Deepspace Interplanetary Navigation Operations Colorado Research EXplorer (DINO C-REx)

DINO C-REx Technical Memorandum

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SYSTEMS ENGINEERING REPORT 4.11: OCCULTATION CHECKS

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Status: Initial Version	
Scope/Contents	
Derivation of a method for removing stars and body facets that are occulted by an ellipsoidal celestial body.	

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Contents

1 Overview

In general, the DINO C-REx camera model works by layering the sources of signal. First stars are added to an image, and then each extended body one at a time in order of distance from the spacecraft (farthest body to closest body). In order to achieve a realistic image, each star and body facet must be checked to ensure that there is truly a line of sight from the spacecraft camera to it, and it must be removed if there is not[1]. This SER describes the derivation of a simple mathematical relation that can quickly be run to perform such a check and its implementation within DINO C-REx.

2 Derivation

The following derivation provides an easily calculated metric to check if there is a line of sight between two points given by position vectors $\vec{a}$ and $\vec{b}$ given an opaque ellipsoid that may lie between them[2]. The derivation is presented in n-dimensions, though of course DINO C-REx only uses 3.

1. First, recall that the surface of an ellipsoid is given by the set of points $\vec{x} = \{x_1, x_2, ..., x_n\}$ that satisfy the following relation:
$$\frac{x_1^2}{\alpha_1^2} + \frac{x_2^2}{\alpha_2^2} + \cdots + \frac{x_n^2}{\alpha_n^2} = 1 \tag{1}$$
Or, in matrix notation:
$$\vec{x}^T A \vec{x} = 1 \tag{2}$$
Where $\vec{x}$ is any point on the surface of the ellipse, and A is[3]:
$$A = \begin{bmatrix} \alpha_1^2 & 0 & \cdots & 0 \\ 0 & \alpha_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_n^2 \end{bmatrix} \tag{3}$$

2. Next, note that in n-dimensional space, the equation of a line is given by:
$$\vec{y} = \vec{y}_0 + \vec{v}t \tag{4}$$
Where $\vec{y}$ is a point the line given $\vec{y}_0$ as an anchor point, also on the line. $\vec{v}$ is any vector that points in the direction of the line, and t is a parameterization value. For our line, we define $\vec{y}_0 = \vec{a}$ and $\vec{v} = \vec{b} - \vec{a}$. For the sake of clean notation, here we continue to call them $\vec{y}_0$ and $\vec{v}$.

3. These two equations are useful because there will be no line of sight between $\vec{a}$ and $\vec{b}$ when line in eq. 4 intersects with the ellipsoid given by eq. 3. Such intersection points can be found by setting $\vec{x}$ equal to $\vec{y}$ in eq. 3:
$$(\vec{y}_0 + \vec{v}t)^T A(\vec{y}_0 + \vec{v}t) = 1 \tag{5}$$
Eq. 5 then expands to become[4]:
$$\vec{y}_0^T A\vec{y}_0 + 2\vec{v}^T A\vec{y}_0 t + \vec{v}^T A\vec{v}_0 t^2 = 1 \tag{6}$$

[1] e.g. there is a body in the way

[2] $\vec{a}$ and $\vec{b}$ must be measured from the center of the ellipsoid

[3] In fact, A may be any positive semi-definite matrix, and the equation will still describe an ellipsoid. Although that is not shown here

[4] Note that because $\vec{y}_0^T A\vec{v}t$ is a scalar, it can be transposed to be $\vec{v}^T A\vec{y}_0 t$

4. Noticing that eq. 6 is simply a scalar equation in quadratic form, we can use the quadratic formula to find solutions. However, since we are only interested in whether or not there is a solution, and not what that solution actually is, we only need to check that the discriminant[1] is $\geqslant$ zero. If the discriminant is < 0, solutions to the eq. 6 will be imaginary, which implies that there is no intersection between the line and the ellipsoid, there *is* an unobstructed line of sight from the spacecraft to the star/beacon facet, and the star/beacon facet is not occulted. The discriminant generally is:

$$b^2 - 4ac \tag{7}$$

Or in our case:

$$(2\vec{v}^T A\vec{y}_0)^2 - 4(\vec{y}_0^T A\vec{y}_0)(\vec{v}^T A\vec{v}_0) \tag{8}$$

5. Finally, after simplification we have the a condition for which a star or body facet will be in view:

$$(\vec{v}^T A\vec{y}_0)^2 - (\vec{y}_0^T A\vec{y}_0)(\vec{v}^T A\vec{v}_0) \geqslant 0 \tag{9}$$

3 DINO C-REx Implementation

DINO C-REx implements occultation checks through the function image.removeOccultations(). It is called once per body by image.updateState(), which has sorted its list of simulated bodies from farthest to closest just before the loop where it calls image.removeOccultations(). Once image.removeOccultations() is called for a body, lightSimFunctions.lightSim() is called to add facets for that body. image.removeOccultations() stacks position vectors for each star and body facet as rows so many computations can be made with a single linear algebra step.

$$\vec{v} = \begin{bmatrix} \vec{n_1}^T \\ \vec{n_2}^T \\ \vdots \\ \vec{n_m}^T \end{bmatrix} = \begin{bmatrix} n_{1,1} & n_{1,2} & n_{1,3} \\ n_{2,1} & n_{2,2} & n_{2,3} \\ \vdots & \vdots & \vdots \\ n_{m,1} & n_{m,2} & n_{m,3} \end{bmatrix} \tag{10}$$

This way the result $\vec{v}^T A\vec{y}_0$ is an m-length column vector where each entry is the solution to some $\vec{n}_*^T A\vec{y}_0$. Furthermore, stacking unit vectors in this way allows us to use the powerful numpy.einsum() function. In particular:

```
vTAv = einsum('ij,ji->i',v.T,A.dot(v))
```

is used to calculate *only* the diagonal of $\vec{v}^T A\vec{v}$, and returns a column vector where each entry is $\vec{n}_*^T A\vec{n}_*$. With these column vectors in hand, we can calculate the discriminant for *all* stars and facets in a single step with:

```
discriminant = vTAy_0**2 - vTAv*(y_0TAy_0 - 1)
```

Finally, we find a boolean array with the following:

```
occCheck = logical_or(discriminant < 0,v.T.dot(y_0) > 0)
```

The need for discriminant < 0 is discussed above. The check for v.T.dot(y_0) > 0 ensures that stars on the opposite side of the celestial sphere to the occulting body are not removed as well as the math will catch both. The array occCheck returns True in positions where the star/facet is occulted and False where it is not.

[1] The part of the quadratic equation under the radical