HW7

Problem 1.

Population proportion

$$H_o: p = 0.5$$

$$H_a: p \neq 0.5$$

Type I error:

$$P(type\ I\ error) = P(x \leq 7\ or\ x \geq 18\ when\ p = 0.5) =$$

$$= P(x \leq 7\ when\ p = 0.5) + P(x \geq 18\ \ when\ p = 0.5) =$$

$$= P(x \leq 7\ when\ p = 0.5) + 1 - P(x \leq 17\ \ when\ p = 0.5) =$$

$$= \sum_{k=0}^{7}\left[\binom{25}{k} * 0.5^k * (1-0.5)^{25-k}\right] + 1 - \sum_{k=0}^{17}\left[\binom{25}{k} * 0.5^k * (1-0.5)^{25-k}\right] =$$

$$= 0.0216 + 1 - 0.9784 = 0.0216 + 0.0216 = 0.0432$$

$$P(type\ I\ error) = 0.0432$$

Type II error:

$$P(type\ II\ error) = P(8 \leq\ x \leq 17\ when\ p = 0.3) =$$

$$= P(\ x \leq 17\ when\ p = 0.3) - P(\ x \leq 7\ \ when\ p = 0.3) =$$

$$= \sum_{k=0}^{17}\left[\binom{25}{k} * 0.3^k * (1-0.3)^{25-k}\right] - \sum_{k=0}^{7}\left[\binom{25}{k} * 0.3^k * (1-0.3)^{25-k}\right] =$$

$$= 1 - 0.5118 = 0.4882$$

$$P(type\ II\ error) = 0.4882$$

Problem 2.

Values: 85, 77, 82, 68, 72, 69

Test statistic:

$$T = \frac{\bar{X} - \mu_o}{\frac{s}{\sqrt{n}}}$$

$$\bar{X} = \frac{85 + 77 + 82 + 68 + 72 + 69}{6} = 75.5$$

$$S = \sqrt{\frac{(85-75.5)^2+(77-75.5)^2+(82-75.5)^2+(68-75.5)^2+(72-75.5)^2+(69-75.5)^2}{6-1}}$$

$$S = \sqrt{\frac{245.5}{5}} = 7.007$$

$$T = \frac{\bar{X}-\mu_o}{\frac{s}{\sqrt{n}}} = \frac{75.5-70}{\frac{7.007}{\sqrt{6}}} = 1.922$$

$$p-value = 2*(1-F_{6-1}(1.922)) = 0.1126$$

$$p-value = 0.1126$$

Problem 3.

$$H_o: \mu = 10$$

$$H_a: \mu = 5$$

From page 509 from "Mathematical Statistics with Applications by Wackerly, Mendenhall and Scheaffer (7th Edition)"

$$z_\alpha = \frac{k-\mu_o}{\sigma/\sqrt{n}}$$

$$-z_\beta = \frac{k-\mu_a}{\sigma/\sqrt{n}}$$

$$z_\alpha = \frac{k-\mu_o}{\sigma/\sqrt{n}} => z_{0.025} = \frac{k-10}{\sqrt{25}/\sqrt{n}} => k = 10 + 1.96\frac{\sqrt{25}}{\sqrt{n}}$$

$$-z_\beta = \frac{k-\mu_a}{\frac{\sigma}{\sqrt{n}}} => -z_{0.025} = \frac{k-5}{\frac{\sqrt{25}}{\sqrt{n}}} => k = 5 - 1.96\frac{\sqrt{25}}{\sqrt{n}}$$

$$k = 10 + 1.96\frac{\sqrt{25}}{\sqrt{n}} = 5 - 1.96\frac{\sqrt{25}}{\sqrt{n}} => n = \frac{(1.96+1.96)^2 * \left(\sqrt{25}\right)^2}{(5-10)^2} = 15.3664$$

$$n = 15.3664$$

We need n to be 16 observations

Problem 4.

$$H_o: \sigma_1^2 = \sigma_2^2$$

$$H_a: \sigma_1^2 \neq \sigma_2^2$$

Test statistic

$$F = \frac{S_1^2}{S_2^2}$$

$$F = \frac{2.78095}{0.17143} = 16.22207$$

$$F_{14}^{14} = 2.4837 \; for \; \alpha = 0.05$$

$$F > F_{14}^{14} \; for \; \alpha = 0.05$$

That means that we reject H_o so we can't conclude that the underlying populations have the same variance

Runing "pf(q=16.22207, df1=14, df2=14, lower.tail=FALSE)" on R, we get

```
> pf(q=16.22207, df1=14, df2=14, lower.tail=FALSE)
[1] 2.794777e-06
```

Thus, the p-value is 0.000002794777

Problem 5.

$$Line\ 1: n_1 = 294$$

$$Line\ 2: n_2 = 276$$

$$Line\ 3: n_3 = 238$$

$$Line\ 4: n_4 = 192$$

$$Total: n = 1000$$

$$H_o: p_1 = p_2 = p_3 = p_4 = 1/4$$

$$H_a: otherwise$$

Test Statistic:

$$\chi^2 = \sum_{I=1}^{4} \frac{(N_i - n * p_i)^2}{n * p_i}$$

$$\chi^2 = \frac{(294 - 1000 * 0.25)^2}{1000 * 0.25} + \frac{(276 - 1000 * 0.25)^2}{1000 * 0.25} + \frac{(238 - 1000 * 0.25)^2}{1000 * 0.25} + \frac{(192 - 1000 * 0.25)^2}{1000 * 0.25}$$

$$\chi^2 = 7.744 + 2.704 + 0.576 + 13.456 = 24.48$$

The p-value is:

```
> pchisq(24.48, 3, lower.tail = FALSE)
[1] 1.983098e-05
```

$p - value = .00001983098$

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Since the p-value is lower than the significance level of alpha=0.05, then we reject the H_0.

So, we can say that we have sufficient evidence to indicate that some lanes are preferred over others.