ASSIGNMENT 4

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January 16, 2021

Question:

Find the equation of the line satisfying the following conditions:

a. passing through the point $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ with slope 1/2.

b. passing through the point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ with slope m.

c. passing through the point $\begin{pmatrix} 2 \\ 2\sqrt{3} \end{pmatrix}$ and inclined with the x-axis at an angle of 75$\circ$.

d. Intersecting the x-axis at a distance of 3 units to the let of the origin with slope -2.

e. Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30$\circ$ with the positive direction of the x-axis.

f. passing through the points $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

g. perpendicular distance from the origin is 5 and the angle made by the perpendicular with the positive x-axis is 30$\circ$.

SOL:

(a). A=$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and m=1/2

The direction vector is k=$\begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$

Hence Normal Vector

$$\text{n}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\text{k}$$

$$\text{n}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$$

$$\text{n}=\begin{pmatrix} -1/2 \\ 1 \end{pmatrix}$$

Now, the equation of the line in terms of normal vector in obtained as:

$$\text{n}^T(X-A)=0$$

$$\begin{pmatrix} -1/2 & 1 \end{pmatrix}\text{X - }\begin{pmatrix} -1/2 & 1 \end{pmatrix}\begin{pmatrix} -4 \\ 3 \end{pmatrix}=0$$

$$\begin{pmatrix} -1/2 & 1 \end{pmatrix}\mathbf{X = 5}$$

which is the required equation.

(b). A=$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and slope = m.

The direction vector is k=$\begin{pmatrix} 1 \\ m \end{pmatrix}$

Hence Normal vector

$$\text{n}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\text{k}$$

$$\text{n}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ m \end{pmatrix}$$

$$\text{n}=\begin{pmatrix} -m \\ 1 \end{pmatrix}$$

Now, the equation of the line in terms of normal vector in obtained as:

$$\text{n}^T(X-A)=0$$

$$\begin{pmatrix} -m & 1 \end{pmatrix}\text{X - }\begin{pmatrix} -m & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix}=0$$

$$\begin{pmatrix} -m & 1 \end{pmatrix}\mathbf{X = 0}$$

which is the required equation.

(c). A=$\begin{pmatrix} 2 \\ 2\sqrt{3} \end{pmatrix}$ and slope = tan75∘= 2+ $\sqrt{3}$

The direction vector is k=$\begin{pmatrix} 1 \\ 2+\sqrt{3} \end{pmatrix}$

Hence Normal vector

$$\text{n}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\text{k}$$

$$\text{n}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2+\sqrt{3} \end{pmatrix}$$

$$\text{n}=\begin{pmatrix} -2-\sqrt{3} \\ 1 \end{pmatrix}$$

Now, the equation of the line in terms of normal vector in obtained as:

$$\text{n}^T(X-A)=0$$

$$\begin{pmatrix} -2-\sqrt{3} & 1 \end{pmatrix}\text{X - }\begin{pmatrix} -2-\sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 2\sqrt{3} \end{pmatrix}=0$$

$$\begin{pmatrix} -2-\sqrt{3} & 1 \end{pmatrix}\mathbf{X = \text{-}4}$$

which is the required equation.

(d). A=$\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ and slope = -2

The direction vector is k=$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Hence Normal vector

$$\text{n}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\text{k}$$

$$\text{n}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\text{n}=\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Now, the equation of the line in terms of normal vector in obtained as:

$$\text{n}^T(X-A)=0$$

$$\begin{pmatrix}2 & 1\end{pmatrix}\text{X - }\begin{pmatrix}2 & 1\end{pmatrix}\begin{pmatrix}-3\\0\end{pmatrix} = 0$$
$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{X = \text{-}6}$$

which is the required equation.

(e). A=$\begin{pmatrix}0\\2\end{pmatrix}$ and slope = tan30○= $1\sqrt{3}$

The direction vector is k=$\begin{pmatrix}1\\1\sqrt{3}\end{pmatrix}$

Hence Normal vector

$$\text{n=}\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\text{k}$$
$$\text{n=}\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\1\sqrt{3}\end{pmatrix}$$
$$\text{n=}\begin{pmatrix}-1\sqrt{3}\\1\end{pmatrix}$$

Now, the equation of the line in terms of normal vector in obtained as:

$$\text{n}^T(X-A) = 0$$
$$\begin{pmatrix}-1\sqrt{3} & 1\end{pmatrix}\text{X - }\begin{pmatrix}-1\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} = 0$$
$$\begin{pmatrix}-1\sqrt{3} & 1\end{pmatrix}\mathbf{X = 2}$$

which is the required equation.

(f). A=$\begin{pmatrix}-1\\1\end{pmatrix}$ and B=$\begin{pmatrix}2\\-4\end{pmatrix}$

The equation of the line joining the points A and B is obtained by:

$$\mathbf{X = A +}\ \lambda(B-A)$$

Therefore,

$$\text{X} = \begin{pmatrix}-1\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\-5\end{pmatrix}$$

which is the required equation

(g). The perpendicular P intersects the lines M and L at the foot of perpendicular.Thus,

$$\mathrm{n}^T P = c$$
$$\mathrm{P}=\mathrm{A}+\lambda n$$
$$\text{or } \mathrm{n}^T P = n^T A + \lambda ||n||^2 = c$$
$$\text{-}\lambda = \frac{n^T A - c}{||n||^2}$$

Also, the distance between A and L is obtained from:

$$\mathrm{P} = \mathrm{A} + \lambda n$$
$$||P - A|| = |\lambda|||n||$$

Also, n= $\begin{pmatrix} 1 \\ tan30 \end{pmatrix}$, A=0

Thus equation becomes

$$5 = |c|/||n||$$
$$\mathrm{c} = \pm 5\sqrt{1 + tan^2 30}$$
$$\mathrm{c} = \pm 5 sec30$$

Thus the required equation is

$$\begin{pmatrix} 1 & 1/\sqrt{3} \end{pmatrix} \mathbf{c} = \pm 10\sqrt{3}$$