

# Advanced Models and Methods in Operations Research

## Project: Vehicle routing

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For each problem considered, instances and a code skeleton containing an instance parser and a solution checker are provided in the `data/` and `python/` folders of the project.

The algorithms must be implemented in the provided files between the tags `TODO START` and `TODO END`.

They must be tested on all the provided instances with the command: `python3 problem.py -i instance.json -c certificate.json`

And each solution file must be validated by the provided checker: `python3 problem.py -a checker -i instance.json -c certificate.json`

The results must be reproducible.

The deliverable must contain:

- A *short* report describing and justifying the proposed algorithms
- The code implementing the algorithms
- The solution files obtained on the provided instances

## 1 Dynamic Programming

We consider the Elementary shortest path problem with resource constraint and a single slot:

- Input:
  - 1 depot
  - $n - 1$  customers; for each customer  $j = 2, \dots, n$ , a visit interval  $[s_j, e_j[$ ,  $s_j \in \mathbf{N}_+$ ,  $e_j \in \mathbf{N}_+$ ,  $s_j < e_j$
  - an  $n \times n$  symmetric matrix  $t$  specifying the times in  $\mathbf{R}_+$  to travel between each pair of locations

- an  $n \times n$  matrix  $c$  specifying the costs in  $\mathbf{R}$  to travel between each pair of locations

- Problem: find a sub-tour starting and ending at the depot such that
  - each customer is visited at most once
  - the arrival at a customer  $j$  is before  $s_j$
  - the departure from a customer is at  $e_j$  (even if the arrival was before  $s_j$ )
- Objective: minimize the cost of the sub-tour

Note that the costs might be negative.

Propose and implement an algorithm based on Dynamic Programming for this problem.

## 2 Heuristic Tree Search

We consider the Elementary shortest path problem with resource constraint and two slots:

- Input:
  - 1 depot
  - $n - 1$  customers; for each customer  $j = 2, \dots, n$ , two visit intervals (which might overlap)
    - \*  $[s_j^1, e_j^1[, s_j^1 \in \mathbf{N}_+, e_j^1 \in \mathbf{N}_+, s_j^1 < e_j^2$
    - \*  $[s_j^2, e_j^2[, s_j^2 \in \mathbf{N}_+, e_j^2 \in \mathbf{N}_+, s_j^2 < e_j^2$
  - an  $n \times n$  symmetric matrix  $t$  specifying the times in  $\mathbf{R}_+$  to travel between each pair of locations
  - an  $n \times n$  matrix  $c$  specifying the costs in  $\mathbf{R}$  to travel between each pair of locations
- Problem: find a sub-tour starting and ending at the depot such that

- each customer is visited at most once
- the arrival and the departure from a customer include one of its two visit intervals
- Objective: minimize the cost of the sub-tour

Propose and implement an algorithm based on Heuristic Tree Search with Dynamic Programming for this problem.

### 3 Column Generation + Dynamic Programming

We consider the Vehicle routing problem with a single slot:

- Input:
  - 1 depot
  - $n - 1$  customers; for each customer  $j = 2, \dots, n$ , a visit interval  $[s_j, e_j[$ ,  $s_j \in \mathbf{N}_+$ ,  $e_j \in \mathbf{N}_+$ ,  $s_j < e_j$
  - an  $n \times n$  symmetric matrix  $t$  specifying the times in  $\mathbf{R}_+$  to travel between each pair of locations
- Problem: find a set of routes starting and ending at the depot such that
  - each customer is visited exactly once
  - the arrival at a customer  $j$  is before  $s_j$
  - the departure from a customer is at  $e_j$  (even if the arrival was before  $s_j$ )
- Objective: minimize the total traveled distance

Propose an exponential formulation and implement an algorithm based on a Column Generation heuristic for this problem.

### 4 Column Generation + Heuristic Tree Search

We consider the Vehicle routing problem with two slots:

- Input:
  - 1 depot

- $n - 1$  customers; for each customer  $j = 2, \dots, n$ , two visit intervals (which might overlap)
  - \*  $[s_j^1, e_j^1[$ ,  $s_j^1 \in \mathbf{N}_+$ ,  $e_j^1 \in \mathbf{N}_+$ ,  $s_j^1 < e_j^2$
  - \*  $[s_j^2, e_j^2[$ ,  $s_j^2 \in \mathbf{N}_+$ ,  $e_j^2 \in \mathbf{N}_+$ ,  $s_j^2 < e_j^2$
- an  $n \times n$  symmetric matrix  $t$  specifying the times in  $\mathbf{R}_+$  to travel between each pair of locations

- Problem: find a set of routes starting and ending at the depot such that
  - each customer is visited exactly once
  - the arrival and the departure from a customer include one of its two visit intervals

- Objective: minimize the total traveled distance

Propose an exponential formulation and implement an algorithm based on a Column Generation heuristic for this problem.