Chapter 6: ODEs
Due date:

A correct answer without proper explanation will not receive full credit.

6.1 IVPs

3 Use separation of variables to find solutions of the IVP given by $y(0) = 1$ and the following differential equations:

(a) $y' = t$

(b) $y' = 2(t + 1)y$

(c) $1/y^2$

For which of the IVPs in the Exercise 3 does the advanced theorem of Existence and Uniqueness of Solutions guarantee a unique solution?

(a) Find the Lipschitz constants if they exist.

(b) What is the largest interval $[0, b]$ for which the solutions exist?

7 (a) Show that $y = \tan(t + c)$ is a solution of the differential equation $y' = 1 + y^2$ for each c.

(b) For each real number y_0, find c in the interval $(-\pi/2, \pi/2)$ such that the initial value problem $y' = 1 + y^2$, $y(0) = y_0$ has a solution $y = \tan(t + c)$.

Computer Problems. Use the code snippet provided on Canvas

1. Apply Euler's Method with step size $h = 0.1$ on $[0, 1]$ to the initial value problems in Exercise 3. Print a table of the t values, Euler approximations, and error (difference from exact solution) at each step.

2. Plot the Euler's Method approximate solutions for the IVPs in Exercise 3 on $[0, 1]$ for step sizes $h = 0.1, 0.05$, and 0.025, along with the exact solution.

4 For the IVPs in Exercise 3, make a log–log plot of the error of Euler's Method at $t = 1$ as a function of $h = 0.1 \times 2^{-k}$ for $0 \leq k \leq 5$.

7 Plot the Euler's Method approximate solution on $[0, 1]$ for the differential equation $y' = 1 + y^2$ and initial condition $y_0 = 1$, along with the exact solution (see Exercise 7). Use step sizes $h = 0.1$ and 0.05.

6.3 Systems of ODEs

Computer Problems

8 Implement the double pendulum, given in Example 6.17, using Taylor's method. Use $d = 0$, i.e. no friction at the pivot. Can you find an initial condition that gives rise to "sustained non-periodicity"? By this we mean a solution that is not periodic, i.e. does not go back to the initial configuration.

10 Adapt `orbit.m` to solve the three-body problem using Taylor's method. (See equation 6.45) Set the masses to $m_1 = 0.3$, $m_2 = m_3 = 0.03$.

(a) Plot the trajectories with initial conditions

$$\begin{aligned} (x_1, y_1) &= (2, 2), & (x_1', y_1') &= (0.2, -0.2), \\ (x_2, y_2) &= (0, 0), & (x_2', y_2') &= (0, 0), \\ (x_3, y_3) &= (-2, -2), & (x_3', y_3') &= (-0.2, 0.2). \end{aligned}$$

(b) Change the initial condition of x_1' to 0.20001, and compare the resulting trajectories. This is a striking visual example of sensitive dependence.