Intuitionistic Approach to Matroids

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Abstract

Matroids are an important interest to (combinatorial) optimization - where each "step" made strictly approaches the optimal solution. Here we assert the properties and definition of matroids in an intuitionistic setting, to better formalize and prove matroids as correct and important.

1 Overview

As one definition, a Matroid M consists of a **ground** set E and a "family" I of **independent** subsets of E, with the following properties:

$$E : \forall \sigma.\ Set\ \sigma,\ \ I : \forall \sigma.\ Set\ (Set\ \sigma) \quad \text{where}$$

$$\begin{aligned}
& I \not\equiv \emptyset \iff \emptyset \in I && \text{(MATROID-NONEMPTY)} \\
& \forall a \in A.\ a \subseteq E && \text{(MATROID-SUBSET)} \\
& \quad \forall \alpha \subseteq a.\ \alpha \in E && \text{(MATROID-HEREDITARY)} \\
& \forall i_n,\ i_{n+1} \in I.\ \exists! e \in i_{n+1} - i_n && \\
& \quad \forall i \in I.\ i \cup \{e\} \in I && \text{(MATROID-GROWTH)}
\end{aligned}$$

Where in MATROID-GROWTH, i_n denotes the element i of I such that its size $|i_n| = n$; if there is *one* element difference between the two sets, then the differing element exists in all independent sets.