Special Topics Comp Stat & Pro MAT5999 and Computational Stats & Prob. AIM 5002
Written Assignment 1 (25 points)
1/26/22

Solutions to be submitted on Canvas by the beginning of class on Wednesday, 2/2/22.

1. **(5 points)** A forester studying the effects of fertilization on certain pine forests in the Southeast is interested in estimating the average basal area of pine trees. In studying basal areas of similar trees for many years, he has discovered that these measurements (in square inches) are normally distributed with standard deviation approximately 4 square inches. If the forester samples $n = 9$ trees, find the probability that the sample mean will be within 2 square inches of the population mean.

2. **(5 points)** Suppose the forester in Exercise 1 would like the sample mean to be within 1 square inch of the population mean, with probability .9. How many trees must he measure in order to ensure this degree of accuracy?

3. **(5 points)** A binary communication channel transmits a sequence of "bits" (0s and 1s). Suppose that for any particular bit transmitted, there is a 10% chance of a transmission error (a 0 becoming a 1 or a 1 becoming a 0). Assume that bit errors occur independently of one another.

   (a) Consider transmitting 1000 bits. What is the approximate probability that at most 125 transmission errors occur?

   (b) Suppose the same 1000-bit message is sent two different times independently of one another. What is the approximate probability that the number of errors in the first transmission is within 50 of the number of errors in the second?

4. **(10 points)** Exercise in R.

   Write a code in a script file named RlabHW1.R that generates a sample from the exponential distribution with parameter $\lambda = 2$, $N = 2000$ times. For each sample, compute the sample mean $\bar{X}$ and create a histogram. Do this simulation for different sample sizes $n = 5; 10; 20; 40$. Turn in the script file and your 4 histograms electronically (one possibility is to save the histograms as a pdf file as we did on class).

   Hint: You can recycle the piece of code in lines 83 - 94 in the file CLT.R that we discussed on 1/26. Use **rexp** to generate exponential random numbers (type in **help(rexp)** to see the help file if you wish). Do not forget to use the proper standardization: the exponential distribution with parameter λ has mean $1/\lambda$ and variance $1/\lambda^2$. In the notation of R, λ is called rate (see the help file).

The following problems form the extra homework. They will not contribute to your final grade and are only included for your entertainment.

5. Compute

$$\lim_{n\to\infty} \sum_{k=0}^{n} \frac{n^k}{k!} e^{-n}.$$

Hint: Try to use the central limit theorem for poisson distribution.