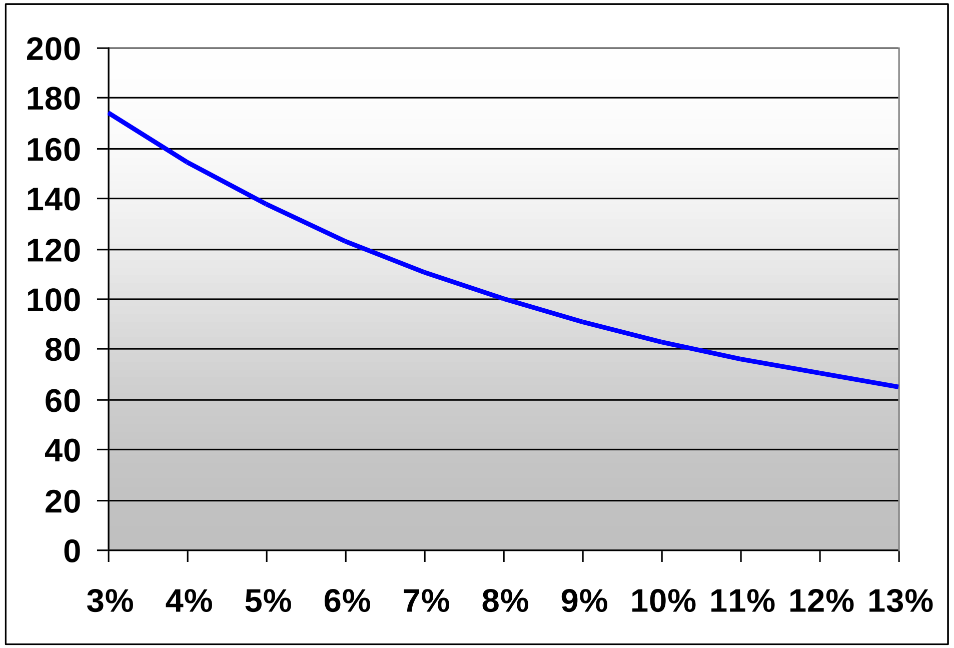

As this slide will show later in the lecture, even U.S. Treasury Bonds have market risk.

Bond Values and Interest Rates

... a contrary relationship

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Topical (earlier today)

Fed Cuts Rate for Third Time This Year, Signals Pause

FOMC statement indicates higher bar for future rate reductions

Federal Reserve Chair Jerome Powell **PHOTO:** MANDEL NGAN/AGENCE FRANCE-PRESSE/GETTY IMAGES

By Nick Timiraos

Updated Oct. 30, 2019 2:58 pm ET

WASHINGTON—The Federal Reserve cut interest rates for the third time this year and began to downplay expectations of further cuts for now.

“The current stance of policy is likely to remain appropriate as long as incoming information about the economy remains broadly consistent” with the Fed’s outlook for moderate growth, said Fed Chairman Jerome Powell at a press conference after Wednesday’s meeting.

From the last slide set:

This flood of liquidity justified by recovery from the last deep recession has made it very easy to fund business expansion and international projects with junk bonds .. meaning bonds rated BB or below. Companies like Frontier Communications and projects like shale-oil drilling were funded with these bonds.

Because of the explosive rallies in nearly all markets in 2017, the effective market yields of these bonds, and the ETFs made up of these bonds, dipped far below any reasonable threshold defined by the risk ratings described in this lecture, despite the fact that default rates were clearly rising. In 2018, rates are higher but still not very high by historical standards.

There will be a day of reckoning. I also said this last year and nothing has happened.

But I also said that no-one is very good at predicting turning points, including me.

?

From the last slide set:

Warning!!

These bonds and the ETFs that hold them may prove illiquid in a crisis!

From the iShares iBoxx High-Yield Corporate Bond (HYG) Prospectus, page S3, July 1, 2016.

Authorized Participant Concentration Risk. Only an Authorized Participant (as defined in the *Creations and Redemptions* section of the Fund's prospectus (the "Prospectus")) may engage in creation or redemption transactions directly with the Fund. The Fund has a limited number of institutions that may act as Authorized Participants on an agency basis (*i.e.*, on behalf of other market participants). To the extent that those Authorized Participants exit the business or are unable to proceed with creation and/or redemption orders with respect to the Fund and no other Authorized Participant is able to step forward to create or redeem Creation Units (as defined in the *Purchase and Sale of Fund Shares* section of the Prospectus), Fund shares may be more likely to trade at a premium or discount to NAV and possibly face trading halts and/or delisting.

YBFA yields

- **Two types of payouts**
  - Coupon payment (bonds and notes
  - Discount (bills, where yield is implicit)
- **Three yield calculations that you must know**
  - Coupon yield (not calculated, stated)

1. **Discount yield** and discount price(bills only)
2. **Current yield**
3. **Yield to maturity** (also called **Ask Yield**)

Coupon Yield / Coupon Payment
U.S. Treasury Notes and Bonds

A note or bond's coupon rate is the rate declared when the bond is issued, expressed as a percentage of par. U.S. Treasury Notes and Bonds pay interest twice per year, according to the following formula:

$$\text{Coupon payment} = \left(\text{par value} \times \text{coupon rate}\right) / 2$$

The interest payment for the 3.750 Feb 2043 bond would be

(100 X 0.03750)/2 = $1.875 semi-annually per 100

Maturity	Coupon	Bid	Asked	Chg	Asked yield
5/15/2043	2.875	92.7109	92.7422	0.5234	3.309
8/15/2043	3.625	105.2891	105.3203	0.5156	3.309
11/15/2043	3.750	107.4844	107.5156	0.6016	3.306
2/15/2044	3.625	105.3984	105.4297	0.6016	3.306
5/15/2044	3.375	101.1563	101.1875	0.5859	3.306

Asked yield is equivalent to our ytm.

1. Discount yield

for bills (money market financial assets)

Just remember this .. common sense approach

Discounting MMFAs: Buy at a discounted price (below par, 100), sell at par, interest is implicit in the appreciation.

$$Yield = \frac{100 - price}{price} X \frac{365}{days\ to\ maturity}$$

For example, a 26-week bill selling at 96.16 yields 8%:

$$8\% = \frac{100 - 96.16}{96.16} X \frac{365}{182}$$

Discount price (when yield is known)

for money market financial assets (continued)

The formula for determining price when the yield is known:

$$P = \frac{Par}{\left(\left[yield \times \left(\frac{dtm}{365}\right)\right]\right) + 1}$$

Same example:

$$\text{Price} = \frac{100}{\left(\text{yield} \times \left(\frac{\text{dtm}}{365}\right)\right) + 1} = \frac{100}{\left(.08 \times \left(\frac{182}{365}\right)\right) + 1} = 96.16$$

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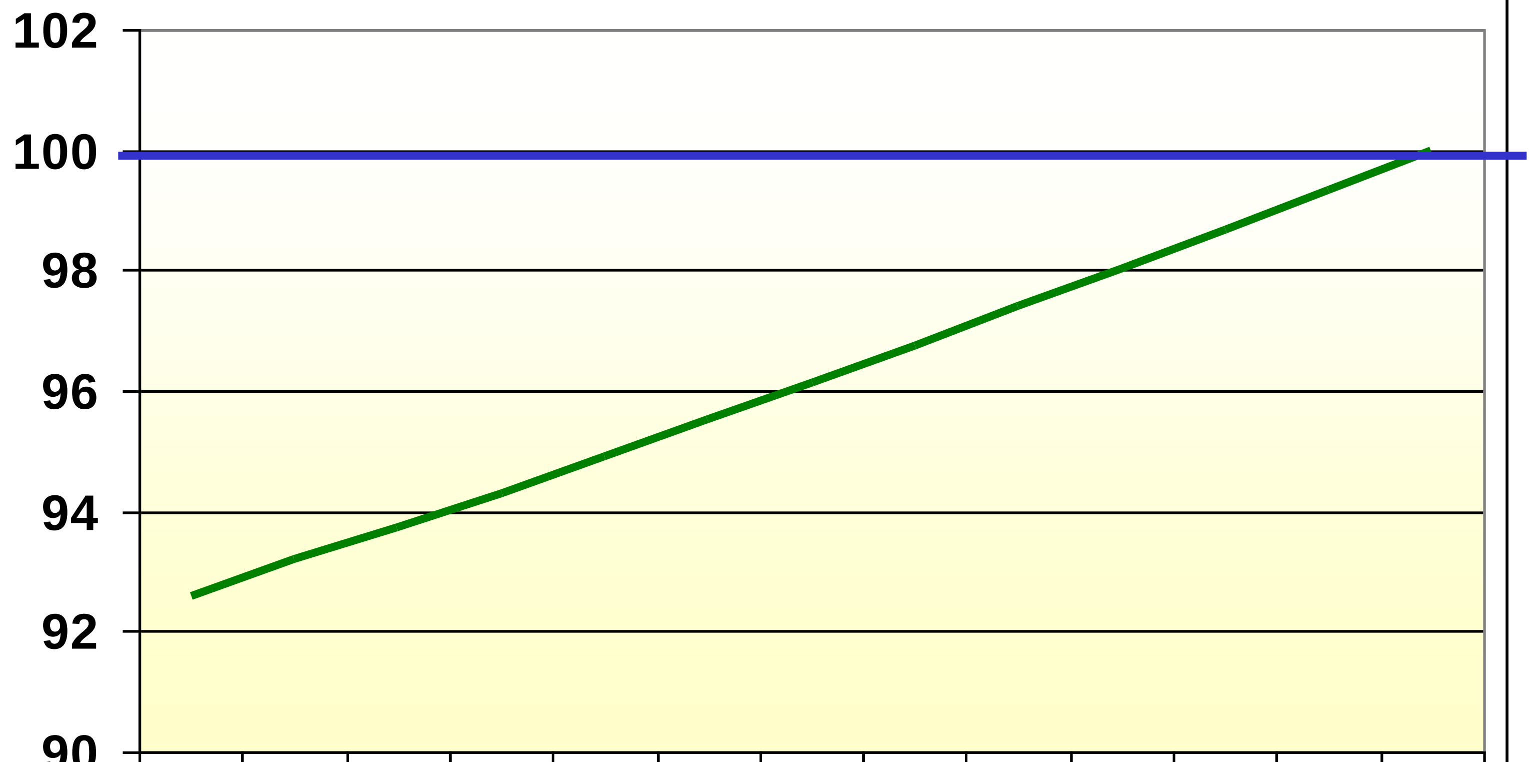
Projected Prices – 52 week bill @ 8%



	Jan			Apr			July			Oct			Jan
Price	92.5	93.1	93.7	94.3	94.9	95.5	96.1	96.7	97.4	98.0	98.6	99.3	100.
Discount	7.41	6.82	6.28	5.69	5.10	4.48	3.88	3.24	2.60	1.98	1.32	0.67	0

2. Current yield

Because you will not be paying par value for the bond or note that you buy, your short-term yield is current yield:

$$\text{Current yield} = \frac{\text{coupon rate} \times \text{par}}{\text{ask price}}$$

Using our example of the November bond with a coupon of 3.750 priced at 118.27 [ask] :

$$3.171\% = 3.750 / 118.27$$

Maturity	Coupon	Bid	Asked	Chg	Asked yield
2/15/2043	3.125	106.4609	106.4922	0.8359	2.767
5/15/2043	2.875	101.7891	101.8203	0.8281	2.775
8/15/2043	3.625	115.7344	115.7656	0.8516	2.767
11/15/2043	3.750	118.2422	118.2734	0.8672	2.762
2/15/2044	3.625	115.9531	115.9844	0.8594	2.766
5/15/2044	3.375	111.2422	111.2734	0.8516	2.773

Note that in this example current yield is above ytm. Do you know why?

A question ....??

- Ten years ago you bought a new bond with the following features:
  - **$1000** even (par 100)
  - 30 year bond (20 years remaining)
  - Coupon rate was **5%** ($50 yearly)
- Today 20 year bonds are yielding **10%**
- Question: Can you resell your bond today ..
  - at par (**$1,000**)?
  - at any price?

Price/Yield Tradeoff

... an example

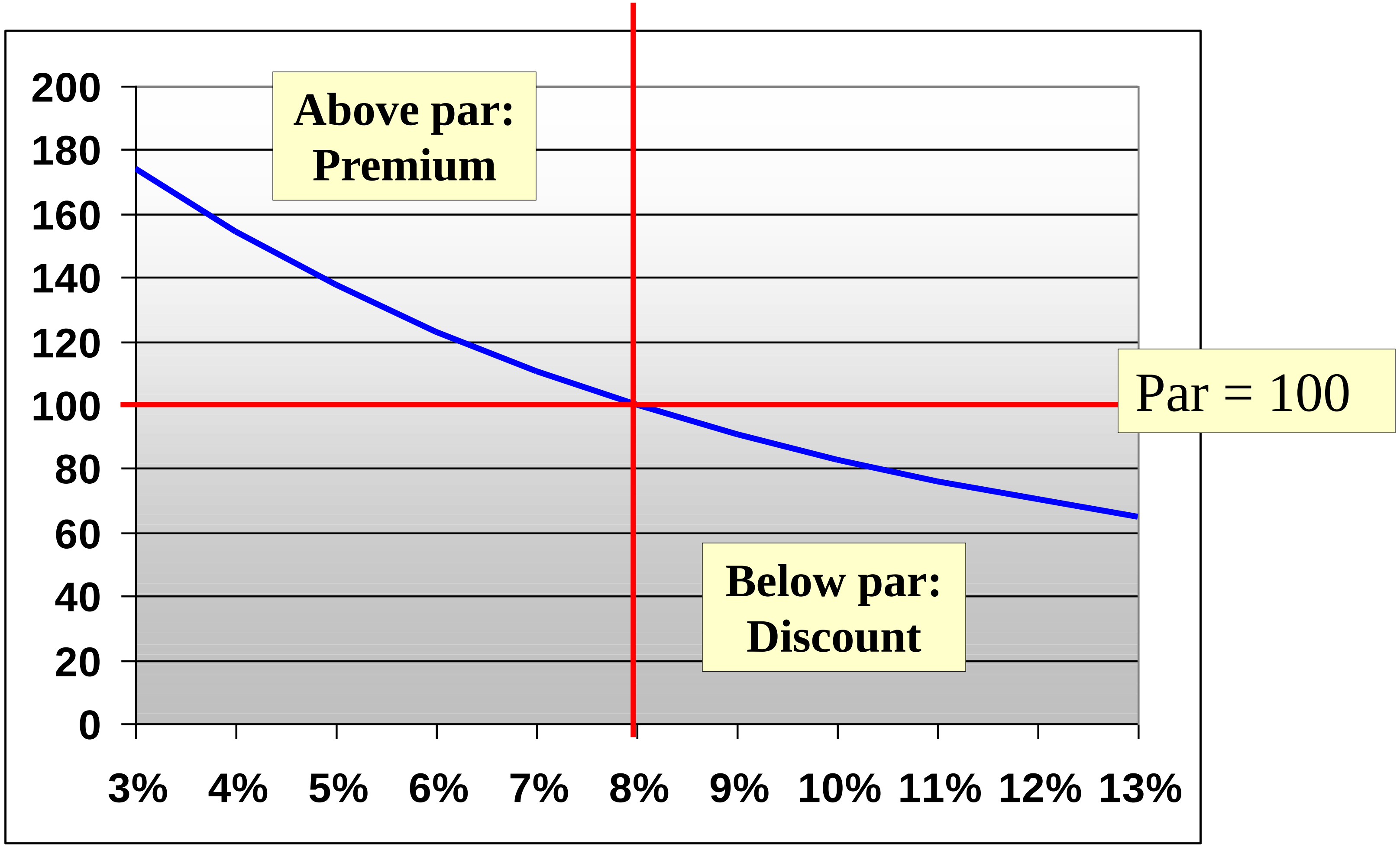
Suppose you buy a newly-issued 30-year bond which has a coupon of **8%**.

Ten years pass. Your 30-bond is now the equivalent of a 20-year bond (it has the same cashflow characteristics). Therefore if you wish to sell it on the secondary market, it must offer an **ask yield**, also called **yield to maturity**, that is competitive with a newly-issued 20-year bond.

What if the 20-year bond coupon is **6%**? Or **11%**? What will your old 30-year bond be worth?

The range of possibilities is shown on the next slide.

Possible Values



This shows the full range of ask-yields (ytm) possible for an original 30-year bond with 8% coupon 10 years later when compared to a 20-year bond with possible yields shown on the bottom axis.

What is a bond or note worth??

A bond or note is nothing more than a future stream of cash. **A bond or note is worth the present discounted value of all future cash payments.** That's it! And this is not an approximation, this is a mathematical certainty that defines the relationship between a bond's market price and its effective yield.

Sometimes the value of future cash payments is known (Treasuries) and sometimes estimated (bonds with default possibilities).

Therefore, to understand the pricing of bonds and notes, you must understand the math of compounding and present value calculations.

Valuing a bond (simple example)

To value a bond (or note) on the secondary market, you will need to know the coupon rate (cr), the time remaining in the life of the bond (t), and the present day yield (y) of equivalent bonds.

What would be today's value of a 30-year bond issued 10 years ago at a coupon rate of 12% (cr) if other bonds with 20 years (t) remaining until maturity today yield only 6% (y)? (Assume annual rather than semi-annual interest payments).

$$PV = \sum_{i=1}^{20} \frac{cr \times 100}{(1+y)^i} + \frac{100}{(1+y)^{20}} = \sum_{i=1}^{20} \frac{12}{1.06^i} + \frac{100}{1.06^{20}}$$

The simple bond valuation formula

$$MV = \sum_{i=1}^{n} \frac{C}{(1+r)^i} + \frac{Par}{(1+r)^n}$$

where
MV = market value *presently* of the bond
n = number of years to maturity
C = coupon payment (**par times the coupon interest rate**)
r = *present* yield of this bond (market determined)

This formula assumes that there is only one interest payment per year and that this bond was priced on the day after the most recent interest payment was made.

… another way of writing it

$$MV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n} + \frac{Par}{(1+r)^n}$$

The present discounted value of each of the coupon interest payments.

The present discounted value of the redemption value of the bond at par.

A yield-bearing financial asset is worth the present discounted value of its future cashflow, which consist of interest payments and redemption value.

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The elementary bond formula (reduced form)

$$MV = (CR \times 100) \times \left(\frac{1 - \left[\frac{1}{(1+r)^n} \right]}{r} \right) + \left(100 \times \frac{1}{(1+r)^n} \right)$$

where
MV: the present market value of the bond
CR: coupon rate (original yield) of the bond
r: current market rate on equivalent bonds
n: number of remaining years to maturity

The original formula is a geometric series and this is a reduced-form equation. To see its justification and derivation, read Appendix B in Chapter 8.

Note: This formula cannot be used to value an actual bond because it assumes only one interest payment per year and that the bond is being bought on the day of the coupon payment. For actual bond pricing a more complicated version of this is shown at the end of this lecture.

… a bond at premium

$$137.39 = (.08 \times 100) \times \left(\frac{1 - \left[\frac{1}{(1+.05)^{20}} \right]}{.05} \right) + \left(100 \times \frac{1}{(1+.05)^{20}} \right)$$

where
MV: **137.39**
CR: 8%
r: 5%
n: 20

This shows the current market value (137.39) of a 30-year bond that was issued 10 years ago (and hence has 20 years of life left) and pays a coupon interest rate of 8% per year (determined at its time of issue) given that interest rates now on an equivalent bond (a 20-year bond) are only 5%.

… a bond at discount

$$82.97 = (.08 \times 100) \times \left(\frac{1 - \left[\frac{1}{(1+.10)^{20}} \right]}{.10} \right) + \left(100 \times \frac{1}{(1+.10)^{20}} \right)$$

where
MV: **82.97**
CR: 8%
r: 10%
n: 20

This shows the current market value (82.97) of a 30-year bond that was issued 10 years ago (and hence has 20 years of life left) and pays a coupon interest rate of 8% per year (determined at its time of issue) given that interest rates now on an equivalent bond (a 20-year bond) are now 10%.

The formula when interest is paid semi-annually

$$MV = \sum_{i=1}^{m} \frac{C}{\left(1+r/2\right)^{i}} + \frac{Par}{\left(1+r/2\right)^{m}}$$

where

MV = market value

C = coupon interest payment (**par X r/2**)

r = present market yield

m = number of coupon payments remaining (years X 2)

3. The final formula … ask yield (ytm)

$$MV = \frac{C}{m}\sum_{i=1}^{n}\frac{1}{\left(1+\frac{r}{m}\right)^{i-a/p}} + \frac{Par}{(1+r)^{(n-1)/m+(p-a)/365}} - \left(\frac{C}{m}\times\frac{a}{p}\right)$$

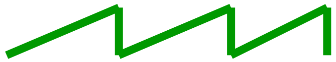
This term represents an adjustment for accrued interest.

MV = market value, the quoted *ask* price of the bond,
C = the annual coupon payment, equal to the coupon rate times par,
Par = 100
r = the prevailing annual market yield, expressed as ***ask yield*** or ***yield to maturity***,
m = the number of coupon payments per year,
n = the number of remaining coupon payments,
p = the number of days in *this* coupon period (between 181-184, use 182 if unknown),
a = the number of days between the last coupon payment and the settlement day.

What is this term?

$$\left[\frac{C}{m} X \frac{a}{p} \right]$$

When you buy a bond, you will owe accrued interest to the seller. It will be the interest accrued since the last coupon payment and it will be equal to the value of a single coupon times the portion of the coupon period since the last coupon was paid. This value is added to the bonds quoted value so that bond prices will not exhibit a saw-tooth pattern, rising as the coupon payment date approaches then plunging the day after.



an example

You are buying a 30-yr bond that was issued on September 15, 1999:

Purchase date: February 8, 2009 (**146** days since last coupon payment, **36** to next),
Next coupon date: March 15, 2009 (the first of 42 coupons remaining),
Redemption date: September 15, 2029 (for par and last coupon),
Coupon rate/amount: **8%** yielding **$4** per coupon payment,
Present market rate (*ask* yield): **10%**.

$$\frac{a}{p} = \frac{146}{182} = 0.8022$$

$$MV = \frac{8}{2}\sum_{i=1}^{42} \frac{1}{\left(1+\frac{.10}{2}\right)^{i-0.8022}} + \frac{100}{(1+.10)^{20.5+0.099}} - \left(\frac{8}{2}\times 0.8022\right)$$

$$MV = 4\sum_{i=1}^{42} \frac{1}{(1.05)^{i-0.8022}} + \frac{100}{(1.10)^{20.60}} - (3.21) = 83.31$$

Reduced-form Version of the Complex Formula

$$MV = \frac{C}{m}\left[\frac{1-\dfrac{1}{\left(1+r/m\right)^{n}}}{\dfrac{r/m}{\left(1+r/m\right)^{a/p}}}\right] + \left(Par \times \frac{1}{\left(1+r\right)^{(n-1)/m+(p-a)/365}}\right) - \left(\frac{C}{m} \times \frac{a}{p}\right)$$

Solving the same problem from the last slide (rounding error explains the difference):

$$MV = 4\left[\frac{1-\dfrac{1}{(1.05)^{42}}}{\dfrac{0.05}{(1.05)^{0.8022}}}\right] + \left(100 \times \frac{1}{(1.10)^{20.5+0.099}}\right) - \left(4 \times 0.8022\right) = 83.31$$

The 10-year note yield history .. (1962 – 2018a)

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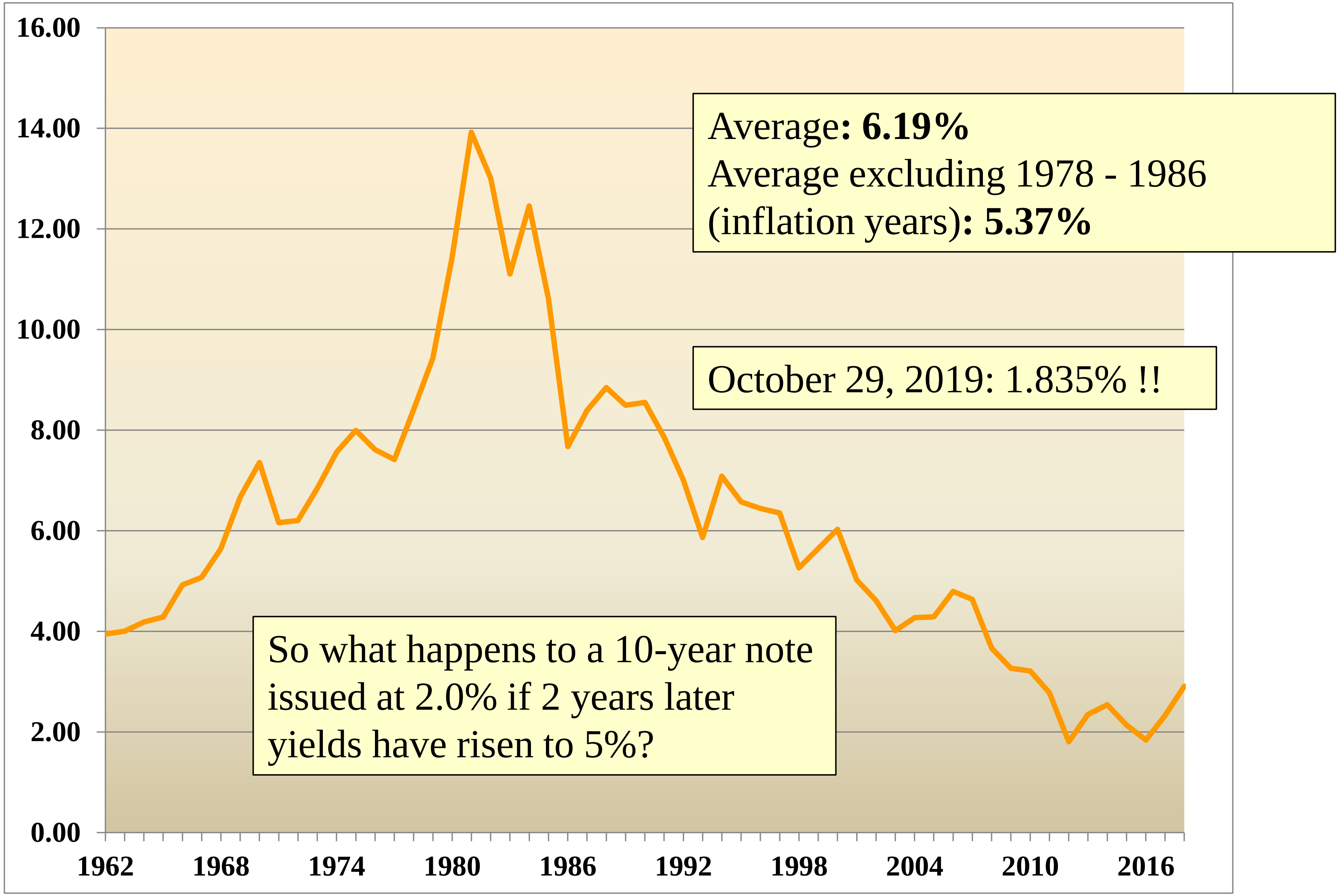

Source: Federal Reserve H.15 data

The 30-year bond history ... (1977 - 2018a)

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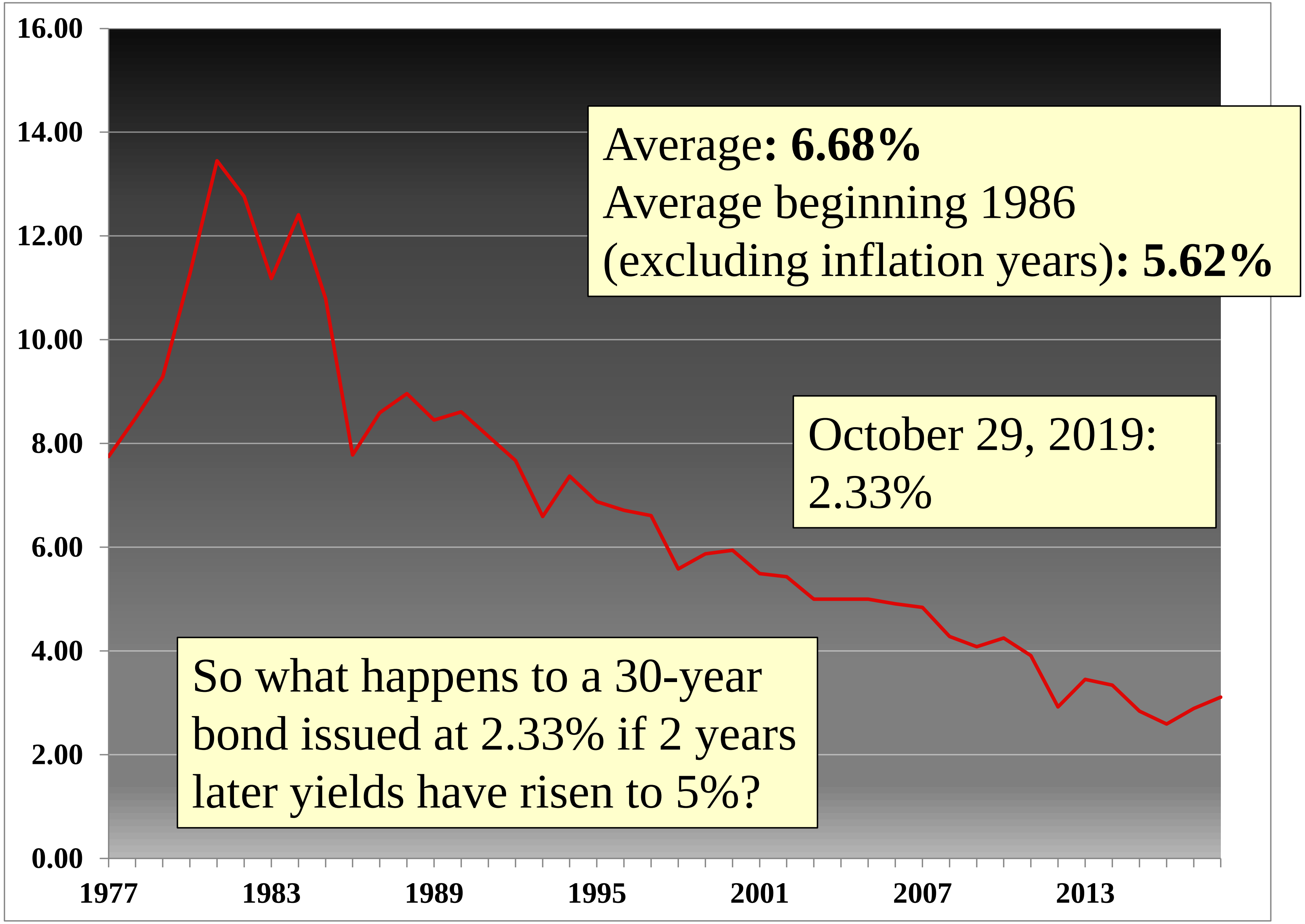

Source: Federal Reserve H.15 data

U.S. Treasury Quotes Tuesday, October 29, 2019

Treasury Notes & Bonds | Treasury Bills

Treasury note and bond data are representative over-the-counter quotations as of 3pm Eastern time. For notes and bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

MATURITY	COUPON	BID	ASKED	CHG	ASKED YIELD
11/15/2043	3.750	126.2500	126.2700	0.0960	2.292
2/15/2044	3.625	124.1940	124.2140	0.1040	2.294
5/15/2044	3.375	119.3160	120.0160	0.1040	2.301
8/15/2044	3.125	115.1120	115.1320	0.1060	2.305
11/15/2044	3.000	112.3160	113.0160	0.0960	2.311
2/15/2045	2.500	103.1260	103.1460	0.1060	2.318
5/15/2045	3.000	113.0740	113.0940	0.7980	2.308
8/15/2045	2.875	110.2460	110.2660	0.1120	2.315
11/15/2045	3.000	113.1220	113.1420	0.1180	2.310
2/15/2046	2.500	103.1160	103.1360	0.1140	2.325
5/15/2046	2.500	103.1240	103.1440	0.1200	2.325
8/15/2046	2.250	98.1140	98.1340	0.8020	2.329
11/15/2046	2.875	111.0240	111.0440	0.7980	2.318
2/15/2047	3.000	113.2540	113.2740	0.1200	2.312
5/15/2047	3.000	113.2220	113.2420	0.1120	2.321
8/15/2047	2.750	108.1700	108.1900	0.1160	2.328
11/15/2047	2.750	108.1820	108.2020	0.1120	2.329
2/15/2048	3.000	113.2740	113.2940	0.1180	2.326
5/15/2048	3.125	116.1920	116.2120	0.1200	2.323
8/15/2048	3.000	114.0060	114.0260	0.7960	2.326
11/15/2048	3.375	122.0800	122.1000	0.8060	2.315
2/15/2049	3.000	114.0960	114.1160	0.7960	2.321
5/15/2049	2.875	111.2160	111.2360	0.1140	2.323
8/15/2049	2.250	98.0940	98.1140	0.8000	2.327

Some questions:

- When was this bond issued?
- What was the 30-year rate in November 2018?
- Why is this bond, which isn't even one year old and issued at par, trading at a premium of 22?
- What will this bond be worth in 2023 if 25-year rates are 5%

Bond Valuation - no documentation - adjustable date

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This model allows the user to calculate the secondary market value of a bond, given a new market interest rate. This model conforms to formula 23 of **Chapter 8 - Bond Valuation and Interest Rate Formulas** of the online book **Introduction to Financial Markets and Instruments** used in Economics 104 and Economics 136 at Harvey Mudd College. This version is the same as the original bondpricing.html, but with no documentation.
This version allows the user to put in the relevant trading date rather than use today's date.
Whatever coupon date is used must be consistent with the date entered (must be within 6 months before).

In [21]:
```
import math
from datetime import date
```

Figuring this out with a Python Bond Value Calculator using our Formula 23

Assign initial values and dates:

In [22]:
```
bond_id = "15 Nov 2048"
bond_desc = "30-year Treasury"
coupon_rate = float(3.375)
market_ytm = (5.0)
par = float(100.0)
mat_year = int(2048)
mat_month = int(11)
mat_day = int (15)
trade_year = int(2023)
trade_month = int(10)
trade_day = int(30)
cou_year = int(2023)   # Most recent coupon date. This must be consistent with what is chosen as the relevant current date.
cou_month = int(5)
cou_day = int (11)
year = float(365.25)
hyear = float(year/2)
```

Import daysto method and set parameters:

In [23]:
```
def daysto(eyear,emonth,eday,tyear,tmonth,tday):
    trade = date(tyear,tmonth,tday)
    expiry = date(eyear, emonth, eday)
    days2expiry = abs(expiry - trade)

    return float(days2expiry.days)
crate = coupon_rate/100.0
ytm = market_ytm/100.0
coupon = crate*par/2
```

Calculate formula 23:

In [24]:
```
rty = daysto(mat_year,mat_month,mat_day,trade_year,trade_month,trade_day)/year
ac_days = int(daysto(cou_year,cou_month,cou_day,trade_year,trade_month,trade_day))
coup_rem = int(2* (rty -(rty%1))+round((rty%1)+1,0))
cpnum = coupon*(1 - (1/((1 + (ytm/2))**coup_rem)))
cpdenom = (ytm/2)/((1 + (ytm/2))**(ac_days/hyear))
pv_coupon = cpnum/cpdenom
expo1 = (coup_rem - 1)/2
expo2 = (hyear - ac_days)/year
expo = expo1 + expo2
pv_redem = par/((1 + ytm)**expo)
bond_value = pv_coupon + pv_redem
acc_int = coupon*(ac_days/hyear)
net_value = bond_value - acc_int
```

Print output:

In [25]:
```
print ()
print ()
print ( "Bond identity:", bond_desc, " ", bond_id)
print ( "Coupon rate:", '{:.3f}'.format(coupon_rate))
print ( "Market rate:", '{:.3f}'.format(market_ytm))
print ( "Remaining time in years:", '{:.3f}'.format(rty))
print ( "Accrued days since last coupon:", ac_days)
print ( "Coupon payments remaining:", coup_rem)
print ( "Coupon amount (per par):", '{:.3f}'.format(coupon))
print ( "Present value of coupons:", '{:.3f}'.format(pv_coupon))
print ( "Present value of redemption:", '{:.3f}'.format(pv_redem))
print ( "Bond Market Value:", '{:.3f}'.format(bond_value))
print ( "Accrued interest:", '{:.3f}'.format(acc_int))
print ( "Net Bond Value:", '{:.3f}'.format(net_value))
```

```
Bond identity: 30-year Treasury   15 Nov 2048
Coupon rate: 3.375
Market rate: 5.000
Remaining time in years: 25.046
Accrued days since last coupon: 172
Coupon payments remaining: 51
Coupon amount (per par): 1.688
Present value of coupons: 49.478
Present value of redemption: 29.488
Bond Market Value: 78.966
Accrued interest: 1.589
Net Bond Value: 77.377
```

77.377