

Special Topics Comp Stat & Pro MAT5999 and Computational Stats & Prob. AIM 5002
Written Assignment 7 (25 points)

3/24/22

Solutions to be returned by the beginning of class on Wednesday, 3/30. All problems are worth 5 points.

1. Two different companies have applied to provide cable television service in a certain region. Let p denote the proportion of all potential subscribers who favor the first company over the second. Consider testing $H_0 : p = 0.5$ versus $H_a : p \neq 0.5$ based on a random sample of 25 individuals. Let X denote the number in the sample who favor the first company and x represent the observed value of X .

Assume a test with rejection region

$$\{x : x \leq 7 \text{ or } x \geq 18\}$$

is performed. Find the type I error probability. Find the type II error probability in case $p = 0.3$.

2. A spectrophotometer used for measuring CO concentration [ppm (parts per million) by volume] is checked for accuracy by taking readings on a manufactured gas (called span gas) in which the CO concentration is very precisely controlled at 70 ppm. If the readings suggest that the spectrophotometer is not working properly, it will have to be recalibrated. Assume that if it is properly calibrated, measured concentration for span gas samples is normally distributed. On the basis of the six readings - 85, 77, 82, 68, 72 and 69 - is recalibration necessary? Carry out a test of the relevant hypotheses and report the P-value
3. For a normal distribution with mean μ and variance $\sigma^2 = 25$, an experimenter wishes to test $H_0 : \mu = 10$ versus $H_a : \mu = 5$. Find the sample size n for which the most powerful test will have $\alpha = \beta = .025$.
4. An experiment published in The American Biology Teacher studied the efficacy of using 95% ethanol and 20% bleach as disinfectants for removing bacterial and fungal contamination when culturing plant tissues. The experiment was repeated 15 times with each disinfectant, using eggplant as the plant tissue cultured (Source: Michael Brehm, J. Buguliskis, D. Hawkins, E. Lee, D. Sabapathi, and R. Smith, 'Determining Differences in Efficacy of Two Disinfectants Using t tests'; The American Biology Teacher 58(2), (1996): 111.). Five cuttings per plant were placed on a petri dish, disinfected using each agent, and stored at 25C for 4 weeks. The observations reported were the number of uncontaminated eggplant cuttings after the 4 weeks of storage. Relevant data is given in the following table. Are you willing to assume that the underlying population variances are equal? Report the P-value of your test

Disinfectant	95% Ethanol	20% bleach
Sample Mean	3.73	4.80
Sample Variance	2.78095	0.17143
n	15	15

5. A city expressway with four lanes in each direction was studied to see whether drivers preferred to drive on the inside lanes. A total of 1000 automobiles were observed during the heavy early-morning traffic, and their respective lanes were recorded. The results are: 294 cars used lane 1, 276 cars used lane 2, 238 cars used lane 3, 192 cars used lane 4. Do the data present sufficient evidence to indicate that some lanes are preferred over others? Test the hypothesis that $p_1 = p_2 = p_3 = p_4 = 1/4$, using $\alpha = .05$. Also report the P-value.