# Modeling the Neuron components

BT6270 Introduction to Computational Neuroscience

# Single Neuron Modeling

Action Potential generation

Hodgkin-Huxley Model

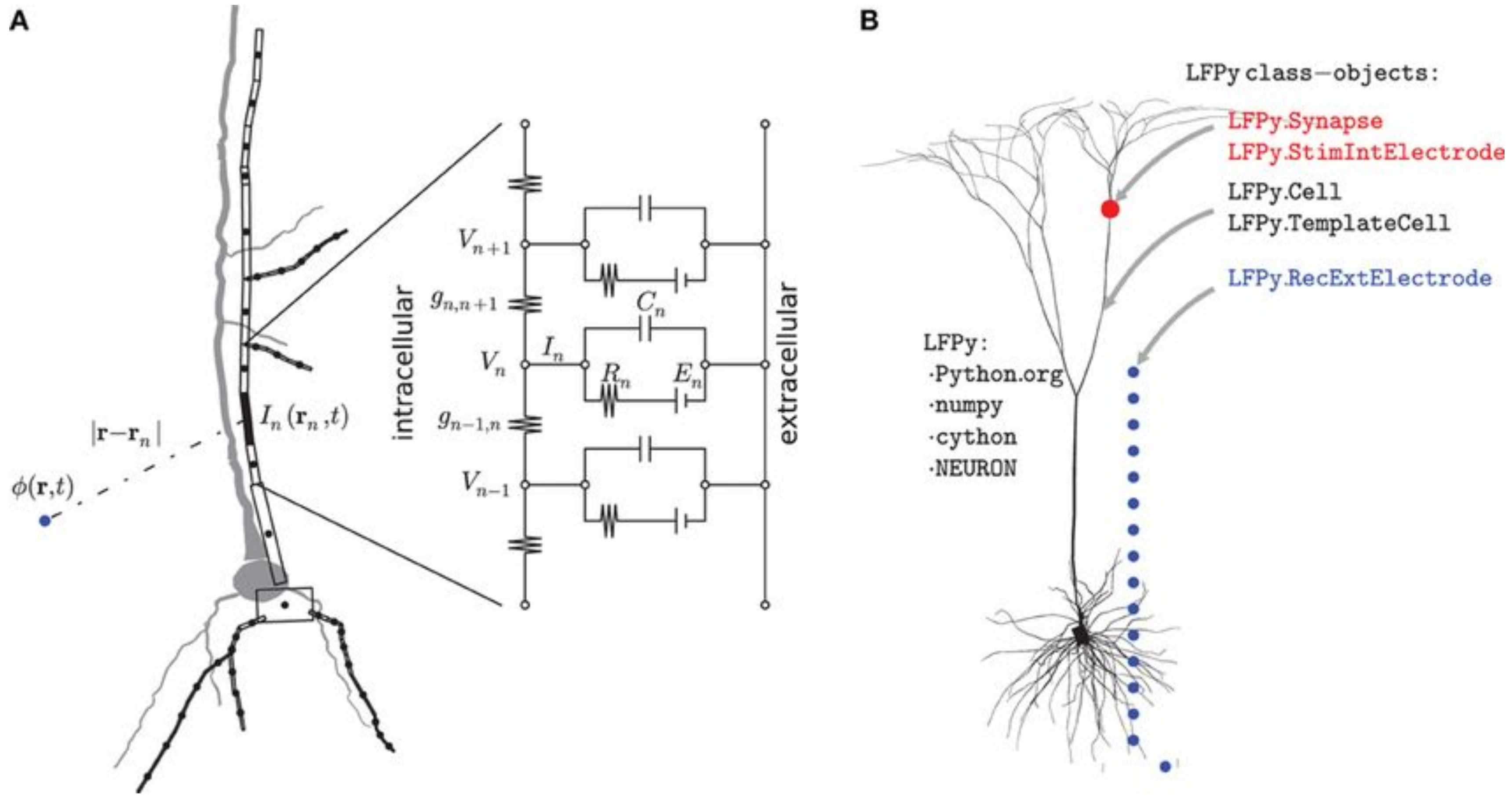
More Detailed

<u>Biophysical Modeling</u>
Dendritic Processing
Axonal Processing
Synaptic Transmission

Less Detailed

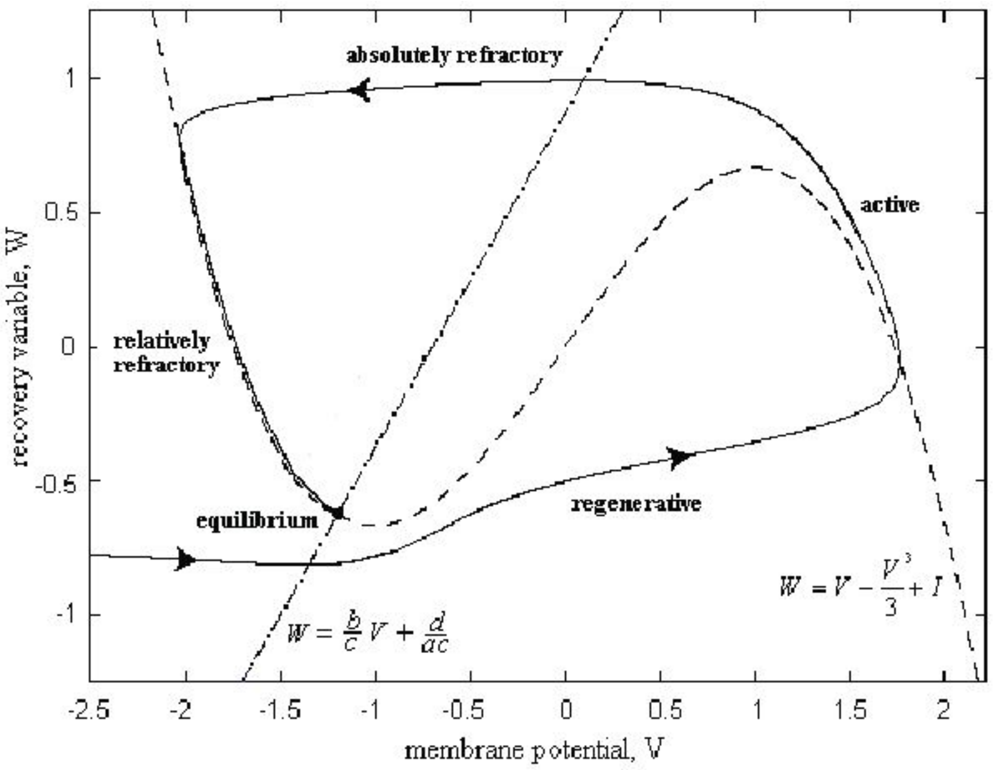
<u>Simplified Neuron Models</u>
FitzHugh-Nagumo model
Morris-Lecar Model
Izhikevich Models
Integrate and fire neuron
McCulloch-Pitts neuron

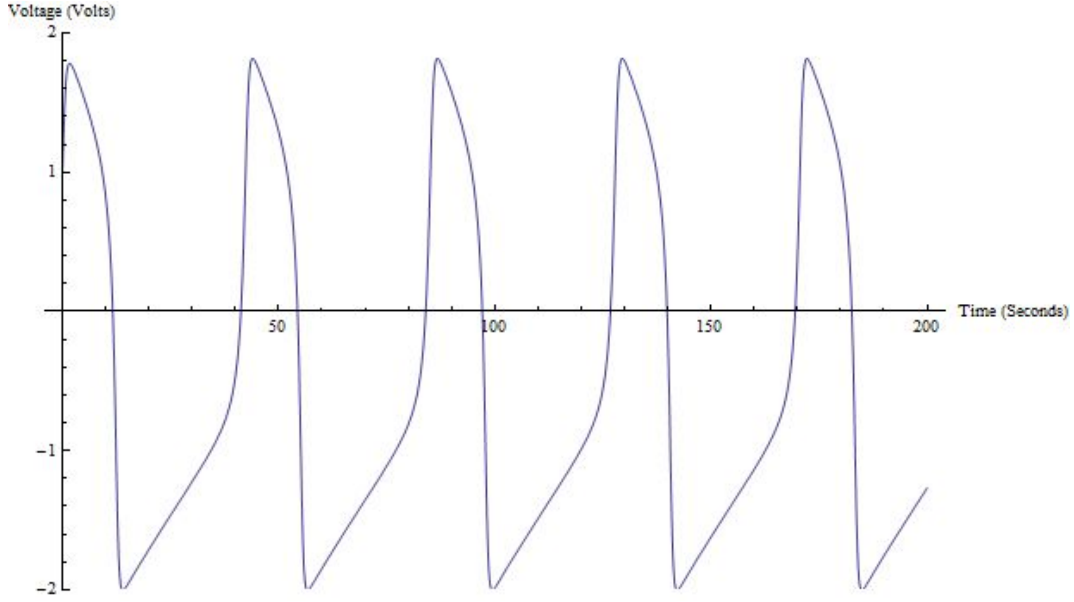
# Biophysical modeling



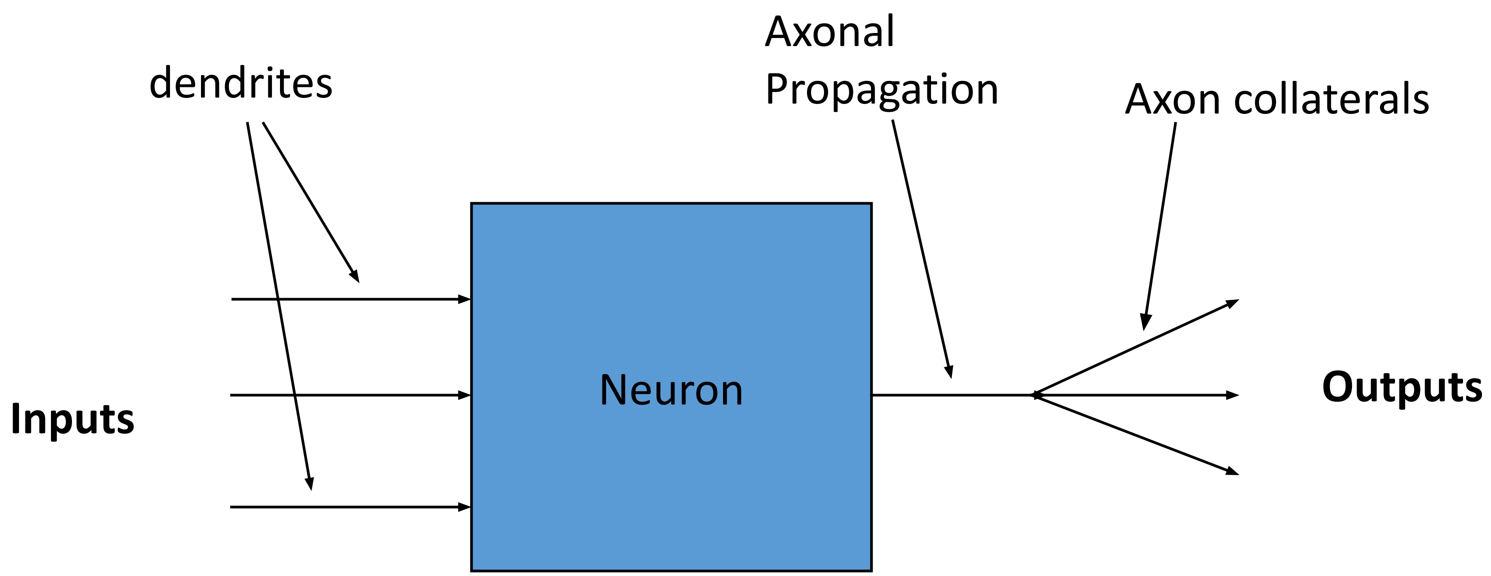

https://www.researchgate.net/publication/259961755_LFPy_A_tool_for_biophysical_simulation_of_extracellular_potentials_generated_by_detailed_model_neurons/figures?lo=1

# Simplified (2-variable) neuron models



# Biophysical modeling: Neuron Signaling components

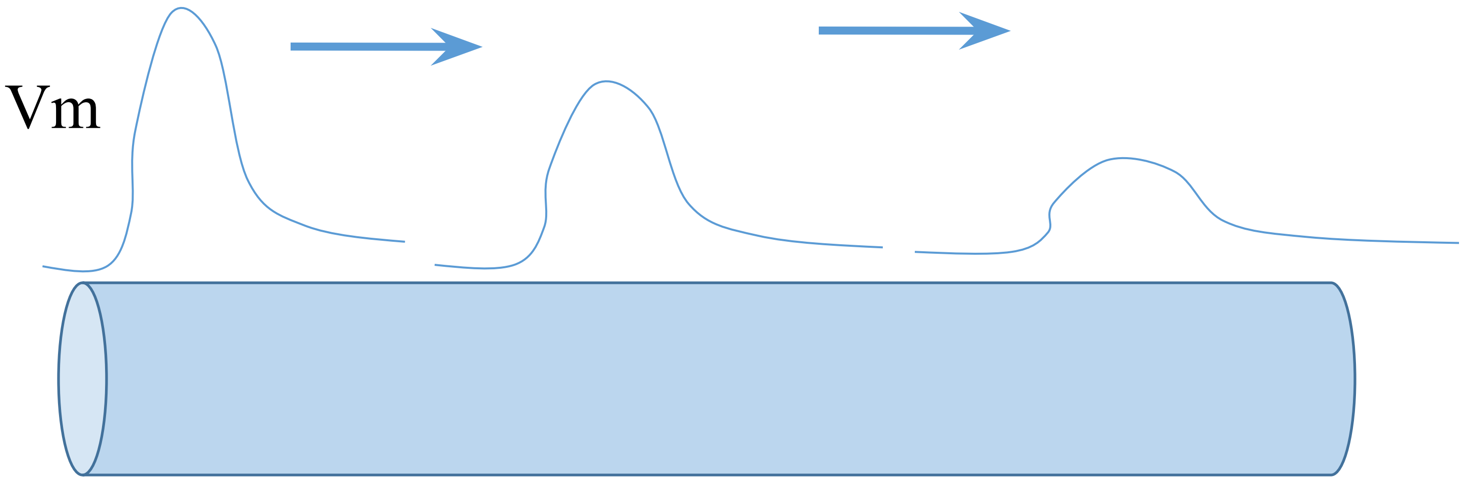


# The 4 Neuron Signaling components

- Dendritic Processing: Signal propagation along dendrites is mostly passive, as along an electrical cable;
- Action Potential Generation: Summation occurs in the axon hillock; action potential is generated
- Axonal Propagation: action potential propagates down the axon without losing amplitude because it is charged all along the way by voltage-sensitive channels;
- Neurotransmission occurs across a synapse – as though there is a “hotline” from axon terminal A to apical dendrite B – via chemical means;
- This whole sequence of events occurs in a neat unidirectional fashion from the apical dendrites to axon terminals.

# Dendritic propagation

# Wave propagation over a dendrite

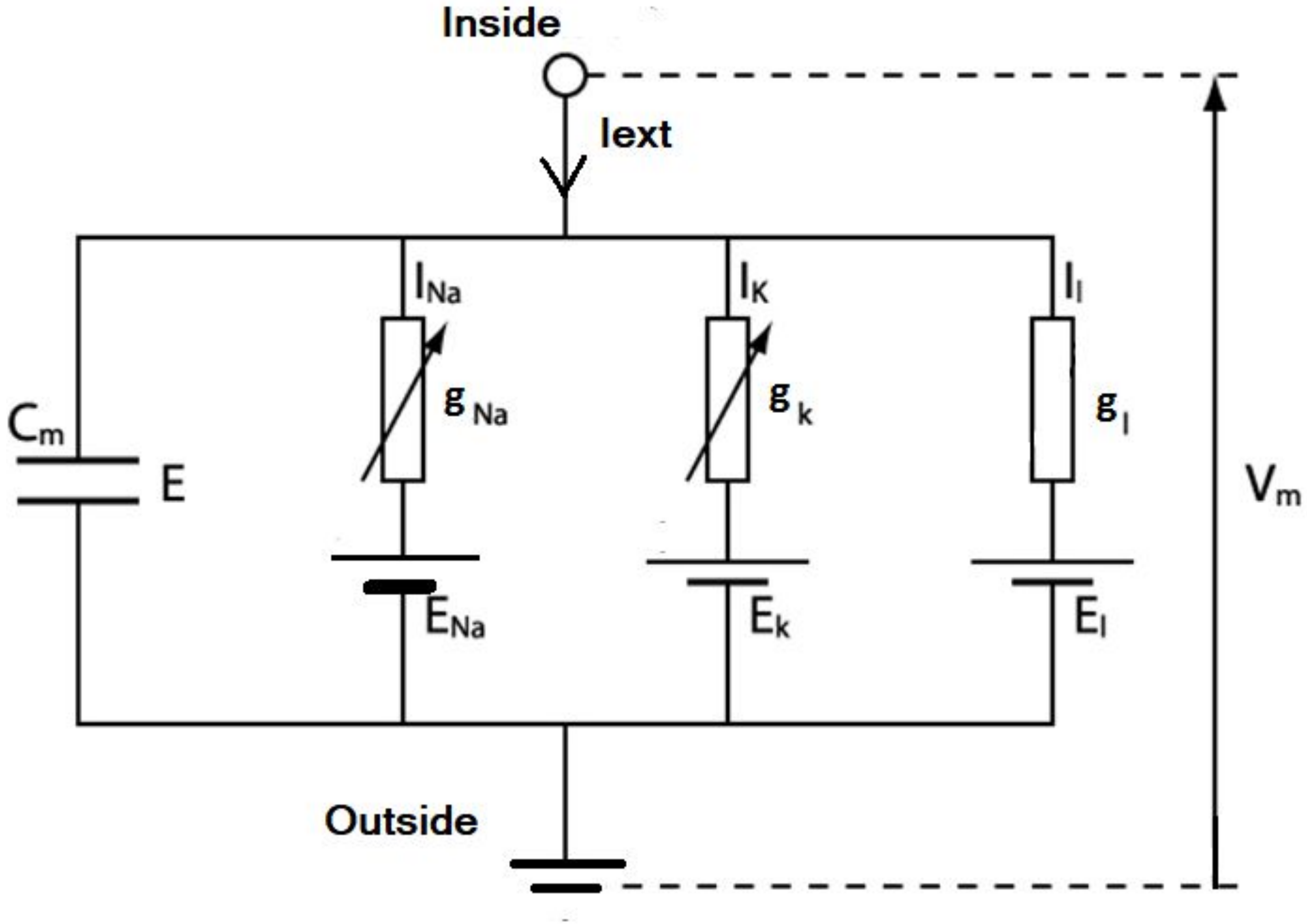



As the wave propagates down the dendrite:

📫 It loses amplitude

📫 It spreads in time

# Membrane with voltage-sensitive channels



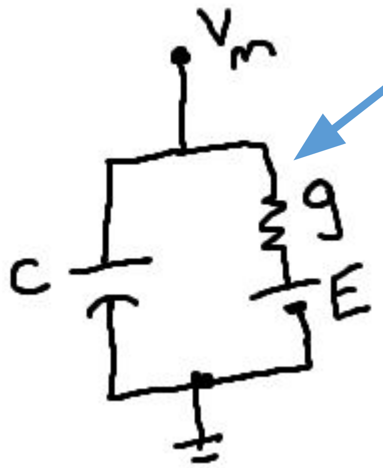
# Dendritic processing: Passive cable

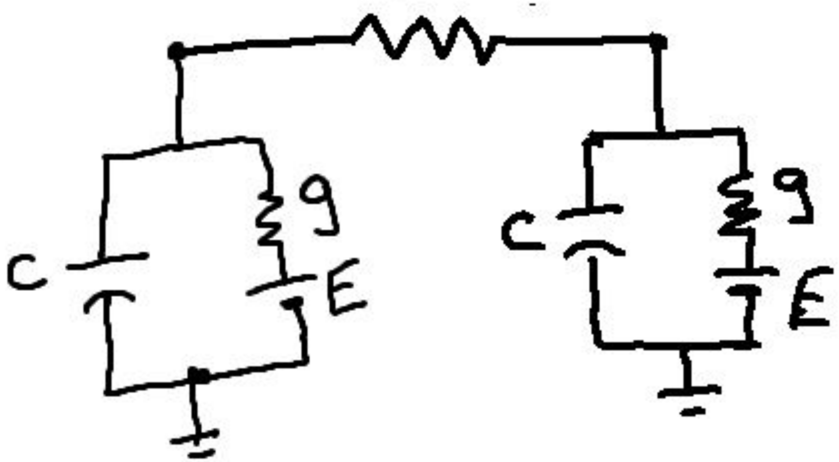
No active elements

No voltage-dependent ion channels

All the conductance branches merged to one

No “arrow” on g




At a single point on a membrane

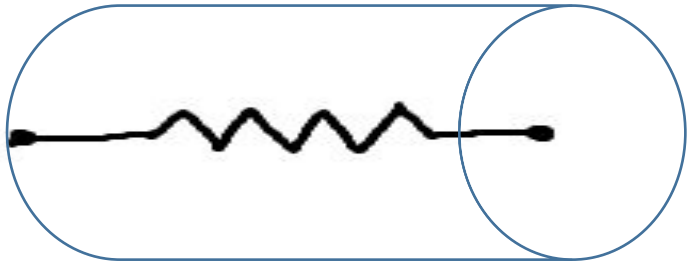
Along the length of the membrane

Reference:
C. Koch, Biophysics of Computation.

# Formulating the cable equation

# Axial resistance

- Resistance offered by the intracellular compartment per unit length of the cable of diameter, d. The resistivity of the intracellular medium is $R_i$.
- We now relate the resistivity, $R_i$, which is an intrinsic property of the intracellular medium, to axial resistance, $r_a$, which is resistance per unit length of the cable.



# Axial resistance

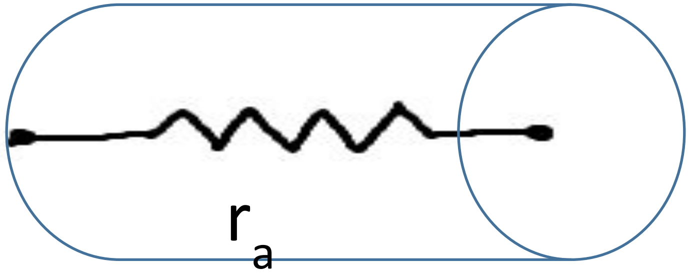
- In general the resistance, R, and resistivity, ρ, of a pipe of area of cross-section, A, and length, L,  are related as:

  R = ρ L/A, and

- Resistance per unit length (L=1) of the pipe is:

  R = ρ/A

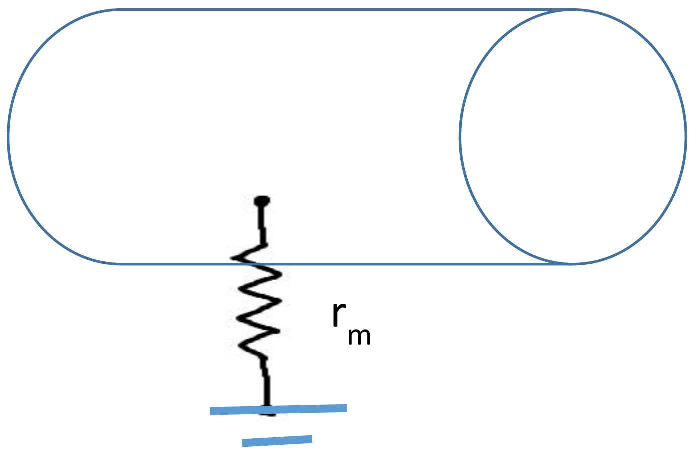
- A similar relation for our cable is:




$$r_a = \frac{R_i}{A} = \frac{R_i}{\pi d^2/4} = \frac{4R_i}{\pi d^2} \quad (\Omega/\mathrm{cm})$$

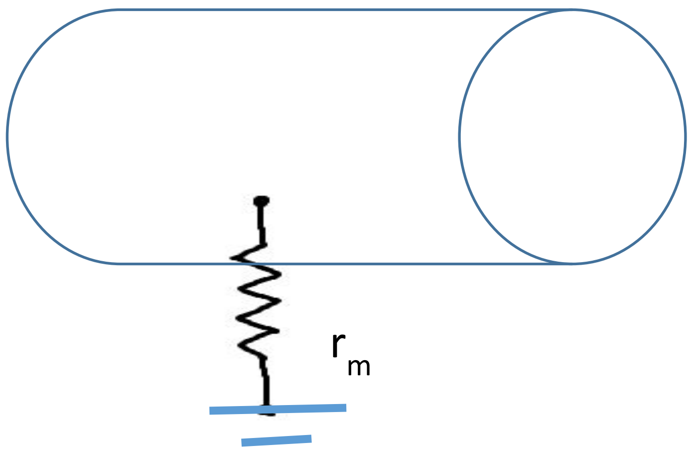
# Membrane resistance

- The membrane offers resistance for flow of current between the intracellular compartment and the extracellular space. This resistance is inversely proportional to the surface area of the membrane.



# Membrane resistance

- Therefore, if $R_m$ is the resistance of a membrane patch of unit area, a quantity referred to as specific resistance, the total resistance offered by a cylinder of diameter, d, and unit length (L=1), is given as:




$g_m = G_m A = G_m (\pi d L)$

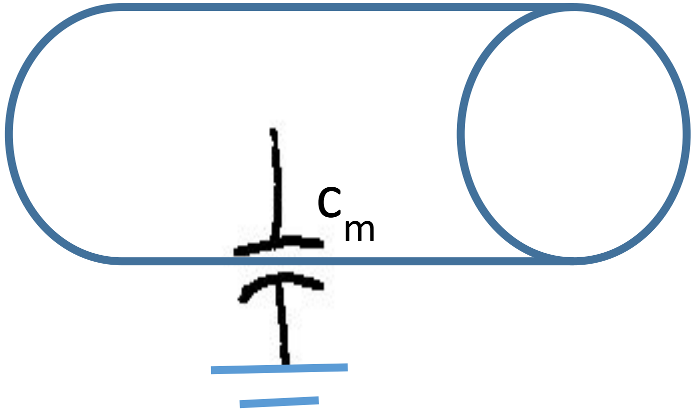
$g_m = 1/ r_m; G_m = 1/R_m$

$$r_m = \frac{R_m}{A} = \frac{R_m}{\pi d.1} = \frac{R_m}{\pi d} \qquad (\Omega \text{ cm})$$

# Membrane capacitance

- The plasma membrane has a specific capacitance, $C_m$, of about $10^{-6}$ F/cm$^2$.
- Therefore, capacitance of the cable of unit length, $c_m$, is,

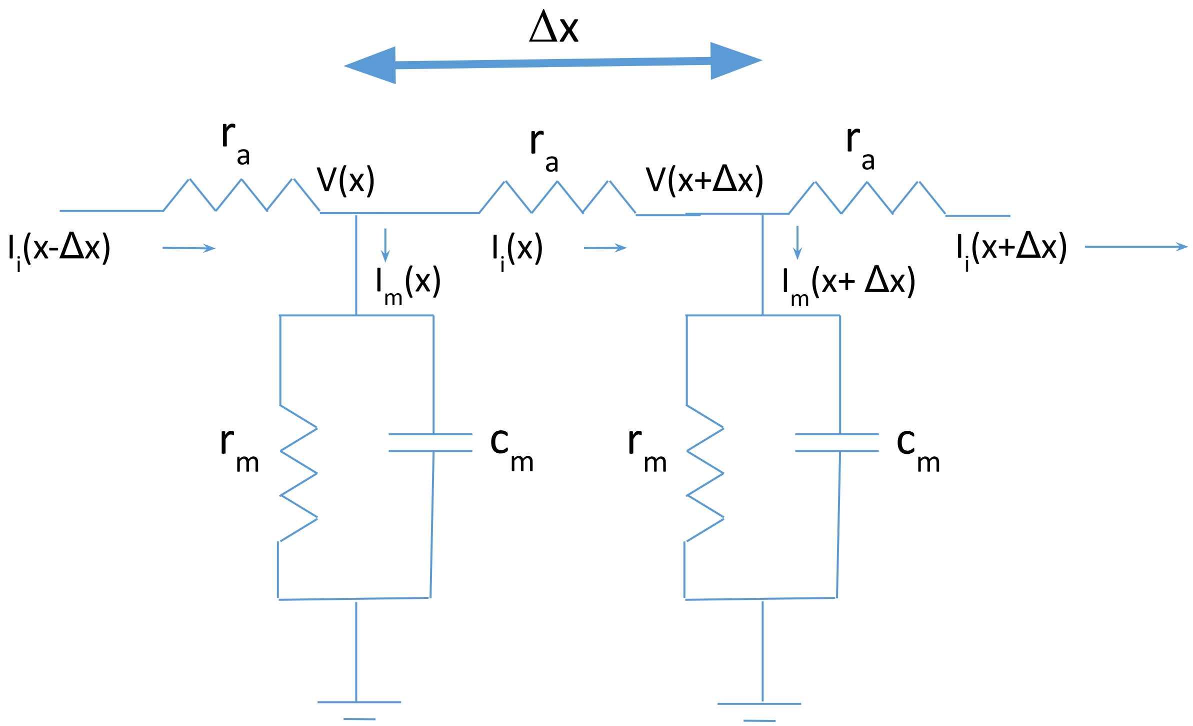
$$c_m = C_m \pi d \quad \text{(F/cm)}$$



# The cable as an electric circuit

- Using the electrical parameters defined above, we can now represent the cable as an electric circuit.
- In this circuit, the continuous cable is represented as a series of discrete circuit elements, in which each element approximates a short length of the cable, say, of length, $\Delta x$.

# Circuit equivalent for a dendrite



# Ohm’s law for axial resistance

- Applying Ohm’s law to one of the axial resistances, $r_a \Delta x$,

$$V_m(x,t) - V_m(x+\Delta x,t) = \Delta x\, r_a I_i(x,t)$$

$$-\frac{\partial V_m}{\partial x} = r_a I_i(x,t) \qquad (1)$$

# Kirchoff’s current law at the nodes

- Applying the law of continuity of current at a given node in the circuit

$$I_i(x,t) - I_i(x-\Delta x,t) = -\Delta x\, I_m(x,t)$$

$$-\frac{\partial I_i}{\partial x} = I_m(x,t) \qquad (2)$$

- Combining (1) and (2),

$$\frac{1}{r_a}\frac{\partial^2 V_m}{\partial x^2} = I_m(x,t) \qquad (3)$$

- Now, the membrane current, $I_m$, can be resolved into three components:

    1) current through membrane capacitance,

    2) current through membrane resistance,

    3) externally injected current, $I_{ext}$, if any.

# Linear Cable Equation

$$I_m(x,t) = \frac{V_m - V_{rest}}{r_m} + c_m \frac{\partial V_m}{\partial t} - I_{ext} \qquad (4)$$

- Combining (3) and (4), we get equation 5, known as the Linear Cable Equation.

$$\lambda^2 \frac{\partial^2 V_m}{\partial x^2} = V_m - V_{rest} + \tau_m \frac{\partial V_m}{\partial t} - r_m I_{ext} \qquad (5)$$

where

$\tau_m = r_m c_m$ known as the time constant, of the cable.

$\lambda = \sqrt{\frac{r_m}{r_a}}$ known as the space constant, and,

- Eqn. 5 can be further simplified if the membrane voltage, is defined with reference to the resting potential, $V_{rest}$.
- Assuming that the resting potential is the same everywhere along the cable, it only offsets the membrane potential and does not affect the derivative terms in eqn. (5).

# The Cable Equation

- Thus, from now on, if we designate $V_m$ to represent the deviation of membrane potential from the resting potential, $V_{rest}$, the $(V_m - V_{rest})$ term in eqn. (5) can be replaced by, $V_m$, and we have the following simpler form.

$$\lambda^2 \frac{\partial^2 V_m}{\partial x^2} = V_m + \tau_m \frac{\partial V_m}{\partial t} - r_m I_{ext} \qquad (6)$$

# The cable equation ???!!!