HW1

Problem 1.

Normally distributed with standard deviation of 4 square inches.

Forest sample n=9 trees.

P(Population Mean-2 < sample mean < Population Mean+2)=¿?

$$P(\,\mathrm{E(x)} - 2 < \frac{x_1 + x_2 + \cdots + x_9}{9} < \mathrm{E(x)} + 2)$$

$$P(\frac{(\mathrm{E(x)} - 2) - \mathrm{E(x)}}{SD(x)} < \frac{\frac{x_1 + x_2 + \cdots + x_9}{9} - \mathrm{E(x)}}{SD(x)} < \frac{(\mathrm{E(x)} + 2) - \mathrm{E(x)}}{SD(x)})$$

$$P\left(\frac{-2}{4/\sqrt{9}} < \frac{\frac{x_1 + x_2 + \cdots + x_9}{9} - \mathrm{E(x)}}{4/\sqrt{9}} < \frac{2}{4/\sqrt{9}}\right) =$$

$$= P\left(-1.5 < \frac{\frac{x_1 + x_2 + \cdots + x_9}{9} - \mathrm{E(x)}}{4/\sqrt{9}} < 1.5\right)$$

By the CLT

$$\int_{-1.5}^{1.5} \varphi(x)dx$$

One way is 1 minus the two tails

$$1 - \phi(-1.5) - (1 - \phi(1.5))$$

$$1 - 0.0668 - (1 - 0.9331) = 0.8663$$

The other option is to the ϕ(1.5) subtract the other tail

$$\phi(1.5) - \phi(-1.5) = 0.9331 - 0.0668 = 0.8663$$

So, the probability is 0.8663 for a sample size of 9

Problem 2.

P(Population Mean-1 < sample mean < Population Mean+1)=0.9

$$P\left(\,\mathrm{E(x)} - 1 < \frac{x_1 + x_2 + \cdots + x_n}{n} < \mathrm{E(x)} + 1\right) = 0.9$$

$$P\left(\frac{(\mathrm{E(x)}-1)-\mathrm{E(x)}}{SD(x)}<\frac{\frac{x_1+x_2+\cdots+x_n}{n}-\mathrm{E(x)}}{SD(x)}<\frac{(\mathrm{E(x)}+1)-\mathrm{E(x)}}{SD(x)}\right)=0.9$$

$$P\left(\frac{-1}{4/\sqrt{n}}<\frac{\frac{x_1+x_2+\cdots+x_n}{n}-\mathrm{E(x)}}{4/\sqrt{n}}<\frac{1}{4/\sqrt{n}}\right)=0.9$$

$$\phi\left(\frac{1}{4/\sqrt{n}}\right)-\phi\left(\frac{-1}{4/\sqrt{n}}\right)=0.9$$

$$\frac{1}{4/\sqrt{n}}-\frac{-1}{4/\sqrt{n}}=0.9 => \frac{1}{4/\sqrt{n}}+\frac{1}{4/\sqrt{n}}=0.9 => \frac{2}{4/\sqrt{n}}=0.9$$

$$\frac{2}{4/\sqrt{n}}=0.9 => 2=0.9*\frac{4}{\sqrt{n}} => \sqrt{n}=\frac{0.9*4}{2} => \sqrt{n}=\frac{0.9*4}{2} => \sqrt{n}=1.8 =>$$

$$n=\pm 3.24$$

For a probability of 0.9 we need 3.24 trees.

Problem 3.

a)

We can get the $\mathrm{E(x)} = \mathrm{n} * \mathrm{p}$

And the Standard Deviation as $\mathrm{SD(x)} = \sqrt{n*p*(1-p)}$

So, the mean is =1000*0.1=100

And the standard deviation is $\mathrm{SD(x)} = \sqrt{1000*0.1*(1-0.1)} = 9.4868$

$$P(x < 125) = P\left(\frac{\mathrm{x}-\mathrm{E(x)}}{SD(x)} \le \frac{125-\mathrm{E(x)}}{SD(x)}\right) = P\left(\frac{\mathrm{x}-\mathrm{E(x)}}{SD(x)} \le \frac{125-100}{9.4868}\right)$$

$$P\left(\frac{\mathrm{x}-\mathrm{E(x)}}{SD(x)} \le \frac{25}{9.4868}\right) = P\left(\frac{\mathrm{x}-\mathrm{E(x)}}{SD(x)} \le 2.6352\right)$$

$$\phi(2.6352) = 0.9957$$

The probability is 99.57%

b)

We have two different measurements: A and B, and we want the difference in the numbers of bit flip due to errors to be less than 50. So, we want the $P(-50 \le (A-B) \le 50)$

$$P(-50 \leq (A-B) \leq 50) = P\left(\frac{-50 - E(A-B)}{SD(A-B)} \leq \frac{(A-B) - E(A-B)}{SD(A-B)} \leq \frac{50 - E(A-B)}{SD(A-B)}\right)$$

$$P\left(\frac{-50-0}{9.4868} \leq \frac{(A-B) - E(A-B)}{SD(A-B)} \leq \frac{50-0}{9.4868}\right) = P(-15.81 \leq Z \leq -5.27)$$

$$\phi(-5.27) - \phi(-15.81\,) = (6.821188e - 08) - (1.327334e - 56) = 6.821188e - 08$$

$$P\left(\frac{-50-0}{\sqrt{2 * \sqrt{n * p * (1-p)}^2}} \leq \frac{(A-B) - E(A-B)}{SD(A-B)} \leq \frac{50-0}{\sqrt{2 * \sqrt{n * p * (1-p)}^2}}\right) =$$

$$P\left(\frac{-50-0}{\sqrt{2 * \sqrt{1000 * 0.1 * (1-0.1)}^2}} \leq \frac{(A-B) - E(A-B)}{SD(A-B)}\right.$$

$$\left. \leq \frac{50-0}{\sqrt{2 * \sqrt{1000 * 0.1 * (1-0.1)}^2}}\right) =$$

$$P\left(\frac{-50-0}{13.4164} \leq \frac{(A-B) - E(A-B)}{SD(A-B)} \leq \frac{50-0}{13.4164}\right) =$$

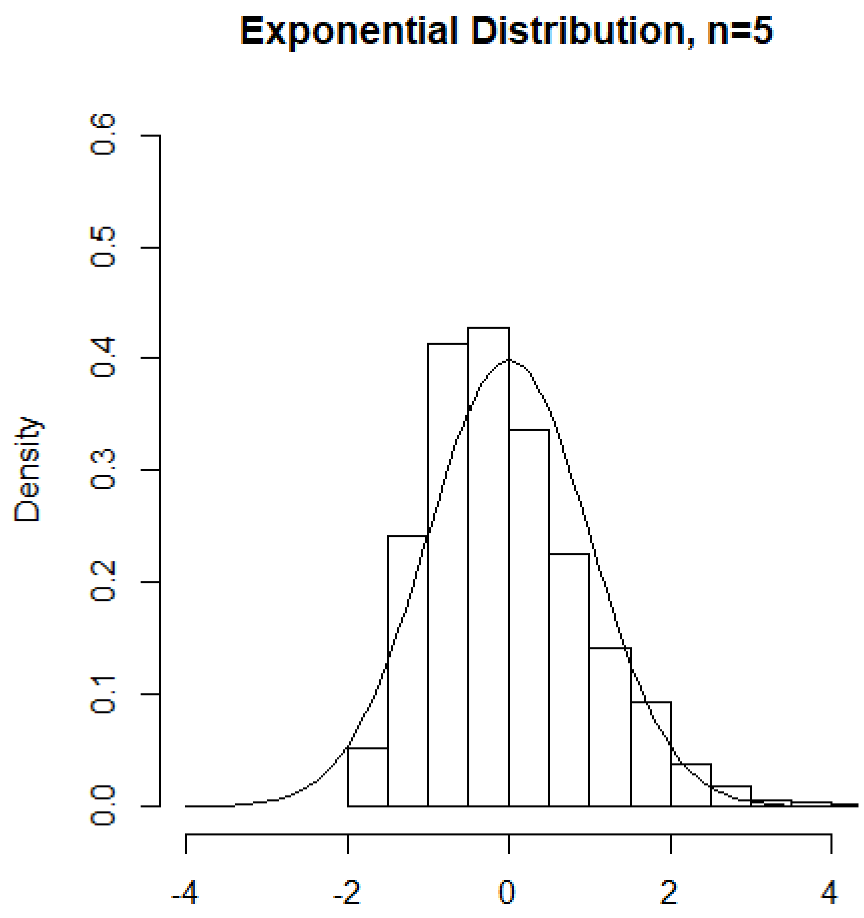
$$P\left(-3.7267 \leq \frac{(A-B) - E(A-B)}{SD(A-B)} \leq 3.7267\right)$$

$$\phi(3.7267) - \phi(-3.7267\,) = (0.99990) - (0.00009) = 0.99981$$

The probability is 99.98%

Problem 4.

n=5



n=10

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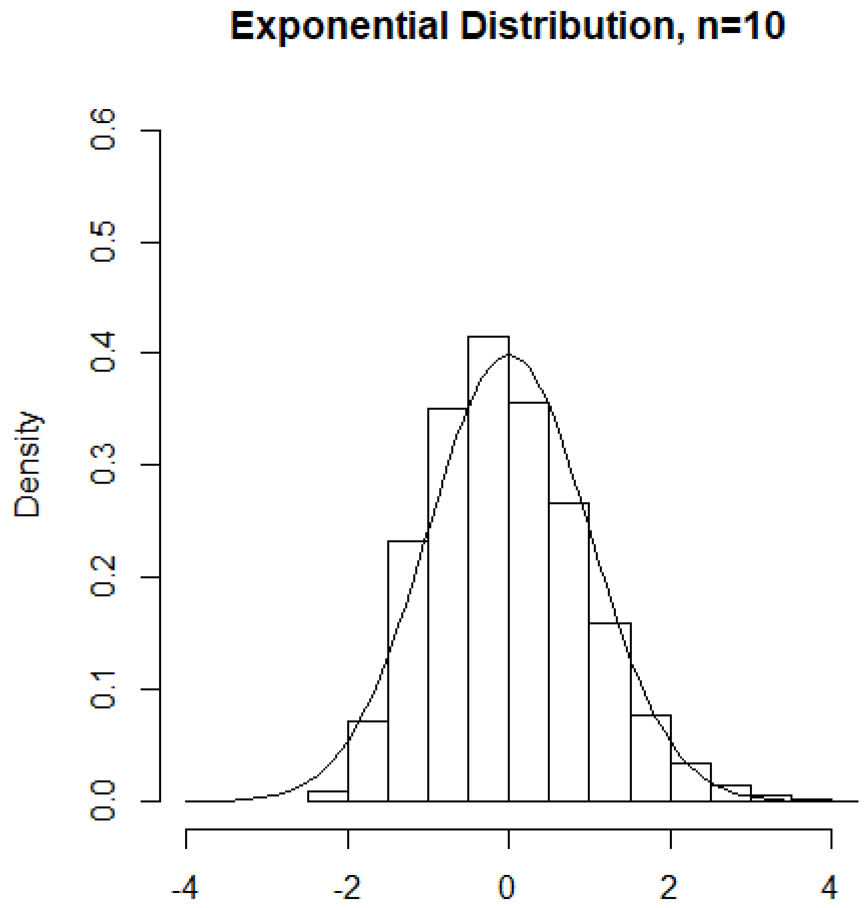
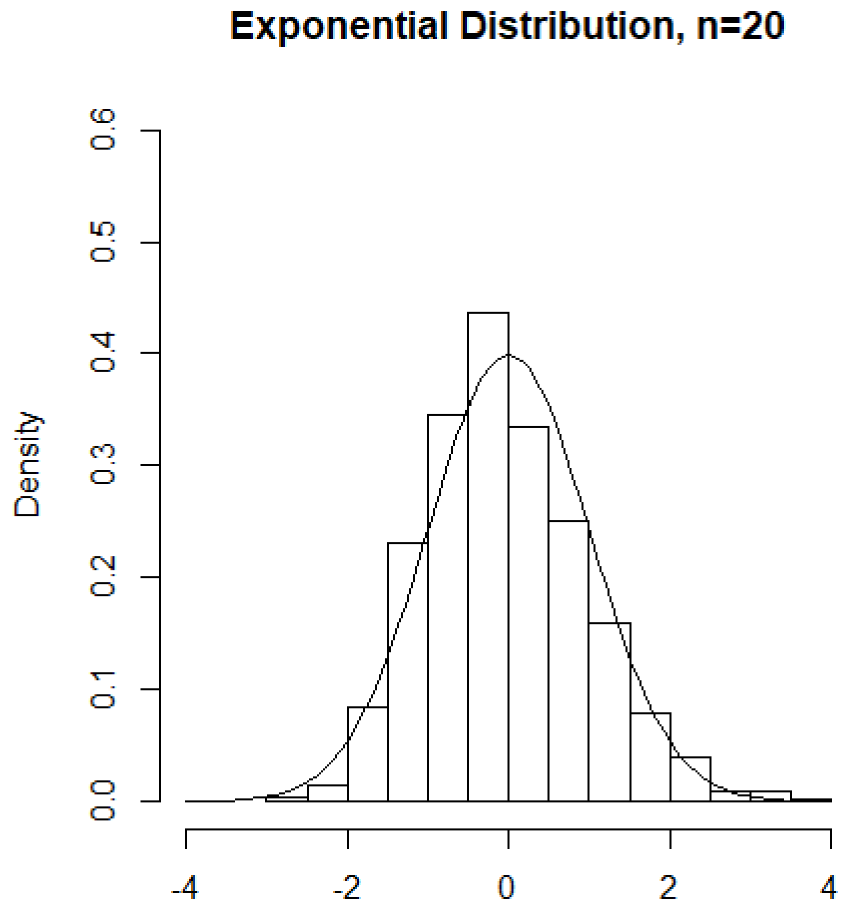

Exponential Distribution, n=10
Density
0.0
0.1
0.2
0.3
0.4
0.5
0.6
-4
-2
0
2
4

n=20



n=40

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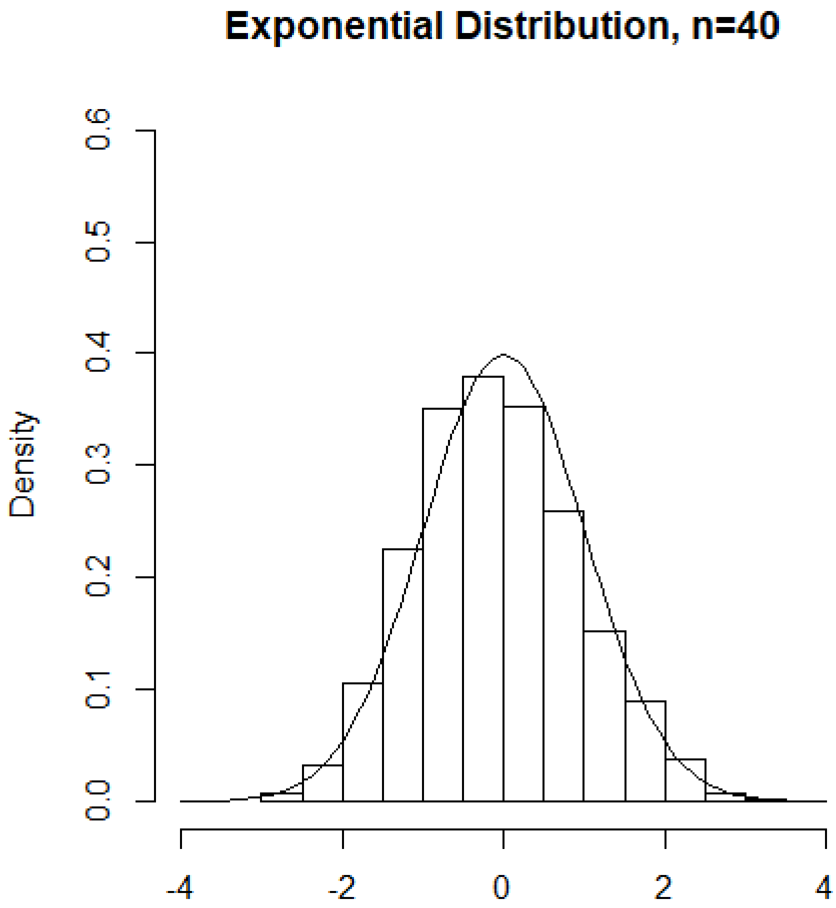

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