HW4

Problem 1.

Problem solve on document: 'HW4_Alex_Parra_Garcia_Code.pdf' or 'HW4_Alex_Parra_Garcia_Code.Rmd

Problem 2.

$$E(Y_1) = E(Y_2 - Y_1) = E(Y_3 - Y_2) = E(Y_4 - Y_3) = E(Y_5 - Y_4) = N - E(Y_5) + 1$$

a)

$$N = (Y_1) + (Y_2 - Y_1) + (Y_3 - Y_2) + (Y_4 - Y_3) + (Y_5 - Y_4) + (N - Y_5 + 1) - 1$$

$$N = E(Y_1) + E(Y_2 - Y_1) + E(Y_3 - Y_2) + E(Y_4 - Y_3) + E(Y_5 - Y_4) + E(N - Y_5 + 1) - 1$$

$$N = E(Y_1) + E(Y_2 - Y_1) + E(Y_3 - Y_2) + E(Y_4 - Y_3) + E(Y_5 - Y_4) + N - E(Y_5) + 1 - 1$$

$$N = E(Y_1) + E(Y_1) + E(Y_1) + E(Y_1) + E(Y_1) + E(Y_1) - 1$$

$$N = 6E(Y_1) - 1$$

b)

$$E(Y_1) = E(Y_2 - Y_1) = E(Y_3 - Y_2) = E(Y_4 - Y_3) = E(Y_5 - Y_4) = N - E(Y_5) + 1$$

We can get:

$$E(Y_1) = E(Y_2 - Y_1) => E(Y_2) = 2E(Y_1)$$

Now we substitute:

$$E(Y_1) = E(Y_3 - Y_2) = E(Y_3) - E(Y_2)$$

$$E(Y_1) = E(Y_3) - E(Y_2) => E(Y_1) = E(Y_3) - 2E(Y_1)$$

$$E(Y_3) = 3E(Y_1)$$

We continue

$$E(Y_1) = E(Y_4 - Y_3) = E(Y_4) - E(Y_3)$$

$$E(Y_1) = E(Y_4) - E(Y_3) => E(Y_1) = E(Y_4) - 3E(Y_1)$$

$$E(Y_4) = 4E(Y_1)$$

Last time:

$$E(Y_1) = E(Y_5 - Y_4) = E(Y_5) - E(Y_4)$$

$$E(Y_1) = E(Y_5) - E(Y_4) => E(Y_1) = E(Y_5) - 4E(Y_1)$$

$$E(Y_5) = 5E(Y_1)$$

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c)

From a)

$$N = 6E(Y_1) - 1$$

From b)

$$E(Y_5) = 5E(Y_1)$$

$$E(Y_5) = 5E(Y_1) => E(Y_1) = \frac{E(Y_5)}{5}$$

Now we substitute on a)

$$\widehat{N} = 6E(Y_1) - 1 => \widehat{N} = 6\frac{E(Y_5)}{5} - 1$$

$$\widehat{N} = \frac{6E(Y_5)}{5} - 1$$

Problem 3.

a)

$$\sigma^2 = \frac{1}{n}\sum\nolimits_{i=1}^{n}(X_i - \mu)^2$$

$$E(s^2) = E\left(\frac{1}{n}\sum\nolimits_{i=1}^{n}(X_i - \mu)^2\right) = E\left(\frac{1}{n}\sum\nolimits_{i=1}^{n}\left(X_i^2 - 2X_i\mu + \mu^2\right)\right) =$$

$$\frac{1}{n}E\left(\sum\nolimits_{i=1}^{n}X_i^2 - \sum\nolimits_{i=1}^{n}2X_i\mu + \sum\nolimits_{i=1}^{n}\mu^2\right) = \frac{1}{n}E\left(\sum\nolimits_{i=1}^{n}X_i^2 - 2\mu\sum\nolimits_{i=1}^{n}X_i + n\mu^2\right) =$$

$$\frac{1}{n}E\left(\sum\nolimits_{i=1}^{n}X_i^2 - 2\mu * n\bar{X} + n\mu^2\right) = \frac{1}{n}\left(\sum\nolimits_{i=1}^{n}E\left(X_i^2\right) - E(2\mu * n\bar{X}) + E(n\mu^2)\right) =$$

$$\frac{1}{n}\left(\sum\nolimits_{i=1}^{n}E\left(X_i^2\right) - 2\mu nE(\bar{X}) + n\mu^2\right) = \frac{1}{n}\left(\sum\nolimits_{i=1}^{n}\sigma^2 + \mu^2 - 2\mu n\mu + n\mu^2\right) =$$

$$\frac{1}{n}\left(\sum\nolimits_{i=1}^{n}\sigma^2 + \mu^2 - 2n\mu^2 + n\mu^2\right) = \frac{1}{n}\left(\sum\nolimits_{i=1}^{n}\sigma^2 + \mu^2 - n\mu^2\right) = \frac{1}{n}(n\sigma^2 + n\mu^2 - n\mu^2) =$$

$$\frac{1}{n}(n\sigma^2) = \sigma^2 * \frac{n}{n} = \sigma^2$$

$$E(S^2) = \sigma^2$$

Is an unbiased estimator for σ^2

b)

$$MSE(S^2) = Var(S^2) = \frac{2\sigma^4}{n-1} = \frac{2\sigma^4}{3-1} = \frac{1}{2}\sigma^4$$

Problem 4.

$$\lambda = \frac{1}{\theta}$$

$$\mu = E(Y_i) = \frac{1}{\lambda}$$

$$\sigma^2 = Var(Y_i) = \frac{1}{\lambda^2}$$

a)

$$E(\hat{\theta}_1) = E(Y_1) = \frac{1}{\lambda} = \theta$$

$$E(\hat{\theta}_2) = E\left(\frac{(Y_1 + 2 * Y_2)}{3}\right) = \frac{1}{3}\left(E(Y_1) + 2E(Y_2)\right) = \frac{1}{3}\left(\frac{1}{\lambda} + 2\frac{1}{\lambda}\right) = \frac{1}{3}(\theta + 2\theta) = \frac{3\theta}{3} = \theta$$

$$E(\hat{\theta}_3) = E(\bar{Y}) = E\left(\frac{\sum Y_i}{n}\right) = \frac{\sum E(Y_i)}{n} = \frac{\sum \frac{1}{\lambda}}{n} = \frac{\sum \theta}{n} = \frac{n\theta}{n} = \theta$$

b)

$$Var(\hat{\theta}_1) = Var(Y_1) = \frac{1}{\lambda^2} = \theta^2$$

$$Var(\hat{\theta}_2) = Var\left(\frac{(Y_1 + 2 * Y_2)}{3}\right) = \frac{1}{9}\left(Var(Y_1) + 4Var(Y_2)\right) = \frac{1}{9}(\theta^2 + 4\theta^2) = \frac{5\theta^2}{9}$$

$$Var(\hat{\theta}_3) = Var(\bar{Y}) = \frac{1}{n}\left(Var(Y_i)\right) = \frac{1}{n} * \frac{1}{\lambda^2} = \frac{1}{n}\theta^2 = \frac{\theta^2}{n} = \frac{\theta^2}{3}$$

$$eff(\hat{\theta}_1, \hat{\theta}_3) = \frac{Var(\hat{\theta}_3)}{Var(\hat{\theta}_1)} = \frac{\frac{\theta^2}{3}}{\theta^2} = \frac{1}{3}$$

$$eff(\hat{\theta}_2, \hat{\theta}_3) = \frac{Var(\hat{\theta}_3)}{Var(\hat{\theta}_2)} = \frac{\frac{\theta^2}{3}}{\frac{5\theta^2}{9}} = \frac{\frac{1}{3}}{\frac{5}{9}} = \frac{9}{15}$$

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