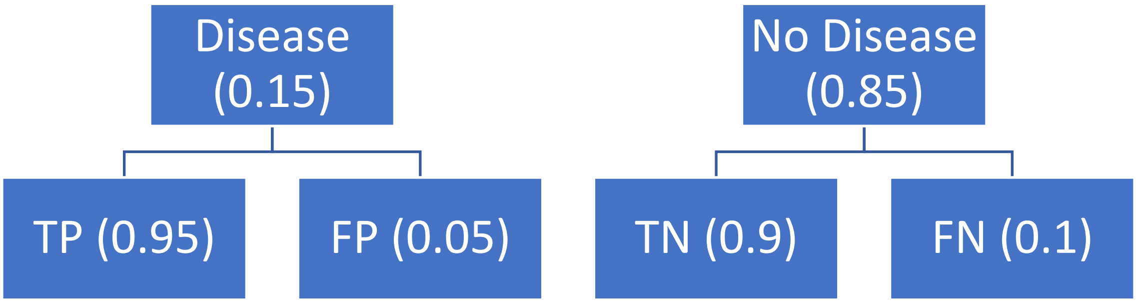
Communication Task 2.2

From the information that we are given 15% of people that take the test have the disease. This means that for the people taking the test the initial probability of having the disease is 15% in other words, P(Disease) = 0.15.

According to the information given by the test company the sensitivity is 95%, that means that of the people with the disease that take the test it identifies 95% of them as having the disease, P(TP|Disease) = 0.95. (TP stands for True Positives)

Also, if we look at the specificity the company says that it is 90%, that means that of the people without the disease that take the test, what is the percentage of them that get a negative result, P(TN|NotDisease) = 0.9. (TN stands for True Negatives)

It is easier to see this information in a graph



The director of the company said that the probability of someone that test positive having the disease is 63%. The first thing we need to do is to check if that 63% is correct or not.

Question 1.

To check if that number is correct, we need to know what we want to calculate. We want to know wats the Probability of having the disease if we have tested positive. In other words, P(Disease|TP).

For that we can apply Bayes rule:

$$\mathrm{P\ (Disease|\ TP)} = \frac{\mathrm{P\ (TP\ |\ Disease)} * \mathrm{P\ (Disease)}}{\mathrm{P(TP)}} = \frac{0.95 * 0.15}{0.95 * 0.15 + 0.1 * 0.85} = 0.6263$$

So, the probability is 62.63%, so if we round the number, we get 63%, so in fact the Director was correct about her claim.

Question 2.

In this case we want to know what value the Specificity will need to be in order to achieve a P(Disease|TP) of at least 80%

In this case we will apply the same formula, but we will be solving for the Specificity, I will call it 'Sp':

$$\mathrm{P\ (Disease|\ TP)} = \frac{\mathrm{P\ (TP\ |\ Disease)} * \mathrm{P\ (Disease)}}{\mathrm{P(TP)}}$$

$$0.80 \leq \frac{0.95 * 0.15}{0.95 * 0.15 + (1 - Sp) * 0.85} => 0.80 \leq \frac{0.1425}{0.1425 + (1 - Sp) * 0.85} =>$$

$$0.80 * (0.1425 + (1 - Sp) * 0.85) \leq 0.1425 => 0.114 + (1 - Sp) * 0.68 \leq 0.1425 =>$$

$$(1 - Sp) * 0.68 \leq 0.0285 => 1 - Sp \leq \frac{0.0285}{0.68} => -Sp \leq \frac{0.0285}{0.68} - 1 =>$$

$$Sp \geq -\left(\frac{0.0285}{0.68} - 1\right) => Sp \geq 0.9580$$

In this case the Specificity needs to be above or equal to 95.80%

Question 3.

As for the previous question we want to make the P(Disease|TP) of at least 80%, but in this case, we want to modify the Sensitivity instead of the Specificity.

We apply the same formula but with a different incognita, in this case the Sensitivity, I will call it 'Se':

$$\mathrm{P\ (Disease|\ TP)} = \frac{\mathrm{P\ (TP\ |\ Disease)} * \mathrm{P\ (Disease)}}{\mathrm{P(TP)}}$$

$$0.80 \leq \frac{Se * 0.15}{Se * 0.15 + 0.1 * 0.85} => 0.80 \leq \frac{Se * 0.15}{Se * 0.15 + 0.085} =>$$

$$0.80 * (Se * 0.15 + 0.085) \leq Se * 0.15 => 0.12 * Se + 0.068 \leq Se * 0.15 =>$$

$$0.068 \leq Se * 0.15 \ - 0.12 * Se => 0.068 \leq Se * 0.03 => Se \geq \frac{0.068}{0.03} =>$$

$$Se \geq 2.266$$

It will need to be 226.66% which is over 100% making it impossible to achieve this kind of Sensitivity

Question 4.

In this question we are asked to figure out what's the probability of the disease from the screening of the patients. To make the probability of having the disease given a positive in the test to be 80%.

I will call this incognita 'Sc' that stands for Screening of patients

$$\text{P (Disease| TP)} = \frac{\text{P (TP | Disease)} * \text{P (Disease)}}{\text{P(TP)}}$$

$$0.80 \leq \frac{0.95 * Sc}{0.95 * Sc + 0.1 * (1 - Sc)} => 0.80 \leq \frac{0.95 * Sc}{0.95 * Sc + 0.1 - 0.1 * Sc} =>$$

$$0.80 \leq \frac{0.95 * Sc}{0.85 * Sc + 0.1} => 0.80 * (0.85 * Sc + 0.1) \leq 0.95 * Sc =>$$

$$0.68 * Sc + 0.08 \leq 0.95 * Sc => 0.08 \leq 0.95 * Sc - 0.68 * Sc =>$$

$$0.08 \leq 0.27 * Sc => Sc \geq \frac{0.08}{0.27} => Sc \geq 0.29629$$

For a chance of 80% of having the disease given a positive result. The previous chance of having the disease for the people taking the test needs to be at least 29.63%