ASSIGNMENT 3

Athar Javed

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- **Question 1 :**

  Find the Inverse and QR Decomposition of the following.

  (a):

$$A = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$$

- **Sol:**

  For Inverse:

  We are given with a matrix

$$A = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$$

  Now,
  DetA=

$$(2 * (-2)) - (1 * (-6))$$
$$-4 + 6 = 2$$

  Also,

$$AdjA = \begin{pmatrix} -2 & 6 \\ -1 & 2 \end{pmatrix}$$

  Now,
  A^{-1}, $can \quad be \quad calculated \quad by \quad the \quad formula,$

A^{-1} $= AdjA/DetA$

Therefore,

$$A^{-1} = \begin{pmatrix} -1 & 3 \\ -1/2 & 1 \end{pmatrix}$$

QR Decomposition:

A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$b = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1 \qquad\qquad b = s_1 u_1 + t_2 u_2$$

where,

$$t_1 = ||a|| \qquad u_1 = a/t_1 \qquad s_1 = u_1^T * b/||u_1||^2 \qquad u_2 = b - s_1 * u_1/||b - s_1 u_1|| \qquad t_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

$$Now,\ using\ the\ equations\ of\ t_1, u_1, s_1, u_2\ and\ t_2\ we\ get\ the\ values\ of\ t_1, u_1, s_1, u_2\ and\ t_2$$

$$t_1 = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$u_1 = 1/\sqrt{5} * \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 2/\sqrt{5} & 1/\sqrt{5} \end{pmatrix} * \begin{pmatrix} -6 \\ -2 \end{pmatrix}$$

$$s_1 = (2/\sqrt{5} * (-6)) + 1/\sqrt{5} * (-2)$$

$$s_1 = -14/\sqrt{5} \qquad\qquad (||u_1||)^2 = 1)$$

$$u_2 = 1/\sqrt{5} * \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$t_2 = \begin{pmatrix} -1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix} * \begin{pmatrix} -6 \\ -2 \end{pmatrix}$$

$$t_2 = 2/\sqrt{5}$$

$$substituting\ \ the\ \ values\ \ of\ \ t_1, u_1, s_1, u_2\ \ and\ \ t_2\ \ in\ \ the\ \ matrix$$
$$\begin{pmatrix} a & b \end{pmatrix} = QR$$
$$we\ \ get\ \ the\ \ required\ \ QR\ \ decomposition\ \ of\ \ A.$$

$$\begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix} * \begin{pmatrix} \sqrt{5} & -14/\sqrt{5} \\ 0 & 2/\sqrt{5} \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$$

- **Sol:**

For Inverse:

We are given with a matrix

$$A = \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$$

Now,

DetA=

$$(6 * (1)) - ((-3) * (-2))$$
$$6 - 6 = 0$$

Since DetA=0
Therefore it's inverse doesn't exist.

QR Decomposition:

A QR decomposition of a real square matrix A is a decomposition of A as,

$$A = QR$$

where Q is the orthogonal matrix and R is the upper triangular matrix.

Given

$$A = \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$$

Let a and b be the column vectors of the given matrix such that,

$$a = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

$$b = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

The above vectors can be expressed as:

$$a = t_1 u_1 \qquad\qquad b = s_1 u_1 + t_2 u_2$$

where,

$$t_1 = ||a|| \qquad u_1 = a/t_1 \qquad s_1 = u_1^T * b/||u_1||^2 \qquad u_2 = b - s_1 * u_1/||b - s_1 u_1|| \qquad t_2 = u_2^T b$$

The Values of a and b can be written as,

$$\begin{pmatrix} a & b \end{pmatrix} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} * \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

and,

$$Q^T * Q = I$$

$Now,\ using\ the\ equations\ of\ t_1, u_1, s_1, u_2\ and\ t_2\ we\ get\ the\ values\ of\ t_1, u_1, s_1, u_2\ and\ t_2$

$$t_1 = \sqrt{6^2 + 2^2} = 2\sqrt{10}$$

$$u_1 = 1/\sqrt{10} * \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$s_1 = \begin{pmatrix} 6/\sqrt{40} & -2/\sqrt{40} \end{pmatrix} * \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$s_1 = (6/\sqrt{40} * (-3)) - (2/\sqrt{5} * (1))$$

$$s_1 = -10/\sqrt{10} \qquad (||u_1||)^2 = 1)$$

$$u_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$t_2 = 0$$

substituting the values of $\mathrm{t}_1, u_1, s_1, u_2$ *and* t_2 *in the matrix*

$$\begin{pmatrix} a & b \end{pmatrix} = QR$$

we get the required QR decomposition of A.

$$\begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 3/\sqrt{10} & 0 \\ -1/\sqrt{10} & 0 \end{pmatrix} * \begin{pmatrix} 2\sqrt{10} & -10/\sqrt{10} \\ 0 & 0 \end{pmatrix}$$