Homework 3

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2/9/2022

Solutions to be submitted on Canvas by the beginning of class on Wednesday, 2/16/22.

(1) The amount of lateral expansion (mils) was determined for a sample of $n = 9$ pulsed-power gas metal arc welds used in LNG ship containment tanks. The resulting sample standard deviation was $s = 2.81$ mils. Assuming normality,

- (a) (**4 points**) derive a 99% CI for σ^2 and for σ. *Hint:* to construct the CI for σ, take the square root of the endpoints of the CI for σ^2.
- (b) (**2 points**) give a 95% upper confidence bound for σ^2.

(2) (**4 points**) A random sample of size $m = 10$ from a normal distribution with unknown expectation μ_1 and variance σ^2 gives $\bar{x} = 15.3$, $s_1 = 2.43$. Another random sample size $n = 15$ from another normal distribution with unknown expectation μ_2 and variance σ^2 (the same σ^2 as before!) gives $\bar{y} = 14.8$, $s_2 = 3.17$. Give a 99% confidence interval for $\mu_1 - \mu_2$.

(3) (**15 points**) Write a function **VarianceCI** in R that does the following.

- (a) takes the number of simulations **N**, the sample size **n**, the parameters **mean** and **sdev**, level of confidence **alpha** as inputs,
- (b) generates random samples of size **n** from the normal distribution with parameters **mean** and **sdev**,
- (c) compute $(100 - alpha)$% CI for the variance $sdev^2$ (you may want to use **qchisq**, the quantile function of the chi-squared distribution),
- (d) repeat (b)-(c) **N** times,
- (e) compute the successful coverage proportion, that is the number of times the CI actually contains $sdev^2$, divided by **N**
- (f) draw a plot for coverage using matplot as we did in the file TDistrCofidenceIntervals.R last week.

Finally, run this function with **mean** = 0 and **sdev** = 1 and with both **N** =10, **N** =100, for both **n** = 5, **n** = 100 (that is, 4 cases in total). Discuss your finding.

Submit your code as well as the resulting plots and discussions in an R Markdown file.

The following problems form the extra homework. They will not contribute to your final grade.

(5) Show that the pdf of the chi-squared distribution with n degrees of freedom is

$$\frac{x^{n/2-1}e^{-x/2}}{2^{n/2}\Gamma(n/2)}$$

for $x > 0$ and 0 for $x \leq 0$. Hint: for $n = 1$ use the transformation of pdf formula that is usually discussed in a first year probability theory class (be careful: you have to distinguish two cases: whether your normal random variable is positive or negative). For $n \geq 1$, use an induction and the convolution formula (also usually discussed in a first year probability class).