# Monte Carlo estimation of option value probabilities Homework for in class

## monte_carlo_stock_price_v1_3

This is a version of the Monte Carlo simulator that is consistent with the modeling contained in the **Monte Carlo Simulations** lecture in Economics 136, assuming **Geometric Brownian Motion**. Prepared by Professor Evans on March 3, 2019, modified in April 2019 and January 6, 2020 (V3). This calculates Ito-adjusted-drift.

In [419]:
```
%matplotlib inline
```

In [420]:
```
import math
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

Set assumptions, including simulation length and the number of simulations. Note that if sims is increased more than 12, the color palettes below must be expanded.

In [421]:
```
days = 18                # default 18
sims = 1000              # default 1000
stock_sym = "HMC"
stock_pr = 100.00        # default 100.0
drift = 0.00041          # our mean, and we could call it that, but it is drift in our model default = 0.00
sigma = 0.0180           # default 0.0180
call_strike = 110.0      # default 110.0
call_price = 0.84        # default 0.84
call_be = call_strike + call_price
# put_strike = 90
# put_price = 0.40
# put_be = put_strike - put_price
```

Set up the numpy arrays for efficiency. We are going to take our random draws for each step in all simulations before we do anything else. Numpy arrays must be typed (often a default is assumed) and the arrays of fixed size, and arrays must be initialized, just like the glory days of Fortran. Order equals 'C' is actually default and unnecessary but it is there to remind you that 'F' is an option.

In [422]:
```
setup = np.arange(sims*days)    #note here that we are creating one very long 1D array
setup = setup.reshape((sims,days),order='C')   #for contiguous columns, order = 'F'
draw = np.zeros_like(setup, dtype="float32")
price = np.zeros_like(setup, dtype="float32")
```

Set the random seed value if you want each simulation to be the same (while debugging or when asking students to submit simulations that m... be identical for grading). To make it more "random," remove the seed command.

In [423]:
```
np.random.seed(742)
draw = np.random.standard_normal([sims,days])
```

Let's do the mean adjustment for the Ito method separately so that we remember that it is necessary:

In [424]:
```
ito_adj_drift = drift - ((sigma**2)/2)
"{:.7f}".format(ito_adj_drift)
```

Out[424]: '0.0002480'

1. Find the student version of our Monte Carlo stock price simulator, part of which is reproduced here, at

   https://www.palmislandtraders.com/econ136/monte_carlo_student_v1_3.html

2. No Python .. we will be using Numpy arrays for everything. Use these realistic defaults for our first run.

3. Without yet understanding why, we need to adjust our drift for Ito's method (you will eventually get an explanation).

4. Put in the proper solution for price right here:

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Let's do the simulation, starting with setting the first value of each simulation equal to the price of the stock:

In [425]:
```
price[0:,:1] = stock_pr
```

Set our counters to zero:

In [426]:
```
eur_itm_count = 0
eur_profit_count = 0
amer_itm_count = 0
amer_profit_count = 0
```

It is possible to use the numpy nditer command to do away with the for loops below but the code is nearly impossible to read, so these old-fashioned for loops are left in. This is much more complicated than array mulitplication - each element must be iterated one element at a time sequentially.

In [427]:
```
for i in range(0,sims):  # rows (each a different simulation)
    for j in range(1,days):    # cols (each a new day!)
        price[i,j] = 14  #student has to figure this out
        # American-style counters go here
        if price[i,j] > call_strike:
            pass
        if price[i,j] > call_be:
            pass
```

European style counters go here (only count the final day)

In [428]:
```
# student has to add this
```

Calculate a reference drift for plotting:

In [429]:
```
# Calculate the reference lines for drift, the call strike, and the break-even
```

Now let's plot it using Seaborn

In [430]:
```
sns.set_style("darkgrid")
sns.palplot(sns.color_palette("coolwarm",12))
sns.set_palette("coolwarm", 12)
sns.set_context("poster")
fig, ax = plt.subplots()
fig.set_size_inches(14,10)
# plt.plot(put, color="green")
plt.title("Monte Carlo Simulation")
for k in range(0,sims):
    plt.plot(price[k,...])
plt.plot(ref, color="black")
plt.plot(call_stk, color="green")
plt.plot(call_bre, color="orange")
```

5. The team must solve for the Monte Carlo mapper,
6. ... then the frequency of being above the strike price and profitable for American-style calls,
7. ... then the frequency of being above the strike price and profitable for European-style calls,
8. Calculate the reference lines for drift (it is not level), the call strike, and the breakeven (maybe use the actual plot below as a hint).

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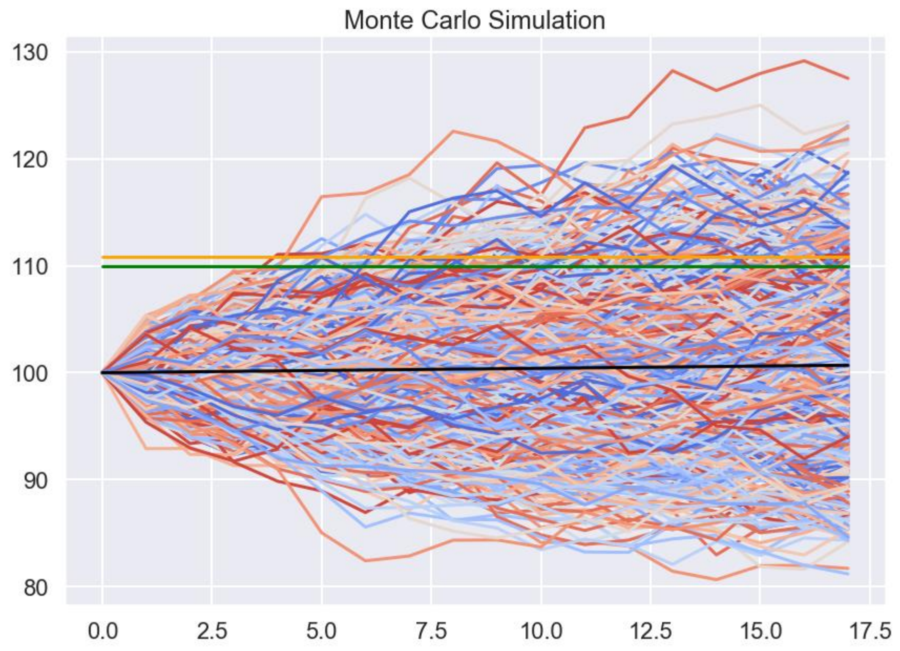


Print ITM and profit frequency:

```
n [431]: print("European in the money {} times at {:.3f} percent.".format(eur_itm_count,((eur_itm_count/sims)*100)))
print("European profitable {} times at {:.3f} percent.".format(eur_profit_count,(eur_profit_count/sims)*100))
print("American in the money {} times in {:,} opportunities".format(amer_itm_count,days*sims))
print("American profitable:", amer_profit_count)
```

```
European in the money 111 times at 11.100 percent.
European profitable 90 times at 9.000 percent.
American in the money 854 times in 18,000 opportunities
American profitable: 681
```

This is what your plot should look like and the answers on the bottom are probably the ones you will get using random seed 742.

9. Note the core drift line to the model, the equivalent of the black line here, is **not** adjusted for half-variance, and you should understand why at some point.

10. Add the means to map a reference line for a put position and a strangle, and a secondary line for a profit line for the strangle.

11. Experiment with this model, using different simulation lengths and numbers and be prepared to use it for real situations. Be prepared to remove the random seed.

12. Your program must count and display the frequency and frequency percentage that your option is
    - in the money at expiry
    - in the money at all
    - profitable at expiry
    - profitable before expiry

9. Keep your model clean and in good shape. We will be back