Special Topics Comp Stat & Pro MAT5999 and Computational Stats & Prob. AIM 5002
Written Assignment 9 (25 points)
4/12/22

Solutions to be returned by the beginning of class on Wednesday, 4/27.

1. The flow rate $y(m^3/min)$ in a device used for air-quality measurement depends on the pressure drop x (in. of water) across the device's filter. Suppose that for x values between 5 and 20, the two variables are related according to the simple linear regression model with true regression line $y = -.12 + .095x$.

   (a) **(2 points)** What is the expected change in flow rate associated with a 1-in. increase in pressure drop when the pressure drop is between 5 and 19? Explain.

   (b) **(3 points)** Assume that $\sigma = .025$ and consider a pressure drop of 10 in. What is the probability that the observed value of flow rate will exceed .835?

2. No-fines concrete, made from a uniformly graded coarse aggregate and a cement-water paste, is beneficial in areas prone to excessive rainfall because of its excellent drainage properties. The article "Pavement Thickness Design for No-Fines Concrete Parking Lots", J. of Trans. Engr., 1995: 476-484) employed a least squares analysis in studying how y =porosity (%) is related to x = unit weight (pcf) in concrete specimens. Consider the following representative data:

x	99.0	101.1	102.7	103.0	105.4	107.0	108.7
y	28.8	27.9	27.0	25.2	22.8	21.5	20.9

x	110.8	112.1	112.4	113.6	113.8	115.1	115.4	120.0
y	19.6	17.1	18.9	16.0	16.7	13.0	13.6	10.8

Relevant summary quantities are $\sum x_i = 1640.1$, $\sum y_i = 299.8$, $\sum x_i^2 = 179849.73$, $\sum x_i y_i = 32308.59$, $\sum y_i^2 = 6430.06$.

   (a) **(2 points)** Obtain the equation of the estimated regression line.

   (b) **(2 points)** Interpret the slope of the least squares line.

   (c) **(2 points)** Use the estimated line to predict porosity when unit weight is 100.

   (d) **(2 points)** Calculate and interpret a point estimate of σ.

   (e) **(2 points)** What proportion of observed variation in porosity can be attributed to the approximate linear relationship between unit weight and porosity?

   (f) **(3 points)** Calculate a confidence interval using confidence level 95% for the expected porosity of a random concrete specimen when $x = 120$.

   (g) **(2 points)** Compute a 95% prediction interval for the porosity of a random specimen with $x = 120$.

3. **(5 points)** You are given a set of data $(x_1, y_1), ..., (x_n, y_n)$ and you use the method of least squares to fit a straight line $\hat{\beta}_0 + \hat{\beta}_1 x$. You assume that your data comes from the linear model and so you estimate the variance of $\hat{\beta}_1$ by $\hat{\sigma}^2 / S_{xx}$ using the properties discussed on class.

   Now assume that someone wants to trick you and duplicates the data set, that is, gives you $(x_1, y_1) ..., (x_{2n}, y_{2n})$, where $(x_1, y_1), ..., (x_n, y_n)$ is exactly as before and $(x_{n+k}, y_{n+k}) = (x_k, y_k)$ for $k = 1, ..., n$. If you perform the same computations as before, how do your results change (that is how do $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}^2 / S_{xx}$ change)?