Assignment 2

January 13, 2021

Question 1: Examine the consistency of the system of given equation

(a)

$$x + 2y = 2$$

$$2x + 3y = 3$$

Sol:

In matrix form, this can be written as **AX=B**

where A = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Now if the **rank of matrix A = rank of augmented matrix (A|B)**, then the system of equation is consistent otherwise inconsistent.

Forming Augmented matrix (A$|B$) = $\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 3 & 3 \end{array}\right]$

Now , In order to find the rank, we use row operations.

$$(A|B) = \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 3 & 3 \end{array}\right]$$

R2 = R2 - 2R1

$$(A|B) = \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -1 & -1 \end{array}\right]$$

As rank of matrix A = rank of Augmented matrix (A$|B) = n(no.of unknowns)$

Hence the system is consistent and has unique solutions.

Now in order to find the solutions, we use the **Guass Jordan Elimination method**. For this, We again use row operations.

. R2 = (-1)R2

$$(A|B) = \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

R1 = R1 - 2R2

$$(A|B) = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right]$$

From above matrix, it is clear that value of **x=0** and value of **y=1.**

(b)

$$2x - y = 5$$
$$x + y = 4$$

Sol:

In matrix form, this can be written as **AX=B**

where A = $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B = $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Now if the **rank of matrix A = rank of augmented matrix (A|B)**, then the system of equation is consistent otherwise inconsistent.

Forming Augmented matrix (A$|B) = \left[\begin{array}{cc|c} 2 & -1 & 5 \\ 1 & 1 & 4 \end{array}\right]$

Now , In order to find the rank, we use row operations.

$$(A|B) = \left[\begin{array}{cc|c} 2 & -1 & 5 \\ 1 & 1 & 4 \end{array}\right]$$

R2 = 2R2 - R1

$$(A|B) = \left[\begin{array}{cc|c} 2 & -1 & 5 \\ 0 & 3 & 3 \end{array}\right]$$

As rank of matrix A = rank of Augmented matrix (A$|B) = n(no.of unknowns)$

Hence the system is consistent and has unique solutions.

Now in order to find the solutions, we use the **Guass Jordan Elimination method**. For this, We again use row operations.

- R2 = R2/3

$$(A|B) = \left[\begin{array}{cc|c} 2 & -1 & 5 \\ 0 & 1 & 1 \end{array}\right]$$

R1 = R1 + R2

$$(A|B) = \left[\begin{array}{cc|c} 2 & 0 & 6 \\ 0 & 1 & 1 \end{array}\right]$$

From above matrix, it is clear that value of **x=3** and value of **y=1.**