HW9

Problem 1.

$$y = -0.12 + 0.095x$$

a)

An increase of 1-in in the pressure drop, changes the y (flow rate) by the slope of the function, that in this case is 0.095.

So, the expected change in flow rate for a 1-in increase in pressure drop is 0.095

b)

$$\sigma = 0.025$$

The expected value when x=10:

$$y = -0.12 + 0.095 * 10 => y = 0.83$$

$$P(Y > 0.835) = P\left(\frac{y-\mu}{\sigma} > \frac{0.835 - 0.83}{0.025}\right) = P\left(\frac{y-\mu}{\sigma} > 0.2\right) = 0.4207$$

$$P(Y > 0.835) = 0.4207$$

Problem 2.

a)

$$S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

$$S_{xx} = 179849.73 - \frac{1640.1^2}{15} = 521.196$$

$$S_{xy} = \sum x_i * y_i - \frac{\sum x_i * \sum y_i}{n}$$

$$S_{xy} = 32308.59 - \frac{1640.1 * 299.8}{15} = -471.542$$

$$\widehat{\beta_1} = \frac{-471.542}{521.196} = -0.90473$$

$$\widehat{\beta_0} = \bar{y} - \widehat{\beta_1} * \bar{x} = \frac{299.8}{15} - (-0.90473) * \frac{1640.1}{15} = 118.9098$$

$$y = 118.9098 - 0.90473 * x$$

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b)

The slope is negative, meaning that there is an inverse relation between the porosity and the unit weight. And, since the slope is almost absolute 1, indicating and almost relation 1 to 1. This means that almost as one variable increases in 1, the other almost decreases in 1

c)

$$y = 118.9098 - 0.90473 * x$$

X=100

$$y = 118.9098 - 0.90473 * 100 = 28.4368$$

$$y = 28.436$$

d)

$$S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 6430.06 - \frac{(299.8)^2}{15} = 438.0573$$

$$SSE = S_{yy} - \widehat{\beta_1} * S_{xy}$$

$$SSE = 438.0573 - (-0.90473) * (-471.542) = 11.4391$$

$$\widehat{\sigma^2} = \frac{SSE}{n-2} = \frac{11.4391}{15-2} = 0.8799$$

$$\hat{\sigma} = 0.938$$

e)

f)

$$\widehat{\beta_0} + \widehat{\beta_1} * x^* \pm t_{\frac{\alpha}{2},n-2} * S_{\hat{y}}$$

$$t_{\frac{0.05}{2},15-2} = 2.160$$

$$S_{\hat{y}} = \hat{\sigma} * \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}} = 0.938 * \sqrt{\frac{1}{15} + \frac{\left(120 - \frac{1640.1}{15}\right)^2}{521.196}} = 0.938 * \sqrt{0.284695}$$

$$\widehat{\beta_0} + \widehat{\beta_1} * x^* \pm t_{\frac{\alpha}{2},n-2} * S_{\hat{y}} = 118.9098 - 0.90473 * 120 \pm 2.160 * 0.938 * \sqrt{0.284695} =$$

$$= 10.3422 \pm 1.081$$

$$CI = [9.2612, 11.4232]$$

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g)

$$\widehat{\beta_0} + \widehat{\beta_1} * x^* \pm t_{\frac{\alpha}{2},n-2} * \hat{\sigma} * \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$$

$$118.9098 - 0.90473 * 120 \pm 2.160 * 0.938 * \sqrt{1 + 0.284695} =$$

$$= 10.3422 \pm 1.13344$$

$$PI = [9.20876, 11.47564]$$

Problem 3.

Since we have duplicated each measure, we know that the new sum is twice the previous one

$$\sum_{i=1}^{2n} x_i = 2 * \sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{2n} y_i = 2 * \sum_{i=1}^{n} y_i$$

$$\sum_{i=1}^{2n} x_i * y_i = 2 * \sum_{i=1}^{n} x_i * y_i$$

$$\sum_{i=1}^{2n} x_i^2 = 2 * \sum_{i=1}^{n} x_i^2$$

$$new\widehat{\beta_1} = \frac{2 * \sum x_i * y_i - \frac{2 * \sum x_i * 2 * \sum y_i}{2 * n}}{2 * \sum x_i^2 - \frac{(2 * \sum x_i)^2}{2 * n}} = \frac{2 * \left(\sum x_i * y_i - \frac{\sum x_i * \sum y_i}{n}\right)}{2 * \left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)} =$$

$$= \frac{\sum x_i * y_i - \frac{\sum x_i * \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} = old\widehat{\beta_1}$$

We can see that $\widehat{\beta_1}$ has not change, this makes also $\widehat{\beta_0}$ the exact same as before. This means that $\widehat{\beta_0}$ and $\widehat{\beta_1}$ do not change if we duplicate the data set.

But what about $Var(\widehat{\beta_1})$

$$Var(\widehat{\beta_1}) = \frac{\sigma^2}{S_{xx}}$$

So, to know the change in the variance we need to know the σ^2 and S_{xx}

First the S_{xx}:

$$S_{xx} = 2 * \left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)$$

$$new\ S_{xx} = 2 * old\ S_{xx}$$

So, we can see the new Sxx is two times the old one

Now theσ^2 :

$$\sigma^2 = \frac{SSE}{n-2}$$

$$SSE = S_{yy} - \widehat{\beta_1} * S_{xy}$$

$$new\ S_{xy} = 2 * old\ S_{xy}$$

$$new\ S_{yy} = 2 * \sum y_i^2 - \frac{(2 * \sum y_i)^2}{2 * n} = 2 * \left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right) = 2 * old\ S_{yy}$$

$$new\ SSE = 2 * old\ S_{yy} - \widehat{\beta_1} * 2 * old\ S_{xy}$$

$$new\ SSE = 2 * \left(old\ S_{yy} - \widehat{\beta_1} * old\ S_{xy}\right) = 2 * old\ SSE$$

Now the σ^2

$$new\ \sigma^2 = \frac{2 * old\ SSE}{2 * n - 2} = \frac{old\ SSE}{n-1} = \frac{old\ SSE}{n-1}$$

$$new\ Var(\widehat{\beta_1}) = \frac{\frac{old\ SSE}{n-1}}{2 * old\ S_{xx}} = 2 * \frac{\frac{old\ SSE}{n-1}}{old\ S_{xx}} =$$

$$= 2 * \frac{\frac{old\ SSE}{n-1}}{old\ S_{xx}} = 2 * \frac{\frac{old\ SSE}{n-1} * \frac{n-2}{n-2}}{old\ S_{xx}} = 2 * \frac{\frac{old\ SSE}{n-2} * \frac{n-2}{n-1}}{old\ S_{xx}} = 2 * \frac{n-2}{n-1} * \frac{\frac{old\ SSE}{n-2}}{old\ S_{xx}} =$$

$$= 2 * \frac{n-2}{n-1} * old\ Var(\widehat{\beta_1})$$

We have seen that $\widehat{\beta_0}$ and $\widehat{\beta_1}$ do not change if we duplicate the data set. And the $Var(\widehat{\beta_1})$ changes by $2 * \frac{n-2}{n-1}$ times

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