ASSIGNMENT 6

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January 22, 2021

- **Question 2.75:**

Discuss the continuity of the function f , where f is defined by,

a)

$$f(x) = \begin{cases} 3 & 0 \leq x \leq 1 \\ 4 & 0 < x \leq 3 \\ 5 & 3 \leq x \leq 10 \end{cases}$$

b)

$$f(x) = \begin{cases} 2x & x < 0 \\ 0 & 0 \leq x \leq 1 \\ 4x & x > 1 \end{cases}$$

c)

$$f(x) = \begin{cases} -2 & x < -1 \\ 2x & -1 \leq x \leq 1 \\ 2 & x > 1 \end{cases}$$

Solution:

a)

$$f(x) = \begin{cases} 3 & 0 \leq x \leq 1 \\ 4 & 0 < x \leq 3 \\ 5 & 3 \leq x \leq 10 \end{cases}$$

As f(x) is the mixture of constant functions it will be continuous at every point except points at which constant functions are changing their behaviour i.e, 1,3.

for 1:

$$\lim_{x\to 1^-} f(x) = 3$$

$$\lim_{x\to 1^+} f(x) = 4$$

or, $$\lim_{x\to 1^-} f(x) \neq lim_{x\to 1^+} f(x)$$

f(x) is not continuous at x=1.

Similarly for x=3.

b)

$$f(x) = \begin{cases} 2x & x < 0 \\ 0 & 0 \leq x \leq 1 \\ 4x & x > 1 \end{cases}$$

for f(x) to be continuous at every point, it should be continuous at 0 and 1.
$for 0$:

$$\lim_{x\to 0^-} f(x) = 2x = 0$$

$$\lim_{x\to 0^+} f(x) = 0$$

or, $$\lim_{x\to 0^-} f(x) = lim_{x\to 0^+} f(x)$$

f(x) is continuous at x=0.

for 1:

$$\lim_{x\to 1^-} f(x) = 0$$

$$\lim_{x\to 1^+} f(x) = 4x = 4$$

or, $$\lim_{x\to 1^-} f(x) \neq lim_{x\to 1^+} f(x)$$

f(x) is not continuous at x=1.

c)

$$f(x) = \begin{cases} -2 & x < -1 \\ 2x & -1 \leq x \leq 1 \\ 2 & x > 1 \end{cases}$$

for f(x) to be continuous at all points it should be continuous at -1 and 1.

for -1:

$$\lim_{x \to -1^-} f(x) = -2$$

$$\lim_{x \to -1^+} f(x) = 2x = -2$$

or, $$\lim_{x \to -1^-} f(x) = lim_{x \to -1^+} f(x)$$

f(x) is continuous at x=-1.

for 1:

$$\lim_{x \to 1^-} f(x) = 2x = 2$$

$$\lim_{x \to 1^+} f(x) = 2$$

or, $$\lim_{x \to 1^-} f(x) = lim_{x \to 1^+} f(x)$$

f(x) is continuous at all points.

- **Question 2.76:**

  Find the relationship between a and b so that the function defined by,

$$f(x) = \begin{cases} ax + 1 & x \leq 1 \\ bx + 3 & x > 3 \end{cases}$$

  is continuous at x=3.

  Solution:

$$f(x) = \begin{cases} ax + 1 & x \leq 1 \\ bx + 3 & x > 3 \end{cases}$$

  for f(x) to be continuous at $x = 3$.

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x).$$

$$b(3) + 3 = a(3) + 1.$$

  Hence, $(a - b) = 2/3$