Problem Set 5.1

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```r
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --

## v ggplot2 3.3.5     v purrr   0.3.4
## v tibble  3.1.6     v dplyr   1.0.7
## v tidyr   1.2.0     v stringr 1.4.0
## v readr   2.1.2     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```r
library(janitor)
```

```
##
## Attaching package: 'janitor'

## The following objects are masked from 'package:stats':
##
##     chisq.test, fisher.test
```

```r
library(rstan)
```

```
## Loading required package: StanHeaders

## rstan (Version 2.21.3, GitRev: 2e1f913d3ca3)

## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)

## Do not specify '-march=native' in 'LOCAL_CPPFLAGS' or a Makevars file

##
## Attaching package: 'rstan'
```

```
## The following object is masked from 'package:tidyr':
##
##     extract
```

```
library(bayesplot)
```

```
## This is bayesplot version 1.8.1

## - Online documentation and vignettes at mc-stan.org/bayesplot

## - bayesplot theme set to bayesplot::theme_default()

##    * Does _not_ affect other ggplot2 plots

##    * See ?bayesplot_theme_set for details on theme setting
```

```
library(bayesrules)
```

Exercise 6.5

Part A

```
# Step 1: Define a grid of 6 pi values
grid_data <- data.frame(pi_grid = seq(from = 0, to = 1, length = 5))

# Step 2: Evaluate the prior & likelihood at each pi
grid_data <- grid_data %>%
  mutate(prior = dbeta(pi_grid, 3, 8),
         likelihood = dbinom(2, 10, pi_grid))

# Step 3: Approximate the posterior
grid_data <- grid_data %>%
  mutate(unnormalized = likelihood * prior,
         posterior = unnormalized / sum(unnormalized))

# Confirm that the posterior approximation sums to 1
grid_data %>%
  summarize(sum(unnormalized), sum(posterior))
```

```
##   sum(unnormalized) sum(posterior)
## 1         0.8765603              1
```

```
sum(grid_data['unnormalized'])
```

```
## [1] 0.8765603
```

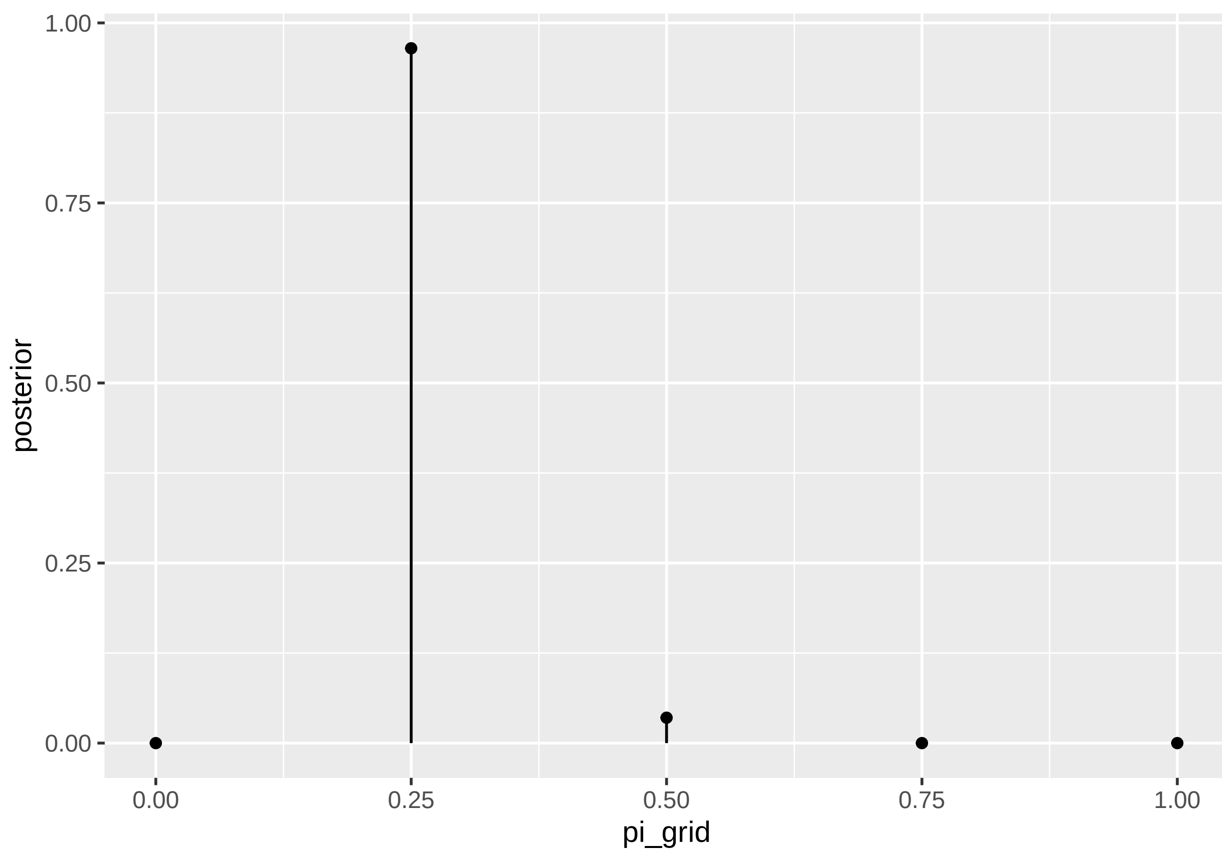
```r
sum(grid_data['posterior'])
```

```
## [1] 1
```

```r
#Table
round(grid_data, 2)
```

```
##   pi_grid prior likelihood unnormalized posterior
## 1    0.00  0.00       0.00         0.00      0.00
## 2    0.25  3.00       0.28         0.85      0.96
## 3    0.50  0.70       0.04         0.03      0.04
## 4    0.75  0.01       0.00         0.00      0.00
## 5    1.00  0.00       0.00         0.00      0.00
```

```r
# Plot the grid approximated posterior
ggplot(grid_data, aes(x = pi_grid, y = posterior)) +
  geom_point() +
  geom_segment(aes(x = pi_grid, xend = pi_grid, y = 0, yend = posterior))
```



Part B

```r
# Step 1: Define a grid of 6 pi values
grid_data <- data.frame(pi_grid = seq(from = 0, to = 1, length = 201))
```

```r
# Step 2: Evaluate the prior & likelihood at each pi
grid_data <- grid_data %>%
  mutate(prior = dbeta(pi_grid, 3, 8),
         likelihood = dbinom(2, 10, pi_grid))

# Step 3: Approximate the posterior
grid_data <- grid_data %>%
  mutate(unnormalized = likelihood * prior,
         posterior = unnormalized / sum(unnormalized))

# Confirm that the posterior approximation sums to 1
grid_data %>%
  summarize(sum(unnormalized), sum(posterior))
```

```
##   sum(unnormalized) sum(posterior)
## 1          41.79567              1
```

```r
sum(grid_data['unnormalized'])
```

```
## [1] 41.79567
```

```r
sum(grid_data['posterior'])
```

```
## [1] 1
```

```r
#Table
round(grid_data, 2)
```

```
##     pi_grid prior likelihood unnormalized posterior
## 1      0.00  0.00       0.00         0.00      0.00
## 2      0.00  0.01       0.00         0.00      0.00
## 3      0.01  0.03       0.00         0.00      0.00
## 4      0.01  0.07       0.01         0.00      0.00
## 5      0.02  0.13       0.02         0.00      0.00
## 6      0.03  0.19       0.02         0.00      0.00
## 7      0.03  0.26       0.03         0.01      0.00
## 8      0.04  0.34       0.04         0.01      0.00
## 9      0.04  0.43       0.05         0.02      0.00
## 10     0.04  0.53       0.06         0.03      0.00
## 11     0.05  0.63       0.07         0.05      0.00
## 12     0.06  0.73       0.09         0.06      0.00
## 13     0.06  0.84       0.10         0.08      0.00
## 14     0.06  0.95       0.11         0.11      0.00
## 15     0.07  1.06       0.12         0.13      0.00
## 16     0.07  1.17       0.14         0.16      0.00
## 17     0.08  1.29       0.15         0.19      0.00
## 18     0.09  1.40       0.16         0.22      0.01
## 19     0.09  1.51       0.17         0.26      0.01
## 20     0.10  1.62       0.18         0.30      0.01
## 21     0.10  1.72       0.19         0.33      0.01
## 22     0.10  1.83       0.20         0.37      0.01
```

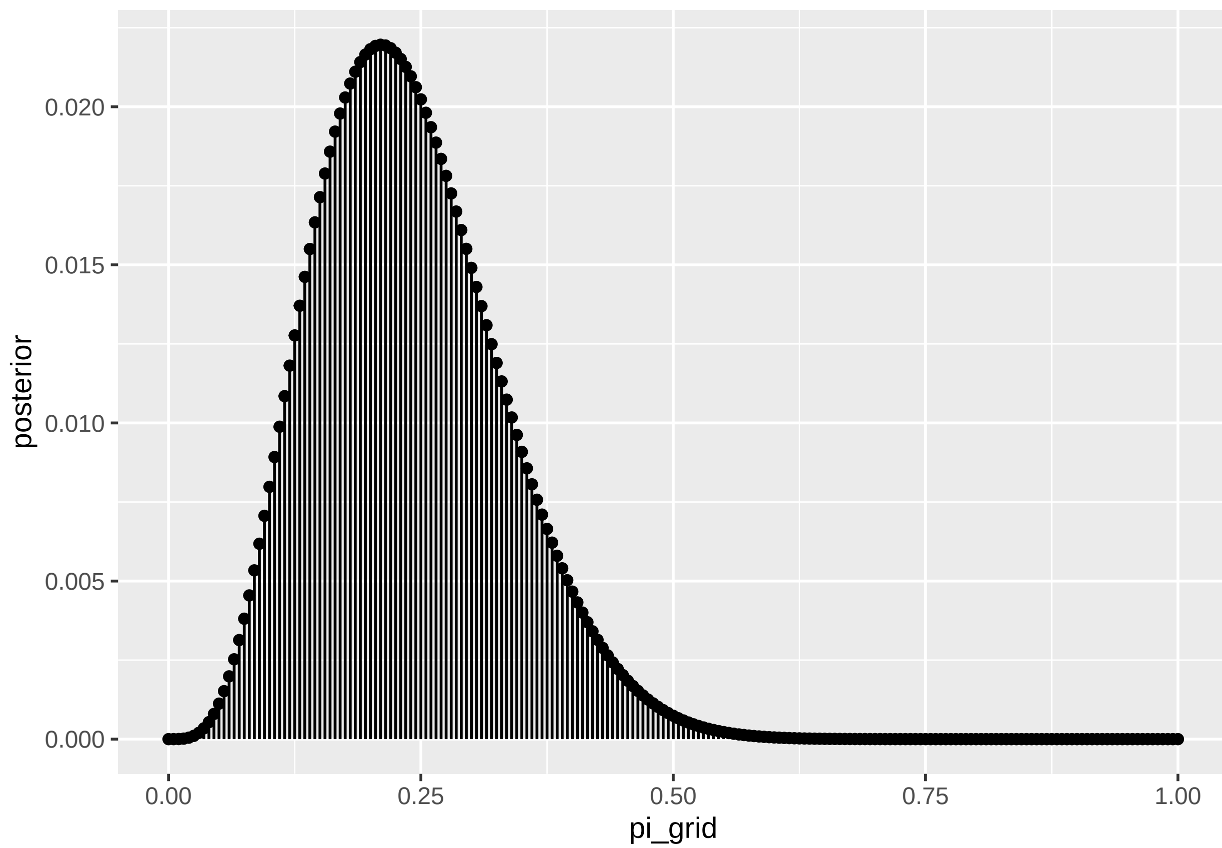
```
## 23      0.11  1.93        0.21          0.41       0.01
## 24      0.12  2.02        0.22          0.45       0.01
## 25      0.12  2.12        0.23          0.49       0.01
## 26      0.12  2.21        0.24          0.53       0.01
## 27      0.13  2.30        0.25          0.57       0.01
## 28      0.14  2.38        0.26          0.61       0.01
## 29      0.14  2.45        0.26          0.65       0.02
## 30      0.14  2.53        0.27          0.68       0.02
## 31      0.15  2.60        0.28          0.72       0.02
## 32      0.16  2.66        0.28          0.75       0.02
## 33      0.16  2.72        0.29          0.78       0.02
## 34      0.16  2.77        0.29          0.80       0.02
## 35      0.17  2.82        0.29          0.83       0.02
## 36      0.18  2.87        0.30          0.85       0.02
## 37      0.18  2.91        0.30          0.87       0.02
## 38      0.18  2.94        0.30          0.88       0.02
## 39      0.19  2.97        0.30          0.89       0.02
## 40      0.20  3.00        0.30          0.90       0.02
## 41      0.20  3.02        0.30          0.91       0.02
## 42      0.21  3.04        0.30          0.92       0.02
## 43      0.21  3.05        0.30          0.92       0.02
## 44      0.22  3.06        0.30          0.92       0.02
## 45      0.22  3.06        0.30          0.91       0.02
## 46      0.22  3.06        0.30          0.91       0.02
## 47      0.23  3.06        0.29          0.90       0.02
## 48      0.24  3.05        0.29          0.89       0.02
## 49      0.24  3.04        0.29          0.88       0.02
## 50      0.24  3.02        0.29          0.86       0.02
## 51      0.25  3.00        0.28          0.85       0.02
## 52      0.26  2.98        0.28          0.83       0.02
## 53      0.26  2.96        0.27          0.81       0.02
## 54      0.26  2.93        0.27          0.79       0.02
## 55      0.27  2.90        0.26          0.77       0.02
## 56      0.28  2.87        0.26          0.74       0.02
## 57      0.28  2.83        0.25          0.72       0.02
## 58      0.29  2.79        0.25          0.70       0.02
## 59      0.29  2.75        0.24          0.67       0.02
## 60      0.30  2.71        0.24          0.65       0.02
## 61      0.30  2.67        0.23          0.62       0.01
## 62      0.30  2.62        0.23          0.60       0.01
## 63      0.31  2.58        0.22          0.57       0.01
## 64      0.32  2.53        0.22          0.55       0.01
## 65      0.32  2.48        0.21          0.52       0.01
## 66      0.32  2.43        0.20          0.50       0.01
## 67      0.33  2.38        0.20          0.47       0.01
## 68      0.34  2.32        0.19          0.45       0.01
## 69      0.34  2.27        0.19          0.43       0.01
## 70      0.35  2.22        0.18          0.40       0.01
## 71      0.35  2.16        0.18          0.38       0.01
## 72      0.36  2.11        0.17          0.36       0.01
## 73      0.36  2.05        0.16          0.34       0.01
## 74      0.36  2.00        0.16          0.32       0.01
## 75      0.37  1.94        0.15          0.30       0.01
## 76      0.38  1.89        0.15          0.28       0.01
```

```
## 77     0.38  1.83       0.14         0.26       0.01
## 78     0.38  1.78       0.14         0.24       0.01
## 79     0.39  1.72       0.13         0.23       0.01
## 80     0.40  1.67       0.13         0.21       0.01
## 81     0.40  1.61       0.12         0.19       0.00
## 82     0.41  1.56       0.12         0.18       0.00
## 83     0.41  1.51       0.11         0.17       0.00
## 84     0.42  1.45       0.11         0.15       0.00
## 85     0.42  1.40       0.10         0.14       0.00
## 86     0.42  1.35       0.10         0.13       0.00
## 87     0.43  1.30       0.09         0.12       0.00
## 88     0.44  1.25       0.09         0.11       0.00
## 89     0.44  1.20       0.08         0.10       0.00
## 90     0.44  1.16       0.08         0.09       0.00
## 91     0.45  1.11       0.08         0.08       0.00
## 92     0.46  1.06       0.07         0.08       0.00
## 93     0.46  1.02       0.07         0.07       0.00
## 94     0.47  0.98       0.07         0.06       0.00
## 95     0.47  0.93       0.06         0.06       0.00
## 96     0.48  0.89       0.06         0.05       0.00
## 97     0.48  0.85       0.06         0.05       0.00
## 98     0.48  0.81       0.05         0.04       0.00
## 99     0.49  0.78       0.05         0.04       0.00
## 100    0.50  0.74       0.05         0.03       0.00
## 101    0.50  0.70       0.04         0.03       0.00
## 102    0.50  0.67       0.04         0.03       0.00
## 103    0.51  0.64       0.04         0.02       0.00
## 104    0.52  0.60       0.04         0.02       0.00
## 105    0.52  0.57       0.03         0.02       0.00
## 106    0.52  0.54       0.03         0.02       0.00
## 107    0.53  0.51       0.03         0.02       0.00
## 108    0.54  0.48       0.03         0.01       0.00
## 109    0.54  0.46       0.03         0.01       0.00
## 110    0.54  0.43       0.02         0.01       0.00
## 111    0.55  0.41       0.02         0.01       0.00
## 112    0.56  0.38       0.02         0.01       0.00
## 113    0.56  0.36       0.02         0.01       0.00
## 114    0.57  0.34       0.02         0.01       0.00
## 115    0.57  0.32       0.02         0.01       0.00
## 116    0.58  0.30       0.02         0.00       0.00
## 117    0.58  0.28       0.01         0.00       0.00
## 118    0.58  0.26       0.01         0.00       0.00
## 119    0.59  0.24       0.01         0.00       0.00
## 120    0.60  0.23       0.01         0.00       0.00
## 121    0.60  0.21       0.01         0.00       0.00
## 122    0.60  0.20       0.01         0.00       0.00
## 123    0.61  0.18       0.01         0.00       0.00
## 124    0.62  0.17       0.01         0.00       0.00
## 125    0.62  0.16       0.01         0.00       0.00
## 126    0.62  0.15       0.01         0.00       0.00
## 127    0.63  0.14       0.01         0.00       0.00
## 128    0.64  0.13       0.01         0.00       0.00
## 129    0.64  0.12       0.01         0.00       0.00
## 130    0.64  0.11       0.00         0.00       0.00
```

```
## 131     0.65  0.10       0.00          0.00       0.00
## 132     0.66  0.09       0.00          0.00       0.00
## 133     0.66  0.08       0.00          0.00       0.00
## 134     0.66  0.08       0.00          0.00       0.00
## 135     0.67  0.07       0.00          0.00       0.00
## 136     0.68  0.06       0.00          0.00       0.00
## 137     0.68  0.06       0.00          0.00       0.00
## 138     0.69  0.05       0.00          0.00       0.00
## 139     0.69  0.05       0.00          0.00       0.00
## 140     0.70  0.04       0.00          0.00       0.00
## 141     0.70  0.04       0.00          0.00       0.00
## 142     0.70  0.03       0.00          0.00       0.00
## 143     0.71  0.03       0.00          0.00       0.00
## 144     0.72  0.03       0.00          0.00       0.00
## 145     0.72  0.03       0.00          0.00       0.00
## 146     0.72  0.02       0.00          0.00       0.00
## 147     0.73  0.02       0.00          0.00       0.00
## 148     0.74  0.02       0.00          0.00       0.00
## 149     0.74  0.02       0.00          0.00       0.00
## 150     0.74  0.01       0.00          0.00       0.00
## 151     0.75  0.01       0.00          0.00       0.00
## 152     0.76  0.01       0.00          0.00       0.00
## 153     0.76  0.01       0.00          0.00       0.00
## 154     0.76  0.01       0.00          0.00       0.00
## 155     0.77  0.01       0.00          0.00       0.00
## 156     0.78  0.01       0.00          0.00       0.00
## 157     0.78  0.01       0.00          0.00       0.00
## 158     0.78  0.00       0.00          0.00       0.00
## 159     0.79  0.00       0.00          0.00       0.00
## 160     0.80  0.00       0.00          0.00       0.00
## 161     0.80  0.00       0.00          0.00       0.00
## 162     0.80  0.00       0.00          0.00       0.00
## 163     0.81  0.00       0.00          0.00       0.00
## 164     0.82  0.00       0.00          0.00       0.00
## 165     0.82  0.00       0.00          0.00       0.00
## 166     0.83  0.00       0.00          0.00       0.00
## 167     0.83  0.00       0.00          0.00       0.00
## 168     0.84  0.00       0.00          0.00       0.00
## 169     0.84  0.00       0.00          0.00       0.00
## 170     0.84  0.00       0.00          0.00       0.00
## 171     0.85  0.00       0.00          0.00       0.00
## 172     0.86  0.00       0.00          0.00       0.00
## 173     0.86  0.00       0.00          0.00       0.00
## 174     0.86  0.00       0.00          0.00       0.00
## 175     0.87  0.00       0.00          0.00       0.00
## 176     0.88  0.00       0.00          0.00       0.00
## 177     0.88  0.00       0.00          0.00       0.00
## 178     0.88  0.00       0.00          0.00       0.00
## 179     0.89  0.00       0.00          0.00       0.00
## 180     0.90  0.00       0.00          0.00       0.00
## 181     0.90  0.00       0.00          0.00       0.00
## 182     0.90  0.00       0.00          0.00       0.00
## 183     0.91  0.00       0.00          0.00       0.00
## 184     0.92  0.00       0.00          0.00       0.00
```

```
## 185     0.92  0.00        0.00          0.00       0.00
## 186     0.92  0.00        0.00          0.00       0.00
## 187     0.93  0.00        0.00          0.00       0.00
## 188     0.94  0.00        0.00          0.00       0.00
## 189     0.94  0.00        0.00          0.00       0.00
## 190     0.95  0.00        0.00          0.00       0.00
## 191     0.95  0.00        0.00          0.00       0.00
## 192     0.96  0.00        0.00          0.00       0.00
## 193     0.96  0.00        0.00          0.00       0.00
## 194     0.96  0.00        0.00          0.00       0.00
## 195     0.97  0.00        0.00          0.00       0.00
## 196     0.98  0.00        0.00          0.00       0.00
## 197     0.98  0.00        0.00          0.00       0.00
## 198     0.98  0.00        0.00          0.00       0.00
## 199     0.99  0.00        0.00          0.00       0.00
## 200     1.00  0.00        0.00          0.00       0.00
## 201     1.00  0.00        0.00          0.00       0.00
```

```r
# Plot the grid approximated posterior
ggplot(grid_data, aes(x = pi_grid, y = posterior)) +
  geom_point() +
  geom_segment(aes(x = pi_grid, xend = pi_grid, y = 0, yend = posterior))
```



problem 6.13

a)

```
# STEP 1: DEFINE the model
bb_model <- "
  data {
    int<lower = 0, upper = 10> Y;
  }
  parameters {
    real<lower = 0, upper = 1> pi;
  }
  model {
    Y ~ binomial(10, pi);
    pi ~ beta(3, 8);
  }
"

# STEP 2: SIMULATE the posterior
bb_sim <- stan(model_code = bb_model, data = list(Y = 2),
               chains = 3, iter = 12000, seed = 1)
```

```
##
## SAMPLING FOR MODEL '2d032ece7c158ae32f246bd4866ed7ab' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:     1 / 12000 [  0%]  (Warmup)
## Chain 1: Iteration:  1200 / 12000 [ 10%]  (Warmup)
## Chain 1: Iteration:  2400 / 12000 [ 20%]  (Warmup)
## Chain 1: Iteration:  3600 / 12000 [ 30%]  (Warmup)
## Chain 1: Iteration:  4800 / 12000 [ 40%]  (Warmup)
## Chain 1: Iteration:  6000 / 12000 [ 50%]  (Warmup)
## Chain 1: Iteration:  6001 / 12000 [ 50%]  (Sampling)
## Chain 1: Iteration:  7200 / 12000 [ 60%]  (Sampling)
## Chain 1: Iteration:  8400 / 12000 [ 70%]  (Sampling)
## Chain 1: Iteration:  9600 / 12000 [ 80%]  (Sampling)
## Chain 1: Iteration: 10800 / 12000 [ 90%]  (Sampling)
## Chain 1: Iteration: 12000 / 12000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.1 seconds (Warm-up)
## Chain 1:                0.128 seconds (Sampling)
## Chain 1:                0.228 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL '2d032ece7c158ae32f246bd4866ed7ab' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 2: Adjust your expectations accordingly!
```

```
## Chain 2:
## Chain 2:
## Chain 2: Iteration:     1 / 12000 [  0%]  (Warmup)
## Chain 2: Iteration:  1200 / 12000 [ 10%]  (Warmup)
## Chain 2: Iteration:  2400 / 12000 [ 20%]  (Warmup)
## Chain 2: Iteration:  3600 / 12000 [ 30%]  (Warmup)
## Chain 2: Iteration:  4800 / 12000 [ 40%]  (Warmup)
## Chain 2: Iteration:  6000 / 12000 [ 50%]  (Warmup)
## Chain 2: Iteration:  6001 / 12000 [ 50%]  (Sampling)
## Chain 2: Iteration:  7200 / 12000 [ 60%]  (Sampling)
## Chain 2: Iteration:  8400 / 12000 [ 70%]  (Sampling)
## Chain 2: Iteration:  9600 / 12000 [ 80%]  (Sampling)
## Chain 2: Iteration: 10800 / 12000 [ 90%]  (Sampling)
## Chain 2: Iteration: 12000 / 12000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.097 seconds (Warm-up)
## Chain 2:                0.113 seconds (Sampling)
## Chain 2:                0.21 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL '2d032ece7c158ae32f246bd4866ed7ab' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:     1 / 12000 [  0%]  (Warmup)
## Chain 3: Iteration:  1200 / 12000 [ 10%]  (Warmup)
## Chain 3: Iteration:  2400 / 12000 [ 20%]  (Warmup)
## Chain 3: Iteration:  3600 / 12000 [ 30%]  (Warmup)
## Chain 3: Iteration:  4800 / 12000 [ 40%]  (Warmup)
## Chain 3: Iteration:  6000 / 12000 [ 50%]  (Warmup)
## Chain 3: Iteration:  6001 / 12000 [ 50%]  (Sampling)
## Chain 3: Iteration:  7200 / 12000 [ 60%]  (Sampling)
## Chain 3: Iteration:  8400 / 12000 [ 70%]  (Sampling)
## Chain 3: Iteration:  9600 / 12000 [ 80%]  (Sampling)
## Chain 3: Iteration: 10800 / 12000 [ 90%]  (Sampling)
## Chain 3: Iteration: 12000 / 12000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.071 seconds (Warm-up)
## Chain 3:                0.081 seconds (Sampling)
## Chain 3:                0.152 seconds (Total)
## Chain 3:
```

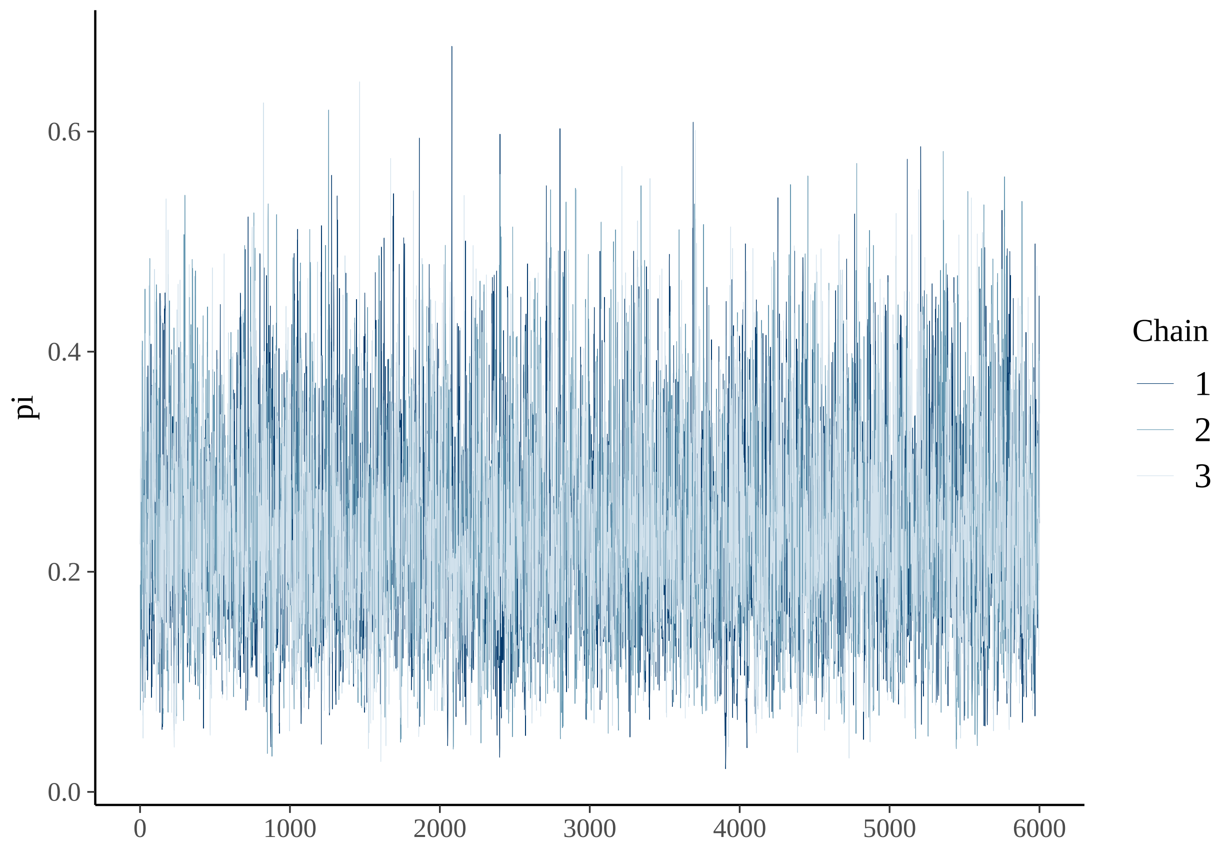
b)

```
as.array(bb_sim, pars = "pi") %>%
  head(4)
```

```
## , , parameters = pi
##
##             chains
```

```
## iterations   chain:1    chain:2   chain:3
##        [1,] 0.2249787 0.18837278 0.2935260
##        [2,] 0.2764489 0.07439545 0.2059931
##        [3,] 0.2323178 0.16303609 0.2013071
##        [4,] 0.2869850 0.12158335 0.3316874
```

```
mcmc_trace(bb_sim, pars = "pi", size = 0.1)
```

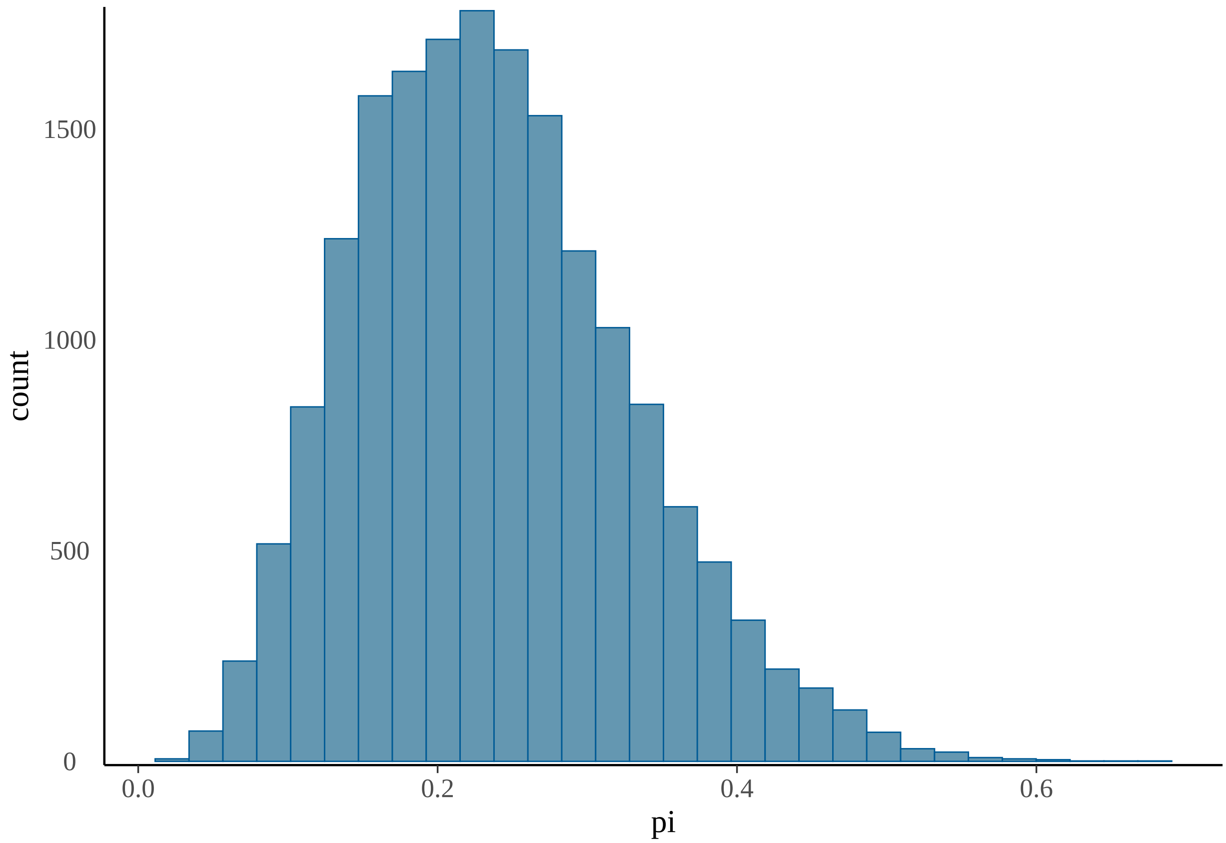


c) The first 50% of the MCMC process there are always remove known as "burn-in" or "warm-up" samples. The second half are kept as the final Markov chain sample. that is the reason why there are only 6,000. (12,000/2)

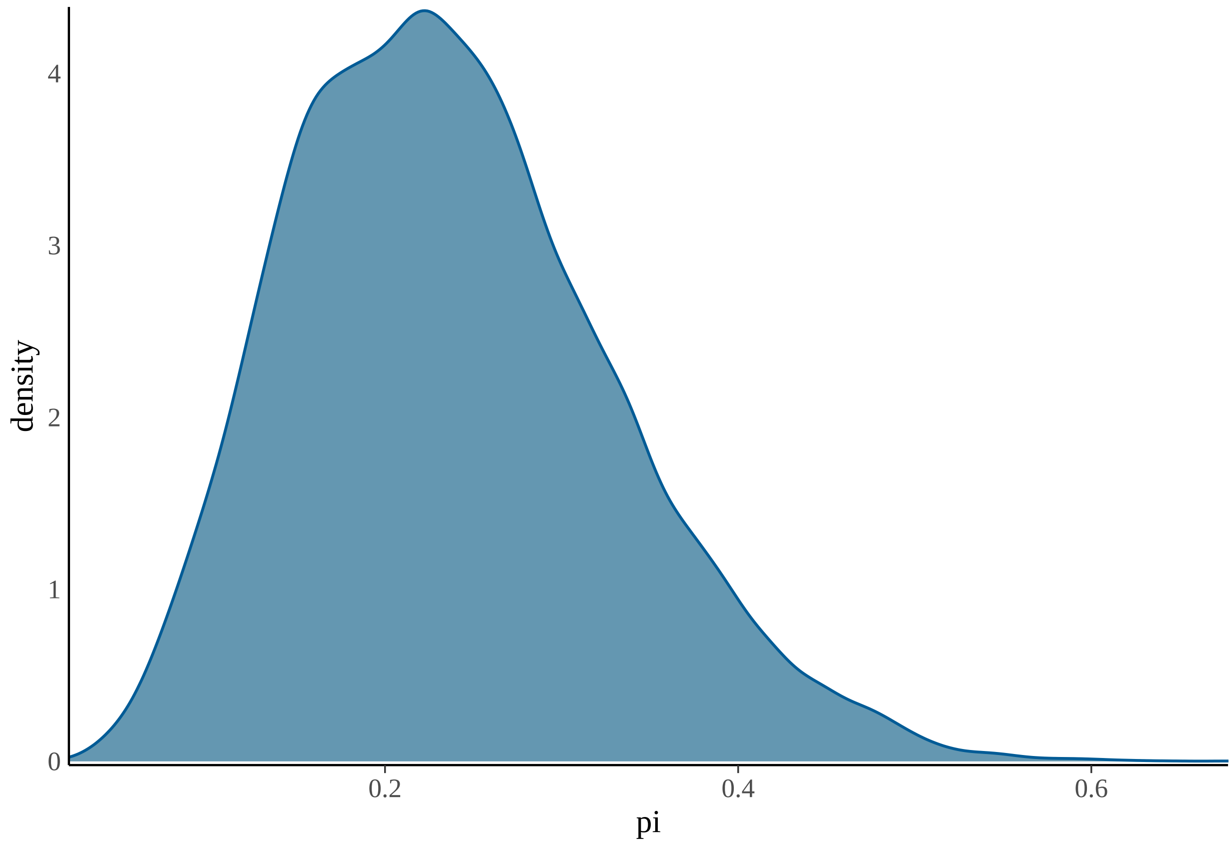
d)

```
# Histogram of the Markov chain values
mcmc_hist(bb_sim, pars = "pi") +
  yaxis_text(TRUE) +
  ylab("count")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

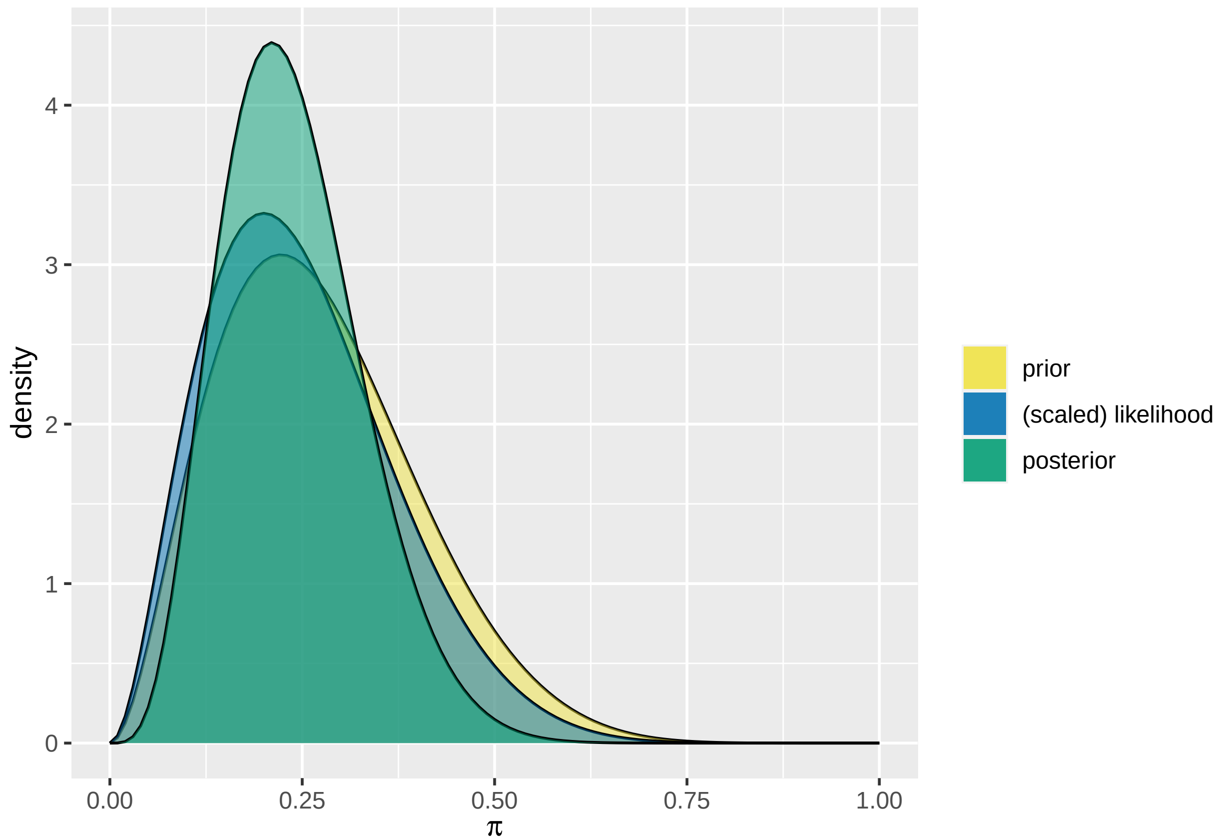



```r
# Density plot of the Markov chain values
mcmc_dens(bb_sim, pars = "pi") +
  yaxis_text(TRUE) +
  ylab("density")
```



e)

```
plot_beta_binomial(alpha = 3, beta = 8, y = 2, n = 10)
```



They look similar, the majority of the pdf is around 0.2 and 0.24. So they look preaty similar

problem 6.14

```
# STEP 1: DEFINE the model
bb_model <- "
  data {
    int<lower = 0, upper = 12> Y;
  }
  parameters {
    real<lower = 0, upper = 1> pi;
  }
  model {
    Y ~ binomial(12, pi);
    pi ~ beta(4, 3);
  }
"

# STEP 2: SIMULATE the posterior
bb_sim <- stan(model_code = bb_model, data = list(Y = 4),
               chains = 3, iter = 12000, seed = 1)
```

```
##
## SAMPLING FOR MODEL 'b61b4553aa22e133ec03c9d7da030f7b' NOW (CHAIN 1).
```

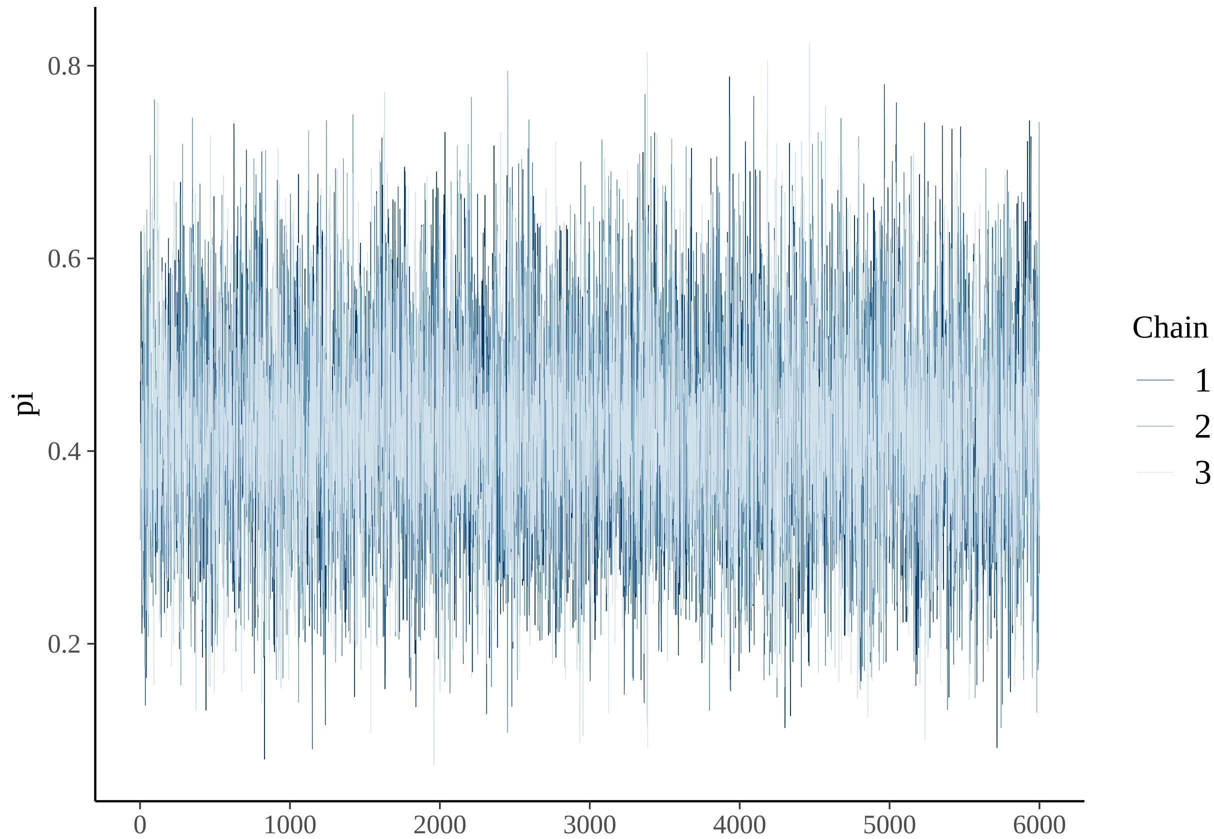
```
## Chain 1:
## Chain 1: Gradient evaluation took 0 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:     1 / 12000 [  0%]  (Warmup)
## Chain 1: Iteration:  1200 / 12000 [ 10%]  (Warmup)
## Chain 1: Iteration:  2400 / 12000 [ 20%]  (Warmup)
## Chain 1: Iteration:  3600 / 12000 [ 30%]  (Warmup)
## Chain 1: Iteration:  4800 / 12000 [ 40%]  (Warmup)
## Chain 1: Iteration:  6000 / 12000 [ 50%]  (Warmup)
## Chain 1: Iteration:  6001 / 12000 [ 50%]  (Sampling)
## Chain 1: Iteration:  7200 / 12000 [ 60%]  (Sampling)
## Chain 1: Iteration:  8400 / 12000 [ 70%]  (Sampling)
## Chain 1: Iteration:  9600 / 12000 [ 80%]  (Sampling)
## Chain 1: Iteration: 10800 / 12000 [ 90%]  (Sampling)
## Chain 1: Iteration: 12000 / 12000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.073 seconds (Warm-up)
## Chain 1:                0.078 seconds (Sampling)
## Chain 1:                0.151 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'b61b4553aa22e133ec03c9d7da030f7b' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:     1 / 12000 [  0%]  (Warmup)
## Chain 2: Iteration:  1200 / 12000 [ 10%]  (Warmup)
## Chain 2: Iteration:  2400 / 12000 [ 20%]  (Warmup)
## Chain 2: Iteration:  3600 / 12000 [ 30%]  (Warmup)
## Chain 2: Iteration:  4800 / 12000 [ 40%]  (Warmup)
## Chain 2: Iteration:  6000 / 12000 [ 50%]  (Warmup)
## Chain 2: Iteration:  6001 / 12000 [ 50%]  (Sampling)
## Chain 2: Iteration:  7200 / 12000 [ 60%]  (Sampling)
## Chain 2: Iteration:  8400 / 12000 [ 70%]  (Sampling)
## Chain 2: Iteration:  9600 / 12000 [ 80%]  (Sampling)
## Chain 2: Iteration: 10800 / 12000 [ 90%]  (Sampling)
## Chain 2: Iteration: 12000 / 12000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.074 seconds (Warm-up)
## Chain 2:                0.072 seconds (Sampling)
## Chain 2:                0.146 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'b61b4553aa22e133ec03c9d7da030f7b' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0 seconds.
## Chain 3: Adjust your expectations accordingly!
```

```
## Chain 3:
## Chain 3:
## Chain 3: Iteration:     1 / 12000 [  0%]  (Warmup)
## Chain 3: Iteration:  1200 / 12000 [ 10%]  (Warmup)
## Chain 3: Iteration:  2400 / 12000 [ 20%]  (Warmup)
## Chain 3: Iteration:  3600 / 12000 [ 30%]  (Warmup)
## Chain 3: Iteration:  4800 / 12000 [ 40%]  (Warmup)
## Chain 3: Iteration:  6000 / 12000 [ 50%]  (Warmup)
## Chain 3: Iteration:  6001 / 12000 [ 50%]  (Sampling)
## Chain 3: Iteration:  7200 / 12000 [ 60%]  (Sampling)
## Chain 3: Iteration:  8400 / 12000 [ 70%]  (Sampling)
## Chain 3: Iteration:  9600 / 12000 [ 80%]  (Sampling)
## Chain 3: Iteration: 10800 / 12000 [ 90%]  (Sampling)
## Chain 3: Iteration: 12000 / 12000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.072 seconds (Warm-up)
## Chain 3:                0.08 seconds (Sampling)
## Chain 3:                0.152 seconds (Total)
## Chain 3:
```

```
as.array(bb_sim, pars = "pi") %>%
  head(4)
```

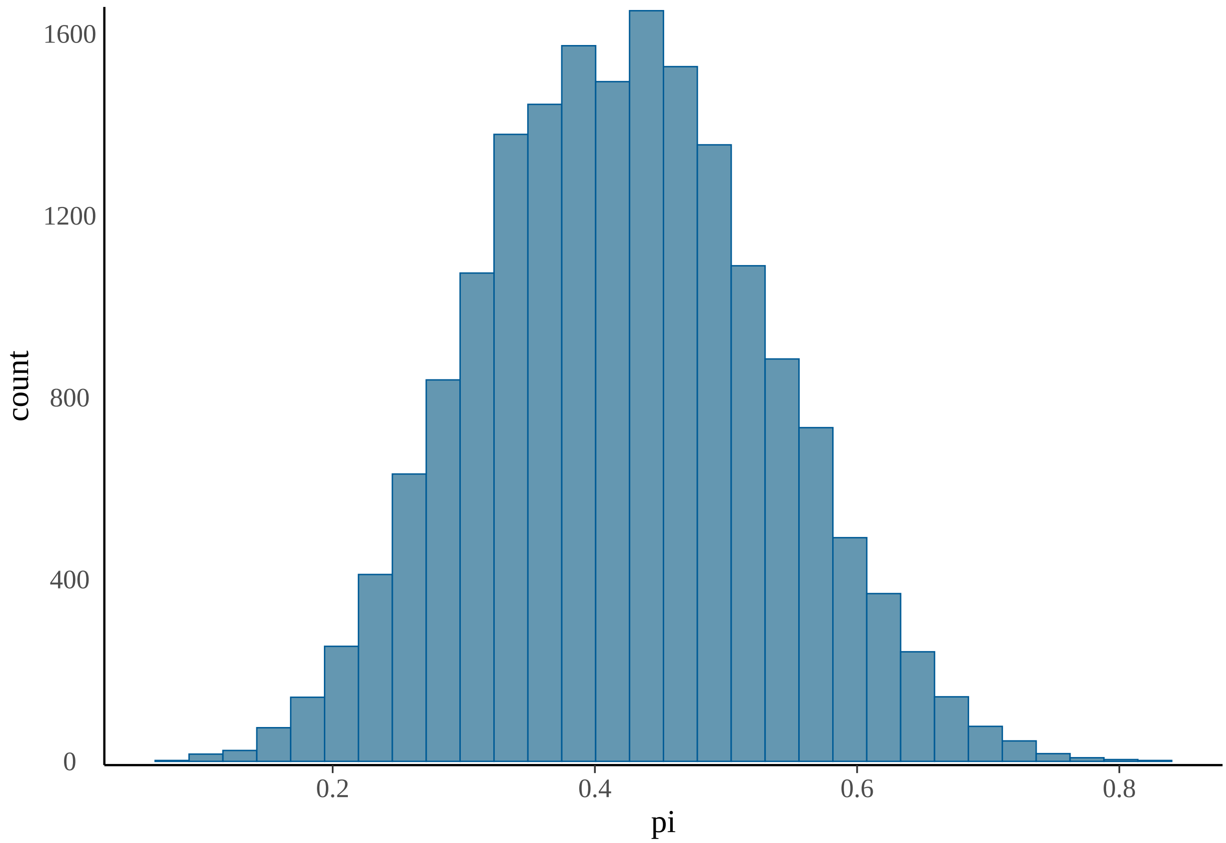
```
## , , parameters = pi
##
##           chains
## iterations   chain:1   chain:2   chain:3
##       [1,] 0.4725206 0.3077464 0.4289977
##       [2,] 0.4688443 0.3743118 0.3724838
##       [3,] 0.4243136 0.3094735 0.3719880
##       [4,] 0.3904008 0.3302203 0.3719880
```

```
mcmc_trace(bb_sim, pars = "pi", size = 0.1)
```

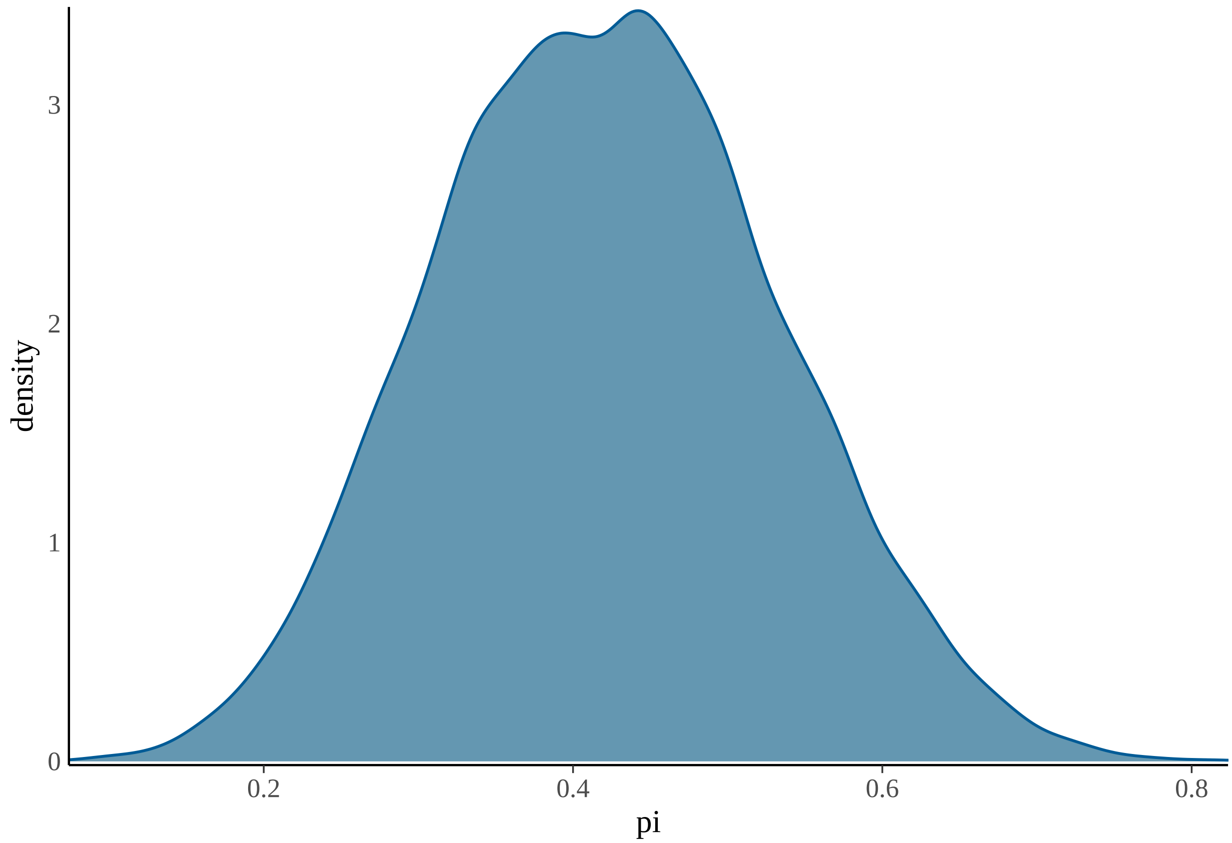



```
# Histogram of the Markov chain values
mcmc_hist(bb_sim, pars = "pi") +
  yaxis_text(TRUE) +
  ylab("count")
```

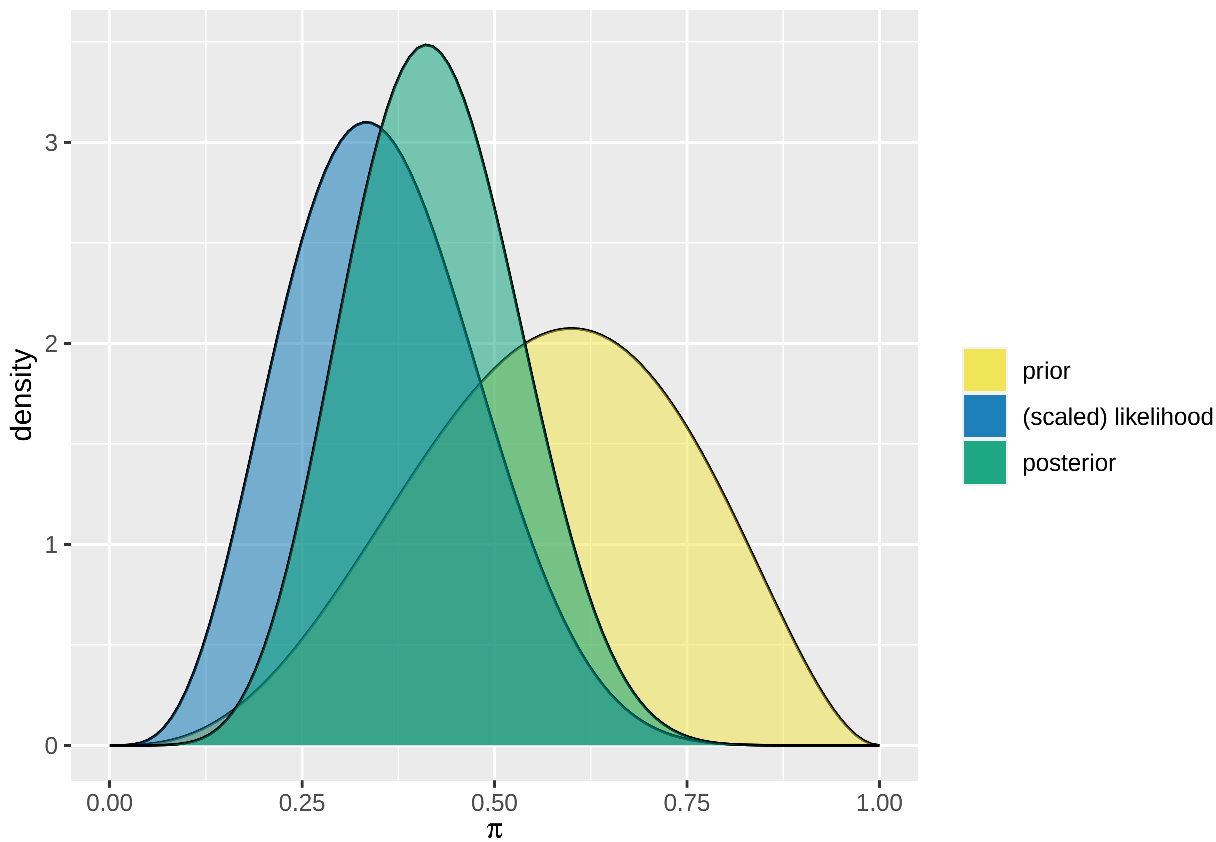
```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```r
# Density plot of the Markov chain values
mcmc_dens(bb_sim, pars = "pi") +
  yaxis_text(TRUE) +
  ylab("density")
```




```
plot_beta_binomial(alpha = 4, beta = 3, y = 4, n = 12)
```



They look similar, the majority of the pdf is around 0.4 and 0.45. So they look preaty similar