Special Topics Comp Stat & Pro MAT5999 and Computational Stats & Prob. AIM 5002
Written Assignment 5 (25 points)
2/23/22

Solutions to be submitted on Canvas by the beginning of class on Wednesday, 3/2/22.

1. **(5 points)** Let $Y_1, ..., Y_n$ be a random sample from normal distribution with unknown μ and known σ. Show that $\bar{Y}$ is a sufficient statistic for μ.

2. **(5 points)** The Pareto distribution of parameters $\alpha, \beta > 0$ is defined by the pdf

$$f(y) = \begin{cases} \alpha\beta^{\alpha} y^{-(\alpha+1)}, & \text{if } y \geq \beta \\ 0 & \text{otherwise} \end{cases}$$

   (The Pareto distribution is often used in economics to model income distributions.) If β is known, show that $\prod_{i=1}^{n} Y_i$ is a sufficient statistic for α.

3. **(5 + 5 points)** Let X denote the proportion of allotted time that a randomly selected student spends working on a certain aptitude test. Suppose the pdf of X is

$$f_\theta(x) = (\theta + 1)x^{\theta} \quad \text{for } 0 < x < 1 \text{ and 0 otherwise,}$$

   where $\theta > -1$ is an unknown parameter. A random sample of 10 students yields data $x_1 = .92, x_2 = .79, x_3 = .90, x_4 = .65, x_5 = .86, x_6 = .47, x_7 = .73, x_8 = .97, x_9 = .84, x_{10} = .67$.

   (a) Use the method of moments to obtain an estimator of θ, and then compute the estimate for this sample.

   (b) Compute the maximum likelihood estimator of θ, and then compute the estimate for this data.

4. **(5 points)** Let $X_1, ..., X_{10}$ be a random sample from a uniform distribution on the interval $[0, \theta]$. Find the likelihood function $L(x_1, ..., x_{10}; \theta)$. Hint: you may wish to distinguish two cases: when $\theta < \max\{x_1, ..., x_{10}\}$ and when $\theta \geq \max\{x_1, ..., x_{10}\}$. Show that the maximum likelihood estimator of θ is $\max\{X_1, ..., X_{10}\}$.

The following problems form the extra homework. They will not contribute to your final grade and are only included for your entertainment.

5. Problem 9.65 from textbook # 1.