Finite Cable

- Consider a finite cable of electrotonic length L.
- Let X denote the electrotonic distance along the cable,
- A general expression for steady state membrane voltage, as a function of X, is given as,

$$X = \frac{x}{\lambda}$$

where,

$$V_m(X) = A\cosh(L - X) + B\sinh(L - X)$$

$$\cosh(x) = (e^x + e^{-x})/2$$

$$\sinh(x) = (e^x - e^{-x})/2$$

Why hyperbolic functions?

Hyperbolic functions

$$\cosh(x) = (e^{x} + e^{-x})/2$$

$$\sinh(x) = (e^{x} - e^{-x})/2$$

$$\frac{d}{dx}\cosh(x) = \sinh(x)$$

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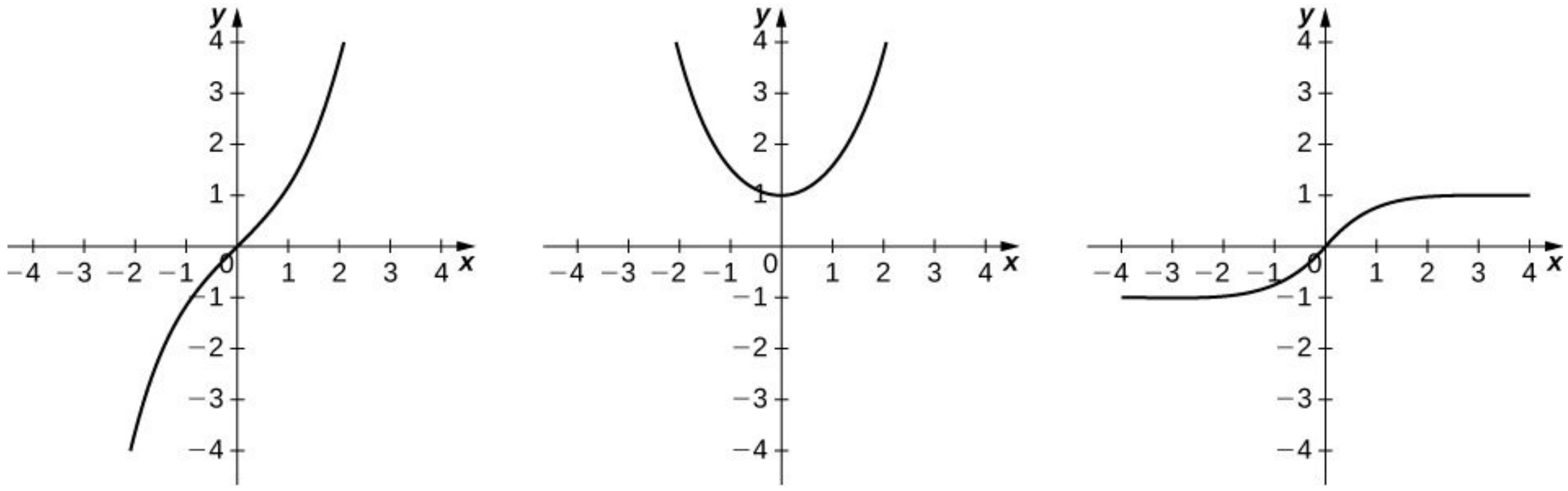


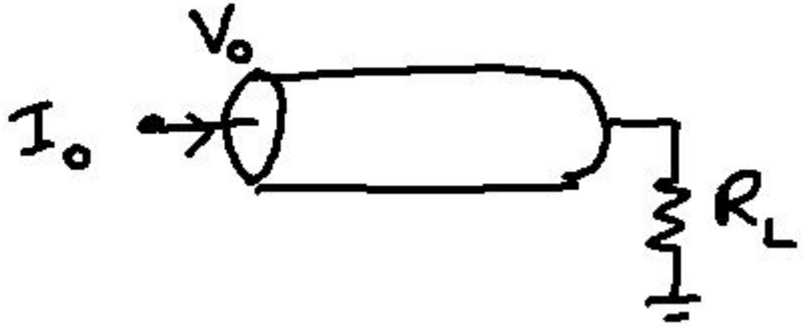
$y = \sinh x$
(a)

$y = \cosh x$
(b)

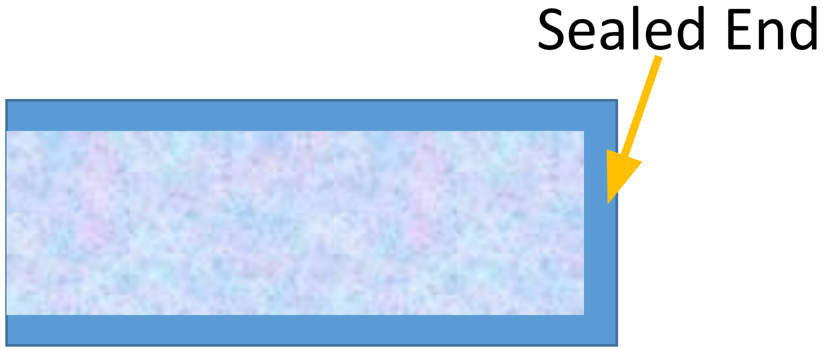
$y = \tanh x$
(c)

- Now we consider three different boundary conditions under which we solve the steady state voltage distribution of a finite length cable:

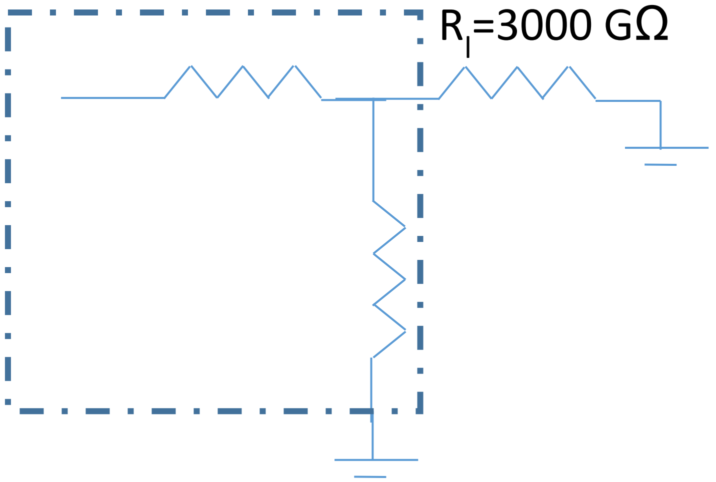
1. Sealed end (R_L = infinity)
2. Killed end (R_L = 0)
3. Arbitrary (R_L = finite)



Sealed-end Boundary Condition



- Physically this refers to the situation when the dendrite is sealed/closed with a membrane patch.
- Electrically this is equivalent to loading the circuit of the dendrite at the far end with a resistance equal to that of membrane patch that sealed the dendritic cable.



- For dendritic cable of diameter, d of 2μm , and R_m of 10^5 Ω.cm^2, the loading resistance R_L is, R_L = 3000G Ω.

$$R_L = \frac{R_m}{\pi d^2/4} = 3000G\Omega$$

- Since the sealed end has high resistance we can approximate it with an open circuit, implying that axial current, Ii =0 at the sealed end (x=L).

- Since the sealed end has high resistance we can approximate it with an open circuit, implying that axial current, I_i =0 at the sealed end (x=L).
- Therefore, from eqn. (1),

$$-\frac{\partial V_m}{\partial x} = r_a I_i(x,t) \qquad \qquad \left.\frac{\partial V_m}{\partial x}\right|_{X=L} = 0$$

- Substituting the above boundary condition in the solution of eqn. (14), we have,

$$\left.\frac{\partial V_m}{\partial x}\right|_{X=L} = (-A\sinh(L-X) - B\cosh(L-X))/\lambda = 0$$

or, B=0. Therefore,

$$V_m(X) = A\cosh(L-X)$$

Assuming V_0 is the voltage at the near-end (x=0=X) of the cable at steady state, the solution can be written as,

$$V_0 = A\cosh(L) \qquad A = V_0/\cosh(L) \qquad V_m(X) = V_0\cosh(L-X)/\cosh(L)$$

- At X=0, I_i is,

$$I_i = -\frac{1}{r_a}\frac{\partial V_m}{\partial x} = \frac{V_0}{\cosh(L)}(-\sinh(L-X)(-\frac{1}{r_a\lambda}) = \frac{V_0}{R_\infty}\frac{\sinh(L-X)}{\cosh(L)}$$

$$= \frac{V_0}{R_\infty}\tanh(L)$$

(At X = 0)

$$\therefore R_{in} = R_\infty \coth(L)$$

Killed End Boundary Condition

- The previous case of ‘sealed-end boundary condition’ refers to the situation when the far end is an open circuit.
- The present case of ‘killed end boundary condition’ the end of the terminal is ‘shorted’ so that **the voltage at the far end is zero**.
- Physically this situation can be created by cutting (“killing”) the far end so that the interior of the dendrite is directly in contact with the extracellular space.

- The form of the solution for this case is again,

$$V_m(X) = A\cosh(L-X) + B\sinh(L-X) \qquad (14)$$

Boundary conditions:

$$V_m(X) = 0$$ at X = L

$$\therefore A = 0$$

$$V_m(X) = V_0$$ at X = 0

$$\therefore B = \frac{V_0}{\sinh(L)}$$

$$V_m(X) = V_0 \frac{\sinh(L-X)}{\sinh(L)}$$

Input resistance

- Calculating Input current:

$$I_i = -r_a \frac{\partial V_m}{\partial x}\bigg|_{x=0}$$

$$\frac{\partial V}{\partial x} = -\frac{V_0}{\lambda}\frac{\cosh(L-X)}{\sinh(L)}$$

$$I_i = -\frac{1}{r_a}\frac{\partial V}{\partial x}\bigg|_{x=0} = \frac{V_0}{\lambda r_a}\coth(L) \qquad (\because\ X = \frac{x}{\lambda})$$

$$\therefore R_{in} = \frac{V_0}{I_i} = \lambda r_a \tanh(L) = R_\infty \tanh(L)$$