Signal Modeling – AR and MA Models

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Variant 9:

- Simulate ARMA(2,1) signal.
- Fit AR(3), MA(3), ARMA(2,1) models.
- Compare residuals and AIC

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
from statsmodels.tsa.arima.model import ARIMA
from scipy.signal import lfilter

np.random.seed(42)
```

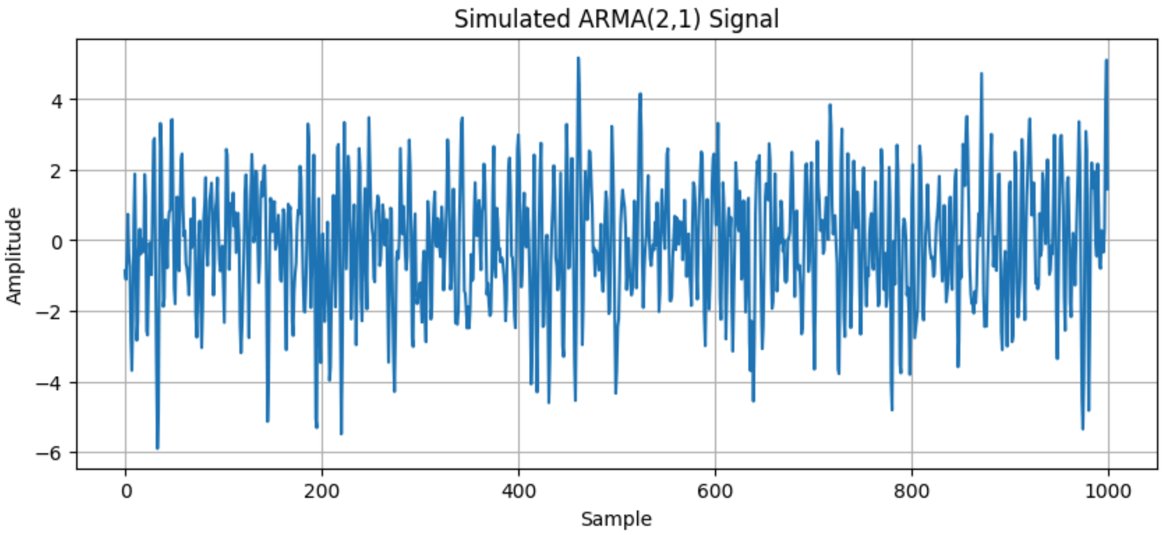
Simulate ARMA(2,1) Signal

In [7]:
```
N = 1000  # Number of samples

# Generate white noise
w = np.random.normal(0, 1, N)

# Simulate ARMA(2,1) process
a = [1, -0.75, 0.5]  # AR coefficients (AR(2))
b = [1.0, 0.5]       # MA coefficients (MA(1))
arma_signal = lfilter(b, a, w)


# Plot the simulated signal
plt.figure(figsize=(10, 4))
plt.plot(arma_signal)
plt.title('Simulated ARMA(2,1) Signal')
plt.xlabel('Sample')
plt.ylabel('Amplitude')
plt.grid(True)
plt.show()
```



Fit different models

In [16]:
```
# AR(3) Model
ar3_model = ARIMA(arma_signal, order=(3, 0, 0))
ar3_results = ar3_model.fit()

# MA(3) Model
ma3_model = ARIMA(arma_signal, order=(0, 0, 3))
ma3_results = ma3_model.fit()

# ARMA(2,1) Model
arma_model = ARIMA(arma_signal, order=(2, 0, 1))
arma_results = arma_model.fit()

# Compare model residuals and AIC
print("Model Comparison:")
print("\nAR(3) Model:")
print(f"AIC: {ar3_results.aic}")
print(f"Residuals Mean: {ar3_results.resid.mean()}")
print(f"Residuals Variance: {ar3_results.resid.var()}")

print("\nMA(3) Model:")
print(f"AIC: {ma3_results.aic}")
print(f"Residuals Mean: {ma3_results.resid.mean()}")
print(f"Residuals Variance: {ma3_results.resid.var()}")

print("\nARMA(2,1) Model:")
print(f"AIC: {arma_results.aic}")
print(f"Residuals Mean: {arma_results.resid.mean()}")
print(f"Residuals Variance: {arma_results.resid.var()}")
```

```
Model Comparison:

AR(3) Model:
AIC: 2832.5123174490395
Residuals Mean: 9.273503070898315e-05
Residuals Variance: 0.9831072637438738

MA(3) Model:
AIC: 2892.924385725415
Residuals Mean: 0.0002003900158066978
Residuals Variance: 1.044415173198735

ARMA(2,1) Model:
AIC: 2806.655068426109
Residuals Mean: 0.0003814027886870317
Residuals Variance: 0.9579774307553867
```

Visualize Residuals

In [17]:

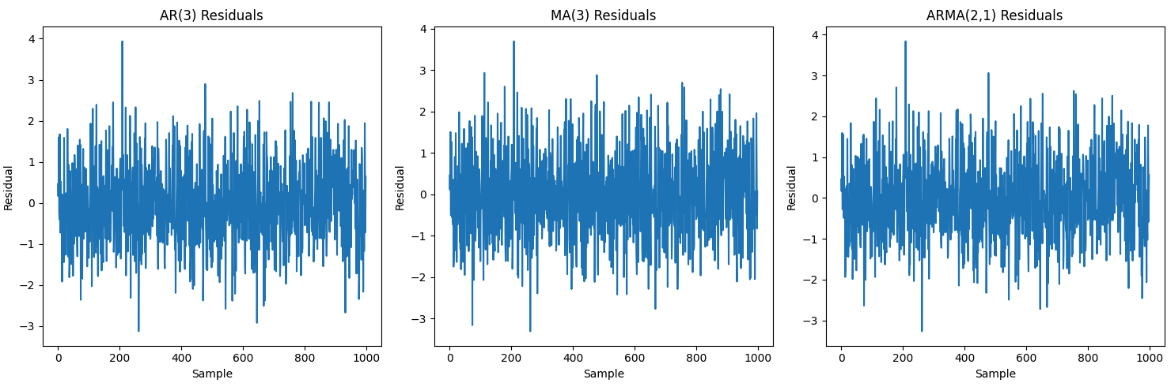
```
plt.figure(figsize=(15, 5))

plt.subplot(1, 3, 1)
plt.plot(ar3_results.resid)
plt.title('AR(3) Residuals')
plt.xlabel('Sample')
plt.ylabel('Residual')

plt.subplot(1, 3, 2)
plt.plot(ma3_results.resid)
plt.title('MA(3) Residuals')
plt.xlabel('Sample')
plt.ylabel('Residual')

plt.subplot(1, 3, 3)
plt.plot(arma_results.resid)
plt.title('ARMA(2,1) Residuals')
plt.xlabel('Sample')
plt.ylabel('Residual')

plt.tight_layout()
plt.show()
```



The AIC values for the three models are:

- AR(3): 2832.51
- MA(3): 2892.92

- ARMA(2,1): 2806.66

The ARMA(2,1) model has the lowest AIC value, suggesting it's the best fit among the three models. The MA(3) model performs the worst with the highest AIC.

Eesidual statistics:

ARMA(2,1) has the lowest residual variance (0.958), indicating it captures the most variation in the data AR(3) has a residual variance of 0.983, slightly higher than ARMA(2,1) MA(3) has the highest residual variance (1.044)

While not explicitly shown in the plots (since they're not fully visible in the document), the residual plots would likely show that the ARMA(2,1) residuals are closest to white noise, with AR(3) being reasonably close and MA(3) showing more structure.