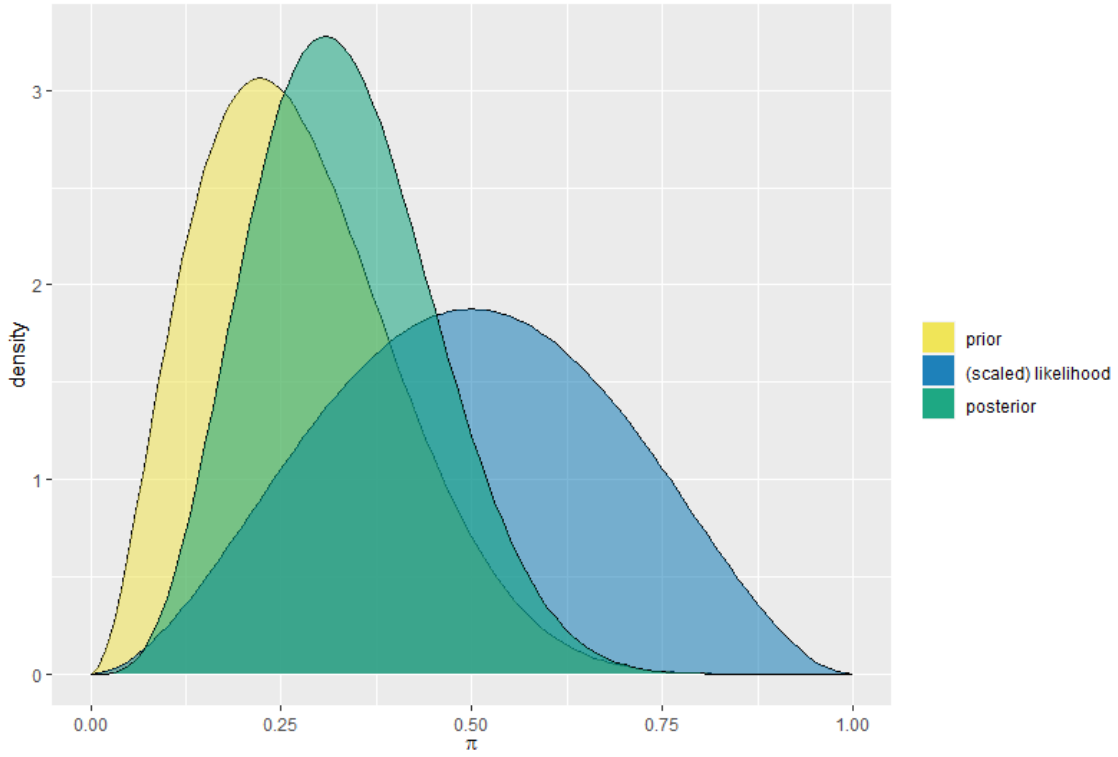
Problem Set 4.2

Exercise 4.2

Solution is: e) alpha = 3, beta = 8, y = 2, n = 4



Exercise 4.10

$$\pi|(Y = y) \sim \text{Beta}(\alpha + y, \beta + n - y)$$

$$\alpha_{prior} + y = \alpha_{post}$$

$$\beta_{prior} + n - y = \beta_{post}$$

a) Prior: Beta (0.5, 0.5), Posterior: Beta (8.5, 2.5)

$$0.5 + y = 8.5$$

$$0.5 + n - y = 2.5$$

$$y = 8$$

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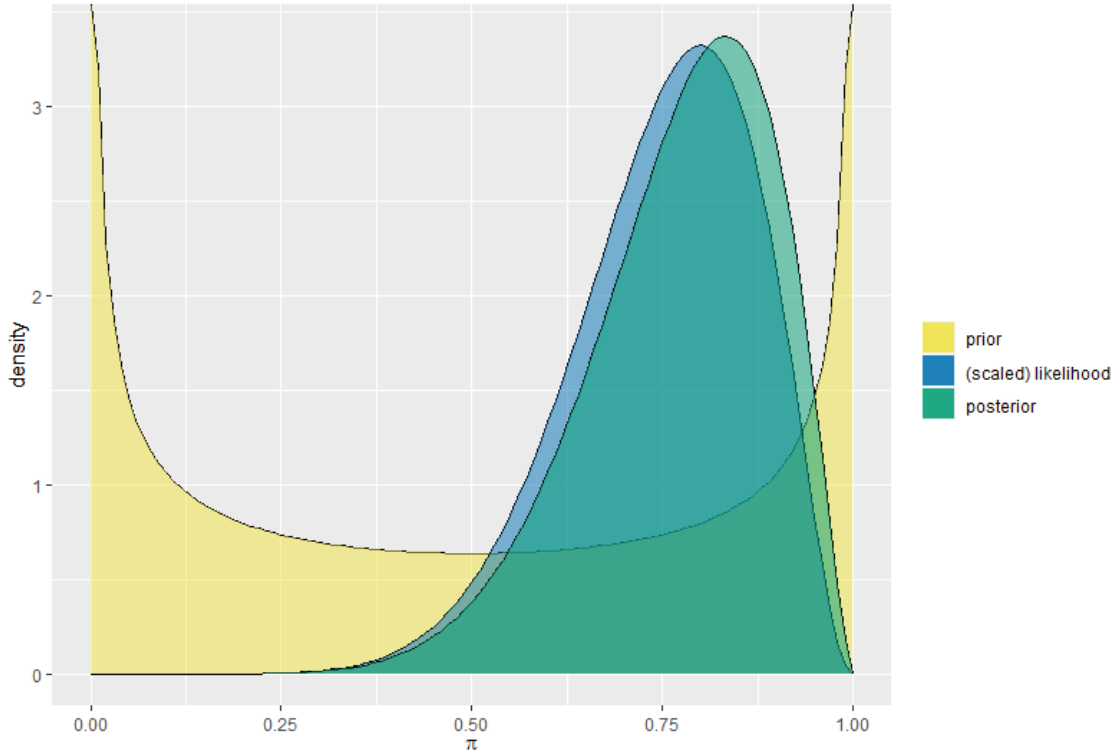
$$0.5 + n - 8 = 2.5 => n = 10$$



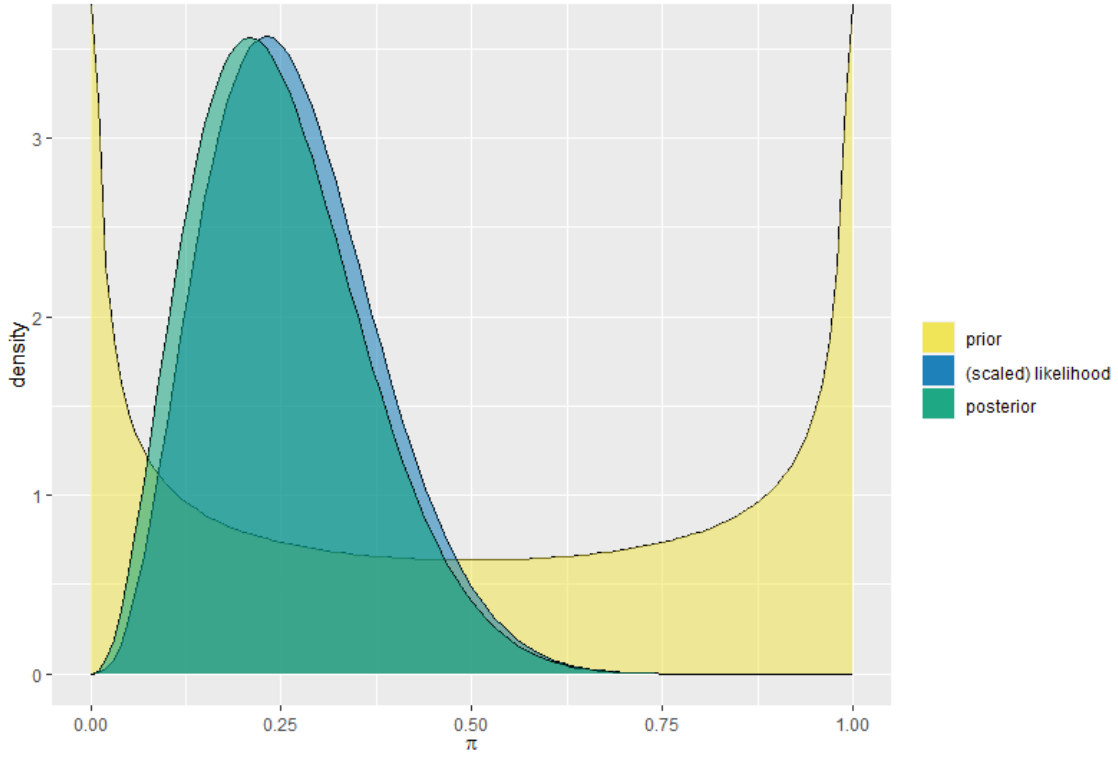
b) Prior: Beta (0.5, 0.5), Posterior: Beta (3.5, 10.5)

$$0.5 + y = 3.5$$

$$0.5 + n - y = 10.5$$

$$y = 3$$

$$0.5 + n - 3 = 10.5 => n = 13$$



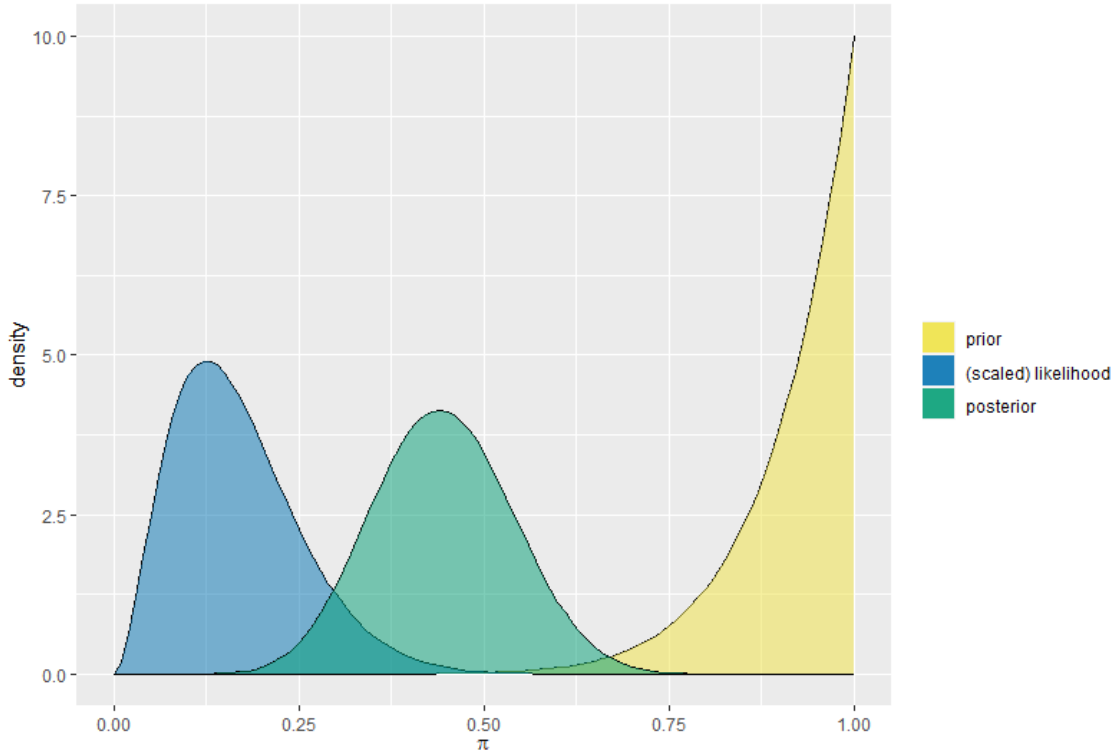
c) Prior: Beta (10, 1), Posterior: Beta (12, 15)

$$10 + y = 12$$

$$1 + n - y = 15$$

$$y = 2$$

$$1 + n - 2 = 15 => n = 16$$



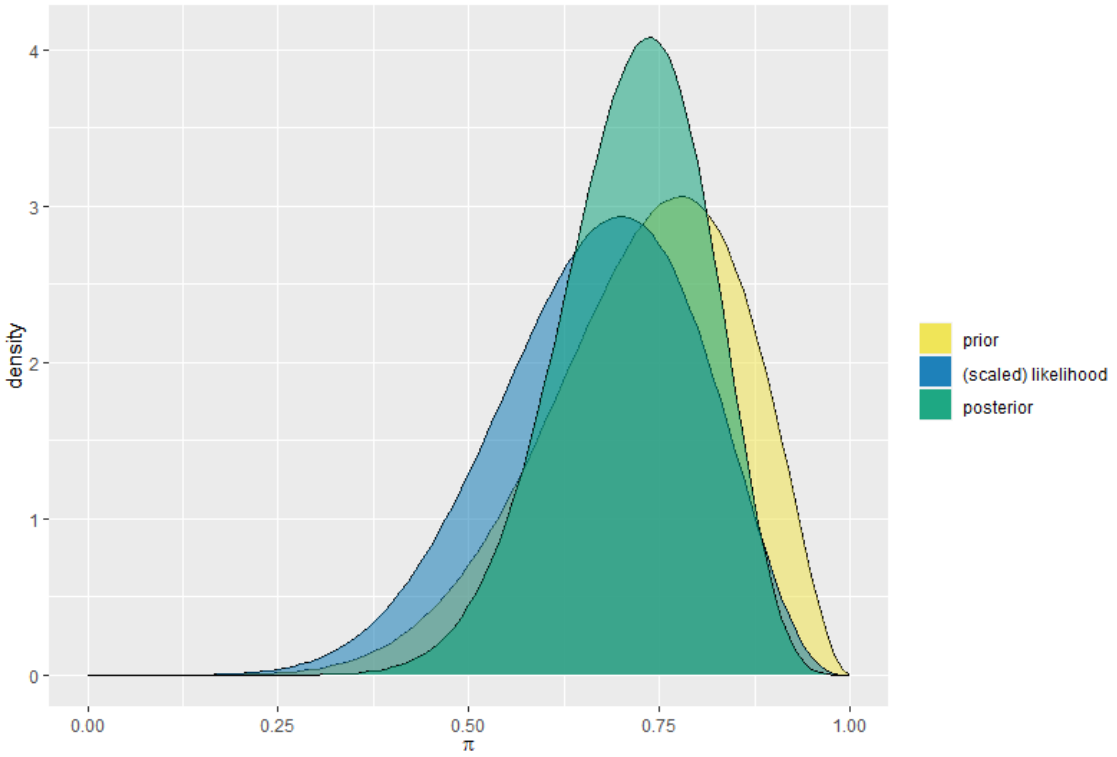
d) Prior: Beta (8, 3), Posterior: Beta (15, 6)

$$8 + y = 15$$

$$3 + n - y = 6$$

$$y = 7$$

$$3 + n - 7 = 6 => n = 10$$



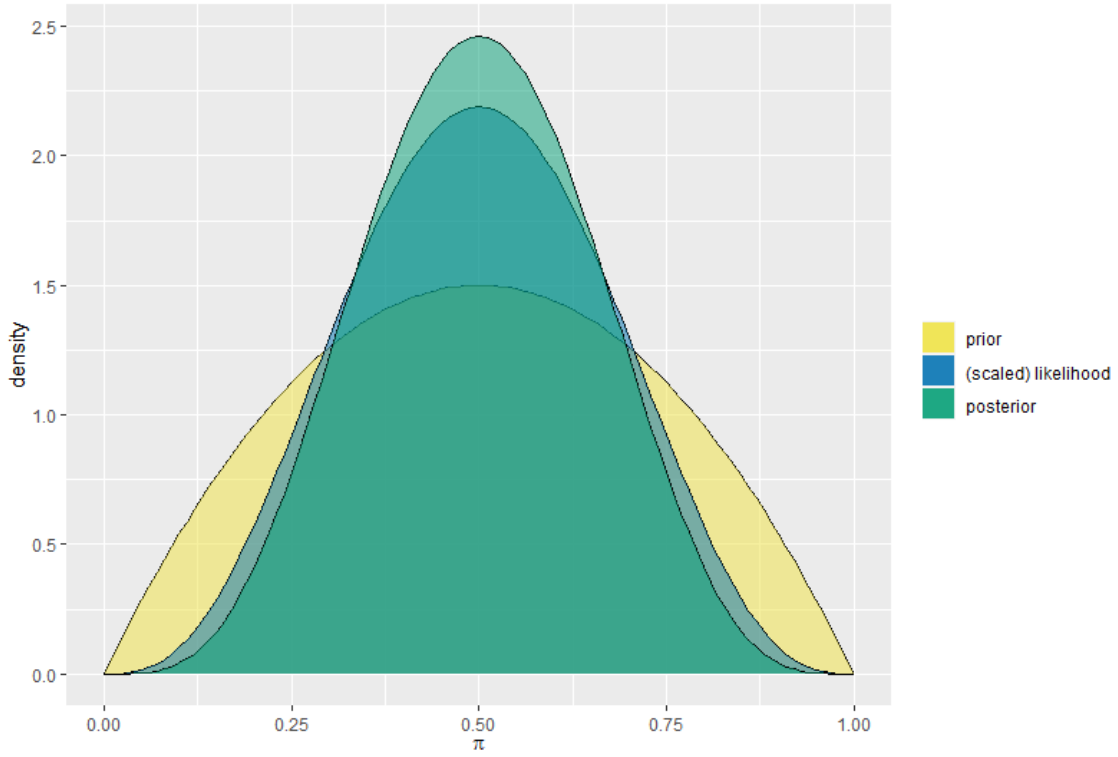
e) Prior: Beta (2, 2), Posterior: Beta (5, 5)

$$2 + y = 5$$

$$2 + n - y = 5$$

$$y = 3$$

$$2 + n - 3 = 5 => n = 6$$



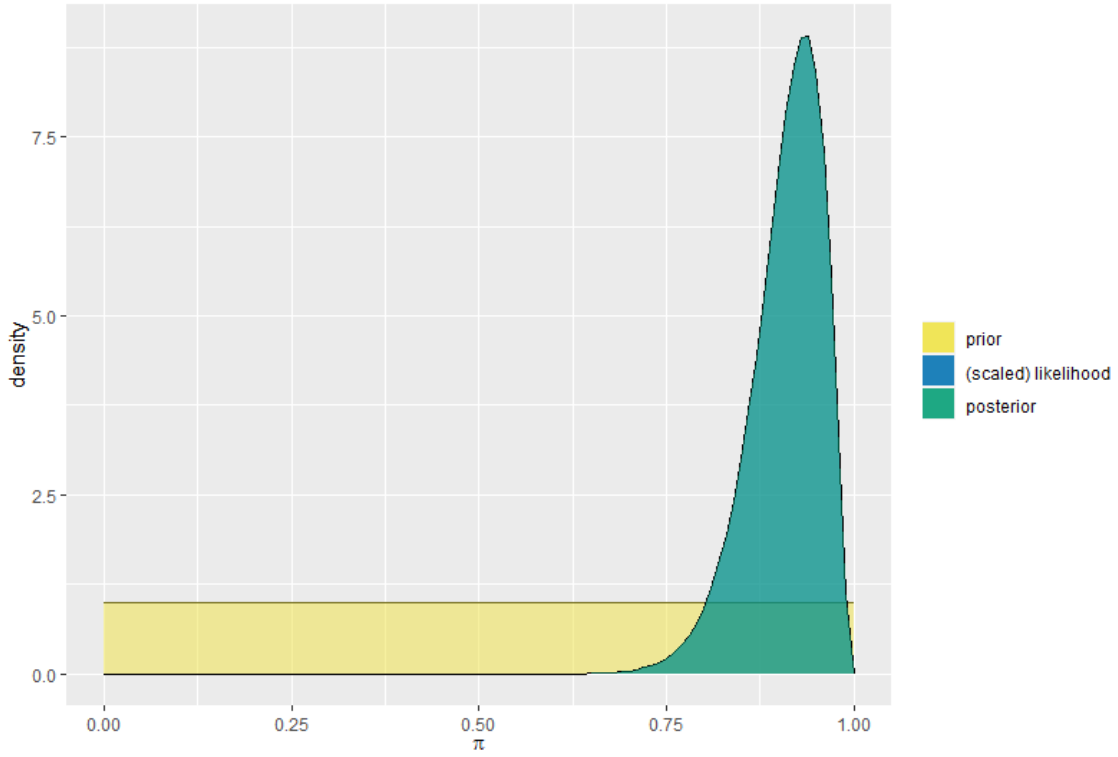
f) Prior: Beta (1, 1), Posterior: Beta (30, 3)

$$1 + y = 30$$

$$1 + n - y = 3$$

$$y = 29$$

$$1 + n - 29 = 3 => n = 31$$



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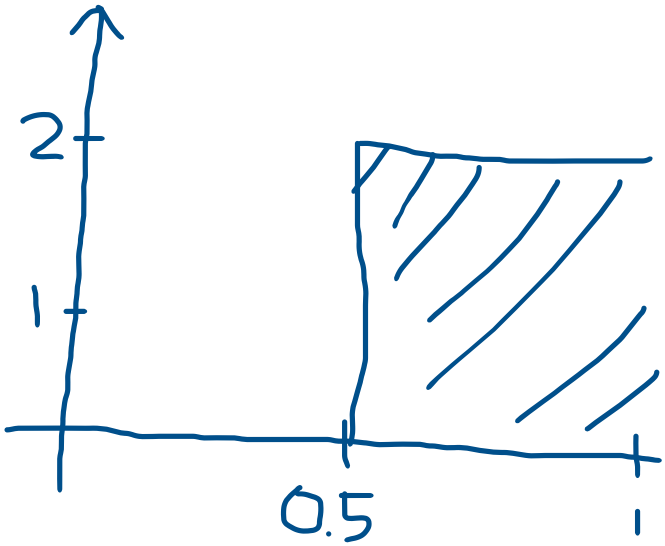
Exercise 4.13

a)

Prior:

$$\pi \sim \text{Unif}(0.5,1)$$



b)

The politician is 100% sur that his approval rating is above 50%.

He doesn't give any change of the approval rating being a failure.

c)

y=0, n=100

$$f(\pi|y=0) = 0 \; for \; \pi \; [0,05]$$

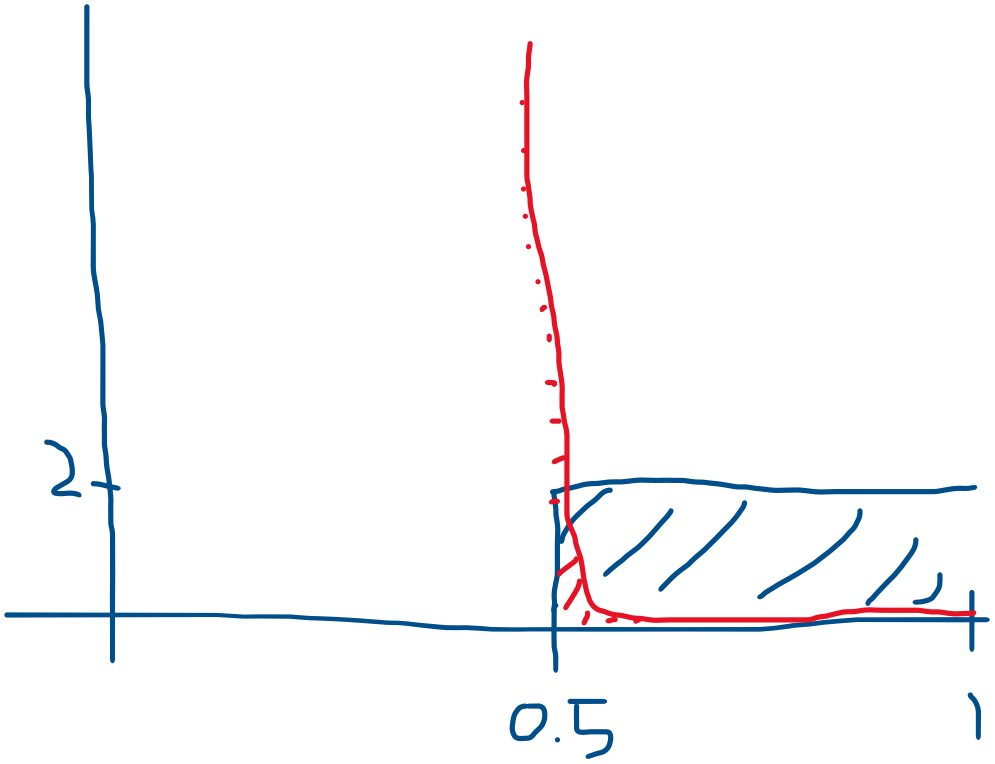
$$f(\pi|y=0) \propto f(\pi)L(\pi|y=0)$$

$$= 2 * \binom{100}{0} \pi^0 (1-\pi)^{(100-0)}$$

$$\propto 2 * \pi^0 (1-\pi)^{100}$$

$$f(\pi|y=0) \begin{matrix} = 0 \; for \; \pi \; [0,0.5] \\ \propto 2 * \pi^0 (1-\pi)^{100} for \; \pi \; [0.5,1] \end{matrix}$$

In red the Posterior pdf of π



d)

If we look now at the posterior, we can say that his approval rate is just above 0.5, meaning that we will barely pass an election. But in return, no one approve him. This is the problem with choosing a prior with a 0 value for some of the values of π, in this case no matter the amount of data you collect the pdf will be 0 for those values where the prior is 0.

Exercise 4.15

$$\pi \sim \text{Beta}(2,3)$$

a) First observation: Success

Prior:

$$\pi \sim \text{Beta}(2,3)$$

posterior:

$$\pi|(Y=1) \sim \text{Beta}(2+1, 3+1-1) = \text{Beta}(3,3)$$

b) Second observation: Success

Prior:

$$\pi \sim \text{Beta}(3,3)$$

posterior:

$$\pi|(Y=1) \sim \text{Beta}(3+1, 3+1-1) = \text{Beta}(4,3)$$

c) Third observation: Failure

Prior:

$$\pi \sim \text{Beta}(4,3)$$

posterior:

$$\pi|(Y=0) \sim \text{Beta}(4+0, 3+1-0) = \text{Beta}(4,4)$$

d) Fourth observation: Success

Prior:

$$\pi \sim \text{Beta}(4,4)$$

posterior:

$$\pi|(Y=1) \sim \text{Beta}(4+1, 4+1-1) = \text{Beta}(5,4)$$