trend Component

- *Long-term tendency*: + Increasing (upward movement) + Decreasing (downward movement)
- Trend can be linear or non-linear (e.g. quadratic trend)

Linear Trend Component For a linear trend, we can write $Y_i = \beta_0 + \beta_1 t_i + \epsilon_i$ *Slope* (β_1) and the *intercept* (β_0) are the unknown parameters, and ϵ_i is the *error term* (or residual, is the distance from point Y_i to the estimated trend, $\beta_0 + \beta_1 t$).

Linear Trend Estimation and Removal

1. Model the trend: find β_0 and β_1.

2. For each observation t remove trend $Y_{detrend_t} = Y_t - (\beta_0 + \beta_1 t)$

Seasonal Component

- Short-term regular wave-like patterns
- Observed within 1 year
- Often monthly or quarterly

Seasonal Trend Estimation Assume the observed series can be represented as $Y_t = \mu_t + X_t$ where $E[X_t] = 0$ For moonthly seasonal data assume 12 parameters as $\mu_t = \beta_1$ for $t = 1, 13, 15, ... = \beta_2$ for $t = 2, 14, 16, ... \ldots = \beta_1 2$ for $t = 12, 24, 36, ...$ The number of seasons may be less than 12.

Seasonal Trend Removal

1. Model the seasonal trend
2. For each observation t remove seasonal trend $Y_{deseason_t} = Y_t - (\sum \beta_s D_{t,s})$

Series with Deterministic Trend

For both linear and polynomial trends, detrending is accomplished by running a regression and obtaining the series of residuals. The residuals will give you the detrended series. Thats what we call **trend-stationarity**.

Series with Stochastic Trend

Some series have what we call **difference-stationarity**. Although trend-stationarity and difference-stationarity are both "trending" over time, the stationarity is achieved by a *distinct procedure*. In the case of difference-stationarity, stationarity is achieved by differencing the series. Sometimes we need to difference the series more than once.

Stationarity Tests

Stationarity Assessment

- **Mann-Kendall Test** - monotonic trend + Commonly employed to detect deterministic trends in series of environmental data, climate data, or hydrological data + *Cannot be applied to seasonal data* + Hypothesis Test: $H_0 : Y_t$ is $i.i.d.(stationarity)$ $H_1 : Y_t$ follow a trend $_{\text{more}}$
- **Spearman's Rank Correlation Test** - monotonic trend + Spearman's correlation coefficient is a statistical measure of the strenth of a monotonic relationship + Unlike Pearson's correlation, the relationship does not need to be linear + In other words, if one variable increases so does the other, it *does not matter the porportion of the increase* + Spearman's will always be larger than Pearson's + To verify a monotonic trend in your data, compute the spearman correlation between your data and the series T + If the correlation is close to 0, then there is no trend + The function to compute spearman correlation is cor() or the cor.test() from package "stats". The latter provides the significance of the coefficient.
- **Dickey-Fuller (ADF) Test** - unit root + The first work on testing for a unit root in time series was done by Dickey and Fuller + Consider the model $Y_t = a + \phi Y_{t-1} + \epsilon_t$ + The objective to test $H_0 : \phi = 1$ (i.e. contain a unit root) $H_1 : \phi < 1$ (i.e. is stationary) + More general case can include more lags, the so called *Augmented Dickey-Fuller (ADF) test* + The ADF test in R is done with the command adf.test() from package "tseries"

Set up

Today we will work with two datasets. The USGS dataset on discharge at the Eno River and a new dataset we haven't explored yet on wind speed. The data file is available at "./Data/Raw/Wind_Speed_PortArthurTX.csv". It contains average wind speed in monthly time steps (elevation = 5 meters). The data is available from

NOAA National Centers for Environmental Information (NCEI) [here][https://www.ncdc.noaa.gov/cdo-web/datasets#GSOM].

```
library(tidyverse)
library(lubridate)
#install.packages("trend")
library(trend)
#install.packages("zoo")
library(zoo)
#install.packages("Kendall")
library(Kendall)
#install.packages("tseries")
library(tseries)

# Set theme
mytheme <- theme_classic(base_size = 14) +
  theme(axis.text = element_text(color = "black"),
        legend.position = "top")
theme_set(mytheme)

#Read Eno river data
EnoDischarge <- read.csv("../Data/Processed/USGS_Site02085000_Flow_Processed.csv",
                         stringsAsFactors = TRUE)
EnoDischarge$datetime <- as.Date(EnoDischarge$datetime, format = "%Y-%m-%d")

#Read wind speed data
wind_data <- read.csv(file="../Data/Raw/Wind_Speed_PortArthurTX.csv",header=TRUE,
                      stringsAsFactors = TRUE)
wind_data$DATE <- ym(wind_data$DATE)
```

Time Series Analysis overview

Time series are a special class of dataset, where a response variable is tracked over time. The frequency of measurement and the timespan of the dataset can vary widely. At its most simple, a time series model includes an explanatory time component and a response variable. Mixed models can include additional explanatory variables (check out the `nlme` and `lme4` R packages). We will cover a few simple applications of time series analysis in these lessons, with references for how to take analyses further.

Opportunities

Analysis of time series presents several opportunities. For environmental data, some of the most common questions we can answer with time series modeling are:

- Has there been an increasing or decreasing **trend** in the response variable over time?
- Can we **forecast** conditions in the future?

Challenges

Time series datasets come with several caveats, which need to be addressed in order to effectively model the system. A few common challenges that arise (and can occur together within a single dataset) are:

- **Autocorrelation**: Data points are not independent from one another (i.e., the measurement at a given time point is dependent on previous time point(s))
- **Data gaps**: Data are not collected at regular intervals, necessitating *interpolation* between measurements.

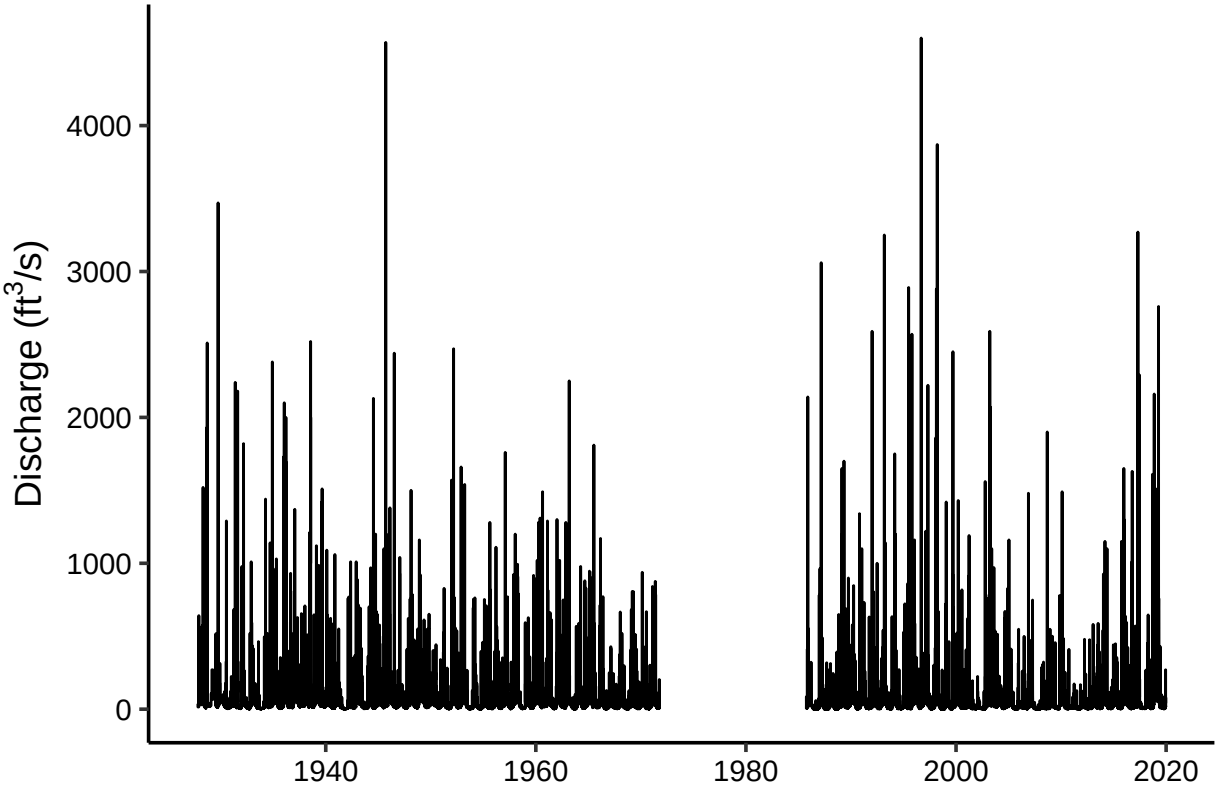
- **Seasonality**: seasonal patterns in variables occur at regular intervals, impeding clear interpretation of a monotonic (unidirectional) trend.
- **Heteroscedasticity**: The variance of the time series is not constant over time
- **Covariance**: the covariance of the time series is not constant over time

Handling data gaps and missing data. Example: Eno River Discharge

River discharge is measured daily at the Eno River gage station. Since we are working with one location measured over time, this will make a great example dataset for time series analysis.

Let's look at what the dataset contains for mean daily discharge.

```
ggplot(EnoDischarge, aes(x = datetime, y = discharge.mean)) +
  geom_line() +
  labs(x = "", y = expression("Discharge (ft"^3*"/s)"))
```



Notice there are missing data from 1971 to 1985. Gaps this large are generally an issue for time series analysis, as we don't have a continuous record of data or a good way to characterize any variability that happened over those years. We will illustrate a few workarounds to address these issues.

Let's start by removing the NAs and splitting the dataset into the early and late years.

```
EnoDischarge.complete <- EnoDischarge %>%
  drop_na(discharge.mean)

EnoDischarge.early <- EnoDischarge.complete %>%
  filter(datetime < as.Date("1985-01-01"))

EnoDischarge.late <- EnoDischarge.complete %>%
  filter(datetime > as.Date("1985-01-01"))
```

Decomposing a time series dataset

A given time series can be made up of several component series:

1. A **seasonal** component, which repeats over a fixed known period (e.g., seasons of the year, months, days of the week, hour of the day)
2. A **trend** component, which quantifies the upward or downward progression over time. The trend component of a time series does not have to be monotonic.
3. An **error** or **random** component, which makes up the remainder of the time series after other components have been accounted for. This component reflects the noise in the dataset.
4. (optional) A **cyclical** component, which repeats over periods greater than the seasonal component. A good example of this is El Niño Southern Oscillation (ENSO) cycles, which occur over a period of 2-8 years.

Example: Eno discharge

We will decompose the EnoDischarge.late data frame for illustrative purposes today. It is possible to run time series analysis on detrended data by subtracting the trend component from the data. However, detrending must be done carefully, as many environmental data are bounded by zero but are not treated as such in a decomposition. If you plan to use decomposition to detrend your data, please consult time series analysis guides before proceeding.

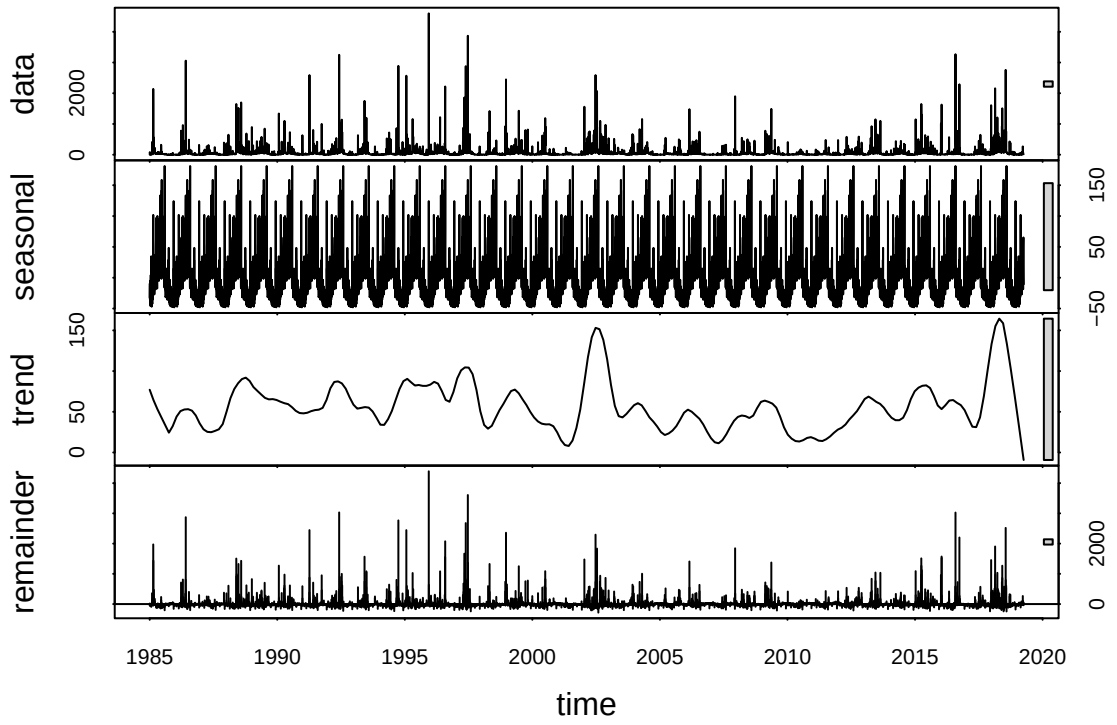
We first need to turn the discharge data into a time series object in R. This is done using the `ts` function. Notice we can only specify one column of data and need to specify the period at which the data are sampled. The resulting time series object cannot be viewed like a regular data frame.

Note: time series objects must be equispaced. In our case, we have daily data with no NAs in the data frame, so we don't need to worry about this. We will cover how to address data that are not equispaced later in the lesson.

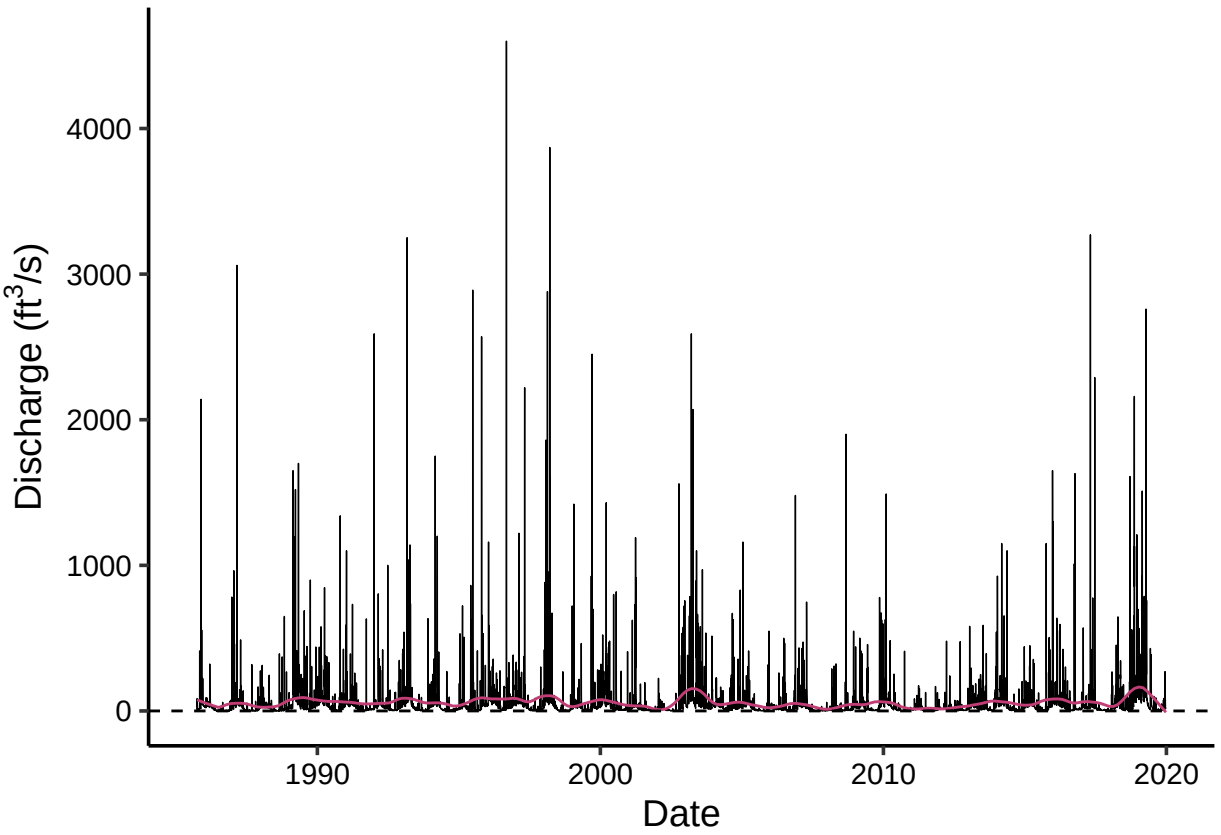
```
EnoDischarge.late_ts <- ts(EnoDischarge.late$discharge.mean, start = c(1985,1), frequency = 365)
```

The `stl` function decomposes the time series object into its component parts. We must specify that the window for seasonal extraction is either "periodic" or a specific number of at least 7. The decomposition proceeds through a loess (locally estimated scatterplot smoothing) function.

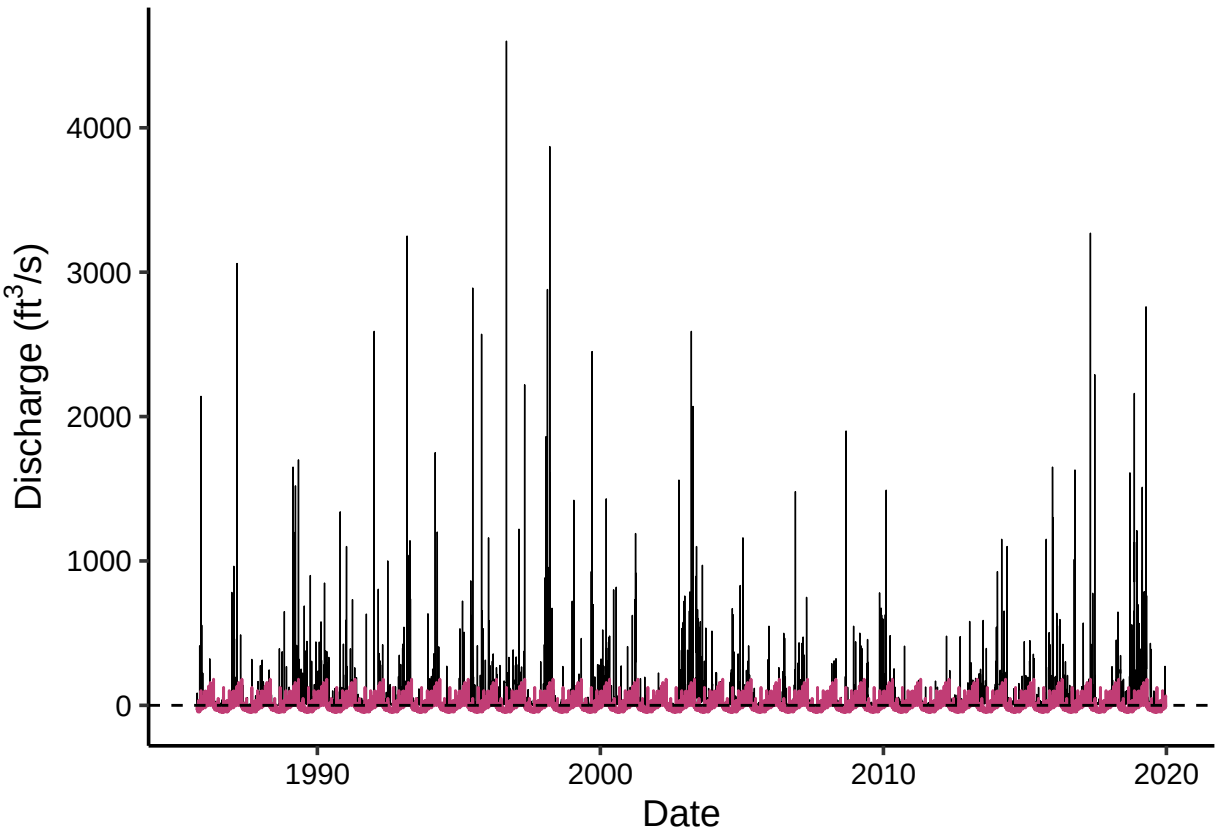
```
?stl
# Generate the decomposition
EnoDischarge.late_Decomposed <- stl(EnoDischarge.late_ts, s.window = "periodic")

# Visualize the decomposed series.
plot(EnoDischarge.late_Decomposed)
```




```
# We can extract the components and turn them into data frames
EnoDischarge.late_Components <- as.data.frame(EnoDischarge.late_Decomposed$time.series[,1:3])

EnoDischarge.late_Components <- mutate(EnoDischarge.late_Components,
        Observed = EnoDischarge.late$discharge.mean,
        Date = EnoDischarge.late$datetime)

# Visualize how the trend maps onto the data
ggplot(EnoDischarge.late_Components) +
  geom_line(aes(y = Observed, x = Date),  size = 0.25) +
  geom_line(aes(y = trend, x = Date), color = "#c13d75ff") +
  geom_hline(yintercept = 0, lty = 2) +
  ylab(expression("Discharge (ft"^3*"/s)"))
```




```
# Visualize how the seasonal cycle maps onto the data
ggplot(EnoDischarge.late_Components) +
  geom_line(aes(y = Observed, x = Date),  size = 0.25) +
  geom_line(aes(y = seasonal, x = Date), color = "#c13d75ff") +
  geom_hline(yintercept = 0, lty = 2) +
  ylab(expression("Discharge (ft"^3*"/s)"))
```



Note that the decomposition can yield negative values when we apply a seasonal adjustment or a trend adjustment to the data. The decomposition is not constrained by a lower bound of zero as discharge is in real life. Make sure to interpret with caution!

Trend analysis

Two types of trends may be present in our time series dataset: **monotonic/deterministic** or **stochastic**. Monotonic trends are a gradual shift over time that is consistent in direction, for example in response to land use change.

A third type of trend we haven't talked about is the **step** trend, also known as a level shift. Step trends are a distinct shift at a given time point, for example in response to a policy being enacted.

Monotonic trend analysis

In general, detecting a monotonic trend requires a long sequence of data with few gaps. If we are working with monthly data, a time series of at least five years is recommended. Gaps can be accounted for, but a gap that makes up more than 1/3 of the sampling period is generally considered the threshold for considering a gap to be too long (a step trend analysis might be better in this situation).

Adjusting the data may be necessary to fulfill the assumptions of a trend test. A common method to replace missing values is **interpolation**. Common interpolation methods:

- **Piecewise constant**: also known as a "nearest neighbor" approach. Any missing data are assumed to be equal to the measurement made nearest to that date (could be earlier or later).
- **Linear**: could be thought of as a "connect the dots" approach. Any missing data are assumed to fall between the previous and next measurement, with a straight line drawn between the known points determining the values of the interpolated data on any given date.

- **Spline**: similar to a linear interpolation except that a quadratic function is used to interpolate rather than drawing a straight line.

Example: interpolation The Eno River discharge data doesn't have any short periods of missing data, so interpolation would not be a good choice for that dataset. We will illustrate a linear interpolation using the wind speed dataset.

```
head(wind_data)
```

```
##       STATION                                        NAME       DATE AWND
## 1 USW00012917 PORT ARTHUR SE TX REGIONAL AIRPORT, TX US 1984-01-01  4.3
## 2 USW00012917 PORT ARTHUR SE TX REGIONAL AIRPORT, TX US 1984-02-01  5.0
## 3 USW00012917 PORT ARTHUR SE TX REGIONAL AIRPORT, TX US 1984-03-01  5.1
## 4 USW00012917 PORT ARTHUR SE TX REGIONAL AIRPORT, TX US 1984-04-01  5.5
## 5 USW00012917 PORT ARTHUR SE TX REGIONAL AIRPORT, TX US 1984-05-01  4.5
## 6 USW00012917 PORT ARTHUR SE TX REGIONAL AIRPORT, TX US 1984-06-01  3.9
```

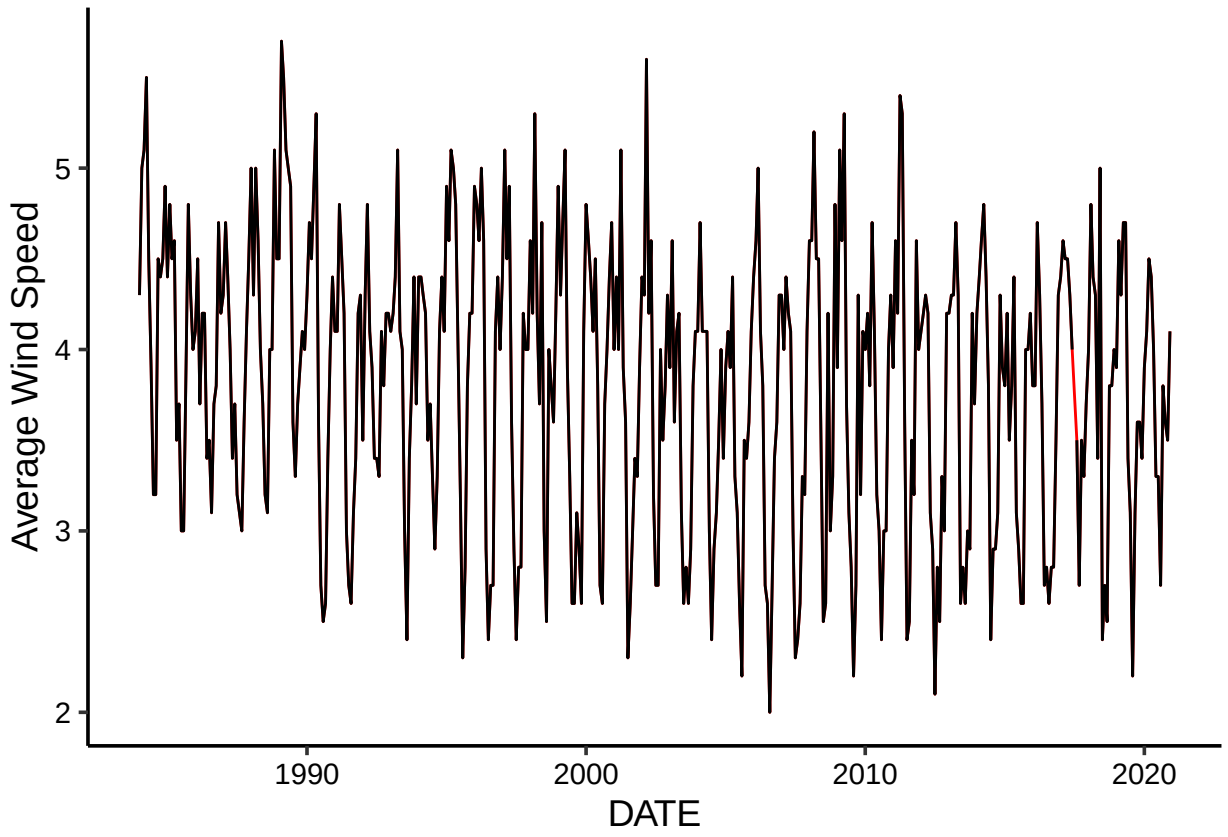
```
summary(wind_data$AWND)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   2.000   3.200   4.000   3.819   4.400   5.700       1
```

```
# Adding new column with no missing obs, just for illustration purpose
# In real applications you will simply replace NAs
wind_data_clean <-
  wind_data %>%
  mutate( AWND.clean = zoo::na.approx(AWND) )

summary(wind_data_clean$AWND.clean)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   2.000   3.200   4.000   3.819   4.400   5.700
```

```
#Note the NA is gone

ggplot(wind_data_clean ) +
  geom_line(aes(x = DATE, y = AWND.clean), color = "red") +
  geom_line(aes(x = DATE, y = AWND), color = "black") +
  ylab("Average Wind Speed")
```



Monotonic trend analysis, continued

Specific tests for monotonic trend analysis are listed below, with assumptions and tips:

- **linear regression**: no seasonality, fits the assumptions of a parametric test. Function: `lm`
- **Mann-Kendall**: no seasonality, non-parametric, missing data allowed. Function: `MannKendall()` (package: Kendall)
- **Seasonal Mann-Kendall**: seasonality, non-parametric `SeasonalMannKendall` (package: Kendall)
- **Spearman Rho**: no seasonality, non-parametric, missing data allowed. Function: `cor.test(method="spearman")` (package: stats)

Specific test for stochastic trend analysis:

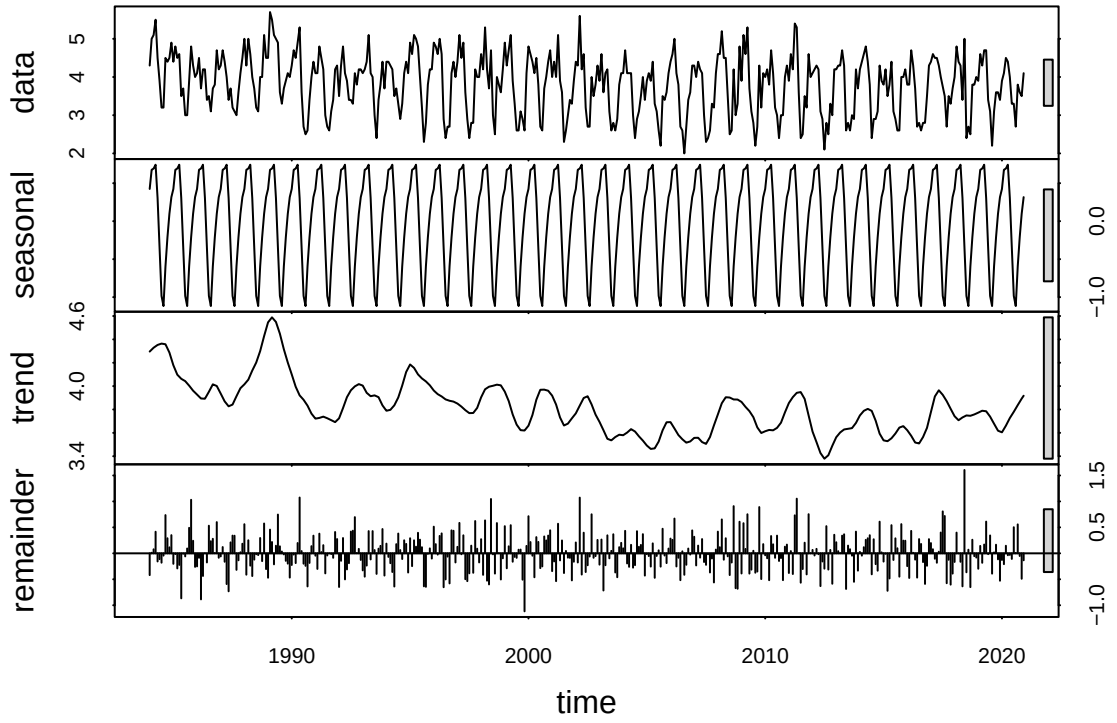
- **Augmented Dickey Fuller**: no seasonality, non-parametric, missing data not allowed. Function: `adf.test()` (package: tseries)

Example: monotonic trend analysis Let's refer to our wind speed data. We already performed interpolation, but we still need to create our time series object and decompose the series to find out which stationarity test we can apply.

Note that wind speed has a seasonal cycle. We might be interested in knowing how (if) speed has changed over the course of measurement while incorporating the seasonal component. In this case, we will use a Seasonal Mann-Kendall test to figure out whether a monotonic trend exists.

```
# Generate time series (trend test needs ts, not data.frame)
f_month <- month(first(wind_data_clean$DATE))
f_year <- year(first(wind_data_clean$DATE))
wind_data_ts <- ts(wind_data_clean$AWND.clean,
                   start=c(f_year,f_month), #first year and first month
                   frequency=12)
```

```r
#decompose
wind_data_decomp <- stl(wind_data_ts,s.window = "periodic")
plot(wind_data_decomp)
```




```r
# Run SMK test
wind_data_trend1 <- Kendall::SeasonalMannKendall(wind_data_ts)

# Inspect results
wind_data_trend1
```

```
## tau = -0.229, 2-sided pvalue =1.7751e-11
```

```r
summary(wind_data_trend1)
```

```
## Score =  -1770 , Var(Score) = 69305.33
## denominator =  7725.182
## tau = -0.229, 2-sided pvalue =1.7751e-11
```

```r
wind_data_trend2 <- trend::smk.test(wind_data_ts)
# Inspect results
wind_data_trend2
```

```
##
##  Seasonal Mann-Kendall trend test (Hirsch-Slack test)
##
## data:  wind_data_ts
## z = -6.7196, p-value = 1.822e-11
## alternative hypothesis: true S is not equal to 0
## sample estimates:
##        S     varS
## -1770.00 69305.33
```

```r
summary(wind_data_trend2)
```

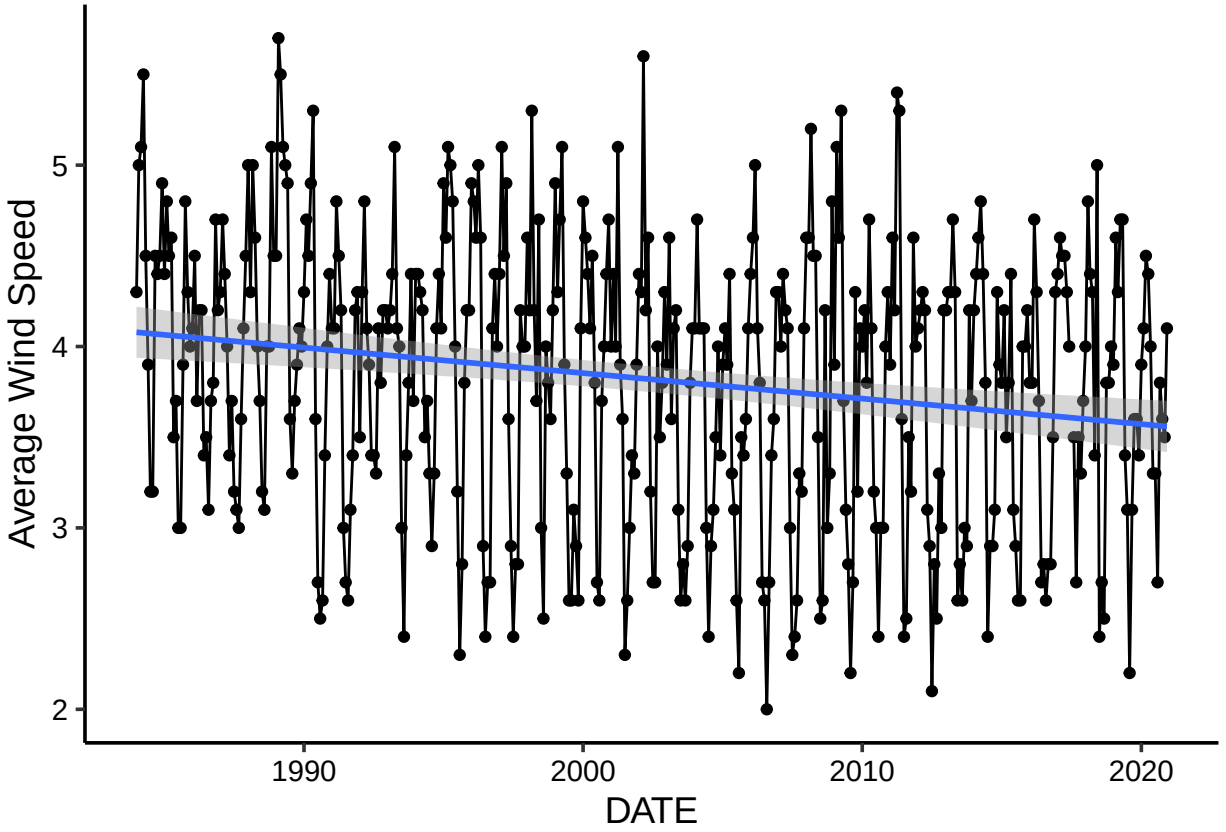
```
##
##  Seasonal Mann-Kendall trend test (Hirsch-Slack test)
##
## data: wind_data_ts
## alternative hypothesis: two.sided
##
## Statistics for individual seasons
##
## H0
##                        S   varS    tau      z   Pr(>|z|)
## Season 1:   S = 0  -203 5753.7 -0.318 -2.663 0.00774363  **
## Season 2:   S = 0  -129 5731.7 -0.203 -1.691 0.09089192   .
## Season 3:   S = 0  -125 5796.3 -0.193 -1.629 0.10337344
## Season 4:   S = 0   -42 5780.0 -0.065 -0.539 0.58968885
## Season 5:   S = 0    -9 5815.7 -0.014 -0.105 0.91645233
## Season 6:   S = 0  -139 5806.3 -0.214 -1.811 0.07013462   .
## Season 7:   S = 0  -148 5777.3 -0.230 -1.934 0.05311469   .
## Season 8:   S = 0  -147 5749.0 -0.230 -1.926 0.05415953   .
## Season 9:   S = 0  -199 5798.3 -0.307 -2.600 0.00931583  **
## Season 10:   S = 0 -216 5783.3 -0.335 -2.827 0.00469638  **
## Season 11:   S = 0 -258 5792.0 -0.399 -3.377 0.00073306 ***
## Season 12:   S = 0 -155 5721.7 -0.245 -2.036 0.04175898   *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
#Visualization
wind_data_plot <-
ggplot(wind_data, aes(x = DATE, y = AWND)) +
  geom_point() +
  geom_line() +
  ylab("Average Wind Speed") +
  geom_smooth( method = lm )
print(wind_data_plot)
```

```
## `geom_smooth()` using formula 'y ~ x'
```

```
## Warning: Removed 1 rows containing non-finite values (stat_smooth).
```

```
## Warning: Removed 1 rows containing missing values (geom_point).
```



What would we conclude based on these findings?

Answer: There is a decreasing trend in wind speed over time.

Forecasting with Autoregressive and Moving Average Models (ARMA)

We might be interested in characterizing a time series in order to understand what happened in the past and to effectively forecast into the future. Two common models that can approximate time series are **autoregressive** and **moving average** models. To classify these models, we use the **ACF (autocorrelation function)** and the **PACF (partial autocorrelation function)**, which correspond to the autocorrelation of a series and the correlation of the residuals, respectively.

Autoregressive models operate under the framework that a given measurements is correlated with previous measurements. For example, an AR1 formulation dictates that a measurement is dependent on the previous measurement, and the value can be predicted by quantifying the lag.

Moving average models operate under the framework that the covariance between a measurement and the previous measurement is zero. While AR models use past forecast *values* to predict future values, MA models use past forecast *errors* to predict future values.

Here are some great resources for examining ACF and PACF lags under different formulations of AR and MA models. https://nwfsc-timeseries.github.io/atsa-labs/sec-tslab-autoregressive-ar-models.html https://nwfsc-timeseries.github.io/atsa-labs/sec-tslab-moving-average-ma-models.html

ARMA models require stationary data. This means that there is no monotonic trend over time and there is also equal variance and covariance across the time series. The function `adf.test` will determine whether our data are stationary. The null hypothesis is that the data are not stationary, so we infer that the data are stationary if the p-value is < 0.05.

While some processes might be easy to identify, it is often complicated to predict the order of AR and MA processes. To get around this issue, it is often necessary to run multiple potential formulations of the

model and see which one results in the most parsimonious fit using AIC. The function `auto.arima` does this automatically.