Infinite Cable - Steady State Analysis:

BT6270 Introduction to Computational Neuroscience

$$\lambda^2 \frac{\partial^2 V_m}{\partial x^2} = V_m + \tau_m \frac{\partial V_m}{\partial t} - r_m I_{ext}$$

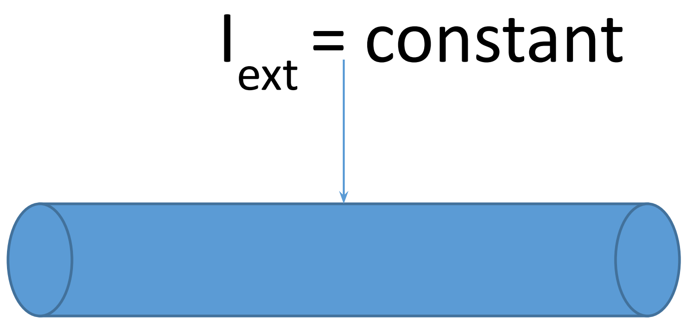
Consider Voltage distribution under steady state conditions

V(x,t) □ V(x)

Infinite Cable - Steady State Analysis:

- Though our ultimate objective is to be able to describe signal transmission along the cables with complex geometries, we begin with a simple situation.

- We consider an infinite cable in which a constant current is injected at a point. The goal is to determine membrane voltage distribution under steady state conditions.



- External Current:

$$I_{ext} = I_0\, \delta(x)\, u(t) \qquad (7)$$

spatially it is a point source; and temporally it is a step function.

- Boundary conditions:

$$V(x,t) = 0, \;\; at \; |x| \to \infty, \forall t \qquad (8.a)$$

- Initial condition:

$$V(x,0) = 0, \;\; x \in (-\infty, \infty) \qquad (8.b)$$

$$\lambda^2 \frac{\partial^2 V_m}{\partial x^2} = V_m + \tau_m \frac{\partial V_m}{\partial t} - r_m I_{ext}$$

- Under steady state conditions,

$$\frac{\partial V_m}{\partial t} = 0$$

- Therefore, eqn. (6) becomes,

$$\lambda^2 \frac{\partial^2 V_m}{\partial x^2} = V_m - r_m I_{ext} \qquad (9)$$

- Since (9) is a II order equation, consider a solution of the form:

$$V_m(x) = Ae^{\lambda x} + Be^{-\lambda x} \qquad (10)$$

- Membrane voltage is represented as, $V_m(x)$.

- Now let us apply the boundary conditions eqn. (8), to the solution eqn. (10).
- Since $V_m(x)$ tends to 0 at +inf, A=0, and since $V_m(x)=0$ at –inf, B = 0.
- This difficulty can be overcome if we let the form of the solution be,

$$V_m(x) = V_0 e^{-|x|/\lambda} \tag{11}$$

where V_0 is the steady state voltage at x=0.

- Let us try to verify that $V_m(x)$ of eqn. (11) satisfies eqn. (9).

$$\frac{\partial V_m(x)}{\partial x} = -\frac{V_0}{\lambda} e^{-|x|/\lambda} sign(x)$$

$$\frac{\partial^2 V_m(x)}{\partial x^2} = \frac{V_0}{\lambda^2} e^{-|x|/\lambda} - 2\frac{V_0}{\lambda} e^{-|x|/\lambda} \delta(x) = \frac{V_0}{\lambda^2} e^{-|x|/\lambda} - 2\frac{V_0}{\lambda} \delta(x)$$

$$\lambda^2 \frac{\partial^2 V_m(x)}{\partial x^2} = V_0 e^{-|x|/\lambda} - 2V_0 \lambda \delta(x)$$

<u>Use:</u>

$(sign(x))^2 = 1$

$d(sign(x))/dx = 2\delta(x)$

$f(x)\delta(x) = f(0)\,\delta(x)$

- Comparing the last equation with eqn. (9), we have

$$V_0 = \frac{I_0 r_m}{2\lambda} \qquad (12)$$

- Therefore, the final form of steady state membrane voltage of an infinite cylinder is,

$$V_m(x) = \frac{I_0 r_m}{2\lambda} e^{-|x|/\lambda} \qquad (13)$$

$$\lambda = \sqrt{\frac{r_m}{r_a}} = \sqrt{\frac{R_m}{R_i} . \frac{d}{4}}$$

Electrotonic Distance and Input Resistance

- Any length, l, can be expressed as electrotonic distance, L, as,

$$L = \frac{l}{\lambda}$$

- Input resistance, R_{in}, is defined as the ratio of voltage at the point of current injection to the magnitude of current injected.

$$R_{in} = \frac{V(x=0)}{I_0} = \frac{r_m}{2\lambda} = \frac{\sqrt{r_a r_m}}{2} = \frac{r_a \lambda}{2}$$

Example:

Consider a infinite cable with the static current I_0= 10 mA injected at point x=2. Plot the static solution for such a cable.

Solution:

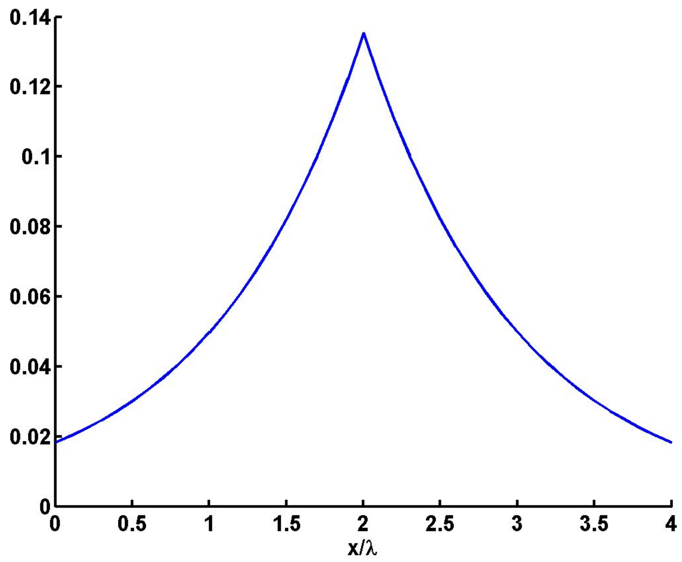
The static solution for the infinite cable is given by: .

$$V_m(x) = \frac{I_0 r_m}{2\lambda} e^{-|x|/\lambda}$$

Since current I_0=10 mA is injected at x=2,

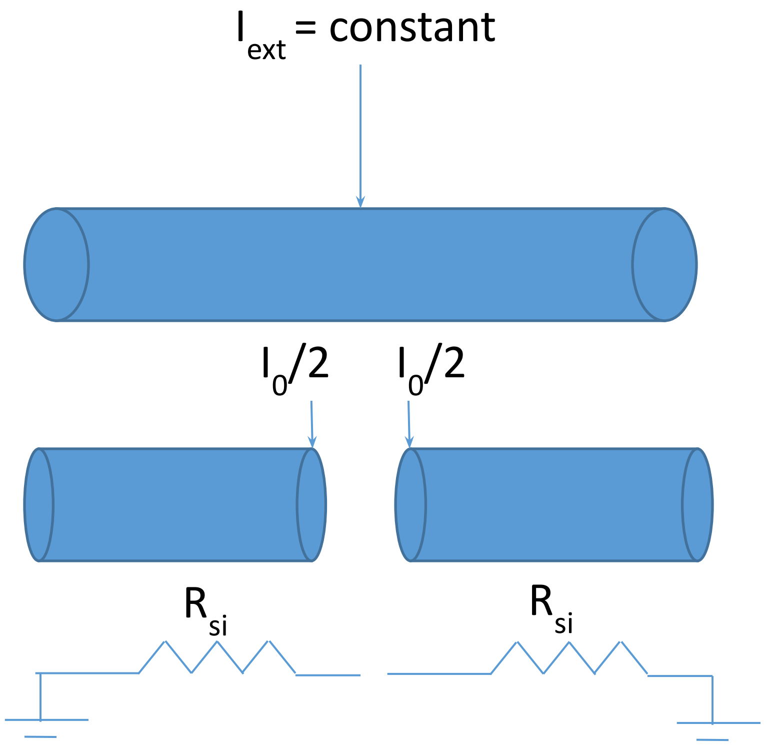
$I_{ext} = I_0\ \delta(x)$;

The plot of $V_m(x)$ vs x/λ would be:



Semi-infinite Cable:

- Let us consider the case of a semi-infinite cable where current of magnitude, I_0, is injected into one end of the cylinder.



- Since an infinite cable can be viewed as two semi-infinite cables in parallel, steady state membrane voltage of a semi-infinite cable is twice that of the infinite cable, and is given as,

$$V_m(x) = \frac{I_0 r_m}{\lambda} e^{-|x|/\lambda}$$

- Similarly, the input resistance, which is naturally twice that of an infinite cable, is,

$$R_{in} = \frac{V(x=0)}{I_0} = \frac{r_m}{\lambda} = \sqrt{r_a r_m} = r_a \lambda \equiv R_\infty$$

R_∞ is called the input resistance of a semi-infinite cable.