Bland's Rule Elaboration

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Abstract

Bland's rule is the most popular method to deciding the next pivot to take when refactoring a system of linear inequalities for more-optimized basic-feasible solutions. Here, we detail Bland's rule, and provide it's dual form for implementation in the Haskell version of the Cassowary constraint solver.

1 Overview

Say we have an objective function in the following form:

$$\omega \;=\; \sum_{0}^{n} \alpha_n x_n \;+\; c_\omega$$

That is, an equation where the unique objective variable ω is in basic-normal form, and the rest of the public n variables are summed with their coefficients - $\alpha_n x_n$, and the constant value c_ω. We use α to denote *objective* coefficients.

This objective function is then maximized over a constraint set with the following form:

$$\left\{ \sum_{0}^{n} \beta_{(m,n)} x_n \;+\; c_m \right\}$$

For each constraint m. We use β to denote *constraint* coefficients. Each constraint m can have any variable x_n in basic-feasible form. Also, note that the slack variables normally introduced in this context are implicit - they are referenced by the unique identifier m per-constraint.

2 Bland's Rule Primal

The primal version of Bland's rule proceeds as follows: Choose a column k out of n, and a row j out of m as the next variable to refactor into basic-normal form over the whole constraint set - this as, x_k will be defined in terms of the constraint j.

2.1 Column Selection

$$\text{Choose } \alpha_k \; := \; \min \{\alpha_n \mid \alpha_n < 0\}$$

We choose the column k to be the minimum *negative* objective coefficient.

2.2 Row Selection

In the primal version of Bland's rule, we need to first solve for k before we can solve for j.

$$\text{Choose } \frac{c_j}{\beta_{(j,k)}} \; := \; \min \left\{ \frac{c_m}{\beta_{(m,k)}} \;\middle|\; \beta_{(m,k)} > 0 \right\}$$

That is, we choose j to be the minimum (primal) Bland ratio - the constant c_m divided by the *strictly positive* coefficient $\beta_{(m,k)}$.

You can find the proofs that this method of row and column selection does not cycle, and is strictly maximizing the basic-feasible solution in the literature.

3 Bland's Rule Dual

The dual version of Bland's rule is very similar to the primal version, except for the swapped roles - this would behave the same as the primal version if we transpose the matrix of the objective function and constraint coefficients.

Here, we will instead need to select the row j **first**, and the column second.

3.1 Row Selection

$$\text{Choose } c_j \; := \; \min \{c_m \mid c_m < 0\}$$

We choose the row j to be the minimum *negative* constraint constant.

3.2 Column Selection

In the dual version of Bland's rule, we need to first solve for j before we can solve for k.

$$\text{Choose } \frac{\alpha_k}{\beta_{(j,k)}} \; := \; \min \left\{ \frac{\alpha_n}{\beta_{(j,n)}} \;\middle|\; \beta_{(j,n)} > 0 \right\}$$

The dual version of Bland's rule will *minimize* the basic-feasible solution for the constraint set and objective function.