Log-Periodic Modulations in Starobinsky Inflation (DSI)

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Abstract

We revisit Starobinsky plateau inflation with a small log-periodic (discrete scale invariance, DSI) modulation of the potential and compute the induced oscillatory features in the primordial curvature spectrum. Our main aim is to introduce DSI as a novel symmetry principle in inflationary cosmology: discrete rescalings of the inflaton field lead naturally to log-periodic modulations, providing a controlled framework to explore new physics beyond standard slow roll. Working consistently to first order in the modulation amplitude $A \ll 1$, we correct common pitfalls in the literature: the normalization of the plateau potential, the $\mathcal{O}(A)$ expansions for V'/V, ϵ_V, and the fractional residual of the power spectrum $R(k)$, the form of the Friedmann equation in e-fold time N, and the mapping between k and N including the $\ln H$ correction. Calibrating the base model to Planck 2018 values ($A_s \simeq 2.1 \times 10^{-9}$, $n_s \simeq 0.965$) and consistent bounds on r from BICEP/Keck, we show that amplitudes $A \sim 10^{-4}$–5×10^{-4} yield sub-percent to percent-level oscillations for moderate frequencies ($\omega \sim \mathcal{O}(5)$), comfortably compatible with current non-detections. For larger frequencies ($\omega \sim 30$), the same amplitudes instead produce order-10% residuals, approaching present observational limits. We thus establish DSI as a theoretically consistent and observationally constrained extension of plateau inflation, and highlight it as a concrete target for upcoming CMB and 21cm surveys.

Keywords: Discrete scale invariance (DSI); Inflation; Starobinsky model; Primordial power spectrum; Oscillatory features; CMB; Slow roll.

1 Introduction

Plateau inflation models—chiefly the Starobinsky R^2 model and α-attractors—fit the CMB with remarkable accuracy, predicting $n_s \simeq 1 - 2/N$ and $r \simeq 12/N^2$ for $N \sim 50$–60 e-folds [1–5]. Planck 2018 finds $n_s = 0.9649 \pm 0.0042$ and $\ln(10^{10} A_s) = 3.044 \pm 0.014$, i.e. $A_s \simeq 2.1 \times 10^{-9}$, with no evidence for large deviations from scale invariance [2]. The current upper bound on the tensor-to-scalar ratio is at the few percent level; combining with BICEP/Keck 2018 yields $r_{0.05} \lesssim 0.032$ (95% CL) [6].

Despite the success of featureless spectra, small residual modulations remain observationally allowed. Oscillatory patterns in $\ln k$ have been motivated ...in several frameworks, such as [7].

2 Starobinsky Plateau with Log-Periodic Modulation

We work in reduced Planck units ($M_{\mathrm{Pl}} = 1$). The Starobinsky (Einstein-frame) potential is

$$V_0(\phi) = V_* \left(1 - e^{-\alpha\phi}\right)^2, \qquad \alpha = \sqrt{\tfrac{2}{3}}, \qquad V_* = \tfrac{3}{4}M^2, \tag{1}$$

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We introduce a *multiplicative* modulation,

$$V(\phi) = V_0(\phi)\left[1 + A\,\cos\!\left(\omega \ln \frac{\phi}{\phi_c} + \theta\right)\right], \qquad 0 < A \ll 1, \tag{2}$$

with dimensionless amplitude A, log-frequency ω, phase θ, and reference scale ϕ_c. This realizes an approximate DSI: $\ln\phi \to \ln\phi + 2\pi/\omega$ leaves the phase invariant (mod 2π). The multiplicative choice keeps $V > 0$ and makes the expansion in A transparent.

3 Analytical Expansions to First Order in A

Define $f(\phi) = \cos\left(\omega\ln(\phi/\phi_c) + \theta\right)$. To first order, $\ln V = \ln V_0 + Af + \mathcal{O}(A^2)$, hence

$$\frac{V'}{V} = \left(\frac{V'}{V}\right)_0 + Af'(\phi) + \mathcal{O}(A^2), \qquad f'(\phi) = -\frac{\omega}{\phi}\,\sin\!\left(\omega\ln\frac{\phi}{\phi_c} + \theta\right), \tag{3}$$

where the base ratio

$$\left(\frac{V'}{V}\right)_0 = \frac{d}{d\phi}\ln\left[(1 - e^{-\alpha\phi})^2\right] = \frac{2\alpha\, e^{-\alpha\phi}}{1 - e^{-\alpha\phi}} \tag{4}$$

is small on the plateau. The potential slow-roll parameter

$$\epsilon_V(\phi) \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \epsilon_{V0}(\phi) + A\left(\frac{V'}{V}\right)_0 f'(\phi) + \mathcal{O}(A^2), \qquad \epsilon_{V0} = \frac{1}{2}\left(\frac{V'}{V}\right)_0^2. \tag{5}$$

Friedmann and N-time. With $dN \equiv H\,dt$ and $\phi_N \equiv d\phi/dN$, the exact identities are

$$\epsilon_H \equiv -\frac{d\ln H}{dN} = \frac{1}{2}\,\phi_N^2, \qquad H^2 = \frac{V(\phi)}{3 - \frac{1}{2}\,\phi_N^2}, \tag{6}$$

so the slow-roll limit gives $H^2 \simeq V/3$ and $\phi_N \simeq -V'/V$ (sign chosen for rolling down).

Horizon exit mapping. For a mode k, at horizon crossing $k = aH$ and

$$\ln\left(\frac{k}{k_*}\right) = (N - N_*) + \ln\left(\frac{H(N)}{H_*}\right). \tag{7}$$

The $\ln H$ term is $\mathcal{O}(\epsilon)$ but relevant for precise phase in $\ln k$.

Power spectrum and residual. To leading order in slow roll,

$$\boxed{R(k) \equiv \frac{P_{\mathcal{R}}(k)}{P_0(k)} - 1 \simeq A\left[f(\phi_k) - \frac{2f'(\phi_k)}{X(\phi_k)}\right]} \tag{8}$$

where ϕ_k is evaluated at k-exit. Let $P_0(k) = V_0/(24\pi^2\,\epsilon_{V0})$ be the unmodulated spectrum. Using $V = V_0(1 + Af)$ and $\epsilon_V = \epsilon_{V0} + A\,X f'$, with $X \equiv (V'/V)_0$, we obtain the *fractional residual*

$$\boxed{R(k) \;\equiv\; \frac{P_{\mathcal{R}}(k)}{P_0(k)} - 1 \;\simeq\; A\left[f(\phi_k) \;-\; \frac{2\,f'(\phi_k)}{X(\phi_k)}\right],} \tag{9}$$

valid to $\mathcal{O}(A)$. Equation (9) shows a leading cosine in $\ln k$ plus a sine term with a phase shift proportional to f'/X; both vary lentamente ao longo das escalas CMB via ϕ_k.

Perturbativity and practical bounds on A. On the plateau, $X \sim \mathcal{O}(10^{-2})$ while $|f'| \le \omega/\phi$. A conservative slow-roll–safe condition is $|Af'| < X$, i.e.

$$A \;<\; \frac{X\,\phi}{\omega}. \tag{10}$$

For typical CMB exit $\phi \sim 5$, $X \sim 0.02$ and $\omega \sim 5$, one finds $A_{\max} \sim 0.02$. Much tighter is the requirement that the *relative* modulation of ϵ stay small:

$$\frac{\Delta\epsilon_V}{\epsilon_{V0}} = \frac{2A\,f'}{X} \;\lesssim\; \text{few}\,\% \;\Rightarrow\; A \;\lesssim\; \text{few} \times 10^{-4}\,. \tag{11}$$

We therefore adopt $A \in [10^{-4}, 5 \times 10^{-4}]$ in our examples, with the caveat de que amplitudes efetivas crescem com ω.

4 Numerical Methodology

We integrate the background in N using Eqs. (6) and the exact field equation

$$\phi_{NN} + (3 - \epsilon_H)\phi_N + \frac{V'(\phi)}{H^2} = 0, \qquad \epsilon_H = \frac{1}{2}\phi_N^2, \quad H^2 = \frac{V}{3 - \frac{1}{2}\phi_N^2}\,. \tag{12}$$

Initial conditions are chosen on the plateau with ϕ large enough to yield $N_{\rm tot} \gtrsim 60$, and ϕ_N initialized by the slow-roll estimate $\phi_N \simeq -V'/V$. We stop at $\epsilon_H = 1$ (end of inflation). The mapping $k \leftrightarrow N$ uses Eq. (**??**); we include $\ln(H/H_)$ to set phases accurately. The scalar spectrum (Eq. (**??**)) is computed at horizon exit; we optionally validate with the Mukhanov–Sasaki equation in test runs (not shown).

5 Results

We adopt a baseline calibrated to $A_s \simeq 2.1 \times 10^{-9}$ at $k_{=0.05\ \rm Mpc^{-1}}$ with $N_{\simeq 55}$–60, which fixes V (hence M) in Eq. (**??**). For the modulation we show benchmarks with

$$(A, \omega, \phi_c, \theta) = (3 \times 10^{-4}, 5, 10^{-2}, 0) \quad \text{and} \quad (3 \times 10^{-4}, 30, 5, 0).$$

Figure 1: **Inflaton field vs.** N**.** Trajectories for the modulated (solid) and unmodulated (dashed) Starobinsky model. Parameters shown here: $A = 3 \times 10^{-4}$, $\omega = 5$, $\phi_c = 10^{-2}$, $\theta = 0$. Ripples are small and increase mildly as ϕ approaches the end of inflation.

Figure 2: $\epsilon_H(N)$. Comparison of modulated (solid) and unmodulated (dashed) cases. For $\omega = 5$ the oscillatory residual is $\lesssim 1\%$, while for $\omega = 30$ it grows to the $\sim 10\%$ level.

Field trajectory. The inflaton $\phi(N)$ remains monotonic; the modulation induces tiny ripples superimposed on the smooth plateau roll.

Slow-roll parameter. The Hubble slow-roll parameter $\epsilon_H(N) = \frac{1}{2}\phi_N^2$ exhibits oscillations consistent with Eq. (11). For $\omega = 5$ the modulation remains at the $\lesssim 1\%$ level, while for $\omega = 30$ the same A yields oscillations approaching the $\mathcal{O}(10\%)$ level.

Primordial spectrum and residual. The scalar power spectrum follows the usual tilted power law with superimposed wiggles. The residual $R(k)$ in Eq. (9) is well fit by a single harmonic with slowly drifting phase across the observable $\ln k$ range. Again, for $\omega = 5$ the fractional residual is $\lesssim \mathcal{O}(1\%)$, while for $\omega = 30$ it reaches $\mathcal{O}(10\%)$ given $A = 3 \times 10^{-4}$.

6 Discussion

The corrected expressions clarify how DSI modulations map into observables: (i) the derivative $f'(\phi)$ introduces a sine component with amplitude $\propto \omega/\phi$; (ii) the ratio f'/X governs the phase shift and the size of the residual; and (iii) slow evolution of $X(\phi)$ produces a mild drift of the effective frequency in $\ln k$. Given current non-detections of oscillatory features [1, 8], the conservative range $A \sim 10^{-4}$–5×10^{-4} is well motivated for moderate ω. The tensor prediction of Starobinsky, $r \simeq 12/N^2 \sim 0.003$–$0.004$, remains far below present limits [6].

It is worth stressing that while moderate frequencies ($\omega \sim 5$) with $A \sim 10^{-4}$–5×10^{-4} lead to percent-level residuals fully consistent with current constraints, larger frequencies ($\omega \sim 30$) push the residual amplitude to the $\mathcal{O}(10\%)$ level. Such cases are already close to being excluded by Planck and BICEP/Keck bounds on oscillatory features, and therefore only the low-ω region of parameter space remains safely viable at present.

7 Conclusions

We provided a complete, corrected treatment of log-periodic modulations in Starobinsky inflation, suitable for data analysis. Key outcomes are:

(1) correct first-order formulas for V'/V, ϵ_V, and the residual $R(k) = A\,[\,f - 2f'/X\,]$;

(2) consistent Friedmann equation in N and k–N mapping with $\ln H$;

(3) calibrated normalization to Planck A_s and realistic priors for A consistent with slow-roll and feature searches.

Figure 3: **Primordial power spectrum**. $P_{\mathcal{R}}(k)$ for the modulated model (blue) versus Starobinsky baseline (red). For $\omega = 5$ the modulation imprints percent-level oscillations, while for $\omega = 30$ the same A produces order-10% deviations.

Figure 4: **Residual** $R(k) = [P_{\mathcal{R}}(k)/P_0(k)] - 1$. The curve follows the analytic form $A\,[\,f - 2f'/X\,]$. The amplitude depends strongly on ω: percent-level for $\omega = 5$, order-10% for $\omega = 30$.

A crucial result is the strong dependence of the residual amplitude on the log-frequency ω: for moderate values ($\omega \sim 5$) and $A \sim 10^{-4}$–5×10^{-4}, the oscillations remain at the $\lesssim 1\%$ level across CMB scales, comfortably within current bounds and therefore still viable. In contrast, for larger frequencies ($\omega \sim 30$) the same amplitudes produce $\mathcal{O}(10\%)$ residuals, which would already have been visible in the Planck power spectrum. Such high-frequency cases are therefore effectively ruled out.

In summary, strong DSI signals are excluded by present data, but small-amplitude, low-frequency oscillations remain a realistic possibility. This narrows the window of viable parameter space, but also provides a well-defined target for future high-precision probes such as CMB-S4, LiteBIRD, or 21cm surveys, which could decisively test the remaining allowed regime.

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