Special Topics Comp Stat & Pro MAT5999 and Computational Stats & Prob. AIM 5002
Written Assignment 5 (25 points)
3/15/22

Solutions to be returned by the beginning of lecture on Wednesday, 3/23/20.

1. **(3 points)** Water samples are taken from water used for cooling as it is being discharged from a power plant into a river. It has been determined that as long as the mean temperature of the discharged water is at most 150F, there will be no negative effects on the river's ecosystem. To investigate whether the plant is in compliance with regulations that prohibit a mean discharge water temperature above 150, 50 water samples will be taken at randomly selected times and the temperature of each sample recorded. The resulting data will be used to test the hypotheses $H_0 : \mu = 150$ versus $H_a : \mu > 150$. In the context of this situation, describe type I and type II errors. Which type of error would you consider more serious? Explain.

2. **(3 points)** The calibration of a scale is to be checked by weighing a 10kg test specimen 25 times. Suppose that the results of different weighings are independent of one another and that the weight on each trial is normally distributed with $\sigma = .200kg$. Let μ denote the true average weight reading on the scale. If $H_0 : \mu = 10$ is tested against $H_a : \mu \neq 10$ and the rejection region

$$\{\bar{x} : \bar{x} \geq 10.1004 \text{ or } \bar{x} \leq 9.8940\} = \{z : z \geq 2.51 \text{ or } z \leq -2.65\}$$

is used, then what is α for this procedure?

3. **(4 points)** A large survey of 1000 students found that freshmen at public universities work on average 12.6 hours/week with a sample standard deviation of 10.5. Does it appear that the true average of working hours is more than 12 hours/week? Test your hypothesis at significance level $\alpha = 0.05$. With your rejection region, what is the probability of making type II error when the true average working hours is 13?

4. (**3 points**) An experiment to compare the tension bond strength of polymer latex modified mortar (Portland cement mortar to which polymer latex emulsions have been added during mixing) to that of unmodified mortar resulted in $\bar{x} = 18.12kgf/cm^2$ for the modified mortar ($m = 40$) and $\bar{y} = 16.87kgf/cm^2$ for the unmodified mortar ($n = 32$). Let μ_1 and μ_2 be the true average tension bond strengths for the modified and unmodified mortars, respectively. Assume that the bond strength distributions are both normal. Assuming that $\sigma_1 = 1.6$ and $\sigma_2 = 1.4$, test $H_0 : \mu_1 - \mu_2 = 0$ versus $H_a : \mu_1 - \mu_2 > 0$ at level .01.

5. (**7 points**) Go to CANVAS Files/RLabs/RLab4 and download Ttest.Rmd

   The function tTestSim computes type I and II error probabilities for lower tail t-test **by simulation**. Complete the code by adding parts for upper-tail and two-sided tests. Test your code on some examples. Turn in your code and examples as a pdf file that you generated from the R markdown file.

6. **(5 points)** A random sample $x_1, ..., x_n$ is given from a normal population with unknown σ. To test

$$H_0 : \sigma^2 = \sigma_0^2 \text{ against } H_a : \sigma^2 > \sigma_0^2,$$

we use the decision rule: reject H_0 at significance level α if $\frac{(n-1)S^2}{\sigma_0^2} \geq \chi^2_{\alpha,n-1}$ (here $\chi^2_{\alpha,n-1}$ is the number where the cdf of the chi-squared distribution with $n-1$ degrees of freedom attains the value $1-\alpha$). Prove that the probability of making a type I error using the above decision rule is α