Problem Set 4.1

Exercise 4.1

a) Beta (1.8,1.8)
   centering π on 0.5

b) Beta (3,2)
   somewhat favoring π > 0.5

c) Beta (1,10)
   strongly favoring π < 0.5

d) Beta (1,3)
   somewhat favoring π < 0.5

e) Beta (17,2)
   strongly favoring π > 0.5

Exercise 4.7

For example, we can see the influence of the prior and the data on the posterior mean:

$$E(\pi|Y=y) = \frac{\alpha + y}{\alpha + \beta + n}$$

$$E(\pi|Y=y) = \frac{\alpha + \beta}{\alpha + \beta + n} * E(\pi) + \frac{n}{\alpha + \beta + n} * \frac{y}{n}$$

We can use the component as weights for the importance of the prior and the data:

For the prior we can use:

$$\frac{\alpha + \beta}{\alpha + \beta + n}$$

For the data we can use:

$$\frac{n}{\alpha + \beta + n}$$

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a) Prior: π ~ Beta (1,4), data: Y=8 successes in n=10 trials

$$Prior\ W: \frac{\alpha+\beta}{\alpha+\beta+n} = \frac{1+4}{1+4+10} = 0.333$$

For the weight of the data, we can use:

$$Data\ W: \frac{n}{\alpha+\beta+n} = \frac{10}{1+4+10} = 0.667$$

Or we can use 1-PriorW=1-0.333=0.667

We can say that in this case the data has more influence on the posterior.

b) Prior: π ~ Beta (20,3), data: Y=0 successes in n=1 trials

$$Prior\ W: \frac{\alpha+\beta}{\alpha+\beta+n} = \frac{20+3}{20+3+1} = 0.9583$$

$$Data\ W: \frac{n}{\alpha+\beta+n} = \frac{1}{20+3+1} = 0.0417$$

We can say that in this case the prior has more influence on the posterior.

c) Prior: π ~ Beta (4,2), data: Y=1 successes in n=3 trials

$$Prior\ W: \frac{\alpha+\beta}{\alpha+\beta+n} = \frac{4+2}{4+2+3} = 0.667$$

$$Data\ W: \frac{n}{\alpha+\beta+n} = \frac{3}{4+2+3} = 0.333$$

We can say that in this case the prior has more influence on the posterior.

Although in this case if we evaluate the mode:

$$Mode(\pi|Y=y) = \frac{\alpha+\beta-2}{\alpha+\beta+n-2} * Mode(\pi) + \frac{n}{\alpha+\beta+n-2} * \frac{y}{n}$$

$$\frac{\alpha+\beta-2}{\alpha+\beta+n-2} = \frac{4+2-2}{4+2+3-2} = 0.5714$$

$$\frac{n}{\alpha+\beta+n-2} = \frac{3}{4+2+3-2} = 0.4286$$

If we look at this the prior still have more influence. But the difference between the prior and data is not so big, meaning that they both contribute to the posterior in almost equal proportion.

d) Prior: π ~ Beta (3,10), data: Y=10 successes in n=13 trials

$$Prior\ W: \frac{\alpha+\beta}{\alpha+\beta+n} = \frac{3+10}{3+10+13} = 0.5$$

$$Data\ W: \frac{n}{\alpha+\beta+n} = \frac{13}{3+10+13} = 0.5$$

We can say that the posterior is an equal compromise between the data and the prior.

e) Prior: π ~ Beta (20,2), data: Y=10 successes in n=200 trials

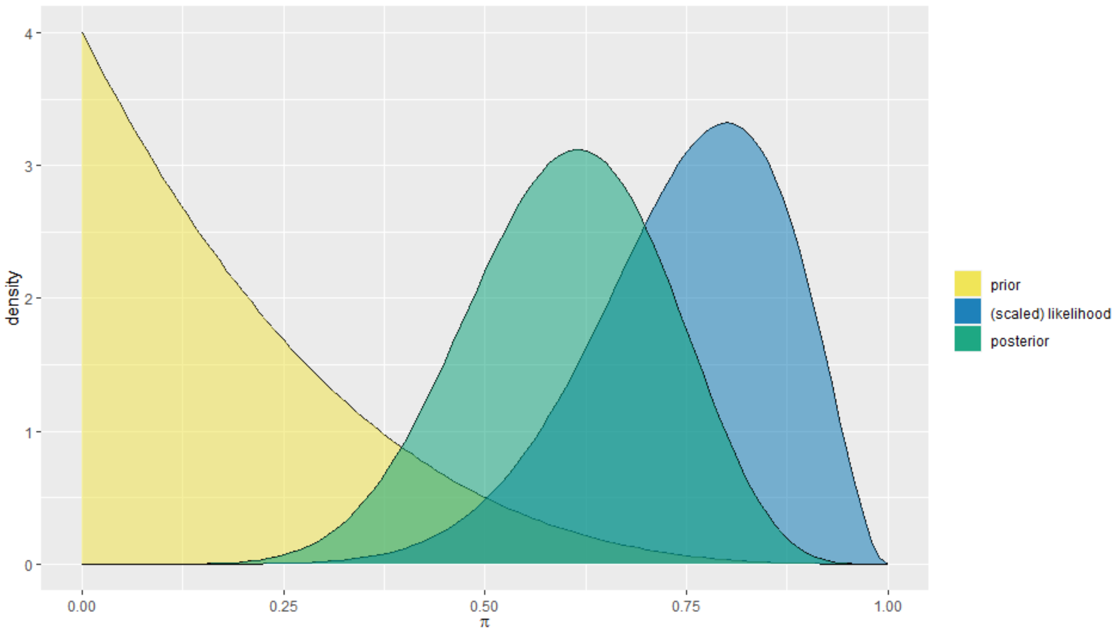
$$Prior\ W: \frac{\alpha+\beta}{\alpha+\beta+n} = \frac{20+2}{20+2+200} = 0.099$$

$$Data\ W: \frac{n}{\alpha+\beta+n} = \frac{200}{20+2+200} = 0.901$$

We can say that in this case the data has more influence on the posterior.

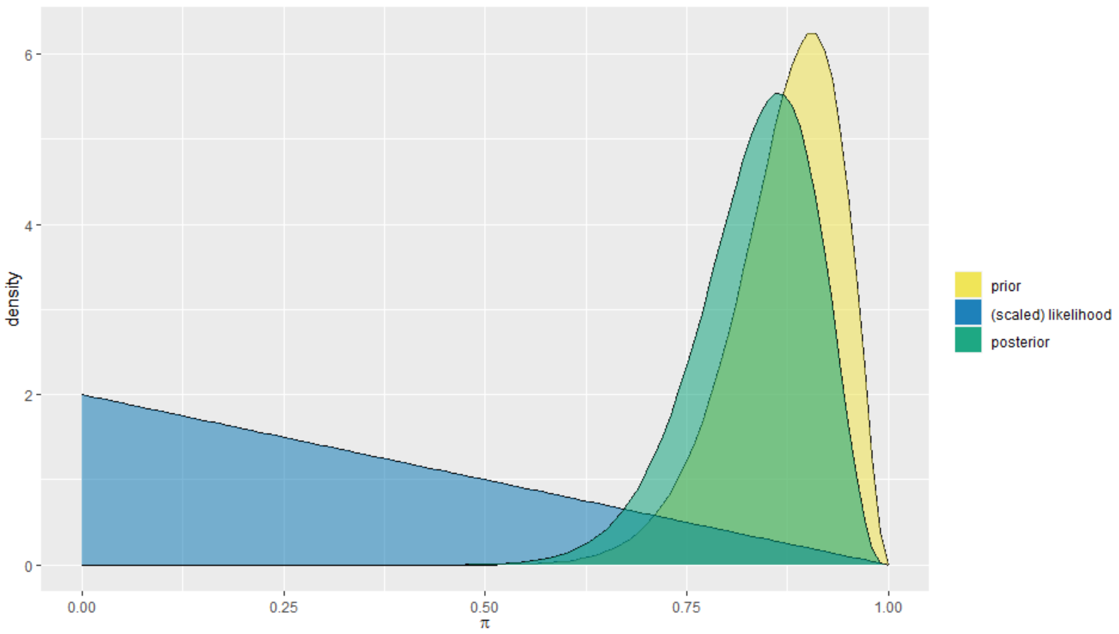
Exercise 4.8

a) Prior: π ~ Beta (1,4), data: Y=8 successes in n=10 trials



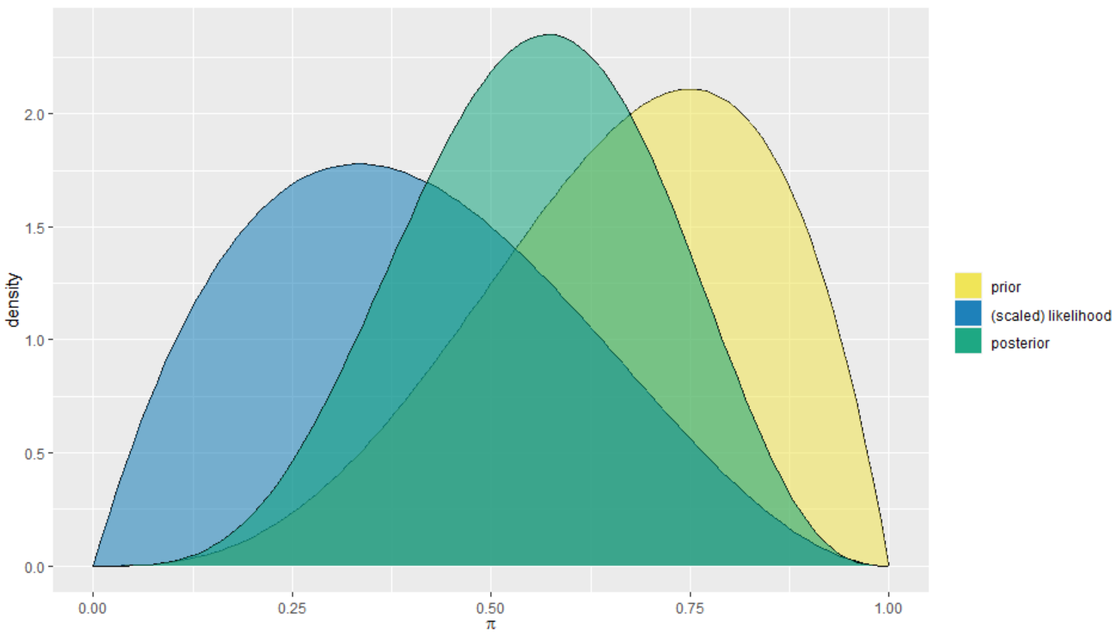
We can see the data (in blue) has more influence on the posterior (green) than prior (yellow)

b) Prior: π ~ Beta (20,3), data: Y=0 successes in n=1 trials



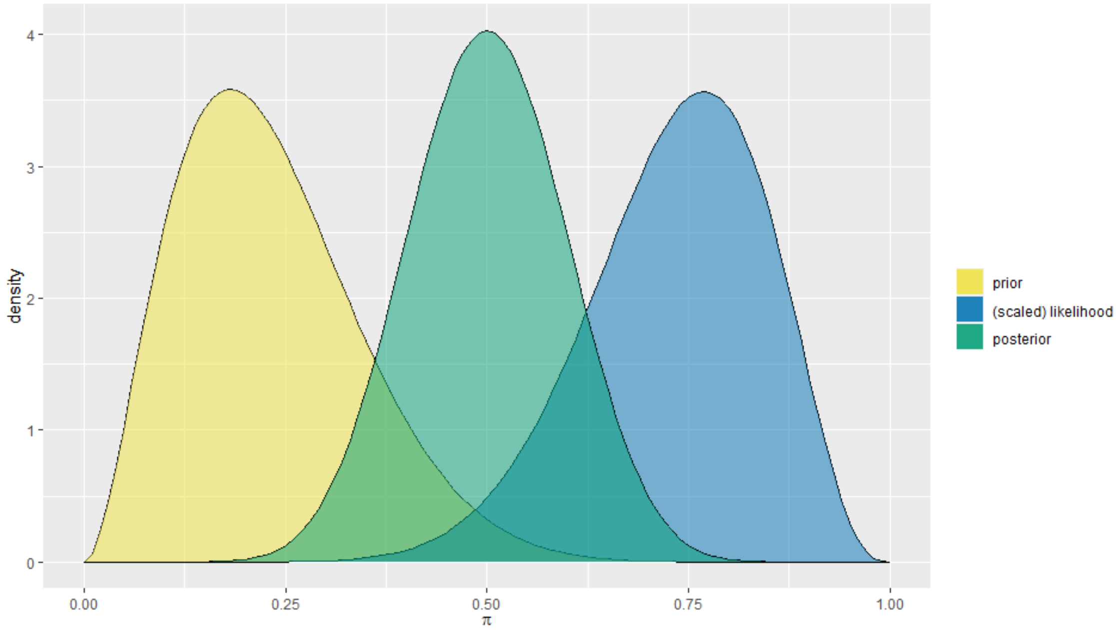
In this case we can see that it's the prior the one that have more influence on the posterior. This case is even greater the influence of one of the sources, as the posterior is almost equal to the prior.

c) Prior: π ~ Beta (4,2), data: Y=1 successes in n=3 trials



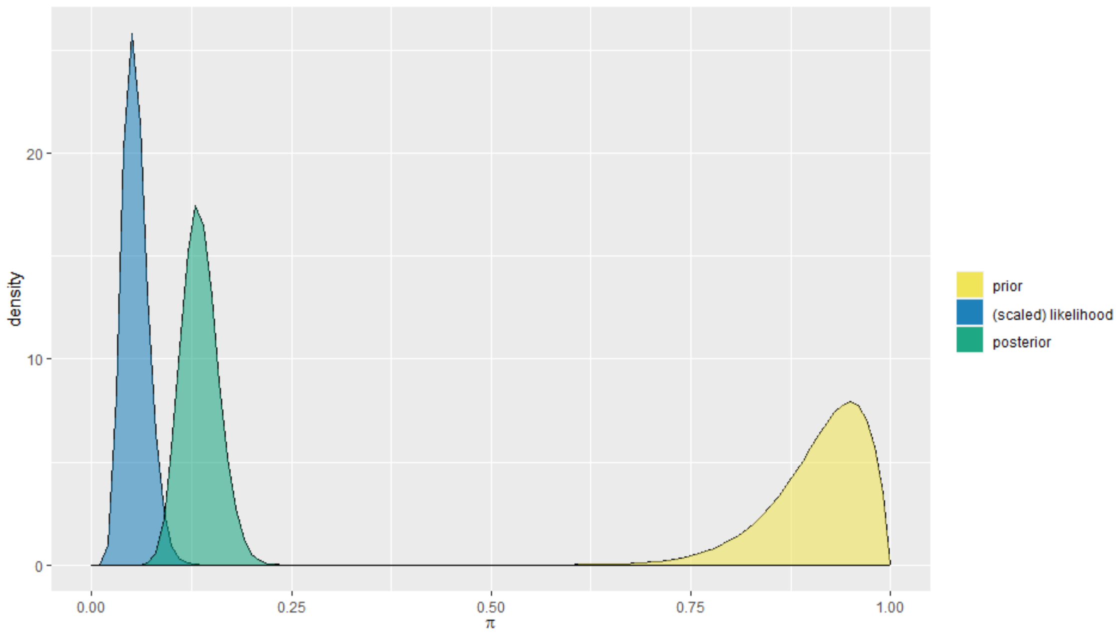
In this case the prior has more influence than the data, but the difference is not so big, meaning that the posterior is somewhat in the middle of both the prior and data.

d) Prior: π ~ Beta (3,10), data: Y=10 successes in n=13 trials



In this case both the prior and the data contributed equally to the posterior, as it sits in the middle of both.

e) Prior: π ~ Beta (20,2), data: Y=10 successes in n=200 trials



This is the most extreme case, where we can see the posterior mostly influence by the data.

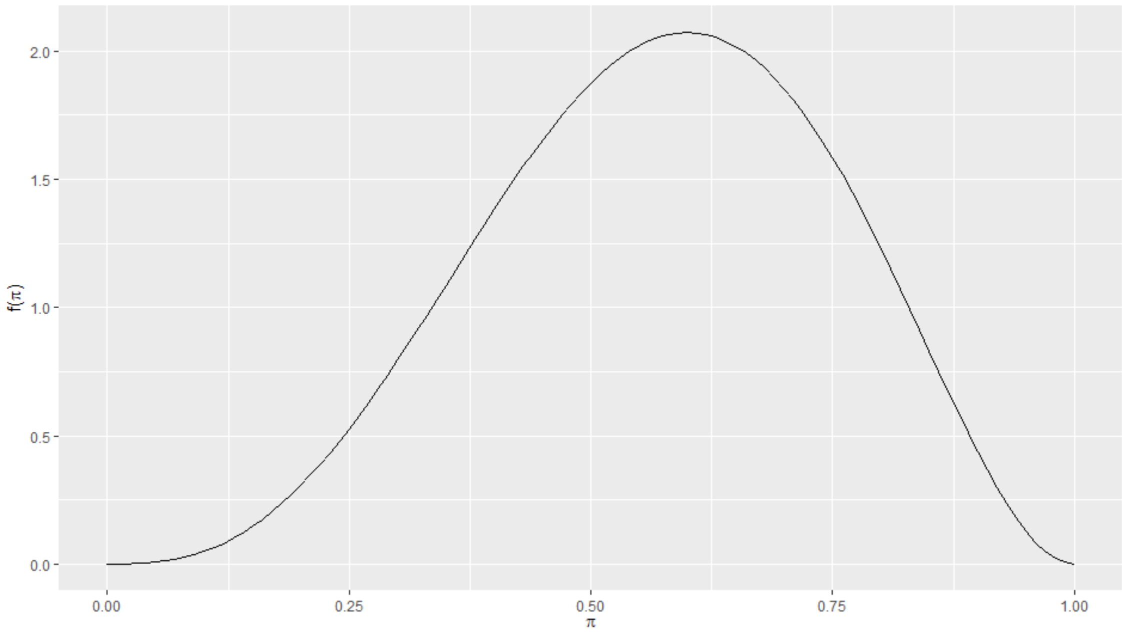
Exercise 4.17

$$\pi \sim \text{Beta}(4,3)$$

3 different studies:

y=0, n=1 | y=3, n=10 | y=20, n=100

a)



The employees prior understanding is, that the bulk is around 0.6 and most of the values are between 0.25 and 0.85, meaning that they are not entirely sure about the results.

b)

First employee: y=0, n=1

$$\text{Beta}(4,4)$$

Second employee: y=3, n=10

$$\text{Beta}(7,10)$$

First employee: y=20, n=100

$$\text{Beta}(24,83)$$

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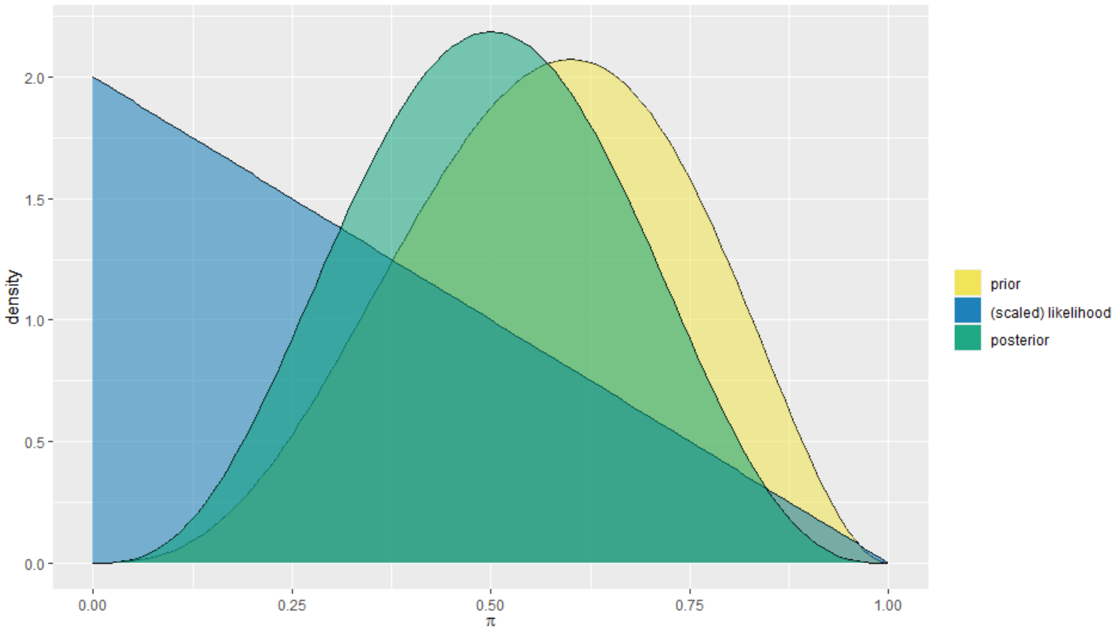
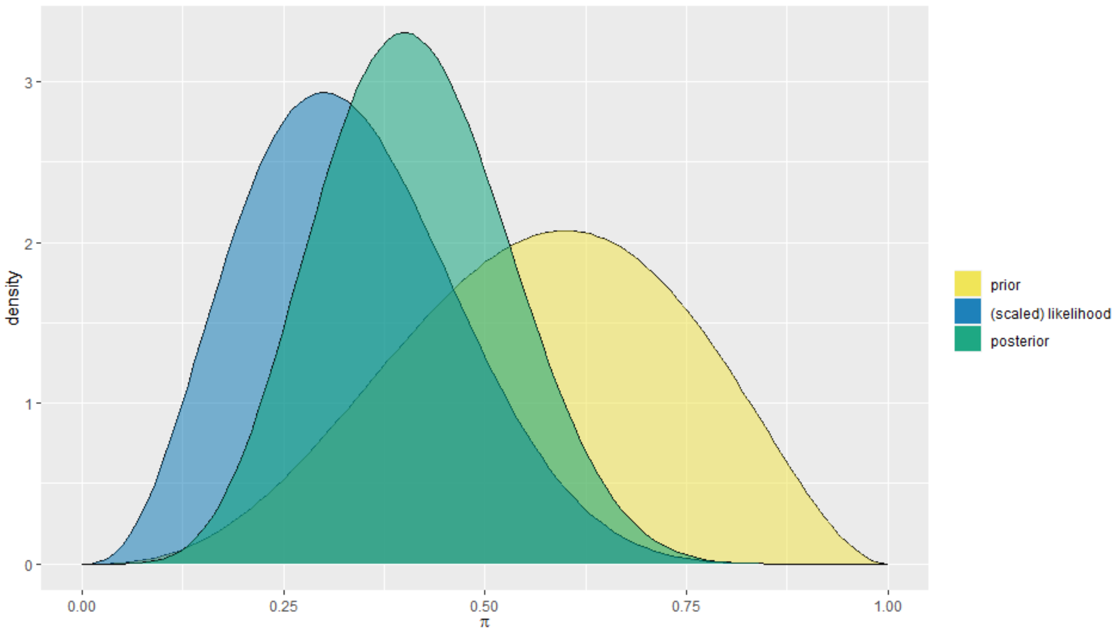
c)

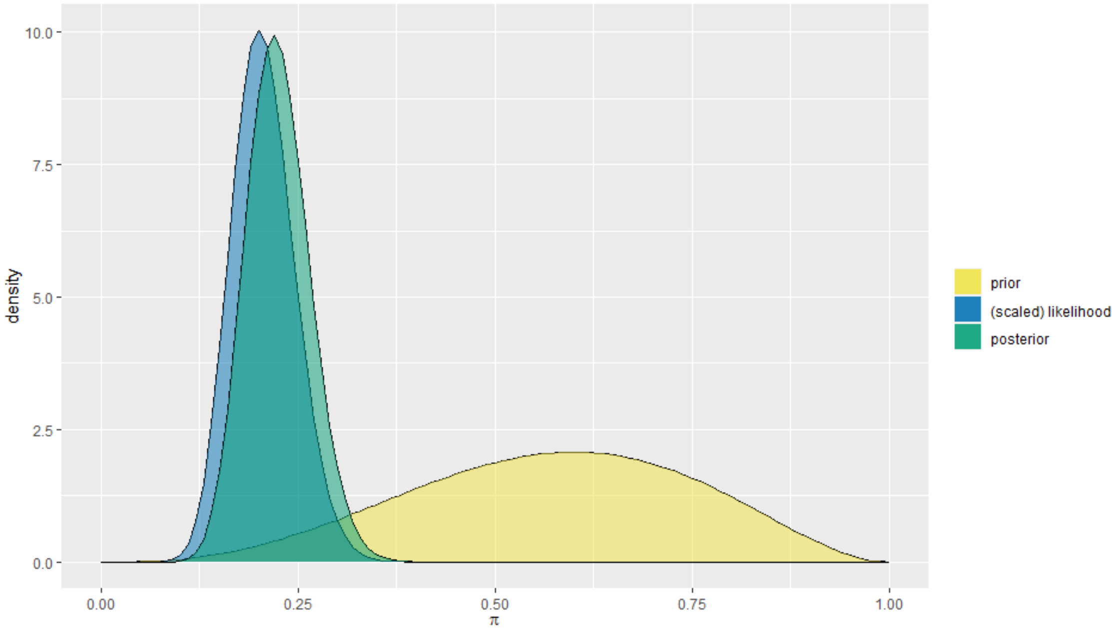
First employee: y=0, n=1



Second employee: y=3, n=10



Third employee: y=20, n=100



d)

As the amount of data collected increase the less importance the prior has. For the first employee the data collected was of only 1 subject, and thus the posterior closely resembles the prior. In the case of the third employee the opposite happens, it has data on 100 subjects, and thus the posterior is almost equal to the data, meaning the influence of the prior is almost non-existent. The model of the second employee fall in between the other extremes, and the posterior is somewhat in between the prior and the data.

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