Problem Set 1.2

Problem 1.

3 letter/number code for each employee

a) How many people for it to be a 25% chance of two employees having the same codename?

26 letters and 10 numbers

$$36^3 = 46656$$

$$P(no\ pair\ has\ same\ code) = \frac{46656!}{46656^n * (46656-n)!} = \frac{n!\binom{46656}{n}}{46656^n}$$

Since we want 25% of probability of a matching pair:

$$1 - \frac{n!\binom{46656}{n}}{46656^n} = 0.25$$

n=164 => 0.2493

n=165 => 0.2519

n=166 => 0.2546

We need 165 people

b) How would the answer change if there are 4 characters?

$$36^4 = 1679616$$

$$1 - \frac{n!\binom{1679616}{n}}{1679616^n} = 0.25$$

n=983 =>0.2498

n=984 =>0.2502

n=985 =>0.2506

We need 984 people

c) Number of people for you to be pretty sure that two have the same code (3-character code)

Pretty sure=95%

n=528 =>0.94988

n=529=>0.95045

n=530=>0.95101

We need 529 people.

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Problem 2.

When you choose (in this case door number two), there is a 0.33 chance of getting the car, while the other two doors have a probability of 0.33 each. Once the show host opens (in this case door number 3 with a goat), the probability of that door goes to the other door (in this case number 1) since we haven't updated our choice. Now when he asks us if we want to change door its beneficial for us to change it since the door number 1 have a 0.66 change of having the car, while our initial door have a change of 0.33.

Another way to see it is as follows:

The first time we choose a door, in this case number 2. There are two possibilities, either the car is in our door, or it isn't. The probability of it being in our door is 1/3 or 0.33, while the probability of the car not being in our door is 2/3 or 0.66.

So now when the show host opens the other door, in this case door number 3, it is guaranteed to be a goat (otherwise the game will sometimes break since the car gets discarded).

The same probability from before still applies, where our initial door had a probability of 1/3 of having the car and a probability of 2/3 of not having the car. Now that we can change door, and there is only ONE other door left, the chances of that door are the opposite of our initial door, a probability of (1-1/3) of having the car and a probability of (1-2/3) of not having the car. And thus, this other door has a 0.66 chance of having the car and a 0.33 change of not having the car.

With this we can conclude that it is better to chance doors.

Problem 3.

a) Calculate the probability that both childs are girls (Mr. Martin)

(boy, boy), (boy, girl), (girl, boy), (girl, girl) => 4 possibilities

In this case the older child is a girl, and we are asked whats is the probability that both are girls.

P(Oldest is a girl): (boy, girl), (girl, girl) => P(Oldest is a girl)=2/4

P(Both are girl): (girl, girl) => P(Both are girl)=1/4

$$\mathrm{P(Both\ are\ girl} \mid \mathrm{Oldest\ is\ a\ girl)} = \frac{\mathrm{P\ (Both\ are\ girl\ \cap\ Oldest\ is\ a\ girl)}}{\mathrm{P(Oldest\ is\ a\ girl)}} = \frac{1/4}{2/4}$$

$$\mathrm{P(Both\ are\ girl} \mid \mathrm{Oldest\ is\ a\ girl)} = \frac{1}{2} = 0.5$$

b) Calculate the probability of both childs are girls (Mrs Gardner)

(boy, boy), (boy, girl), (girl, boy), (girl, girl) => 4 possibilities

In this case we know that at least 1 child is a girl.

P(At least 1 girl): (boy, girl), (girl, boy), (girl, girl) => P(At least 1 girl)=3/4

P(Both are girl): (girl, girl) => P(Both are girl)=1/4

$$P(\text{Both are girl} \mid \text{At least 1 girl}) = \frac{P\ (\text{Both are girl}\ \cap\ \text{At least 1 girl})}{P(\text{At least 1 girl})} = \frac{1/4}{3/4}$$

$$P(\text{Both are girl} \mid \text{At least 1 girl}) = \frac{1}{3} = 0.33$$

c) Identify a different approach to the problem

The original space of possibilities is 4 [(boy, boy), (boy, girl), (girl, boy), (girl, girl)] but since we have some restrictions the space of possibilities gets reduced, in one case to 2 possibilities and in the other to 3.

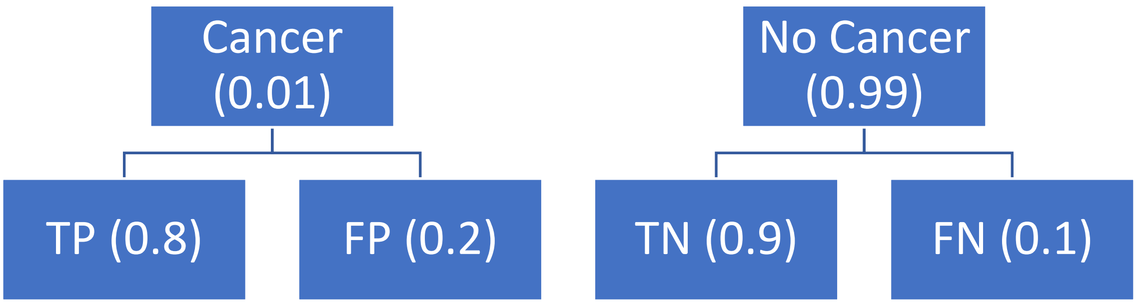
Original	boy, boy	boy, girl	girl, boy	girl, girl	4/4
Mr. Martin	X	boy, girl	X	girl, girl	2/4
Mrs Gardner	X	boy, girl	girl, boy	girl, girl	3/4
Both girls	X	X	X	girl, girl	1/4

Because of this we are looking for only 1 possible option out of 2 (in the case of Mr. Martin) and out of 3 (in the case of Mrs Gardner) the probabilities change for both persons.

Problem 4.

Your test → 0.01 Cancer | 0.99 No Cancer

Mammography → Cancer(TP =0.8 | FP=0.2) | No Cancer(TP =0.9 | FP=0.1)



$$\mathrm{P\ (Cancer\ |\ TP)} = \frac{\mathrm{P\ (TP\ |\ Cancer) * P\ (Cancer)}}{\mathrm{P(TP)}} = \frac{0.8 * 0.01}{0.8 * 0.01 + 0.1 * 0.99} = 0.747$$

New Probability base on new information is 7.47%

Problem 5.

30% of flights depart in Morning | 30% in Afternoon | 40% in evening

15% of all flights are delayed

40% of delayed flights are in Morning | 50% in afternoon | 10% in evening

P(Morning) =0.3

P(Afternoon) =0.3

P(Evening) =0.4

P(Delayed) =0.15

P (Morning | Delayed) =0.4

P (Afternoon | Delayed) =0.5

P (Evening |Delayed) =0.1

a) Mine flight is in the Morning. What's the prob of her flight being delayed?

$$\mathrm{P\ (Delayed\ |\ Morning)} = \frac{\mathrm{P\ (Morning\ |\ Delayed) * P(Delayed)}}{\mathrm{P(Morning)}} = \frac{0.4 * 0.15}{0.3} = 0.2$$

$$\mathrm{P\ (Delayed\ |\ Morning)} = 0.2$$

The probability of her flight being delayed is 0.2

b) Alicia Flight is not delayed. What's the prob of her flight being in the morning?

$$\mathrm{P\ (Morning\ |\ NotDelayed)} = \frac{\mathrm{P\ (Morning\ \cap NotDelayed)}}{\mathrm{P(NotDelayed)}}$$

$$\mathrm{P\ (Morning\ \cap NotDelayed))} = \mathrm{P(Morning\ )} - \mathrm{P\ (Morning|Delayed) * P(Delayed)}$$

$$\mathrm{P\ (Morning\ \cap NotDelayed)} = 0.3 - 0.4 * 0.15$$

$$\mathrm{P\ (Morning\ \cap NotDelayed)} = 0.24$$

	Delayed	NotDelayed	Total
Morning	(0.4*0.15)=0.06	(0.3-0.06)=0.24	0.3
Afternoon	(0.5*0.15)=0.075	(0.3-0.075)=0.225	0.3
Evening	(0.1*0.15)=0.015	(0.4-0.015)=0.385	0.4
Total	0.15	0.85	1

$$P\ (\text{Morning} \mid \text{NotDelayed}) = \frac{P\ (\text{Morning}\ \cap \text{NotDelayed})}{P(\text{NotDelayed})} = \frac{0.24}{0.85} = 0.282$$

$$P\ (\text{Morning} \mid \text{NotDelayed}) = 0.282$$

The probability of her flight being in the morning is 0.282