Yeshiva University

AIM 5009 Bayesian Statistics
Problem Set 1.1

Instructions

This problem set reviews some of the key concepts from the Computational Statistics and Probability course that are used in Bayesian Statistics. This assignment focuses on core concepts around likelihood.

Complete the following problems. Your full solutions should be typed or neatly handwritten and should be submitted as a PDF file by the due date given in Canvas.

Problems

1. In this problem, we review how to find the expected value, variance, and percentiles of a distribution based on its probability density function (PDF). Consider the following PDF:

$$f(x) = \frac{1}{4}e^{-x/4}, \qquad x \geq 0$$

   a. Derive the expected value $E(X)$ from the PDF above.
   b. Derive the variance $\sigma^2(X)$ from the PDF above.
   c. Determine the 5th and 95th percentiles of the PDF above.
   d. Determine the median of the PDF above. Comment on how it compares to the mean.

2. In this problem, we review how to find the maximum likelihood estimator for a probability distribution function. Consider the exponential distribution (familiar from the previous problem above):

$$f(x) = \frac{1}{\lambda}e^{-x/\lambda}, \qquad x \geq 0$$

   a. Write the likelihood function for this distribution assuming a sample size of n.
   b. Write the log-likelihood function for this distribution assuming a sample size of n.
   c. Find the maximum likelihood estimator $\hat{\lambda}$ for this distribution.
   d. Suppose a sample of size $n = 9$ was drawn. The resulting sample observations are given by $X = \{3, 3, 7, 11, 12, 18, 22, 34, 41\}$. What is the maximum likelihood estimate of λ?

3. Here is another problem to review the use of likelihood functions via maximum likelihood estimation. Consider the following distribution:

$$f(x) = \frac{\alpha}{x^{\alpha+1}}, \qquad x \geq 1$$

   a. Verify this is a probability density function by integrating it with respect to x.
   b. Write the likelihood function for this distribution assuming a sample size of n.
   c. Write the log-likelihood function for this distribution assuming a sample size of n.
   d. Find the maximum likelihood estimator $\hat{\alpha}$ for this distribution.

4. Finally, we consider a case where calculus is not useful (or necessary) for finding the maximum likelihood estimate of a parameter. Consider a uniform distribution:

$$f(x) = \frac{1}{\theta}, \qquad 0 \leq x \leq \theta$$

   a. Verify this is a probability density function by integrating it with respect to x.
   b. What is the likelihood function for this distribution assuming a sample size of n?
   c. We are trying to identify the most likely value of θ given a sample of data. If we have a sample of $X = \{2, 7, 19, 24\}$, what do you think we should choose as our maximum likelihood estimate $\hat{\theta}$? Why?

Grading Rubric

For this assignment, the following rubric will be used:

Correctness of Answers	0 – 6
Clarity and Organization of Presentation	0 – 4