

Log-Periodic Modulations in Starobinsky Inflation (DSI)

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Abstract

We revisit Starobinsky plateau inflation with a small log-periodic (discrete scale invariance, DSI) modulation of the potential and compute the induced oscillatory features in the primordial curvature spectrum. Our main aim is to introduce DSI as a novel symmetry principle in inflationary cosmology: discrete rescalings of the inflaton field lead naturally to log-periodic modulations, providing a controlled framework to explore new physics beyond standard slow roll. Working consistently to first order in the modulation amplitude $A \ll 1$, we correct common pitfalls in the literature: the normalization of the plateau potential, the $\mathcal{O}(A)$ expansions for V'/V , ϵ_V , and the fractional residual of the power spectrum $R(k)$, the form of the Friedmann equation in e -fold time N , and the mapping between k and N including the $\ln H$ correction. Calibrating the base model to Planck 2018 values ($A_s \simeq 2.1 \times 10^{-9}$, $n_s \simeq 0.965$) and consistent bounds on r from BICEP/Keck, we show that amplitudes $A \sim 10^{-4}$ – 5×10^{-4} yield sub-percent to percent-level oscillations for moderate frequencies ($\omega \sim \mathcal{O}(5)$), comfortably compatible with current non-detections. For larger frequencies ($\omega \sim 30$), the same amplitudes instead produce order-10% residuals, approaching present observational limits. We thus establish DSI as a theoretically consistent and observationally constrained extension of plateau inflation, and highlight it as a concrete target for upcoming CMB and 21cm surveys.

Keywords: Discrete scale invariance (DSI); Inflation; Starobinsky model; Primordial power spectrum; Oscillatory features; CMB; Slow roll.

1 Introduction

Plateau inflation models—chiefly the Starobinsky R^2 model and α -attractors—fit the CMB with remarkable accuracy, predicting $n_s \simeq 1 - 2/N$ and $r \simeq 12/N^2$ for $N \sim 50$ – 60 e -folds [1–5]. Planck 2018 finds $n_s = 0.9649 \pm 0.0042$ and $\ln(10^{10} A_s) = 3.044 \pm 0.014$, i.e. $A_s \simeq 2.1 \times 10^{-9}$, with no evidence for large deviations from scale invariance [2]. The current upper bound on the tensor-to-scalar ratio is at the few percent level; combining with BICEP/Keck 2018 yields $r_{0.05} \lesssim 0.032$ (95% CL) [6].

Despite the success of featureless spectra, small residual modulations remain observationally allowed. Oscillatory patterns in $\ln k$ have been motivated in several frameworks, such as

2 Starobinsky Plateau with Log-Periodic Modulation

We work in reduced Planck units ($M_{\text{Pl}} = 1$). The Starobinsky (Einstein-frame) potential is

$$V_0(\phi) = V_{(1-e^{-\alpha\phi})^2}, \quad \alpha = \sqrt{\frac{2}{3}}, \quad V_{\frac{3}{4}M^2}(1)$$

where M is the R^2 mass parameter in the Jordan frame [4, 5]. Matching A_s near horizon exit for $N_{\sim 50-60}$ fixes $M \simeq (1.2-1.4) \times 10^{-5}$ and $V_{\sim 10^{-10}}$; $V_{1/4 \sim 3 \times 10^{-3} M_{\text{Pl}} \simeq 7 \times 10^{15} \text{ GeV}}$ [1, 2].

We introduce a *multiplicative* modulation,

$$V(\phi) = V_0(\phi) \left[1 + A \cos \left(\omega \ln \frac{\phi}{\phi_c} + \theta \right) \right], \quad 0 < A \ll 1, \quad (2)$$

with dimensionless amplitude A , log-frequency ω , phase θ , and reference scale ϕ_c . This realizes an approximate DSI: $\ln \phi \rightarrow \ln \phi + 2\pi/\omega$ leaves the phase invariant (mod 2π). The multiplicative choice keeps $V > 0$ e torna a expansão em A transparente.

3 Analytical Expansions to First Order in A

Define $f(\phi) = \cos(\omega \ln(\phi/\phi_c) + \theta)$. To first order, $\ln V = \ln V_0 + Af + \mathcal{O}(A^2)$, hence

$$\frac{V'}{V} = \left(\frac{V'}{V} \right)_0 + Af'(\phi) + \mathcal{O}(A^2), \quad f'(\phi) = -\frac{\omega}{\phi} \sin \left(\omega \ln \frac{\phi}{\phi_c} + \theta \right), \quad (3)$$

where the base ratio

$$\left(\frac{V'}{V} \right)_0 = \frac{d}{d\phi} \ln \left[(1 - e^{-\alpha\phi})^2 \right] = \frac{2\alpha e^{-\alpha\phi}}{1 - e^{-\alpha\phi}} \quad (4)$$

is small on the plateau. The potential slow-roll parameter

$$\epsilon_V(\phi) \equiv \frac{1}{2} \left(\frac{V'}{V} \right)^2 = \epsilon_{V0}(\phi) + A \left(\frac{V'}{V} \right)_0 f'(\phi) + \mathcal{O}(A^2), \quad \epsilon_{V0} = \frac{1}{2} \left(\frac{V'}{V} \right)_0^2. \quad (5)$$

Friedmann and N -time. With $dN \equiv H dt$ and $\phi_N \equiv d\phi/dN$, the exact identities are

$$\epsilon_H \equiv -\frac{d \ln H}{dN} = \frac{1}{2} \phi_N^2, \quad H^2 = \frac{V(\phi)}{3 - \frac{1}{2} \phi_N^2}, \quad (6)$$

so the slow-roll limit gives $H^2 \simeq V/3$ and $\phi_N \simeq -V'/V$ (sign chosen for rolling down).

Horizon exit mapping. For a mode k , at horizon crossing $k = aH$ and

$$\ln \frac{k}{k = (N - N_{\text{H}})_{+\ln \frac{H(N)}{H}} \cdot (7)}$$

The $\ln H$ term is $\mathcal{O}(\epsilon)$ but relevant for precise phase in $\ln k$.

Power spectrum and residual. To leading order in slow roll,

$$P_{\mathcal{R}}(k) \simeq \frac{V(\phi_k)}{24\pi^2 \epsilon_V(\phi_k)}, \quad (8)$$

where ϕ_k is evaluated at k -exit. Let $P_0(k) = V_0/(24\pi^2 \epsilon_{V0})$ be the unmodulated spectrum. Using $V = V_0(1 + Af)$ and $\epsilon_V = \epsilon_{V0} + AXf'$, with $X \equiv (V'/V)_0$, we obtain the *fractional residual*

$$R(k) \equiv \frac{P_{\mathcal{R}}(k)}{P_0(k)} - 1 \simeq A \left[f(\phi_k) - \frac{2f'(\phi_k)}{X(\phi_k)} \right], \quad (9)$$

valid to $\mathcal{O}(A)$. Equation (9) shows a leading cosine in $\ln k$ plus a sine term with a phase shift proportional to f'/X ; both vary lentamente ao longo das escalas CMB via ϕ_k .

Perturbativity and practical bounds on A . On the plateau, $X \sim \mathcal{O}(10^{-2})$ while $|f'| \leq \omega/\phi$. A conservative slow-roll-safe condition is $|Af'| < X$, i.e.

$$A < \frac{X\phi}{\omega}. \quad (10)$$

For typical CMB exit $\phi \sim 5$, $X \sim 0.02$ and $\omega \sim 5$, one finds $A_{\max} \sim 0.02$. Much tighter is the requirement that the *relative* modulation of ϵ stay small:

$$\frac{\Delta\epsilon_V}{\epsilon_{V0}} = \frac{2Af'}{X} \lesssim \text{few \%} \Rightarrow A \lesssim \text{few} \times 10^{-4}. \quad (11)$$

We therefore adopt $A \in [10^{-4}, 5 \times 10^{-4}]$ in our examples, with the caveat de que amplitudes efetivas crescem com ω .

4 Numerical Methodology

We integrate the background in N using Eqs. (6) and the exact field equation

$$\phi_{NN} + (3 - \epsilon_H)\phi_N + \frac{V'(\phi)}{H^2} = 0, \quad \epsilon_H = \frac{1}{2}\phi_N^2, \quad H^2 = \frac{V}{3 - \frac{1}{2}\phi_N^2}. \quad (12)$$

Initial conditions are chosen on the plateau with ϕ large enough to yield $N_{\text{tot}} \gtrsim 60$, and ϕ_N initialized by the slow-roll estimate $\phi_N \simeq -V'/V$. We stop at $\epsilon_H = 1$ (end of inflation). The mapping $k \leftrightarrow N$ uses Eq. (7); we include $\ln(H/H_0)$ to set phases accurately. The scalar spectrum (Eq. (8)) is computed at horizon exit; we optionally validate with the Mukhanov–Sasaki equation in test runs (not shown).

5 Results

We adopt a baseline calibrated to $A_s \simeq 2.1 \times 10^{-9}$ at $k_{=0.05 \text{ Mpc}^{-1}}$ with $N_{\simeq 55-60}$, which fixes V (hence M) in Eq. (1). For the modulation we show benchmarks with

$$(A, \omega, \phi_c, \theta) = (3 \times 10^{-4}, 5, 10^{-2}, 0) \quad \text{and} \quad (3 \times 10^{-4}, 30, 5, 0).$$

Figure 1: **Inflaton field vs. N .** Trajectories for the modulated (solid) and unmodulated (dashed) Starobinsky model. Parameters shown here: $A = 3 \times 10^{-4}$, $\omega = 5$, $\phi_c = 10^{-2}$, $\theta = 0$. Ripples are small and increase mildly as ϕ approaches the end of inflation.

Figure 2: $\epsilon_H(N)$. Comparison of modulated (solid) and unmodulated (dashed) cases. For $\omega = 5$ the oscillatory residual is $\lesssim 1\%$, while for $\omega = 30$ it grows to the $\sim 10\%$ level.

Field trajectory. The inflaton $\phi(N)$ remains monotonic; the modulation induces tiny ripples superimposed on the smooth plateau roll.

Slow-roll parameter. The Hubble slow-roll parameter $\epsilon_H(N) = \frac{1}{2}\phi_N^2$ exhibits oscillations consistent with Eq. (11). For $\omega = 5$ the modulation remains at the $\lesssim 1\%$ level, while for $\omega = 30$ the same A yields oscillations approaching the $\mathcal{O}(10\%)$ level.

Primordial spectrum and residual. The scalar power spectrum follows the usual tilted power law with superimposed wiggles. The residual $R(k)$ in Eq. (9) is well fit by a single harmonic with slowly drifting phase across the observable $\ln k$ range. Again, for $\omega = 5$ the fractional residual is $\lesssim \mathcal{O}(1\%)$, while for $\omega = 30$ it reaches $\mathcal{O}(10\%)$ given $A = 3 \times 10^{-4}$.

6 Discussion

The corrected expressions clarify how DSI modulations map into observables: (i) the derivative $f'(\phi)$ introduces a sine component with amplitude $\propto \omega/\phi$; (ii) the ratio f'/X governs the phase shift and the size of the residual; and (iii) slow evolution of $X(\phi)$ produces a mild drift of the effective frequency in $\ln k$. Given current non-detections of oscillatory features [1, 7], the conservative range $A \sim 10^{-4}$ – 5×10^{-4} is well motivated for moderate ω . The tensor prediction of Starobinsky, $r \simeq 12/N^2 \sim 0.003$ – 0.004 , remains far below present limits [6].

It is worth stressing that while moderate frequencies ($\omega \sim 5$) with $A \sim 10^{-4}$ – 5×10^{-4} lead to percent-level residuals fully consistent with current constraints, larger frequencies ($\omega \sim 30$) push the residual amplitude to the $\mathcal{O}(10\%)$ level. Such cases are already close to being excluded by Planck and BICEP/Keck bounds on oscillatory features, and therefore only the low- ω region of parameter space remains safely viable at present.

7 Conclusions

We provided a complete, corrected treatment of log-periodic modulations in Starobinsky inflation, suitable for data analysis. Key outcomes are:

- (1) correct first-order formulas for V'/V , ϵ_V , and the residual $R(k) = A[f - 2f'/X]$;
- (2) consistent Friedmann equation in N and k – N mapping with $\ln H$;
- (3) calibrated normalization to Planck A_s and realistic priors for A consistent with slow-roll and feature searches.

Figure 3: **Primordial power spectrum.** $P_{\mathcal{R}}(k)$ for the modulated model (blue) versus Starobinsky baseline (red). For $\omega = 5$ the modulation imprints percent-level oscillations, while for $\omega = 30$ the same A produces order-10% deviations.

Figure 4: **Residual** $R(k) = [P_{\mathcal{R}}(k)/P_0(k)] - 1$. The curve follows the analytic form $A[f - 2f'/X]$. The amplitude depends strongly on ω : percent-level for $\omega = 5$, order-10% for $\omega = 30$.

A crucial result is the strong dependence of the residual amplitude on the log-frequency ω : for moderate values ($\omega \sim 5$) and $A \sim 10^{-4}$ – 5×10^{-4} , the oscillations remain at the $\lesssim 1\%$ level across CMB scales, comfortably within current bounds and therefore still viable. In contrast, for larger frequencies ($\omega \sim 30$) the same amplitudes produce $\mathcal{O}(10\%)$ residuals, which would already have been visible in the Planck power spectrum. Such high-frequency cases are therefore effectively ruled out.

In summary, strong DSI signals are excluded by present data, but small-amplitude, low-frequency oscillations remain a realistic possibility. This narrows the window of viable parameter space, but also provides a well-defined target for future high-precision probes such as CMB-S4, LiteBIRD, or 21cm surveys, which could decisively test the remaining allowed regime.

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