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Comp 250 Assignment 6

Question 1 (a)

Algorithm: GreedyChoice(int C[0…k-1], int U[0…k-1], int k, int N)
Input:

- An array C of costs for the k objects
- An array U of utilities of the k objects
- A total budget N

Output: An array Q[0…k-1] of quantities to purchase

The code used for sorting in this question is really similar to the sorting code for question 2. Since there was a lot of code required for writing the sorting algorithm, I have included it here (along with the code to calculate the quantities) to avoid loss of meaning and ambiguity. I have tested all the code and it works.

In this scenario, we have a 2D array whose 1st column refers to the object number, 2nd column to cost, 3rd column for utility and 4th column for cost per utility. We sort the entire 2D array based on sorting the last column which is the cost per utility column. Each row in the array represents a specific object, its cost, utility and cost per utility. We rearrange entire rows so that the rows are sorted by least cost per utility first.

(Most of the code used here is for sorting. The calculation of the quantities themselves didn't require much code)

```
Integer[] costs = new Integer[c.length];
Integer[] util = new Integer[u.length];
int[] q = new int[c.length];
Integer[][] details = new Integer[c.length][4];
Integer[] costsPerUtil = new Integer[c.length];
for(int i = 0; i < costsPerUtil.length; i++) {
    costs[i] = Integer.valueOf(c[i]);
    util[i] = Integer.valueOf(u[i]);
    /* we multiply the costs by 100.0 since the
        sorting method used below can only sort
        integers, not doubles. We multiply by 100.0
        so that we have sufficient number of digits
        to sort the costPerUtil array */
    costsPerUtil[i] = (Integer) ((int)((costs[i] * 100.0)/util[i]));
}
```

```
for(int j = 0; j < details[0].length; j++) {
    for(int i = 0; i < details.length; i++) {
        if(j == 0) {
            details[i][j] = i;
        }
        if(j == 1) {
            details[i][j] = costs[i];
        }
        if(j == 2) {
            details[i][j] = util[i];
        }
        if(j == 3) {
            details[i][j] = costsPerUtil[i];
        }
    }
}

Integer[][] answer = sort(details);

/* the method 'sort' sorts the 2D array 'details' based on
    its last column which is costPerUtil. all the other
    columns also get sorted in such a way so that they
    match the CORRESPONDING entries of the last column

    from this point lies the code used to calculate
    the number of quantities to purchase */

for(int j = 0; j < q.length; j++) {
    int quantity = 0;
    int cost = answer[j][1];
    while(cost * quantity <= n) {
        quantity++;
    }

    if (quantity != 0) {
        quantity -= 1;
    }

    n = n - (int) (cost * quantity);
    int index = answer[j][0];
    q[index] = quantity;
}
return q;
```

The sort method used above has the following code:
Algorithm: sort(Integer[][] details)

Input: A 2D array containing object number, cost, utility and cost per utility
Output: A 2D array with rows sorted according to least cost per utility first

```
int max = 0;
Integer[] costPerUtil = new Integer[details.length];
Integer[][] arrays = new Integer[details.length][details[0].length];
for(int i = 0; i < costPerUtil.length; i++) {
    int num = details[i][details[0].length - 1];
    if(max < num) {
        max = num;
    }
    costPerUtil[i] = num;
}

for(int j = 0; j < details.length; j++) {
    arrays[j] = details[j];
}

int maxNumDigits = (int) Math.log10(max) + 1;
LinkedList<LinkedList<Integer>> bucketsForCPerU = new
LinkedList<LinkedList<Integer>>();
LinkedList<LinkedList<Integer[]>> bucketsForArray = new
LinkedList<LinkedList<Integer[]>>();
for(int p = 0; p < 10; p++) {
    bucketsForCPerU.add(new LinkedList<Integer>());
    bucketsForArray.add(new LinkedList<Integer[]>());
}


for(int j = 1; j <= maxNumDigits; j++) {
    for(int k = 0; k < costPerUtil.length; k++) {
        int number = costPerUtil[k];
        int digit = (int) ((number % Math.pow(10, j)) / Math.pow(10,j-1));
        bucketsForCPerU.get(digit).add(number);
        bucketsForArray.get(digit).add(arrays[k]);
    }

    Integer[] partiallySorted = new Integer[costPerUtil.length];
    Integer[][] parSorted = new Integer[details.length][details[0].length];
    int numAt = 0;

    search:
    for(int m = 0; m < 10; m++) {
        while(!bucketsForCPerU.get(m).isEmpty()) {
            partiallySorted[numAt] = bucketsForCPerU.get(m).pollFirst();
            parSorted[numAt] = bucketsForArray.get(m).pollFirst();
            numAt++;
```

```
        }
        if(numAt == costPerUtil.length) {
            break search;
        }
    }
    costPerUtil = partiallySorted;
    arrays = parSorted;
}

Integer[][] answer = arrays;

return answer;
```

Question 1 (b)

An example in which the greedy algorithm above would not produce the optimal result would be:

Total budget (N) = 8

Object	Cost	Utility	Cost per utility
O_0	3	4	0.75
O_1	8	6	1.33
O_2	5	5	1.00

Since O_0 has the lowest cost per utility, the greedy algorithm would start by picking as many O_0 as possible. It would pick 2 of O_0 for a total cost of (3x2) = 6 and a total utility of 8. After this we don't have enough money left to buy anything else. Therefore, the maximum utility obtained would be **8**.

But this is not the most optimal result possible for these sets of values. If we pick 1 of O_0 and 1 of O_2 for a total cost of (3+5) = 8, we would obtain a total utility of (4+5) = **9** which is higher than what we obtained by applying the greedy algorithm.

Question 1 (c)

Utility(0) = 0

Utility(N) = $\max_p[U[p] + Utility(N - C[p])]$, where p is limited so that $N - C[p] \geq 0$

Algorithm: DynProgChoice(int C[0…k-1], int U[0…k-1], int k, int N)
Input:

- An array C of costs for the k objects

- An array U of utilities of the k objects
- A total budget N

Output: The maximum total utility that can be achieved with a budget N

Again, due to indentation and to avoid confusion and loss of meaning, I am attaching the code that I wrote to test this and it works

```
int[] achievableUtil = new int[n + 1];
achievableUtil[0] = 0;
for(int i = 1; i < achievableUtil.length; i++) {
    LinkedList<Integer> possibleObjects = new LinkedList<Integer>();
    for(int p = 0; p < costs.length; p++) {
        if(i - costs[p] >= 0) {
            possibleObjects.add(p);
        }
    }
    achievableUtil[i] = 0;
    for(int j = 0; j < possibleObjects.size(); j++) {
        int index = possibleObjects.get(j);
        int possibleVal = util[index] + achievableUtil[i - costs[index]];
        if(achievableUtil[i] < possibleVal) {
            achievableUtil[i] = possibleVal;
        }
    }
}
return achievableUtil[n];
```

Question 1 (d)

Total budget (N) = 38

Object	Cost	Utility	Quantities
O_0	2	1	0
O_1	6	5	1
O_2	8	8	4
O_3	10	9	0

Using the Dynamic Programming algorithm above, the maximum total utility that can be achieved with the given budget is **37** with $\{O_0, O_1, O_2, O_3\} = \{0, 1, 4, 0\}$

Question 2 (a)

Since the pseudocode might be hard to understand due to a lot of indentation and to avoid any confusions, I am attaching the code that I wrote to sort the array.

Any number can only consist of the digits 0-9 (10 digits). We make 10 'buckets', using a LinkedList of LinkedList of Integers, one for each digit and proceed in the following way:

We first find the maximum number in the given array and find the number of digits in it. That tells us the number of times we have to perform our sorting. We first sort the numbers by their units' digit and place them in buckets corresponding to their units' digit. For example, if a number ends with '5' it will be placed in the bucket labeled '5'. After this first iteration, we take out the numbers from the buckets from the start of the list to the end of the list, proceeding towards the last bucket.

Then we sort the numbers according to their tens' digits, hundreds' digits and so on. After the last iteration, we will obtain our sorted array.

```
int max = 0;
int[] copied = new int[array.length];
for(int i = 0; i < array.length; i++) {
    int num = array[i];
    if(max < num) {
        max = num;
    }
    copied[i] = num;
}

int maxNumDigits = (int) Math.log10(max) + 1;
LinkedList<LinkedList<Integer>> buckets = new LinkedList<LinkedList<Integer>>();
for(int p = 0; p < 10; p++) {
    buckets.add(new LinkedList<Integer>());
}

for(int j = 1; j <= maxNumDigits; j++) {
    for(int k = 0; k < array.length; k++) {
        int number = copied[k];
        int digit = (int) ((number % Math.pow(10, j)) / Math.pow(10,j-1));
        buckets.get(digit).add(number);
    }

    int[] partiallySorted = new int[array.length];
    int numAt = 0;

    search:
    for(int m = 0; m < 10; m++) {
        while(!buckets.get(m).isEmpty()) {
            partiallySorted[numAt] = buckets.get(m).pollFirst();
```

```
            numAt++;
        }
        if(numAt == array.length) {
            break search;
        }
    }
    copied = partiallySorted;
}
return copied;
```

Question 2 (b)

The running time for the algorithm comes out as:

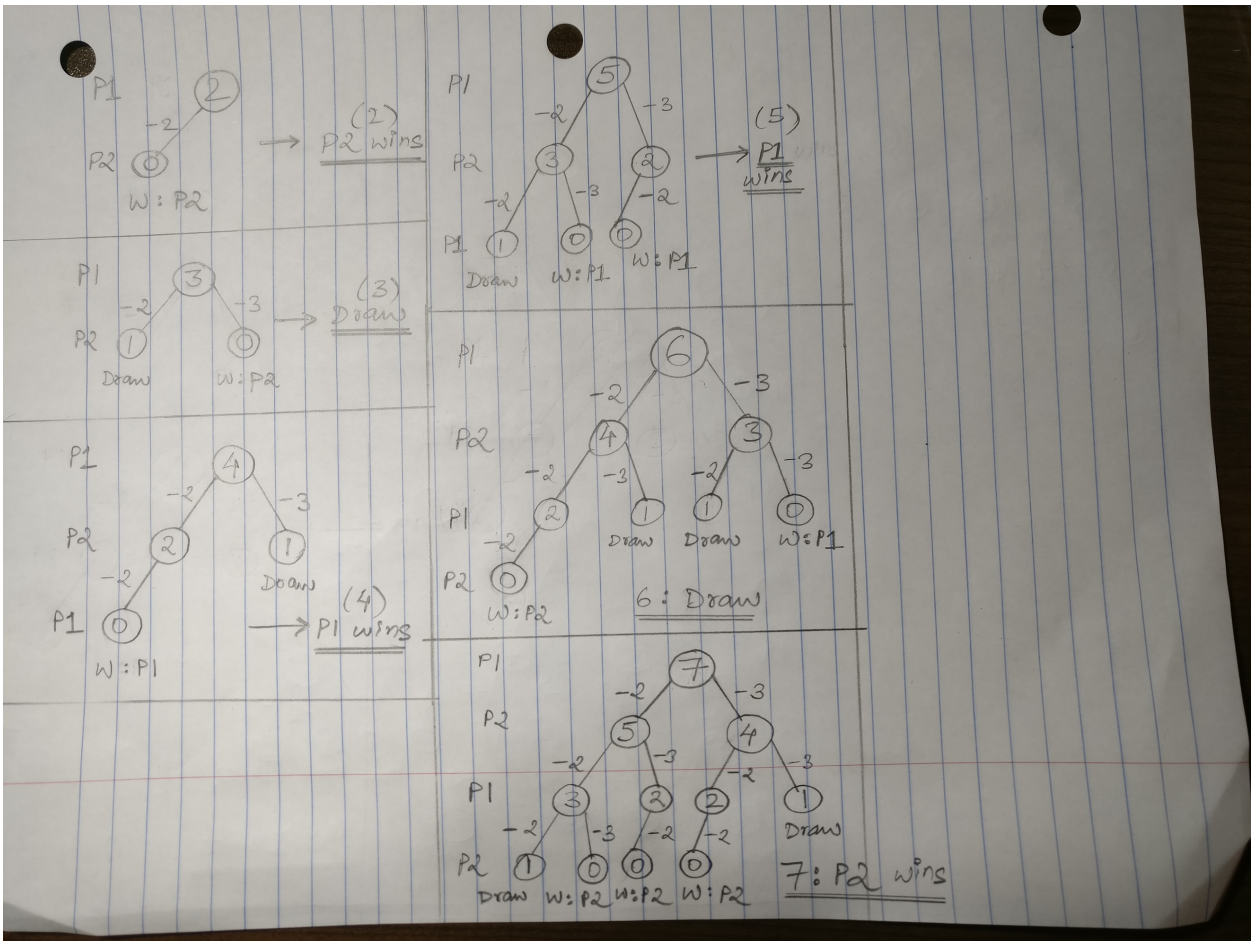
$$T(n) = 9n + 35 + (\lfloor \log_{10}(max) \rfloor + 1) * (12n + 176) \quad \text{-------} > \quad O(n)$$

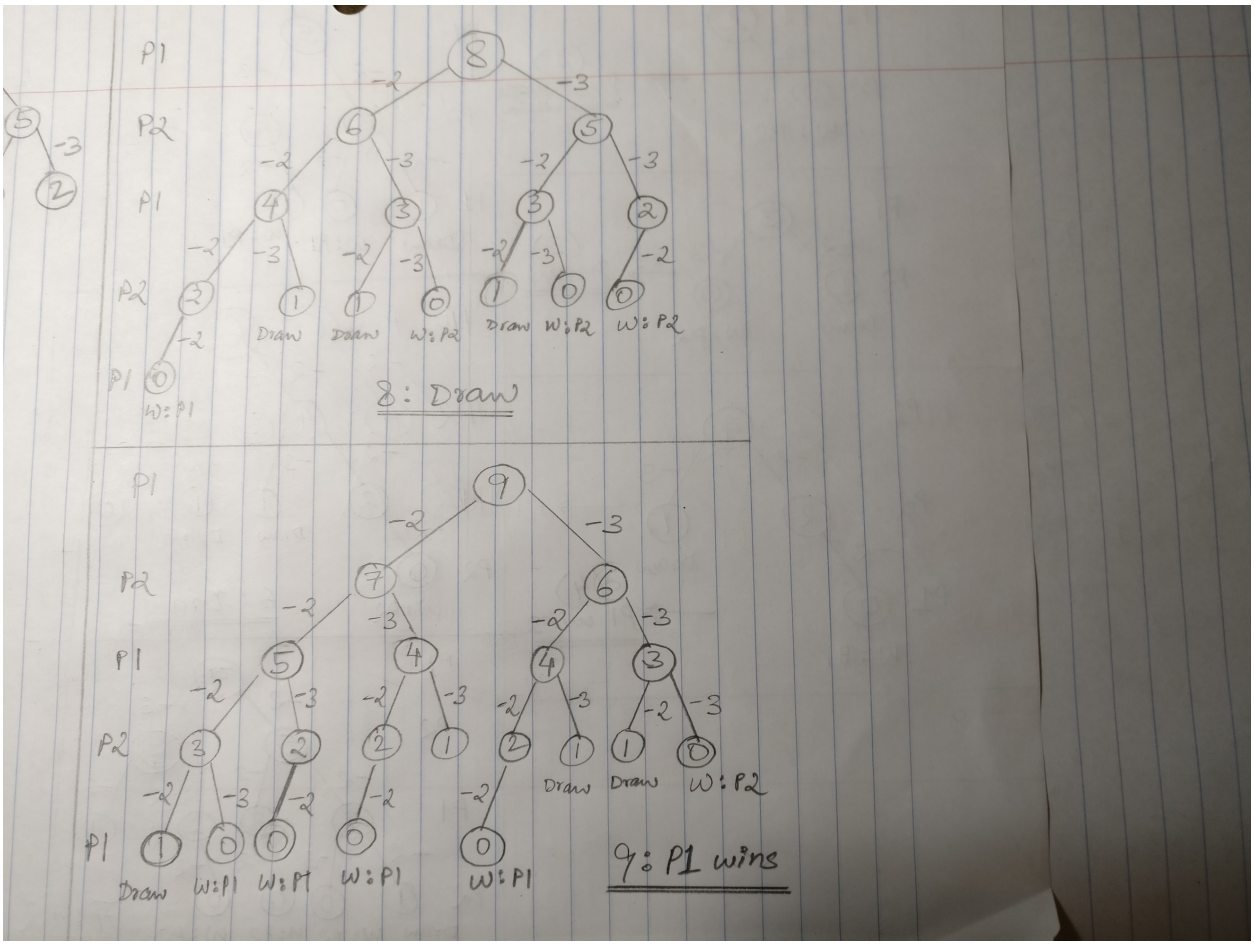
where $\lfloor \log_{10}(max) \rfloor$ denotes the Greatest Integer Value of $\log_{10}(max)$. (Since we are dealing with numbers greater than 1, log will not be negative and so taking the floor is equivalent to casting the value to an integer)

The algorithm cannot be used to sort an **arbitrary** set of numbers because suppose that even though we have a small array (say n = 5 numbers) but the max of those numbers is say 10^64. This would significantly increase the number of steps that we would have to take and the algorithm would waste a lot of time doing extra computations just because we have one number that is super large. This makes the algorithm inefficient for an arbitrary set of integers. This algorithm would be good to use in a situation where the numbers are within some known range

Question 3 (a)

The following pictures show the trees if the number of matches were 2,3,4,…9:


P1
2
-2
P2
0
W: P2
(2)
P2 wins
P1
3
-2
-3
P2
1
0
Draw
W: P2
(3)
Draw
P1
4
-2
-3
P2
2
1
Draw
-2
P1
0
W: P1
(4)
P1 wins
P1
5
-2
-3
P2
3
2
-2
-3
-2
P1
1
0
0
Draw
W: P1
W: P1
(5)
P1 wins
P1
6
-2
-3
P2
4
3
-2
-3
-2
-3
P1
2
1
1
0
Draw
Draw
W: P1
-2
P2
0
W: P2
6: Draw
P1
7
-2
-3
P2
5
4
-2
-3
-2
-3
P1
3
2
2
1
-2
-3
-2
-2
Draw
P2
1
0
0
0
Draw
W: P2
W: P2
W: P2
7: P2 wins



As can be seen from the above picture, **P1 would win for 9 matches**.

If we observe carefully, any higher number of matches involves the tree for a lower number of matches. For example, the tree for 6 matches involves the trees for 4 and 3 matches. Since we already know the results of a game starting with 4 and 3 matches with a certain player, we don't need to calculate the result for these two once again when doing the tree for 6 matches.

The tree for 6 matches could be expressed in a simplified way:

P1 6

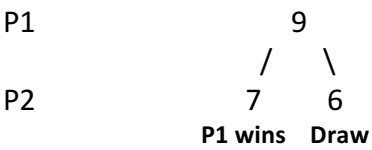
/ \

P2 4 3

P2 wins **Draw**

In the case of 4 matches, the player **who starts** wins the game. Therefore, if the game progresses to 4 matches P2 would win. In the case of 3 matches, the game is a draw. Depending on which of the 2 routes P1 goes down either it will end in a draw or P2 will win. Therefore, P1 will move in the direction such that the game ends in a draw. This result is consistent with what we got by drawing out the entire tree for 6 matches.

Applying the same logic for 9 matches:



For 7 matches, using the tree we already have, we know that the player who gets to **play second** wins, therefore, P1 would win if the game progressed to 7 matches and for 6 matches it would be a draw. Therefore, between the two choices of P1's victory and a draw, P1 goes down its path of victory and therefore **P1 would win the game for 9 matches**

Question 3 (b)

Using the above way of simplified trees, we get the following results up to 16 matches (we could go beyond this as well):

Assuming all the games are started by P1 –

No. of matches	Winner	No. of matches	Winner	No. of matches	Winner
2	P2	**7**	P2	**12**	P2
3	Draw	**8**	Draw	**13**	Draw
4	P1	**9**	P1	**14**	P1
5	P1	**10**	P1	**15**	P1
6	Draw	**11**	Draw	**16**	Draw

It can be clearly seen from the table that the results repeat themselves in cycles of 5. This would continue for further numbers as well and so we can generalize this and find the winner for 'n' number of matches.

No. of matches	Winner
If n is of the form (5k – 3)	P2
If n is of the form (5k – 2)	Draw
If n is of the form (5k – 1)	P1
If n is of the form (5k)	P1
If n is of the form (5k + 1)	Draw

Where k is a natural number

Question 4 (a)

Algorithm: eccentricity(vertex u)

Input: a vertex u from the graph
Output: the eccentricity of u

```
q ← new Queue()
setVisited(u, true)
setDistance(u, 0)
q.enqueue(u)
eccentricity ← 0

while(!q.empty()) do
        w ← q.deque()
        eccentricity ← getDistance(w)
        for all v ∈ getNeighbors(w) do
                if (!getVisited(v)) then
                        setVisited(v, true)
                        setDistance(v, getDistance(w) + 1)
                        q.enqueue(v)

return eccentricity
```

Question 4 (b)

Algorithm: is2colorable(vertex u)
Input: a graph vertex u
Output: true if the graph to which u belongs is 2-colorable, and false otherwise

```
q ← new Queue()
setVisited(u, true)
setColor(u, 0)
q.enqueue(u)

while(!q.empty()) do
        w ← q.deque()
        for all v ∈ getNeighbors(w) do
                if(!getVisited(v)) then
                        setVisited(v, true)
                        setColor(v, 1 – getColor(w))
                        q.enqueue(v)
                else
                        if(getColor(v) == getColor(w)) then
                                return false

return true
```