COMPUTING WITH RHYTHMS:
THE SEARCH FOR OSCILLATORY DEEP NEURAL NETWORKS

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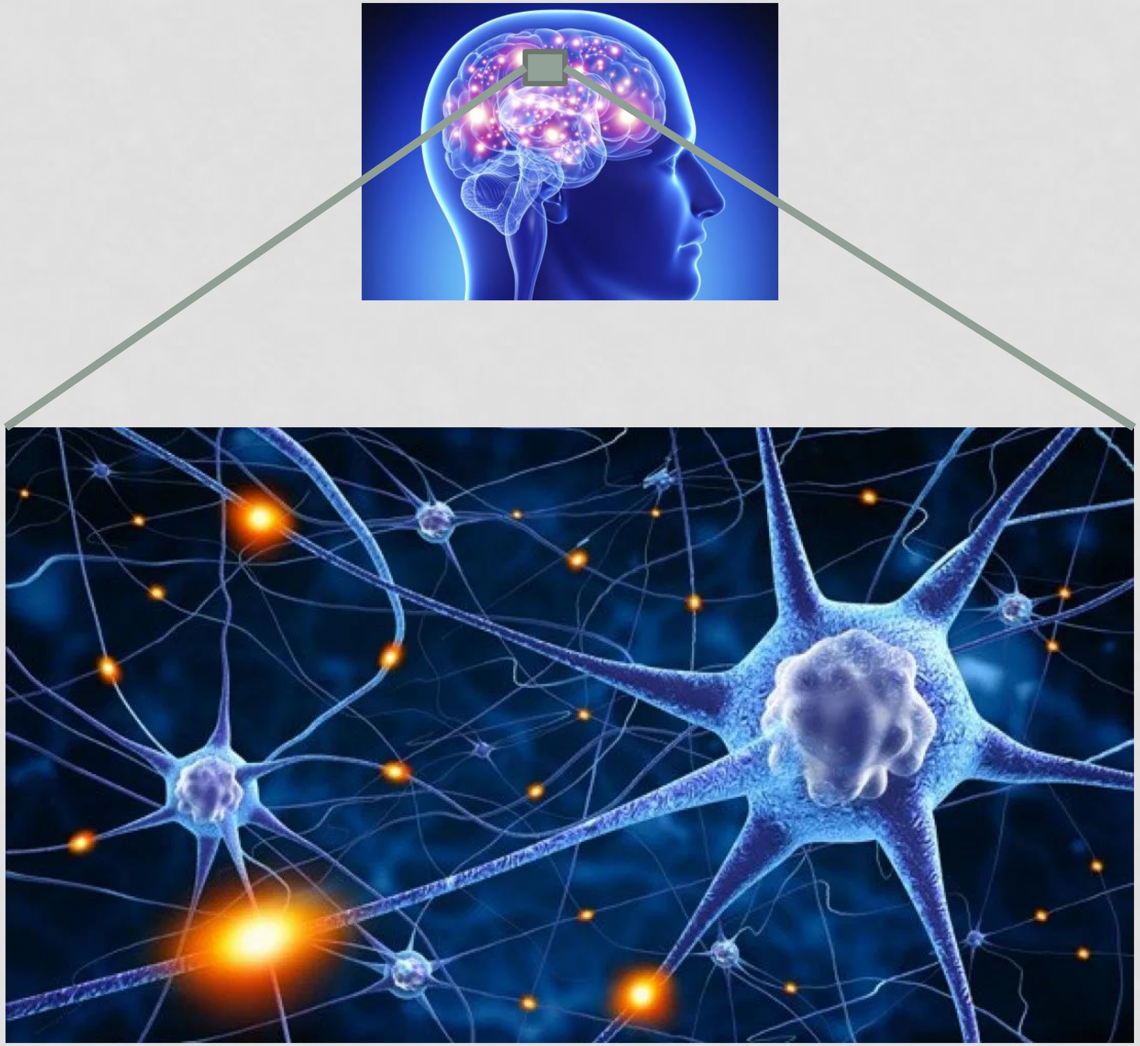
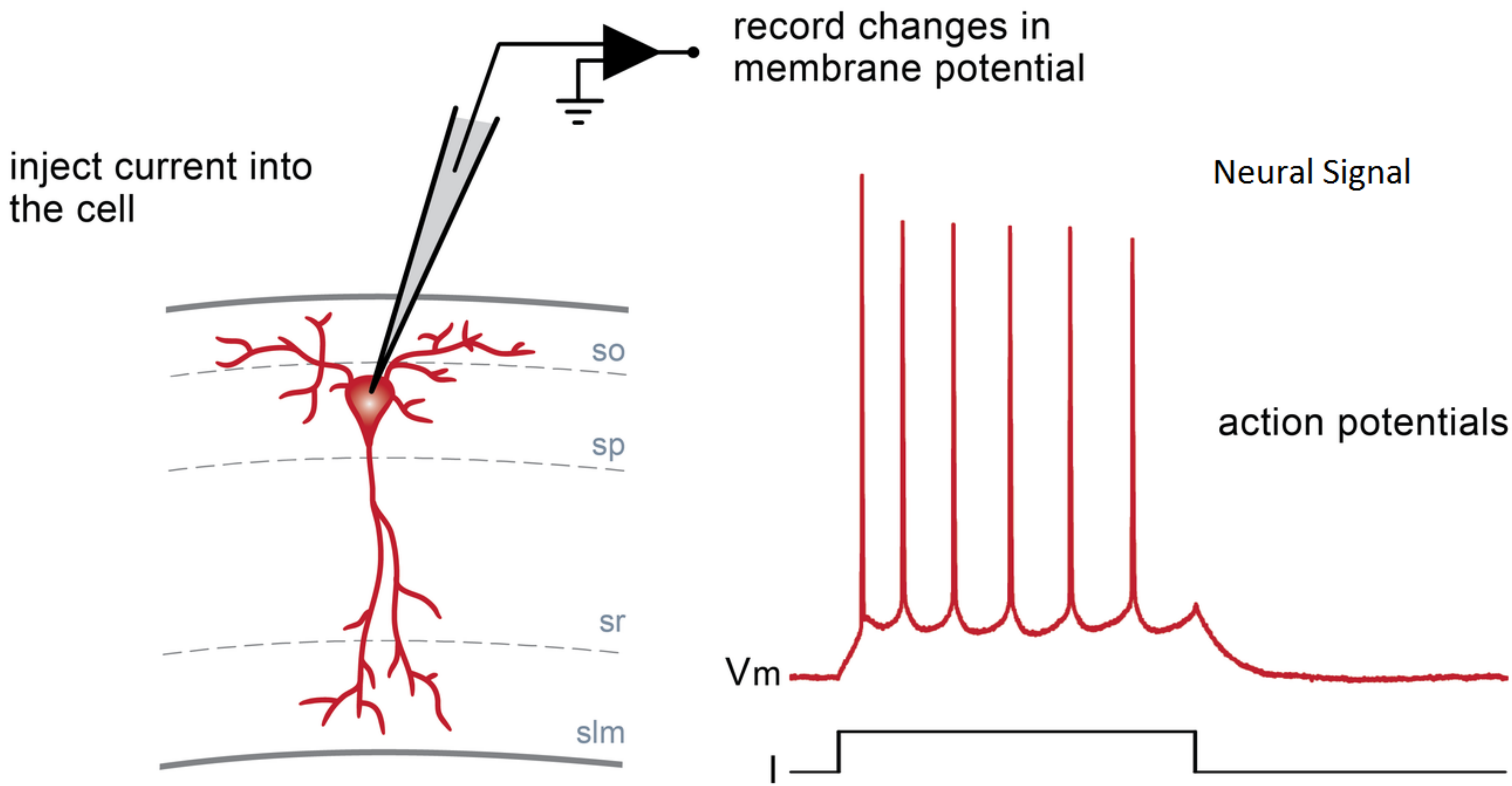
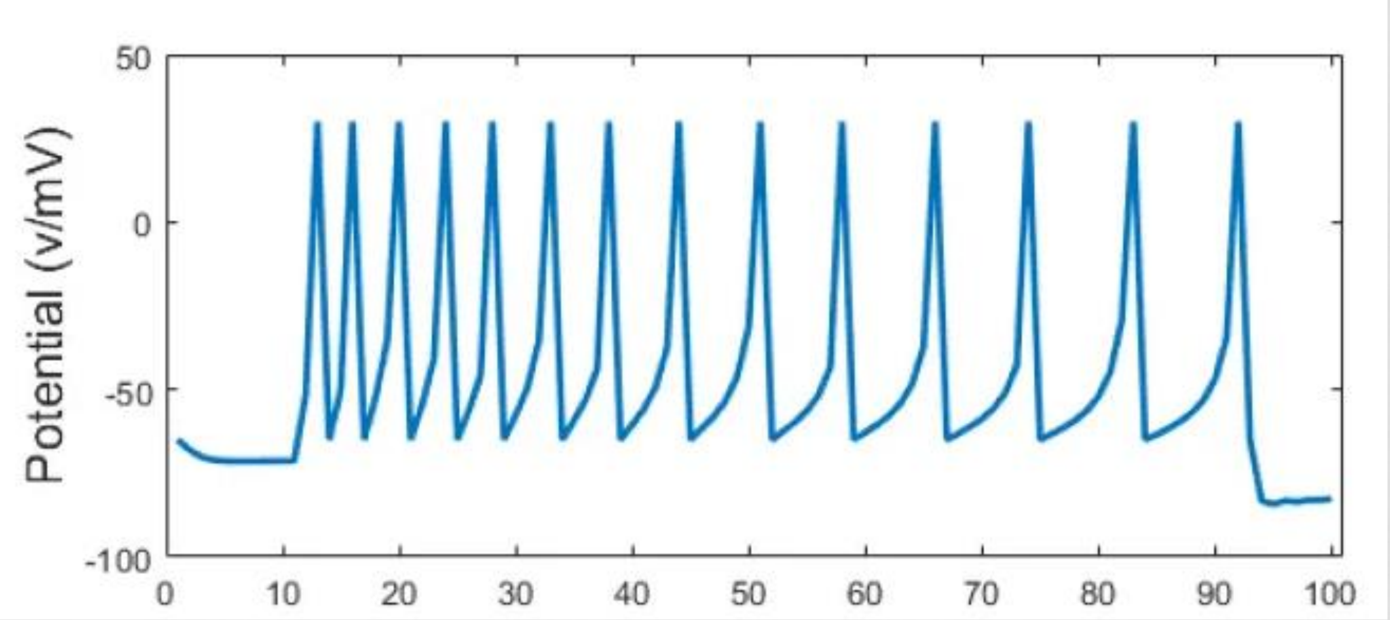


The Brain is a network of Neurons

Neurons communicate with each other by exchanging electrical signals


record changes in
membrane potential
inject current into
the cell
so
sp
sr
slm
Neural Signal
action potentials
Vm
I

WHAT IS THE NEURAL CODE?



How do you mathematically describe the information present in the electrical spiking activity of a neuron?

TWO THEORIES

Spike code

- Information is present in the times of occurrence of the spikes

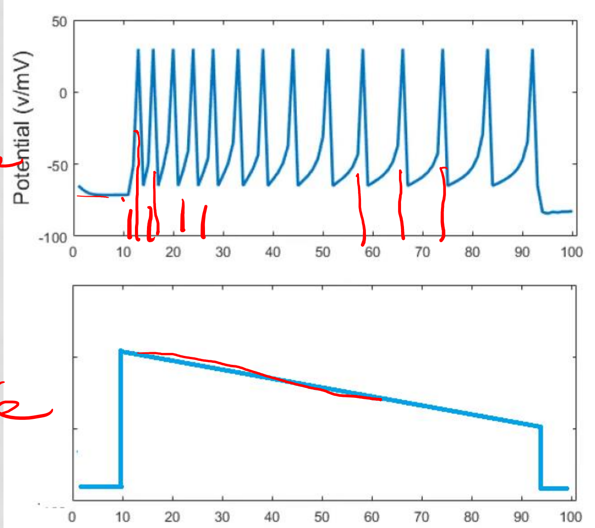
$$s(t) = \sum_i \delta(t - t_i)$$

Rate Code

- Information is present in the frequency (rate) of the spikes

Spike

rate



TWO KINDS OF NEURAL NETWORK MODELS

Spiking neuron networks

- Used often to model brain function

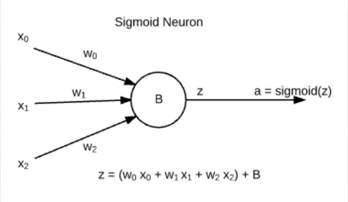
Rate coded neural networks

- Used more in artificial, engineering domains
- Eg. Deep neural networks

RATE-CODED NEURON MODELS

Used in Deep Neural Networks

Sigmoidal neuron - static, rate-coded



Relu neuron - static, rate-coded

$y = x, \; x > 0$
$= 0, \;$ other

Long Short-Term Memory block:

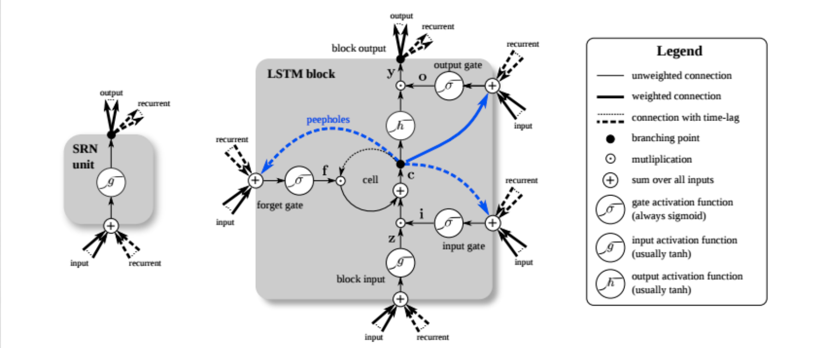


Figure 1. Detailed schematic of the Simple Recurrent Network (SRN) unit (left) and a Long Short-Term Memory block (right) as used in the hidden layers of a recurrent neural network.

DEEP LEARNING/AI

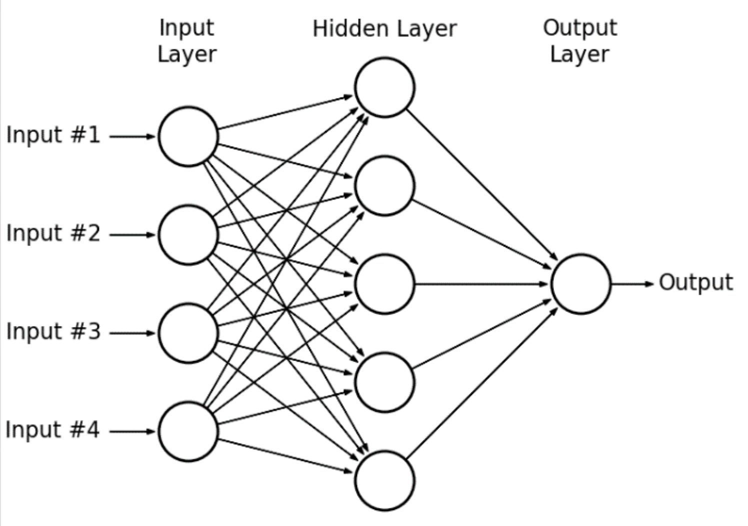
- The aim of Deep learning/AI is to reproduce human intelligence.
- The approach is to achieve it through an implementation of the human brain
- Deep Neural networks are offered as an answer

NNS AND DEEP NNS

- 1980s-1990s: Multilayer perceptrons and backpropagation

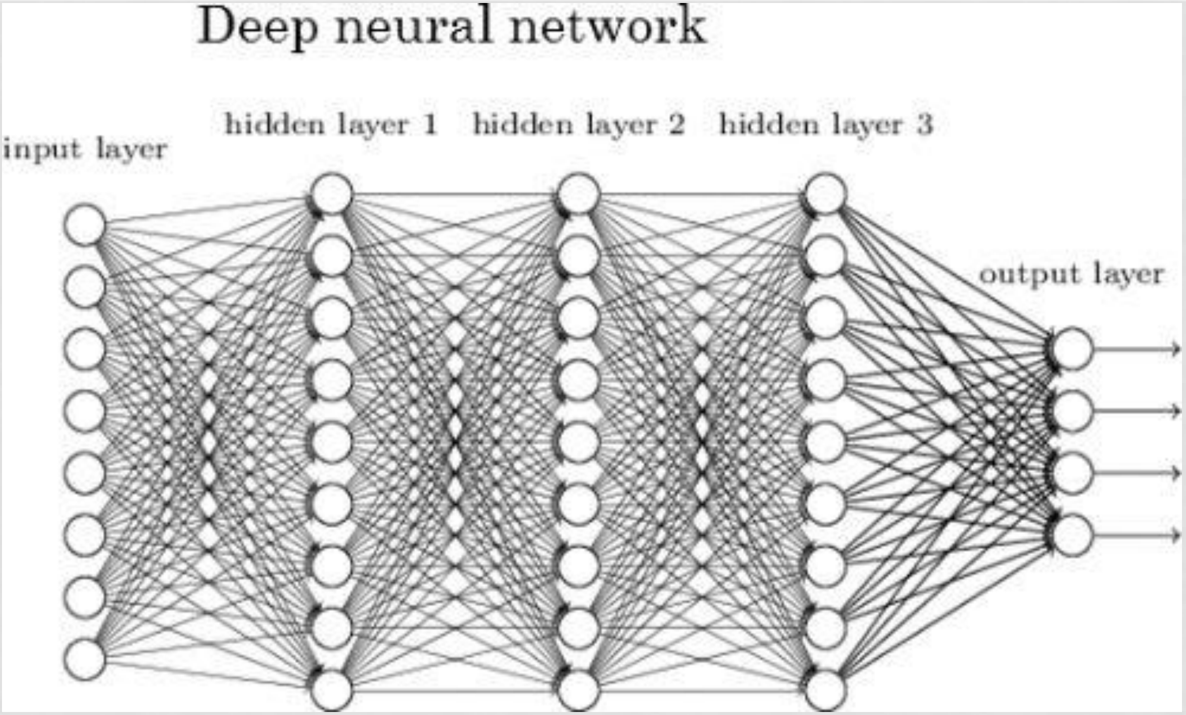
- Deep learning – 2006

- Powerful computational properties.
- Universal approximation results



Multi-layer Perceptron

DEEP NEURAL NETWORK

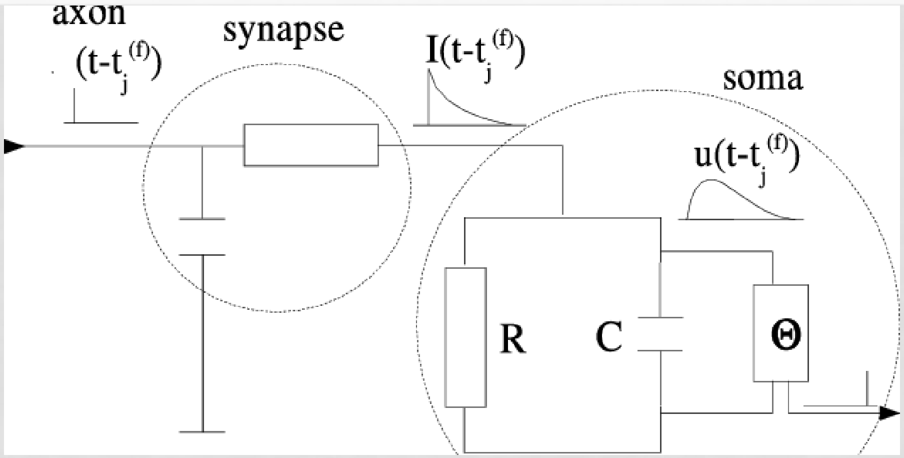


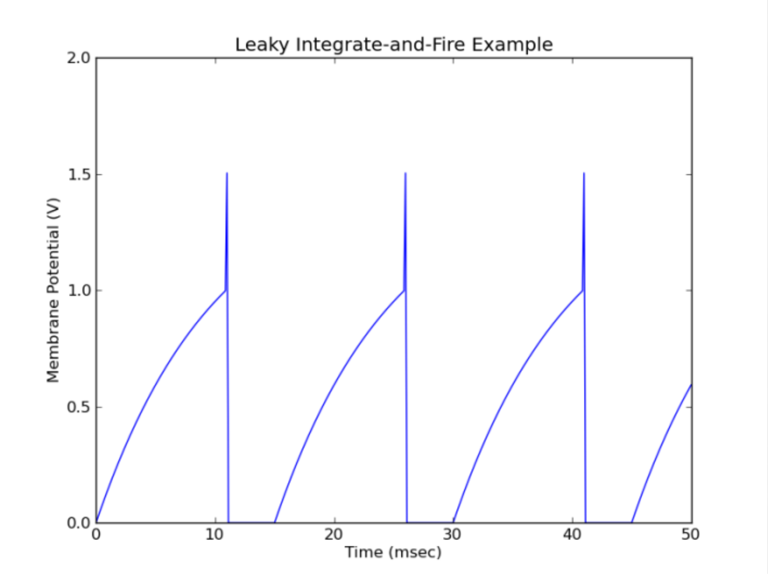
Has Universal Approximation properties
Can learn to map arbitrary input/output
Excellent applications in image pattern recognition,
Signal processing, robotics…

SPIKING NEURON MODELS

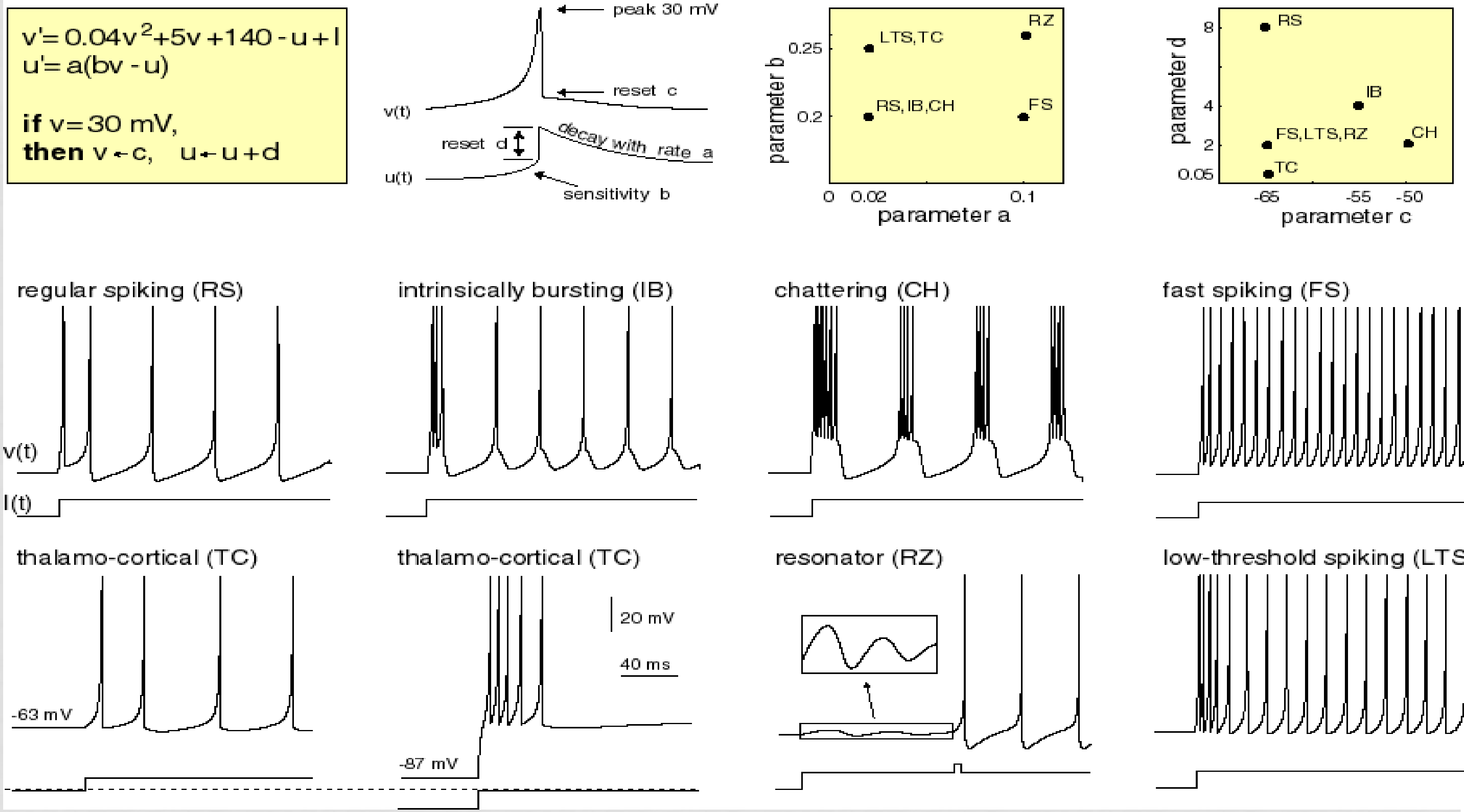
- Leaky Integrate and fire neuron (1 variable)
- Izhikevich neuron (2 variables)
- Hodgkin-Huxley neuron model (4 variables)
- Biophysical neuron models (large number of variables)

LEAKY INTEGRATE AND FIRE NEURON MODEL





IZHIKEVICH NEURON MODEL

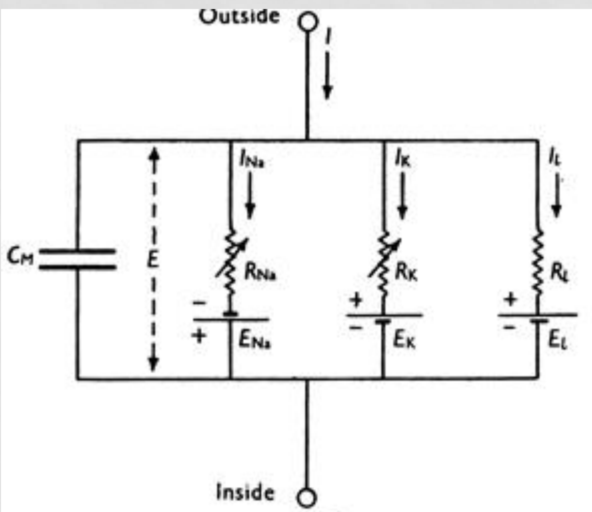


https://www.izhikevich.org/publications/whichmod.htm

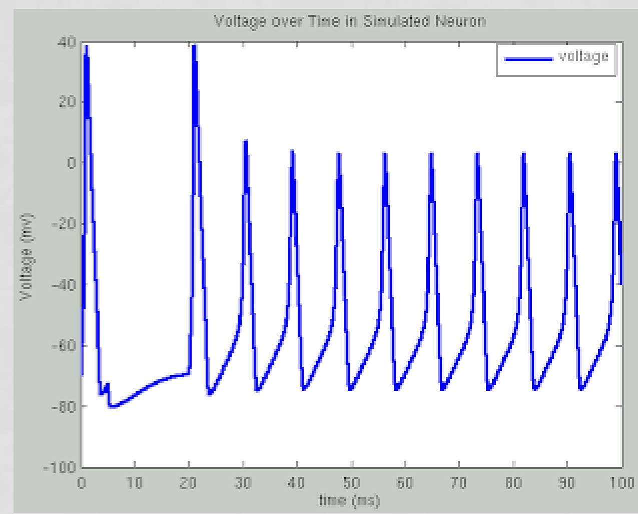
HODGKIN-HUXLEY NEURON MODEL

Hodgkin-Huxley Model (Example of a "point neuron" model)

$$
\begin{aligned}
\frac{dv}{dt} &= \frac{1}{C_m}[I - g_{Na}m^3h(v - E_{Na}) - g_k n^4(v - E_K) \\
&\quad -g_L(v - E_L)] \\
\frac{dm}{dt} &= \alpha_m(v)(1 - m) - \beta_m(v)m \qquad (1) \\
\frac{dm}{dt} &= \alpha_m(v)(1 - m) - \beta_m(v)m \\
\frac{dh}{dt} &= \alpha_h(v)(1 - h) - \beta_h(v)h
\end{aligned}
$$

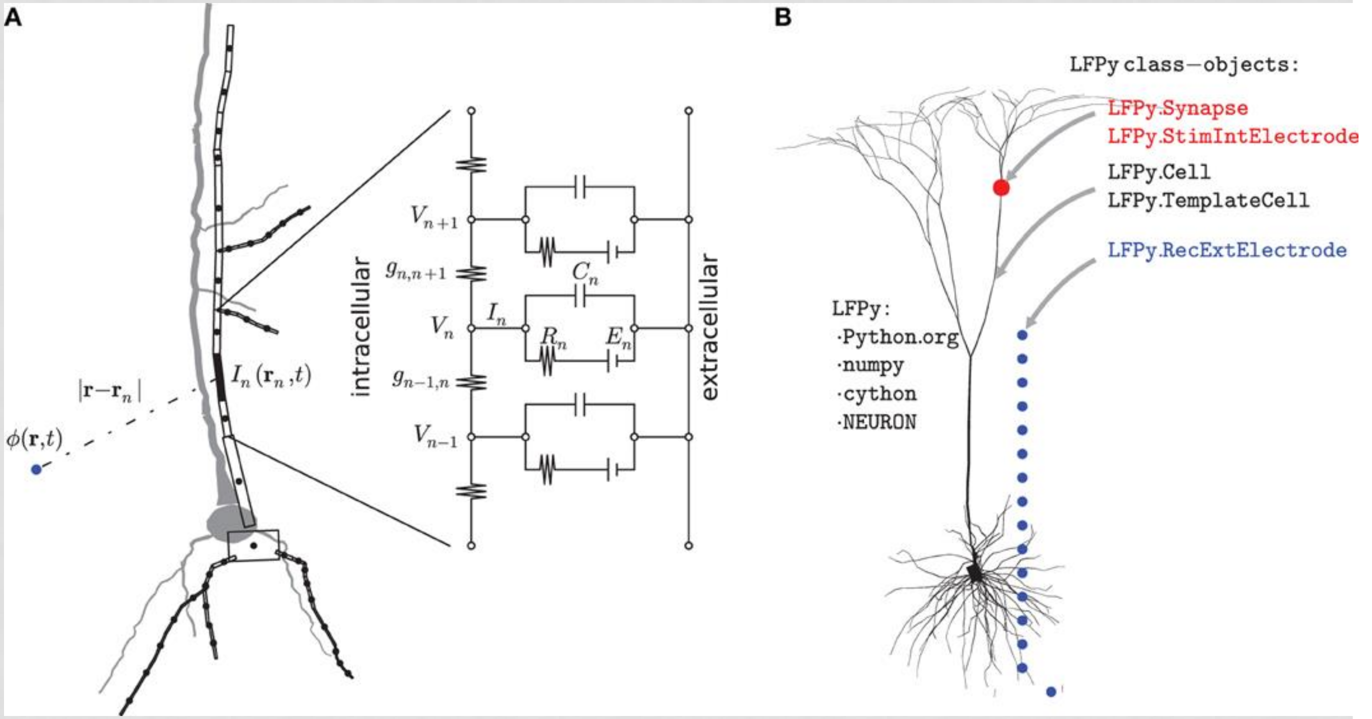


Hodgkin & Huxley
Were awarded the
Nobel Prize in 1963

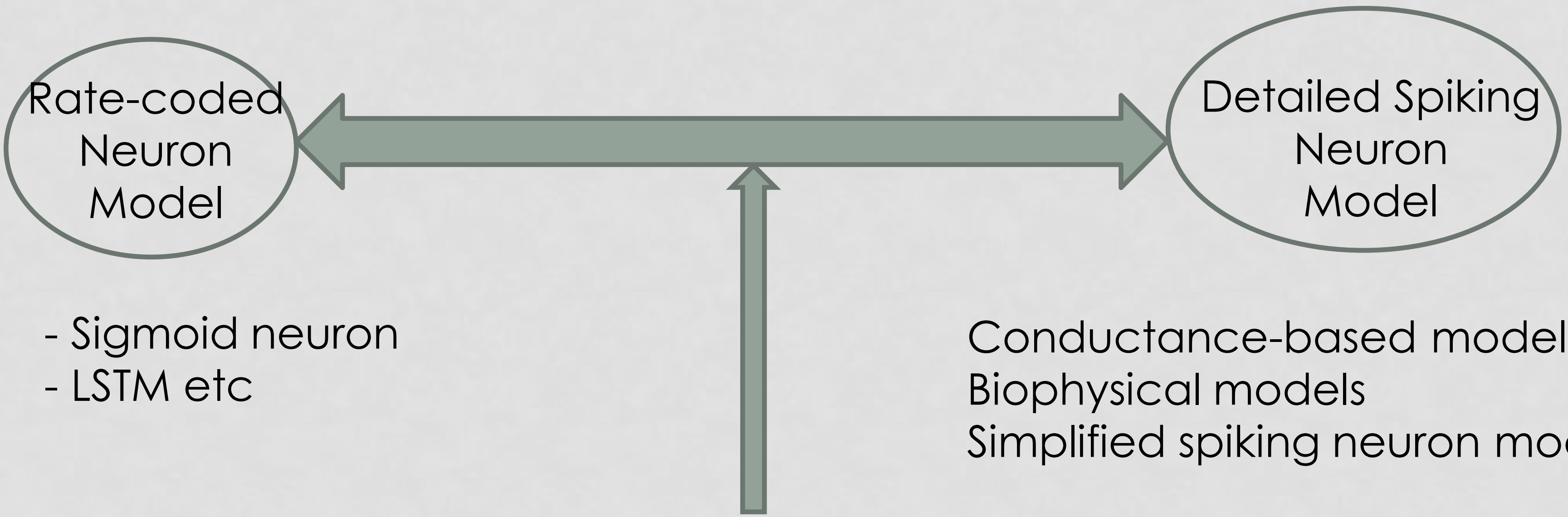


BIOPHYSICAL NEURON MODEL

Simulate the entire neuronal arbor



https://www.frontiersin.org/articles/10.3389/fninf.2013.00041/full



- Sigmoid neuron
- LSTM etc

TOO ARTIFICIAL

Conductance-based models (HH)
Biophysical models
Simplified spiking neuron models

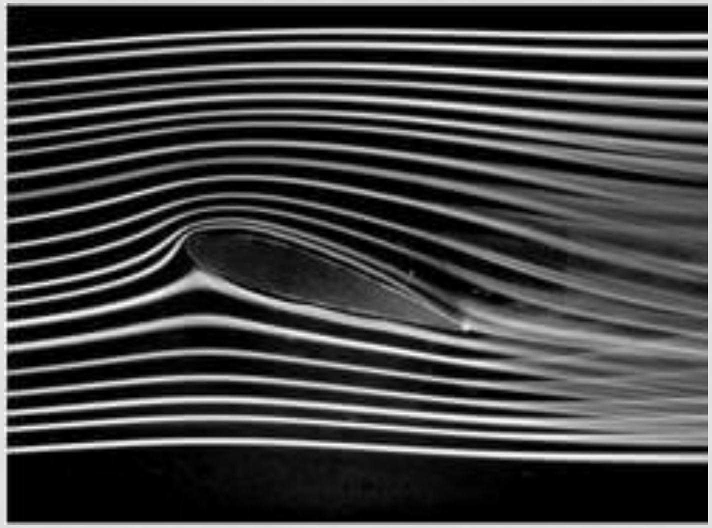
TOO REAL?

Is there a middle ground?

THE QUESTION OF THE RIGHT LEVEL

Example:

Aircraft wing in a wind tunnel



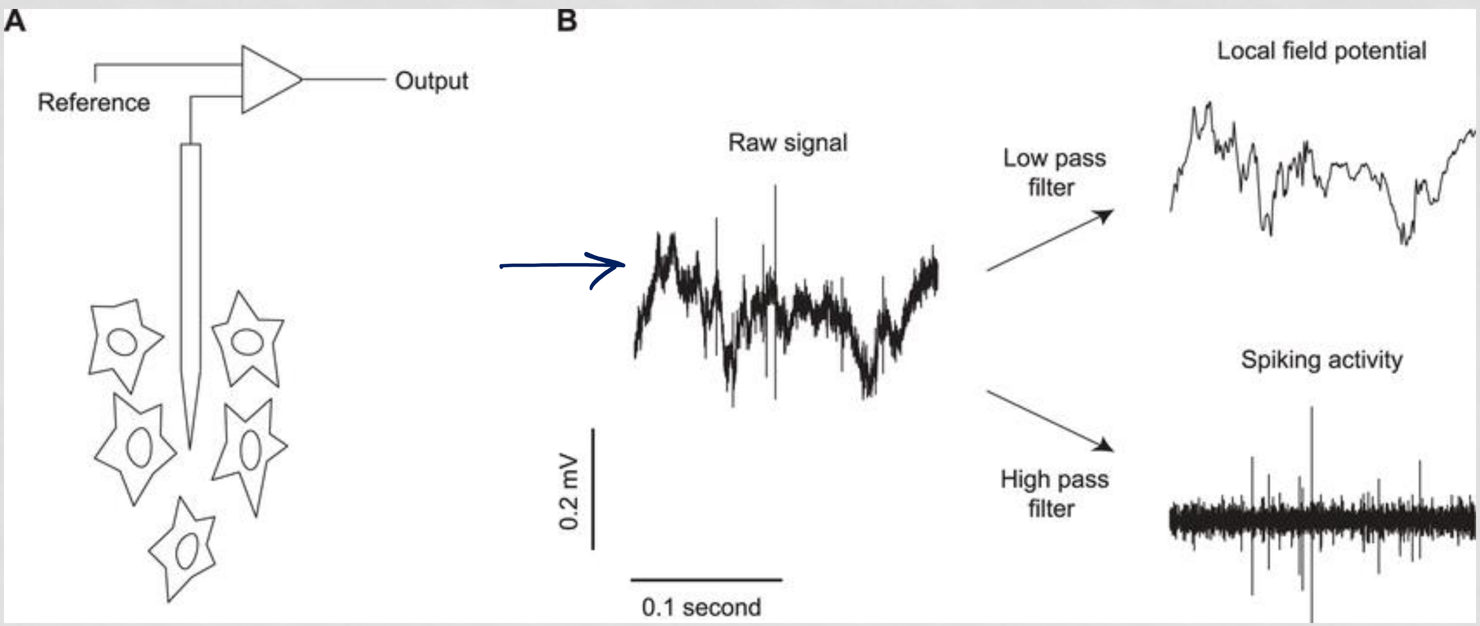
Study fluid flow using Navier Stokes Equation

FLUID is an abstraction

FLUID is NOT molecules

FROM SINGLE NEURON TO NEURAL ENSEMBLE

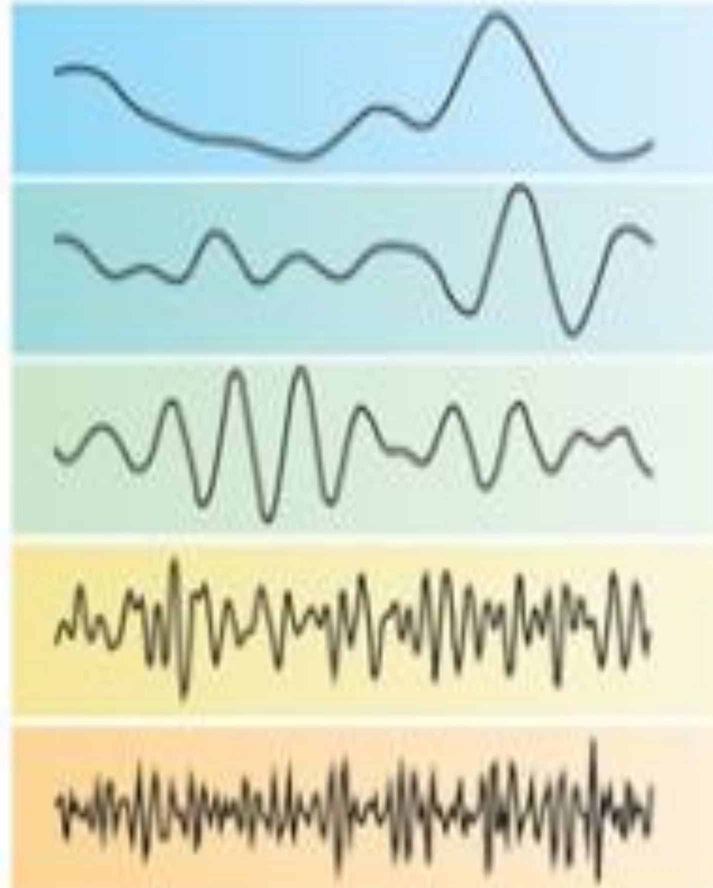
- Local field potential
  - activity of a local neural population
- Neural field models:
  - Coarse-grained models of spatio-temporal evolution of neural tissue (eg cortex)



NEURAL "WAVES"

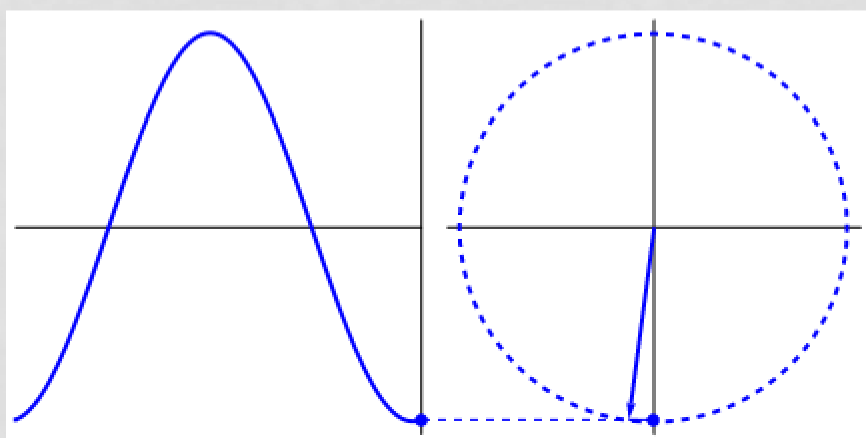
ElectroEncephaloGram (EEG)

Wave	Frequency (hertz)	Properties
Delta	0.2–3	Lowest in frequency and arising in deep, dreamless sleep
Theta	3–8	Important in strengthening synapses during learning
Alpha	8–12	Predominant in a relaxed state when the brain is at rest and the eyes are closed
Beta	12–30	Common when the brain is in an alert state, attentive and concentrating
Gamma	30–120	Associated with information processing in the cerebral cortex, thinking and learning



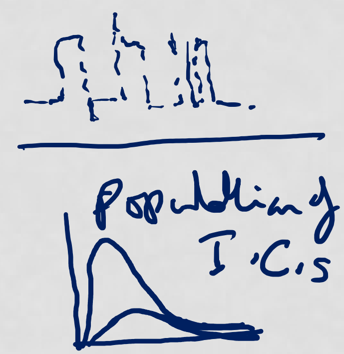
DO MODELS OF NEURAL OSCILLATORS OCCUPY THAT IDEAL MIDDLE GROUND?

"Synchronous activity of oscillating networks is now viewed as the critical 'middle ground' linking single-neuron activity to behavior" - (Buzsaki and Draghun, Science 2004)



Ex

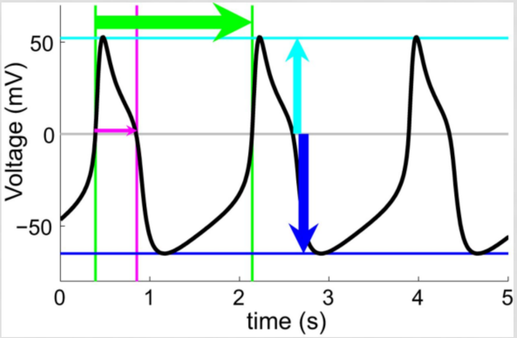
single ion channel



NEURAL OSCILLATOR MODELS (TYPICALLY 2 VARIABLE MODELS)

Wilson-Cowan model

$$\dot{x} = -\alpha x + f(ax - by + \rho_x)$$
$$\dot{y} = -\beta y + f(cx - dy + \rho_y)\,.$$

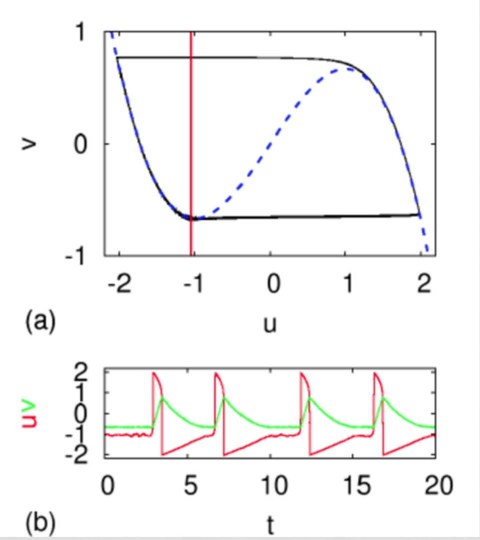


FitzHugh - Nagumo model

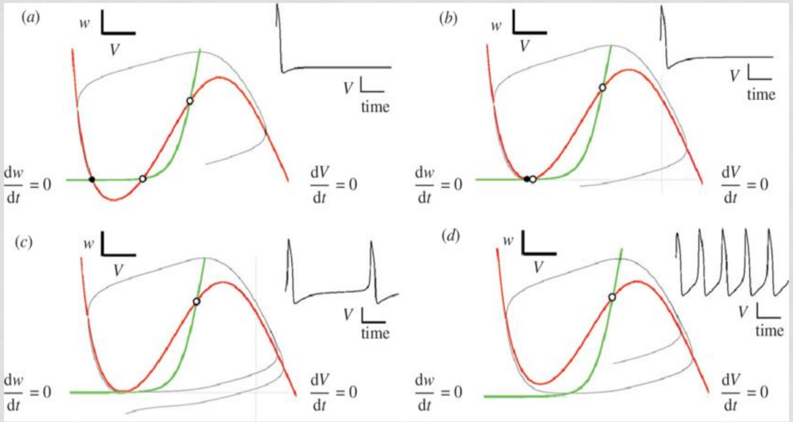
$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} + (a - V)(V-1)V - v \qquad (1)$$

$$\frac{\partial v}{\partial t} = \epsilon(\beta V - \gamma v - \delta) \qquad (2)$$

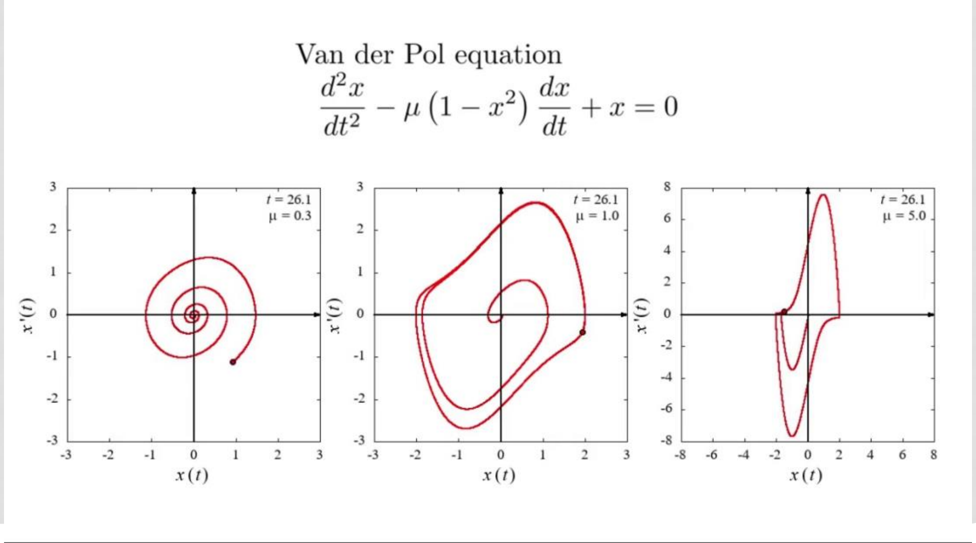
V = Voltage (fast variable)
v = v-gate (slow varibale)



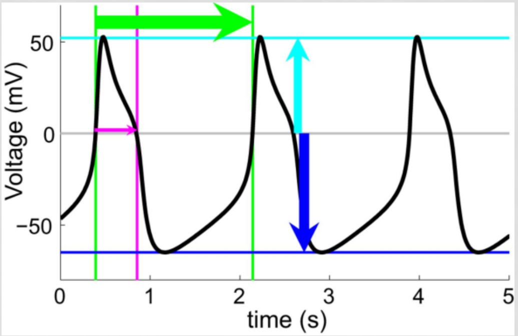
Morris-Lecarr model



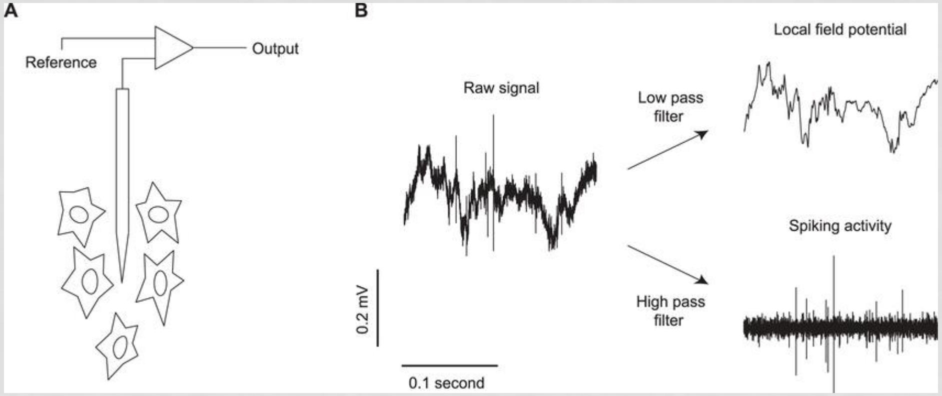
Van Der Pol model



Oscillation as a model of an Action Potential



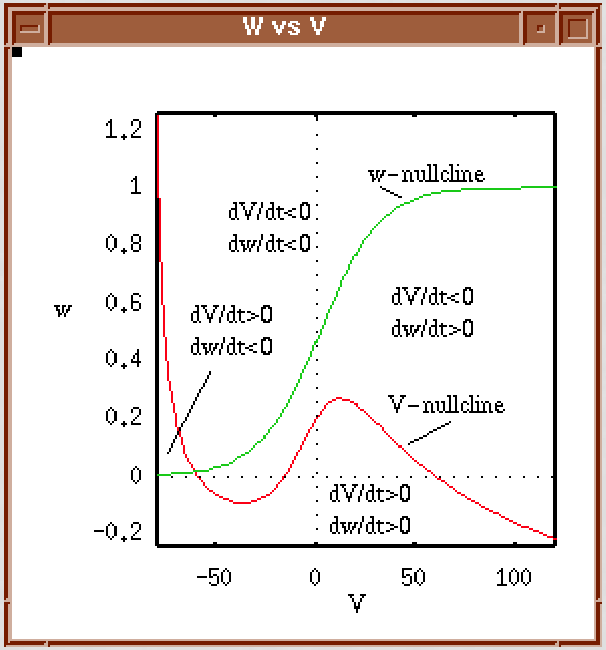
Oscillation as a model of Local Field Potential



The same models have been used for both

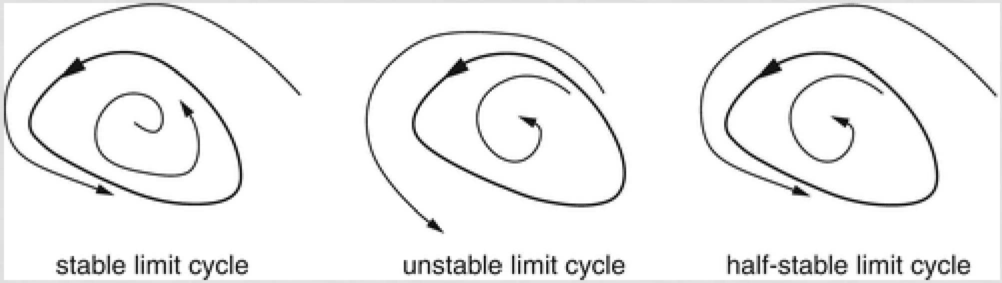
PHASE PLANE ANALYSIS

Two-variable form:
dx/dt = f(x,y)
dy/dt = g(x,y)



Can exhibit limit cycle oscillations

LIMIT CYCLES



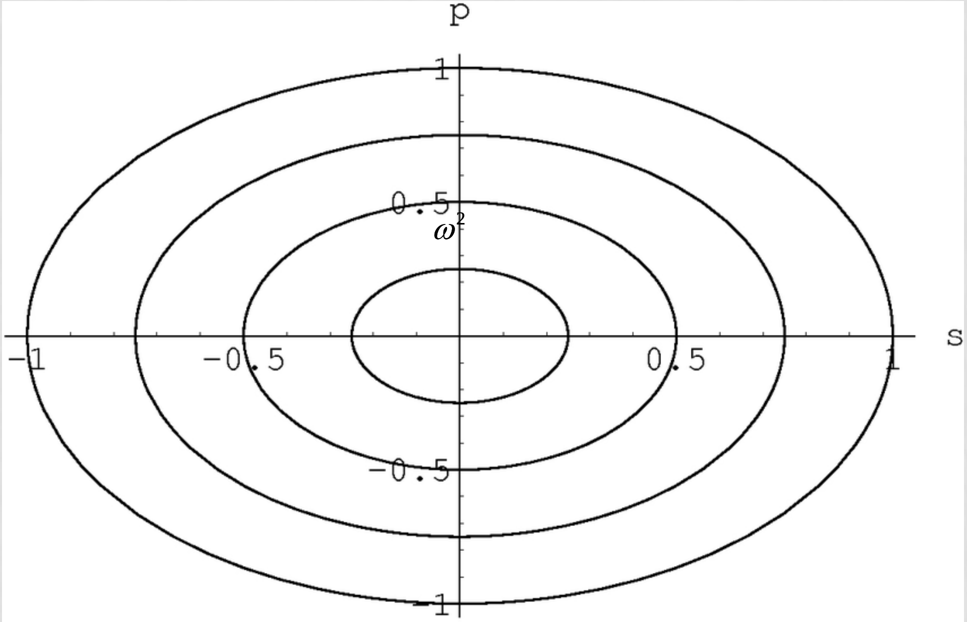
Definition: A *limit cycle* is a closed trajectory in phase space having the property that at least one other trajectory spirals into it either as time approaches infinity or as time approaches negative infinity.

Can only be exhibited by nonlinear systems.

HARMONIC OSCILLATOR

Two-variable form:
$dx/dt = f(x,y) = wy$
$dy/dt = -wx$

$d^2x/dt^2 = -w^2 x$



Solution is strictly dependent on the initial condition

Neural oscillations are Limit Cycle type

HOPF OSCILLATOR

$$\dot{x} = -y + \mu x(1 - x^2 - y^2)$$

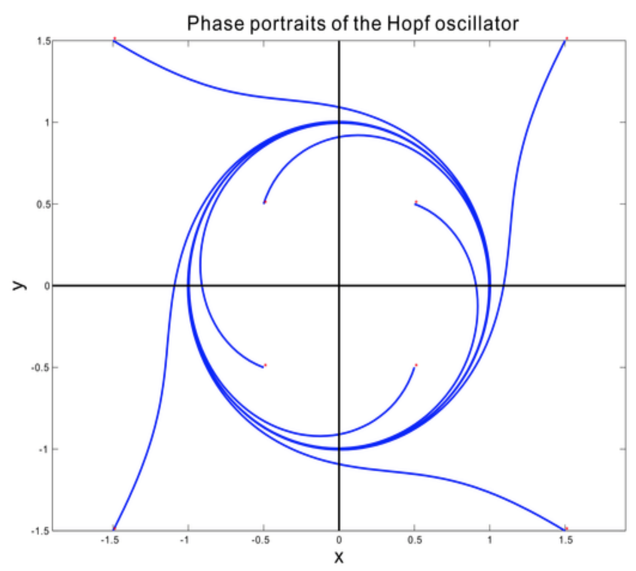
$$\dot{y} = x + \mu y(1 - x^2 - y^2)$$

In polar coordinates the equations have an elegant form.

Let x= r cos(θ), y = r sin(θ)

$$\dot{r} = \mu r(1 - r^2)$$

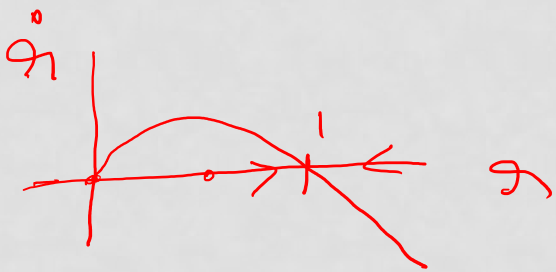
$$\dot{\theta} = 1$$



$$x\dot{x} + y\dot{y} = \mu(x^2 + y^2)(1 - x^2 - y^2)$$
$$= \mu r^2 (1 - r^2)$$

$$\frac{1}{2}\frac{d(r^2)}{dt} = \mu r^2 (1 - r^2)$$

$$\frac{dr}{dt} = \mu r (1 - r^2)$$



Now consider, $\theta = \arctan(y/x)$

- Differentiating both sides,

$$\dot{\theta} = \frac{1}{1+\frac{y^2}{x^2}} \frac{x\dot{y} - y\dot{x}}{x^2}$$

$$x\dot{y} - y\dot{x} = x^2 + y^2 = r^2$$

$\dot{\omega} = 1$

$$\dot{\theta} = \omega$$

IN COMPLEX VARIABLE FORM

$$\frac{dz}{dt} = i\omega z + (1 - |z|^2)z$$

$z = x + iy$

Let $z = re^{i\theta}$

$$\frac{dz}{dt} = e^{i\theta}\frac{dr}{dt} + ire^{i\theta}\frac{d\theta}{dt}$$

$$e^{i\theta}\frac{dr}{dt} + ire^{i\theta}\frac{d\theta}{dt} = i\omega re^{i\theta} + (1 - r^2)re^{i\theta}$$

$cancel\ \ e^{i\theta},$

$$\frac{dr}{dt} + ir\frac{d\theta}{dt} = i\omega r + (1 - r^2)r$$

Equating real and imaginary parts:

$$\frac{dr}{dt} = (1 - r^2)r \qquad \frac{d\theta}{dt} = \omega$$

NO NONLINEARITY → SHM

$$\frac{dz}{dt} = i\omega z + (1 - |z|^2)z$$

Eliminating nonlinearity…

$$\frac{dz}{dt} = i\omega z$$

$$\frac{dx}{dt} + i\frac{dy}{dt} = i\omega(x + iy) = \omega(ix - y)$$

$$\frac{dx}{dt} = \omega y \qquad \frac{dy}{dt} = -\omega x$$

$$A = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}$$

$$\lambda = \pm i\omega$$

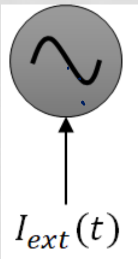
RESEARCH AGENDA

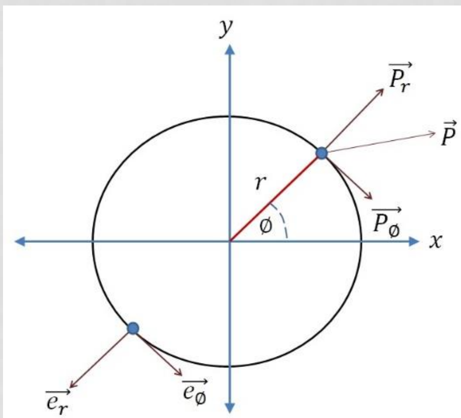
- To construct general oscillatory neural networks using Hopf Oscillators:
  - Oscillatory Hopfield networks (done) ✓
  - Oscillatory Auto Encoders (done) ✓
  - Oscillatory Self-organizing Maps (SOMs) ✓
  - Oscillatory Deep networks ✓
  - Oscillatory Convolutional Networks etc etc

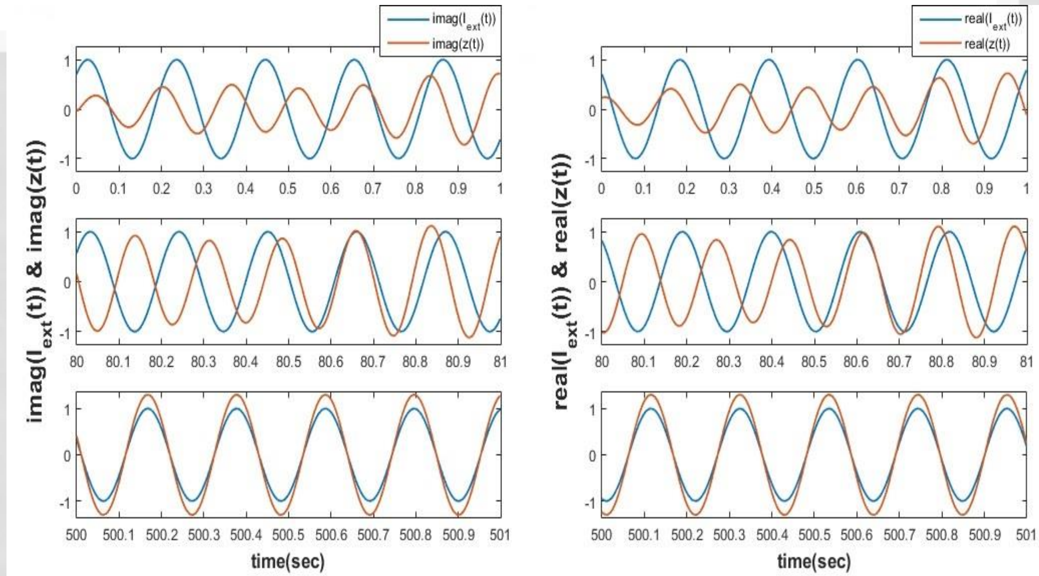
COMPLEX ADAPTIVE HOPF OSCILLATOR:

$$\dot{z} = z(\mu + i\omega - |z|^2) + \varepsilon I_{ext}(t)$$

$$\dot{\omega} = -\varepsilon I_{ext}(t) \sin \emptyset$$



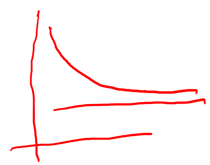


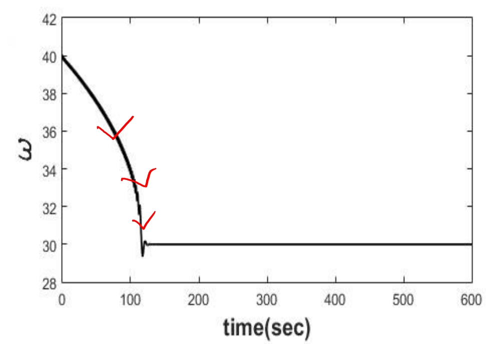


$$I_{ext}(t) = I_0 e^{i(\omega_0 t + \varphi)}$$

$$\dot{\omega} = -\varepsilon\big(real(I_{ext}(t)) \sin \emptyset - img(I_{ext}(t)) \cos \emptyset\big)$$

$$\dot{\omega} = -\varepsilon I_0 \sin(\emptyset - \omega_0 t - \varphi)$$





Righetti et al., 2005

(Biswas & Chakravarthy, 2020)

In this scenario, $I_{ext}(t)$ is a complex sinusoidal signal. It is straight forward to derive the learning rule for the natural frequency of the oscillator if eq. 2a is represented in the Cartesian and polar coordinate forms, respectively, as follows:

$$\begin{aligned} \dot{x} &= \left(\mu - r^2\right) x - \omega y + \varepsilon I_0 \cos(\omega_0 t + \varphi) \\ \dot{y} &= \left(\mu - r^2\right) y + \omega x + \varepsilon I_0 \sin(\omega_0 t + \varphi) \end{aligned} \qquad (2a1)$$

$$\begin{aligned} \dot{r} &= \left(\mu - r^2\right) r + \varepsilon I_0 \cos(\omega_0 t + \varphi - \emptyset) \\ \dot{\emptyset} &= \omega + \frac{\varepsilon I_0}{r} \sin(\omega_0 t + \varphi - \emptyset) \end{aligned} \qquad (2a2)$$

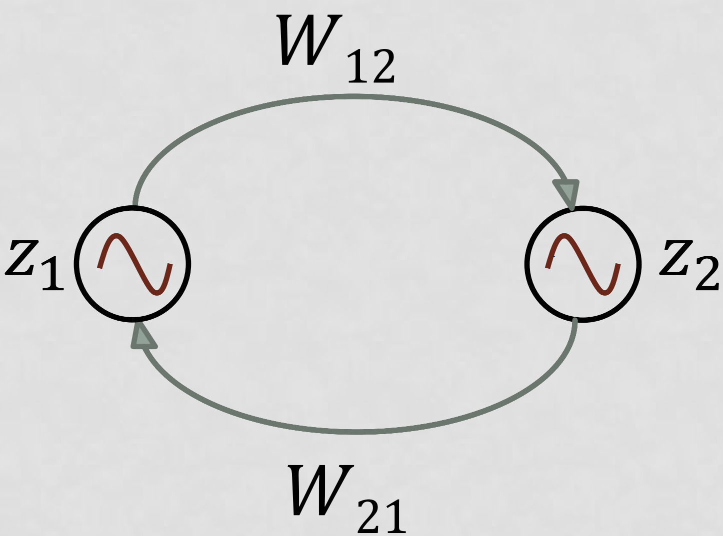
In the phase plane representation, it can be observed from eq. 2a2 that the influence caused by the input perturbation on the oscillator phase is $\frac{\varepsilon I_0}{r}\sin(\omega_0 t - \emptyset)$. Whereas from the Cartesian coordinate system

$$\phi = \omega t + \xi$$

$$\dot{\omega} = -\epsilon \sin(\omega t + \xi - \omega_0 t \cancel{\phi} - \varphi)$$

PROBLEM#1
A PAIR OF COUPLED HOPF OSCILLATORS

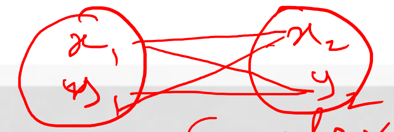
$$\dot{z_1} = (\mu - |z_1|^2)z_1 + i\omega_1 z_1 + W_{21} real(z_2)$$
$$\dot{z_2} = (\mu - |z_2|^2)z_2 + i\omega_2 z_2 + W_{12} real(z_1)$$



What is the effect of the Coupling Strength (W) on the Phase Difference (ψ)?

TWO HOPF OSCILLATORS
SAME FREQUENCY ($\omega_1 = \omega_2$), REAL COUPLING

real



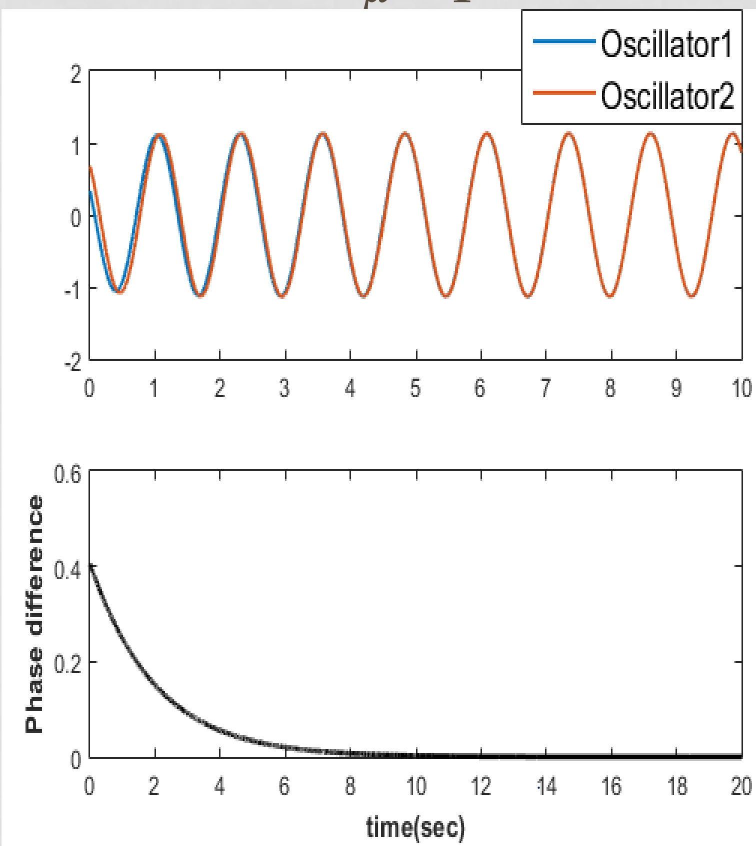
Complex

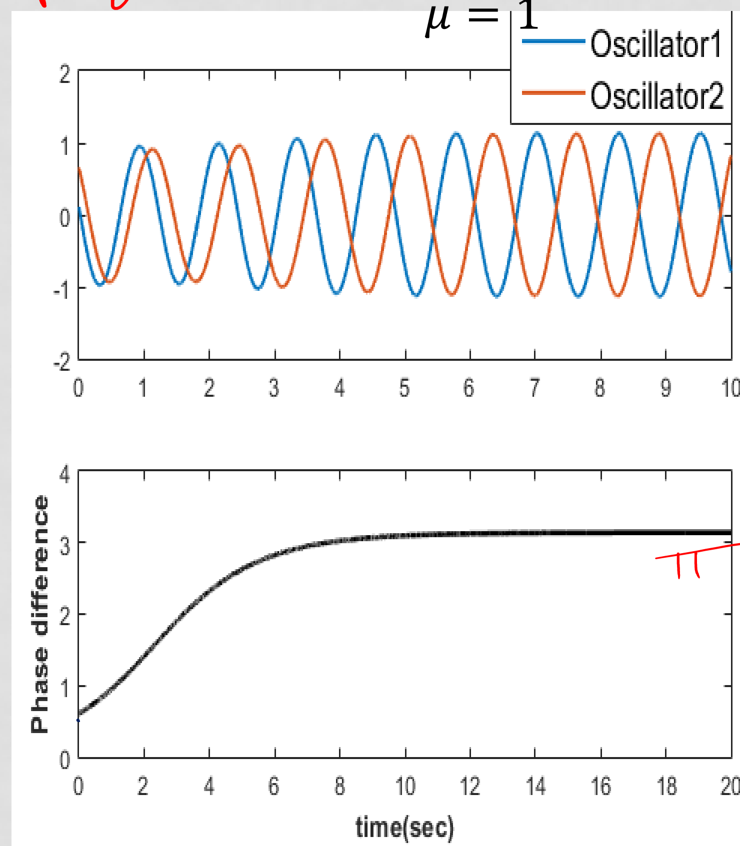
$$\omega_1 = \omega_2 = 5$$
$$W_{21} = W_{12} = 0.5$$
$$\mu = 1$$

$\omega_1 = \omega_2$

Symmetric coupling

$$\omega_1 = \omega_2 = 5$$
$$W_{21} = W_{12} = -0.5$$
$$\mu = 1$$





π

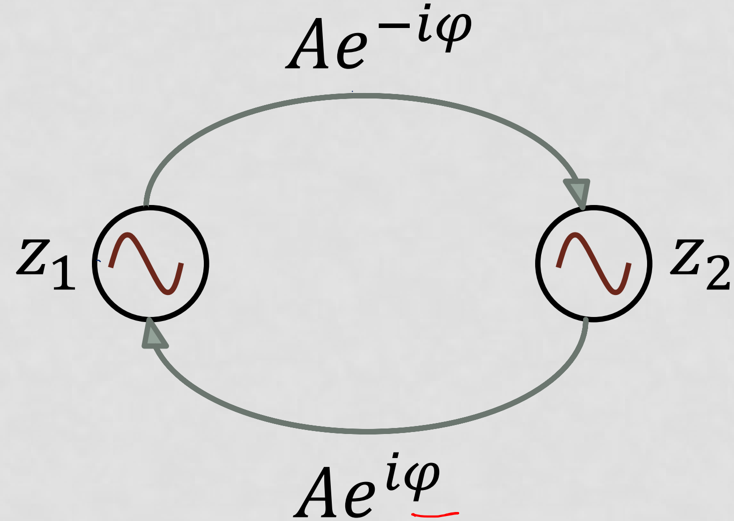
TWO COUPLED HOPF OSCILLATORS: SAME FREQUENCIES ($\omega_1 = \omega_2$), COMPLEX COUPLING

- Complex variable representation of Hopf oscillator.

$$\dot{z} = (\mu - |z|^2)z + i\omega z$$

Where, $z = x + iy = re^{i\theta}$

- Two oscillators coupled through complex lateral weights.



$$\dot{z}_1 = (\mu - |z_1|^2)z_1 + i\omega_1 z_1 + Ae^{i\varphi} z_2$$
$$\dot{z}_2 = (\mu - |z_2|^2)z_2 + i\omega_2 z_2 + Ae^{-i\varphi} z_1$$

THEORETICAL EXPLANATION

- Polar coordinate representation:

$$\dot{r_1} = (\mu - {r_1}^2)r_1 + Ar_2\cos(\theta_2 - \theta_1 + \varphi)$$
$$\dot{\theta}_1 = \omega_1 + A\frac{r_2}{r_1}\sin(\theta_2 - \theta_1 + \varphi)$$
$$\dot{r_2} = (\mu - {r_2}^2)r_2 + Ar_1\cos(\theta_1 - \theta_2 - \varphi)$$

$\psi = \theta_1 - \theta_2$

$$\dot{\theta}_2 = \omega_2 + A\frac{r_1}{r_2}\sin(\theta_1 - \theta_2 - \varphi)$$

$\theta_1 - \theta_2 =$ ~~arbitrary~~ ?

- From which:

$\omega_1 = \omega_2$

$$\dot{\psi} = \dot{\theta}_1 - \dot{\theta}_2 = \underbrace{(\omega_1 - \omega_2)}_{=0} - \frac{A\sin(\psi - \varphi)}{r_1 r_2}({r_1}^2 + {r_2}^2)$$

$r_1 = r_2 = \sqrt{\mu}$

- Assuming $\omega_1 = \omega_2$, at steady state:

$$\dot{\psi} = 2A\sin(\varphi - \psi) = 0$$

- Which ensures the phase difference$(\theta_1 - \theta_2)$ between the two oscillators to be φ: the angle of the complex coupling weight.

NUMERICAL RESULTS

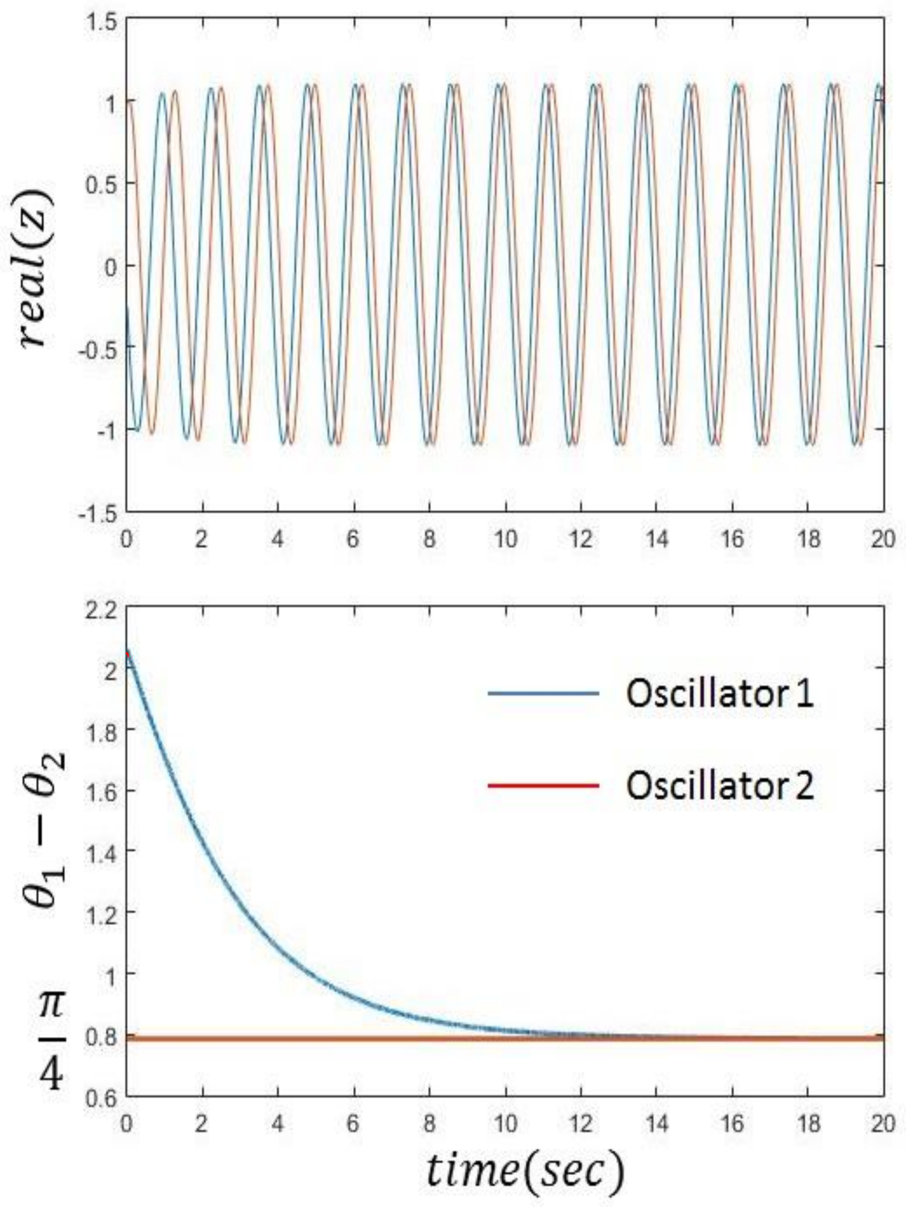
- Parameters:

$$A = 0.2, \varphi = \frac{\pi}{4}$$

$$\omega_1 = \omega_2 = 5$$

Where: $z_1 = r_1 e^{i\theta_1}$ and

$$z_2 = r_2 e^{i\theta_2}$$



COUPLING TWO HOPF OSCILLATORS
NEARBY FREQUENCIES, REAL COUPLING

- When two Hopf oscillators, with slightly different frequencies ($\omega_1 \cong \omega_2$) are coupled, for a sufficiently strong coupling strength, the two oscillators come to a common frequency (ω) such that ($\omega_1 < \omega < \omega_2$), with oscillator #2 leading oscillator #1.

$\omega_1 = \omega_2 + \epsilon \qquad 0 < \epsilon << 1$

$\delta \cdot 1\,\mathrm{Hz}$

θ

α

ρ

$r \; 100\,\mathrm{Hz}$

$\theta_1 > \theta_2$

$\omega_1 \rightarrow \omega$

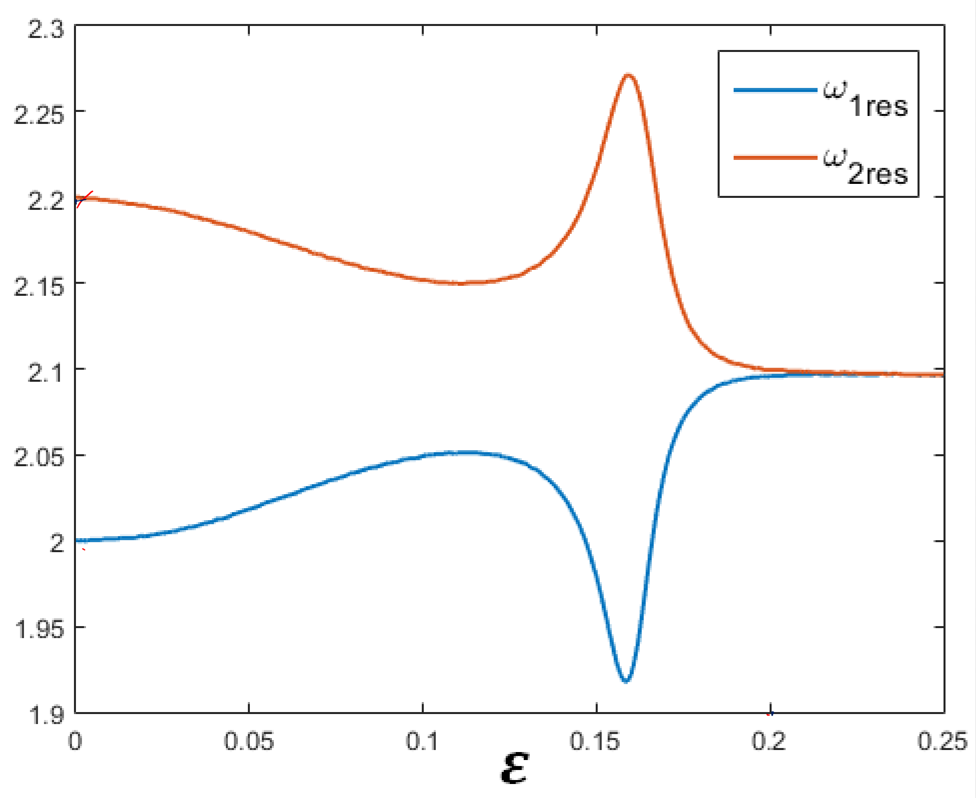
$\omega_2 \nearrow$

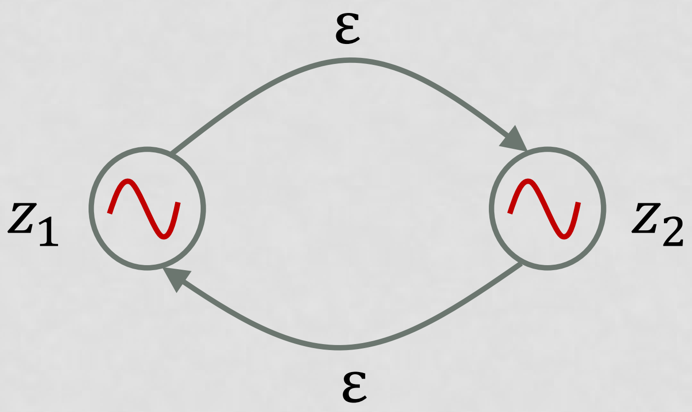
$\omega_1 > \omega > \omega_2$

$$\dot{z_1} = (\mu - |z_1|^2)z_1 + i\omega_1 z_1 + \varepsilon \times real(z_1)$$
$$\dot{z_2} = (\mu - |z_2|^2)z_2 + i\omega_2 z_2 + \varepsilon \times real(z_2)$$

Simulation parameters:

$$\mu = 1, \omega_1 = 2, \omega_2 = 2.2$$



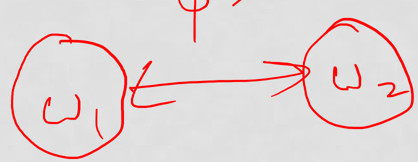


COUPLING TWO HOPF OSCILLATORS WITH ARBITRARILY DIFFERENT FREQUENCIES

- When two oscillators with different intrinsic frequencies are coupled through real coupling weights a new behavior emerges while analyzing their synchronization behavior, called **Arnold Tongues.**

$$\dot{z}_1 = (\mu - |z_1|^2)z_1 + i\omega_1 z_1 + \varepsilon \times real(z_1)$$
$$\dot{z}_2 = (\mu - |z_2|^2)z_2 + i\omega_2 z_2 + \varepsilon \times real(z_2)$$

ψ = desired value

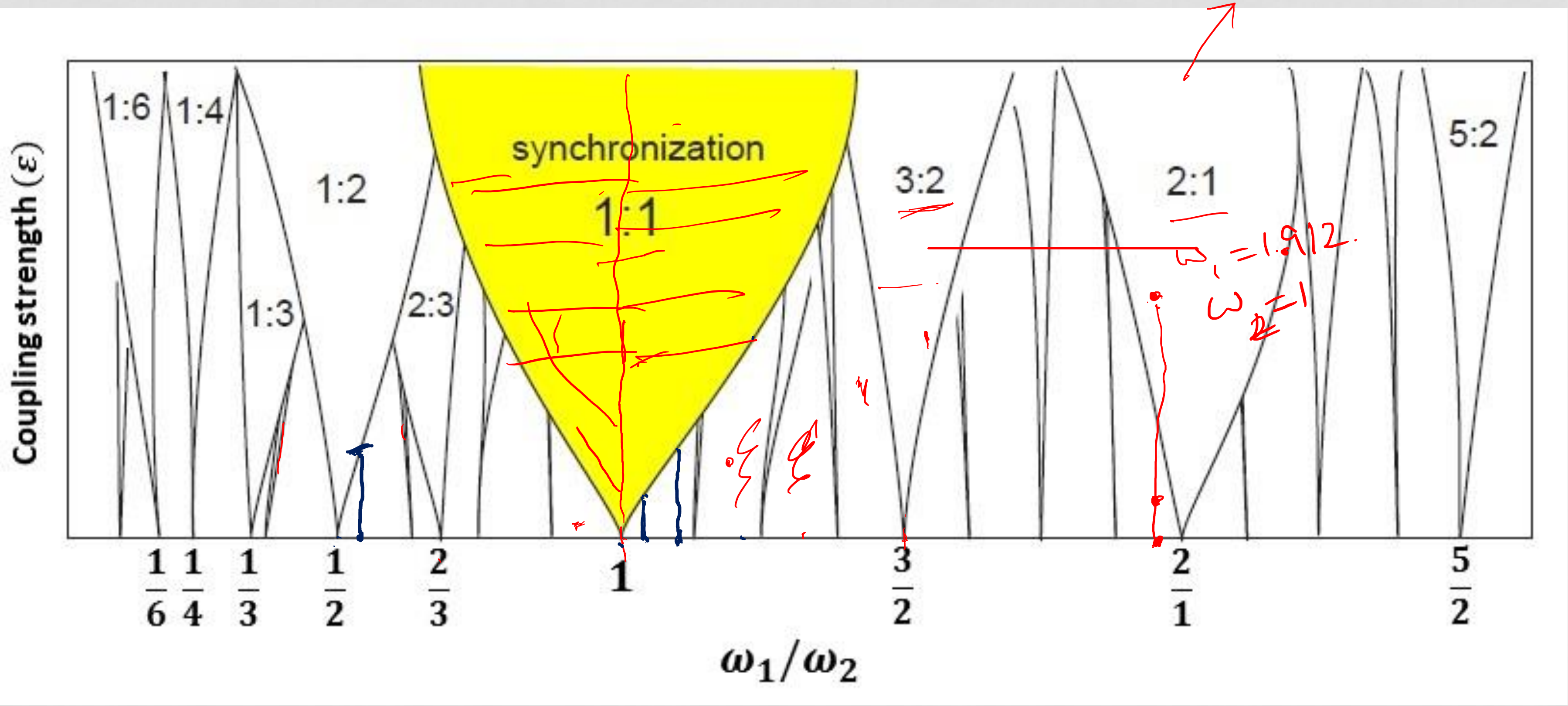


?

COUPLING TWO HOPF OSCILLATORS: ARBITRARILY DIFFERENT FREQUENCIES , REAL COUPLING GENERAL SCENARIO

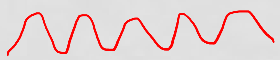
ARNOLD TONGUES

entrainment



NORMALIZED PHASE DIFFERENCE

- When the frequencies are different, the usual phase difference cannot be constant
- But normalized phase difference can be constant



$\frac{\theta_1}{\omega_1}$

$\frac{\theta_2}{\omega_2}$

- normalized phase difference:

$$\psi = \frac{\theta_1}{\omega_1} - \frac{\theta_2}{\omega_2}$$

- This is what is constant when two oscillators are entrained in a simple integral ratio

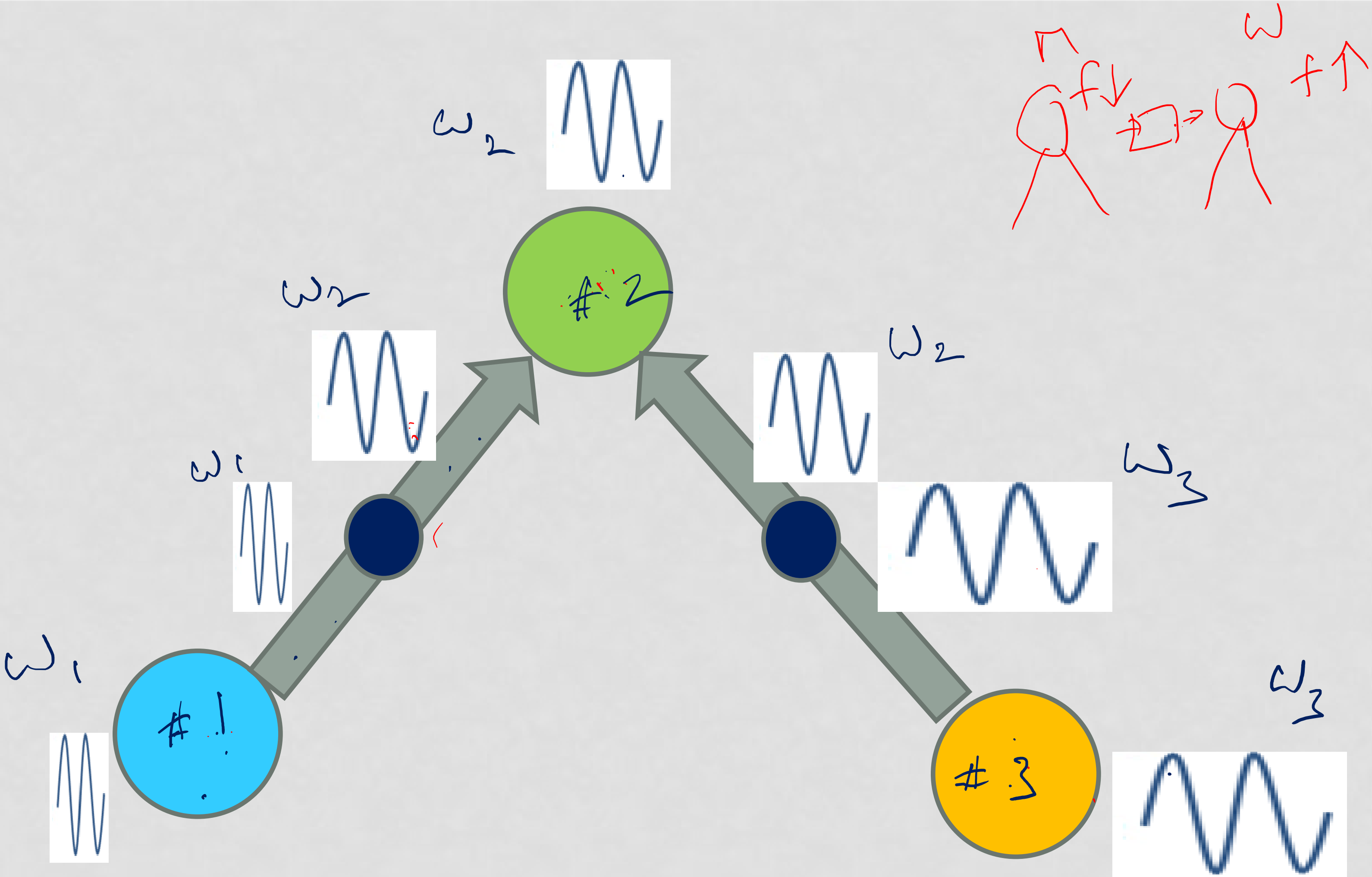
- How to achieve constant normalized phase difference for arbitrary frequencies?

Answer:

It requires a trick called

Power coupling

TRANSFORMING TRANSMITTER FREQUENCIES TO RECEIVER FREQUENCY



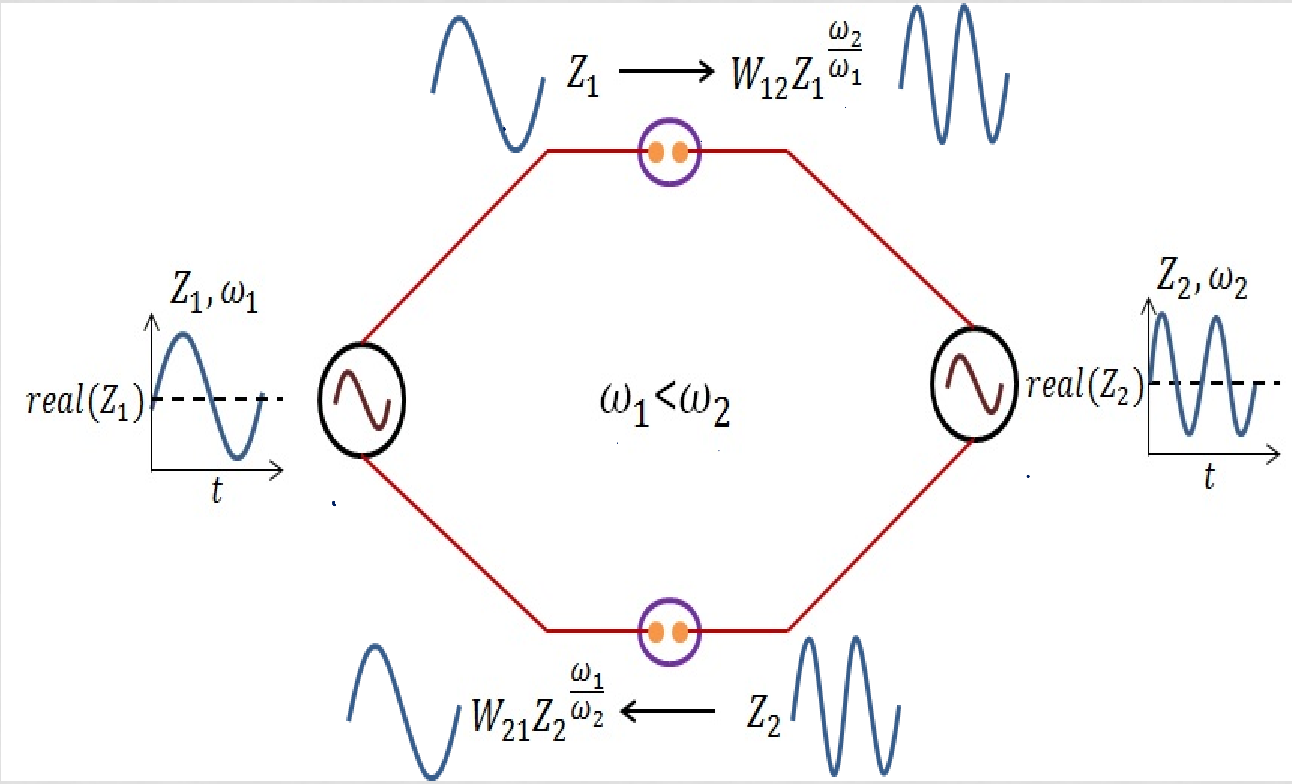
$$z_1 = A_1 e^{i\omega_1 t}$$

How to transform ω_1 to ω_2?

$$(z_1)^{\omega_2/\omega_1} = (A_1 e^{i\omega_1 t})^{\omega_2/\omega_1} = (A_1 e^{i\omega_2 t})$$

POWER COUPLING

- Frequency of oscillator 1 is transformed into frequency of oscillator 2, as the signal passes over the connection



THEORETICAL ANALYSIS

- Two oscillators coupled through "power coupling":

$$\dot{z}_1 = (\mu - |z_1|^2)z_1 + i\omega_1 z_1 + Ae^{i\frac{\varphi}{\omega_2}} z_2^{\frac{\omega_1}{\omega_2}}$$

$$\dot{z}_2 = (\mu - |z_2|^2)z_2 + i\omega_2 z_2 + Ae^{-i\frac{\varphi}{\omega_1}} z_1^{\frac{\omega_2}{\omega_1}}$$

- The angular part of the polar form:

$$\dot{\theta}_1 = \omega_1 + A\frac{r_2^{\frac{\omega_1}{\omega_2}}}{r_1}\sin\omega_1\left(\frac{\theta_2}{\omega_2} - \frac{\theta_1}{\omega_1} + \frac{\varphi}{\omega_1\omega_2}\right)$$

$$\dot{\theta}_2 = \omega_2 + A\frac{r_1^{\frac{\omega_2}{\omega_1}}}{r_2}\sin\omega_2\left(\frac{\theta_1}{\omega_1} - \frac{\theta_2}{\omega_2} - \frac{\varphi}{\omega_1\omega_2}\right)$$

- We denote the normalized phase difference:

$$\psi = \frac{\theta_1}{\omega_1} - \frac{\theta_2}{\omega_2}$$

- The derivative of normalized phase difference w.r.t time:

$$\dot{\psi} = \frac{\dot{\theta}_1}{\omega_1} - \frac{\dot{\theta}_2}{\omega_2}$$

$$\dot{\psi} = \frac{A r_2^{\frac{\omega_1}{\omega_2}}}{\omega_1 r_1} \sin \omega_1 \left(\frac{\varphi}{\omega_1 \omega_2} - \psi \right) + \frac{A r_1^{\frac{\omega_2}{\omega_1}}}{\omega_2 r_2} \sin \omega_2 \left(\frac{\varphi}{\omega_1 \omega_2} - \psi \right)$$

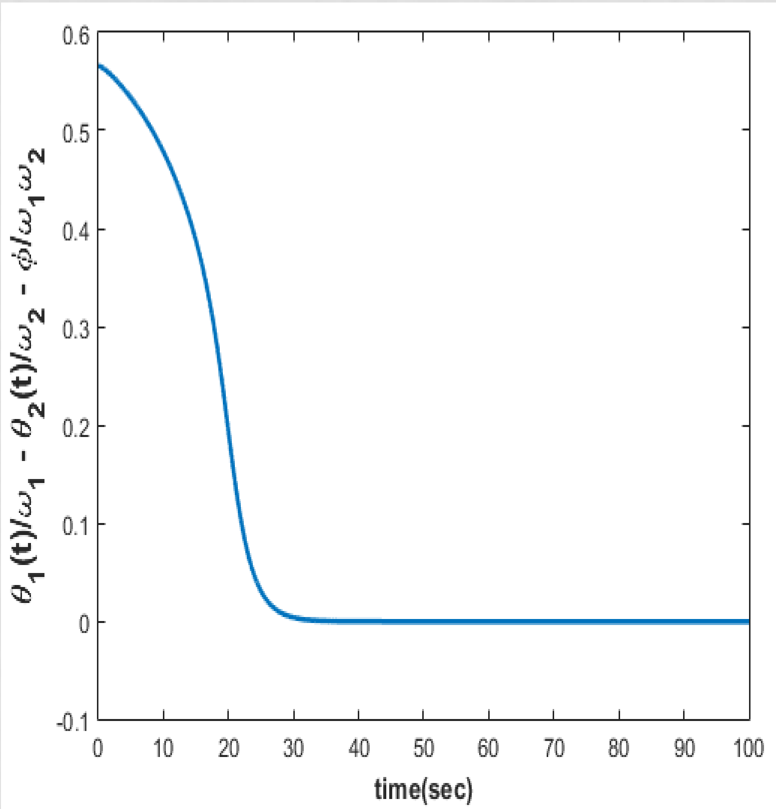
- At steady state; $\psi = constant, \dot{\psi} = 0$ and $r_1 = r_2 = 1$:

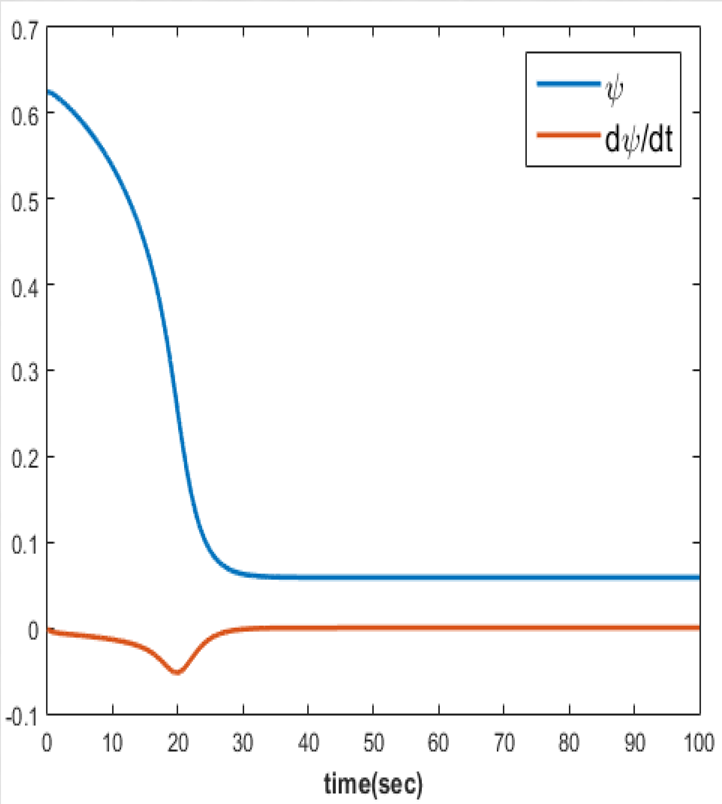
$$\dot{\psi} = \frac{A}{\omega_1} \sin \omega_1 \left(\frac{\varphi}{\omega_1 \omega_2} - \psi \right) + \frac{A}{\omega_2} \sin \omega_2 \left(\frac{\varphi}{\omega_1 \omega_2} - \psi \right) = 0$$

- The steady state normalized phase difference satisfies the equation:

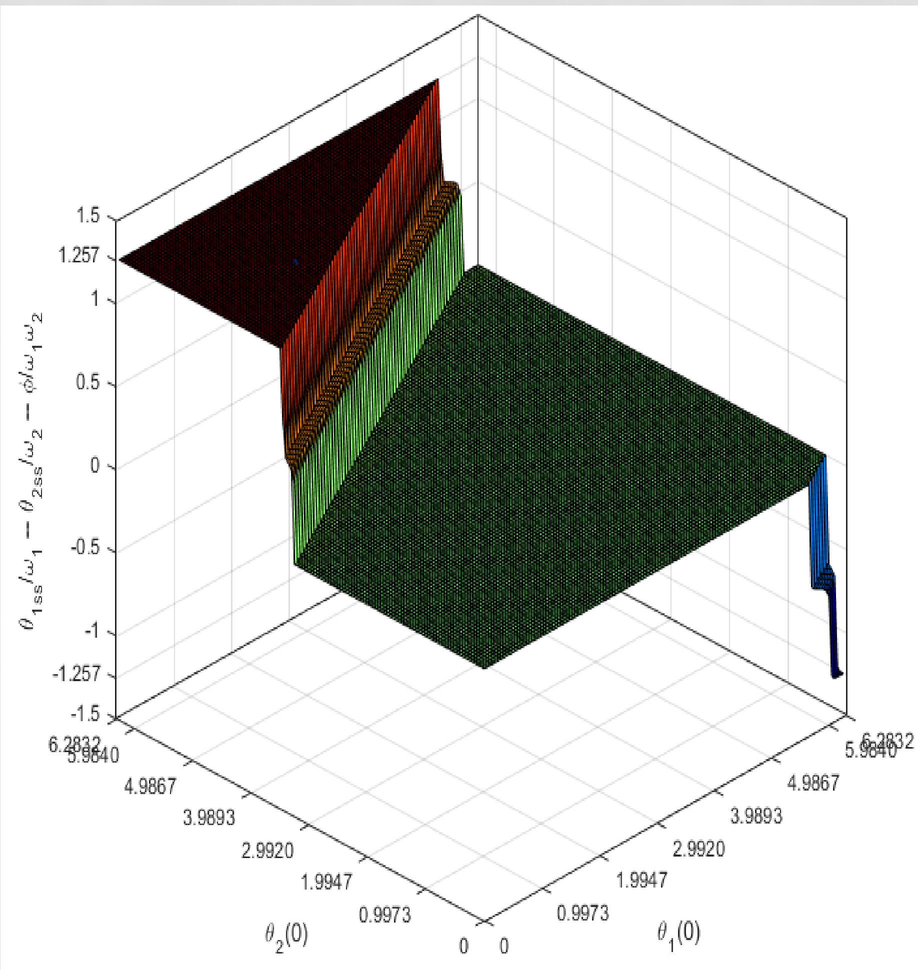
$$\psi = \frac{\theta_1}{\omega_1} - \frac{\theta_2}{\omega_2} = \frac{\varphi}{\omega_1 \omega_2}$$

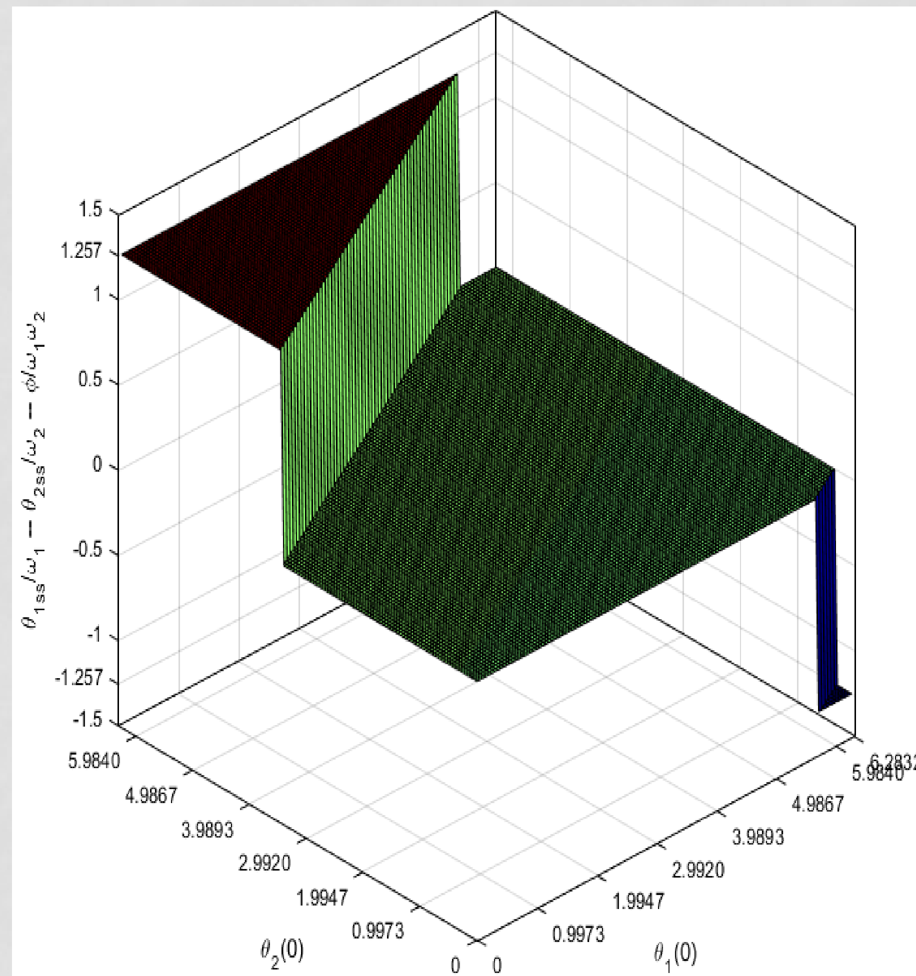
- The $\sigma = \frac{\theta_1(t)}{\omega_1} - \frac{\theta_2(t)}{\omega_2} - \frac{\varphi}{\omega_1\omega_2}$, $\psi(t)$ and $\dot{\psi}(t)$ variation w.r.t time for two 'power coupled' Hopf oscillators with the following parameters.
- $\omega_1 = 5, \omega_2 = 10, A_{12} = A_{21} = 0.2, \varphi = 2.9644.$ φ/50=ψ





- Depending on initial values of θ_1 and θ_2, σ_{ss} can attain any of the solutions of $\dot{\psi_{ss}} = 0$ equation.
- The solutions other than $\sigma_{ss} = 0$ are: $\sigma_{ss} = \frac{2\pi n_1}{\omega_1} = \frac{2\pi n_2}{\omega_2}$, where $\frac{n_1}{n_2} = \frac{\omega_1}{\omega_2}$
- In the simulated scenario the stable solutions are: $\sigma_{ss} = 0, \frac{2n_1\pi}{\omega_1} = \frac{2n_2\pi}{\omega_2}$



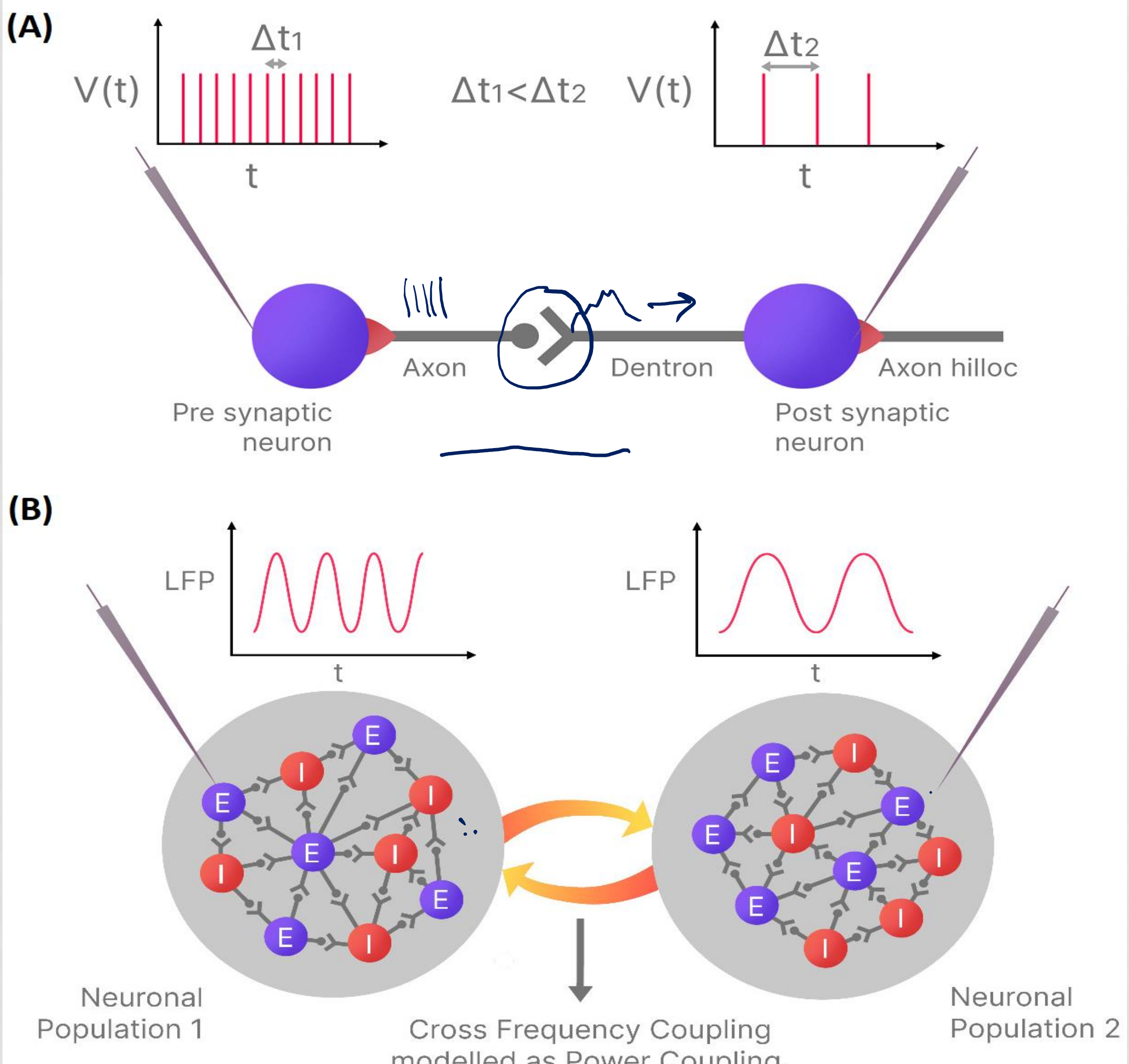


$$\frac{\omega_1}{\omega_2} = \text{rational} \#$$

$$\sim \sin\left(\omega_1\left(\frac{2\pi n_1}{\cancel{\omega_1}}\right)\right) + \sim \sin\left(\omega_2\left(\frac{2\pi n_2}{\cancel{\omega_2}}\right)\right)$$

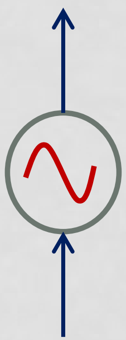
$$= 0$$

BIOLOGICAL INTERPRETATION



FREQUENCY TRACKING

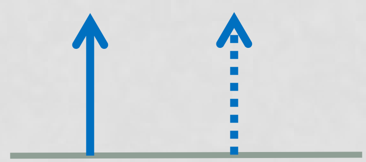
The intrinsic frequency of the oscillator can track the input frequency



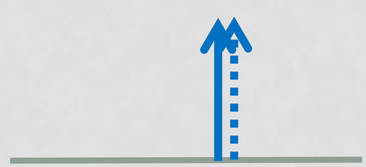
$$F(t) = Ae^{i(\omega t)}$$

$$\dot{x} = (1 - r^2)x - \omega y + \varepsilon F(t)$$
$$\dot{y} = (1 - r^2)y + \omega x$$
$$\dot{\omega} = -\varepsilon F(t)\frac{y}{r}$$

(Righetti et al 2005)



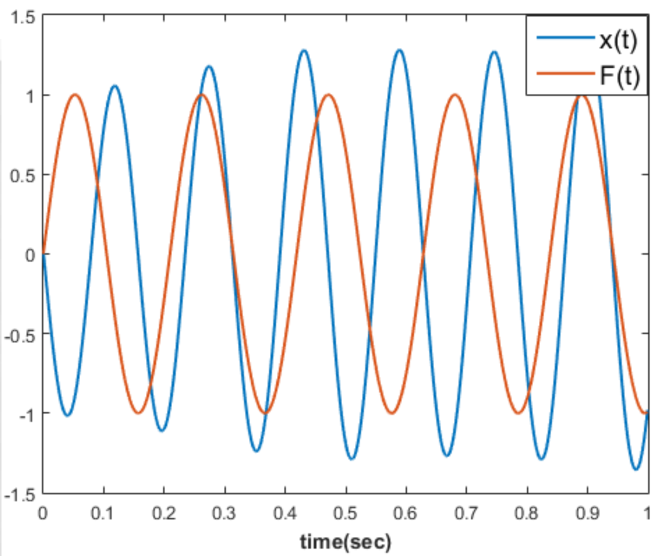
Before Training

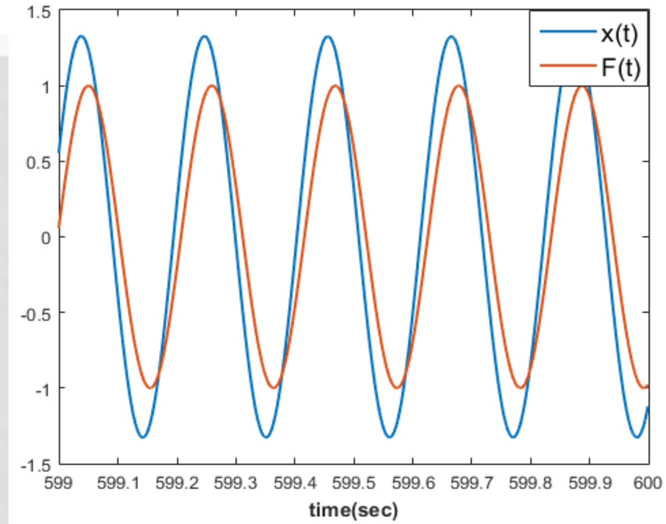


After Training

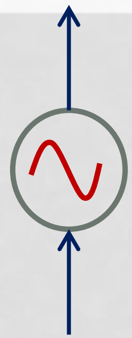
$$\dot{x} = (1 - r^2)x - \omega y + \varepsilon F(t)$$
$$\dot{y} = (1 - r^2)y + \omega x$$
$$\dot{\omega} = -\varepsilon F(t)\frac{y}{r}$$

$$\omega(0) = 40,\ \varepsilon = 0.9,\ F(t) = \sin 30t$$

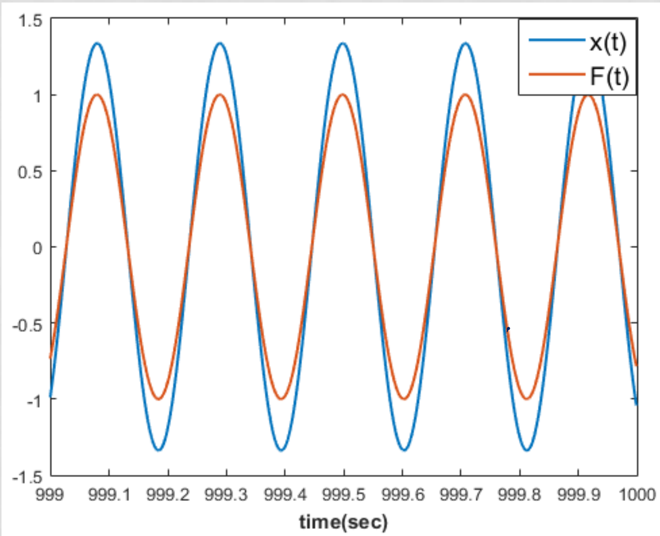


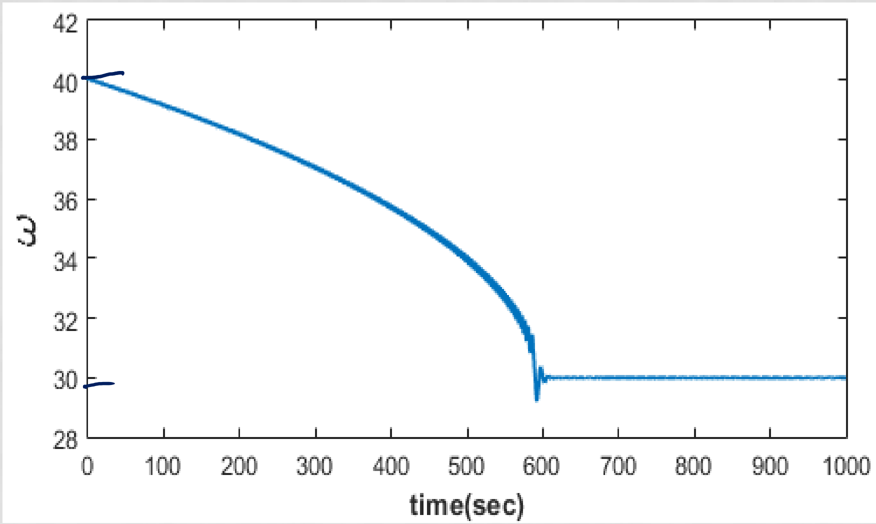


$x(t)$

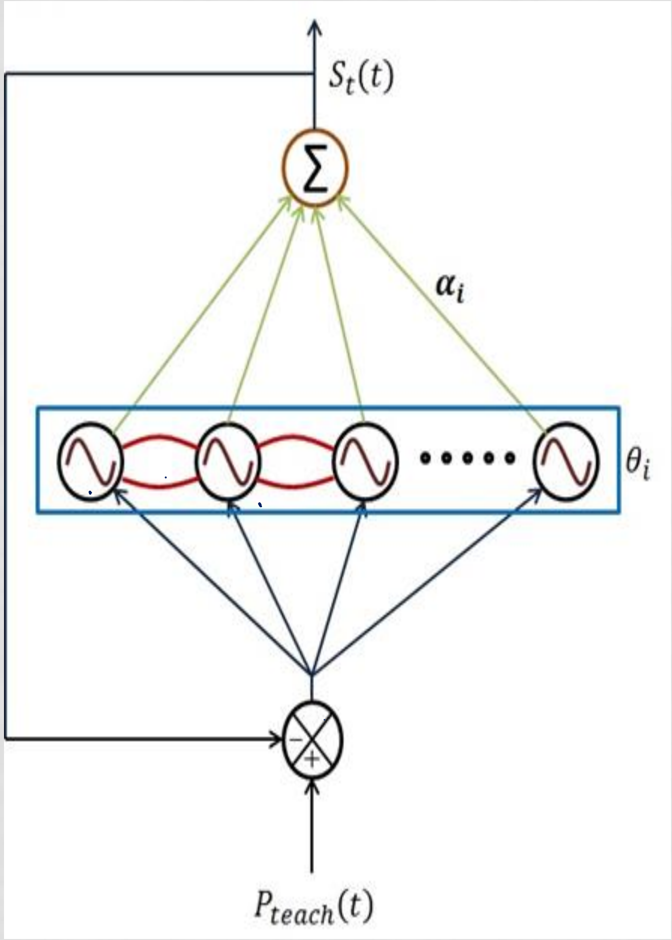


$F(t)$

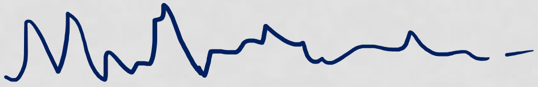




COUPLED N-OSCILLATOR NETWORK



Reconstructing a time series
By taking a weighted sum of outputs of N-oscillator network



RECONSTRUCTING A TIME SERIES

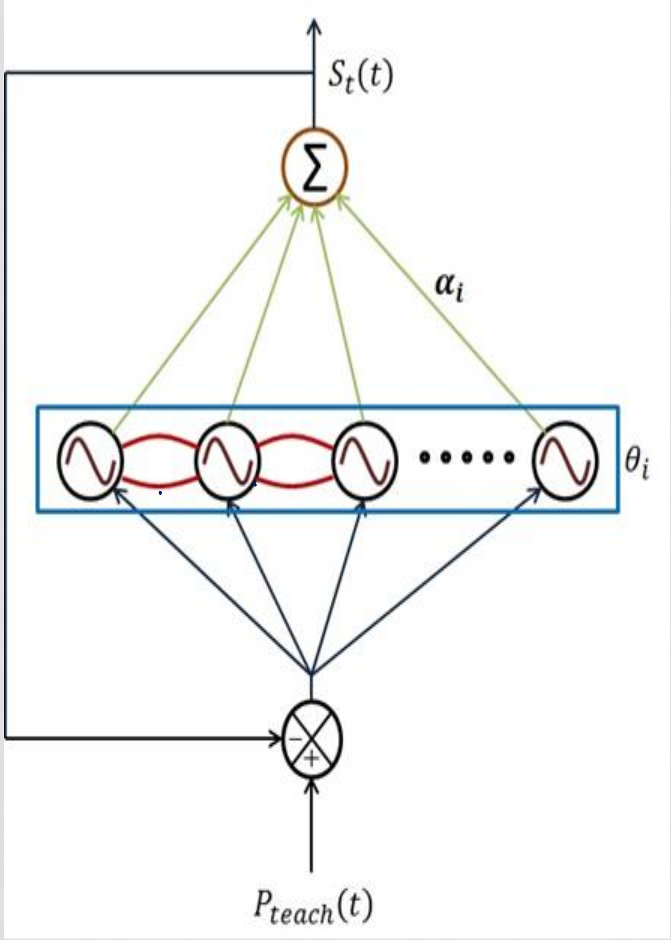
- Network of n no of Kuramoto oscillators can learn Some sort of Fourier decomposition of any $P_{teach}(t)$ signal.

$$\dot{Z}_i = (\mu - |Z_i|^2)Z_i + i\omega Z_i + \sum_j A_{ij} e^{i\frac{\phi_{ij}}{\omega_j}} Z_j^{\frac{\omega_i}{\omega_j}} + \varepsilon\big(P_{teach}(t) - S(t)\big)$$

$$\dot{\theta}_j = \omega_j + \sum_k A_{jk} \sin \omega_j \left(\frac{\theta_k}{\omega_k} - \frac{\theta_j}{\omega_j} + \frac{\phi_{jk}}{\omega_j \omega_k}\right) + \varepsilon\big(P_{teach}(t) - S(t)\big) \sin \theta_j$$

$$\dot{\omega}_j = -\eta_\omega \big(P_{teach}(t) - S(t)\big) \sin \theta_j \quad \checkmark$$

$S(t) = \sum_j \alpha_j \cos \theta_j$ (reconstructed signal)



TEST CASE#1
THE INTRINSIC FREQUENCIES OF THE OSCILLATORS LEARNED THE FREQUENCY COMPONENTS OF THE INPUT SIGNAL

$$P_{teach}(t) = A_1 \cos(\omega_1 t + \emptyset_1) + A_2 \cos(\omega_2 t + \emptyset_2) + A_3 \cos(\omega_2 t + \emptyset_2)$$

Where: $A_1 = 1.5909, A_2 = 1.377, A_3 = 1.3938$

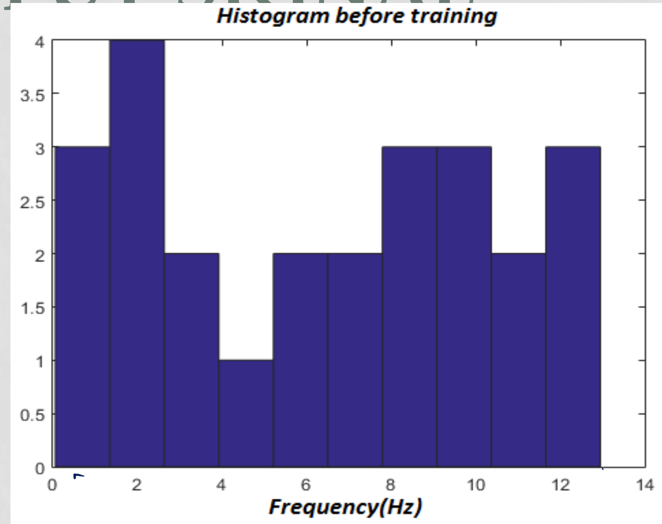
$$\omega_1 = 3, \omega_2 = 7, \omega_3 = 11$$

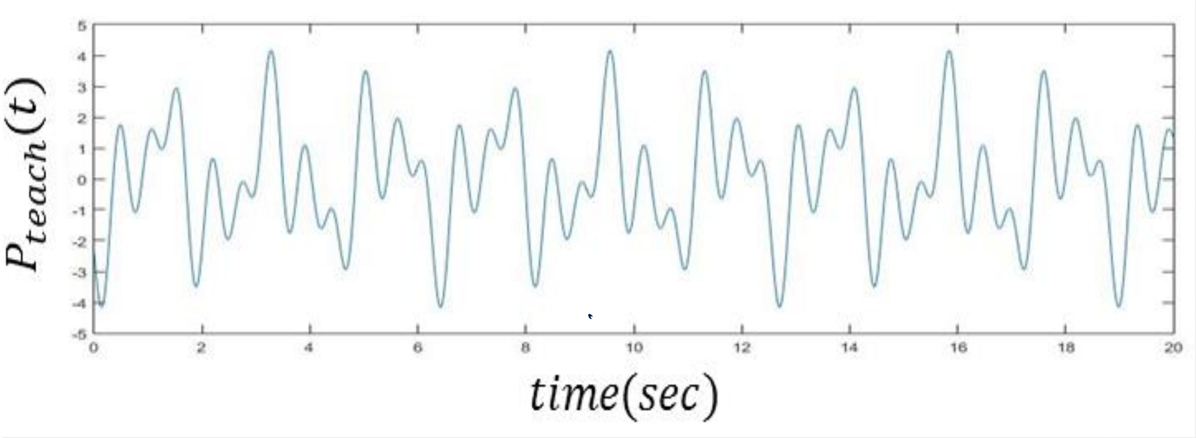
$$\emptyset_1 = 2.6911, \emptyset_2 = 2.6579, \emptyset_3 = 1.3755$$

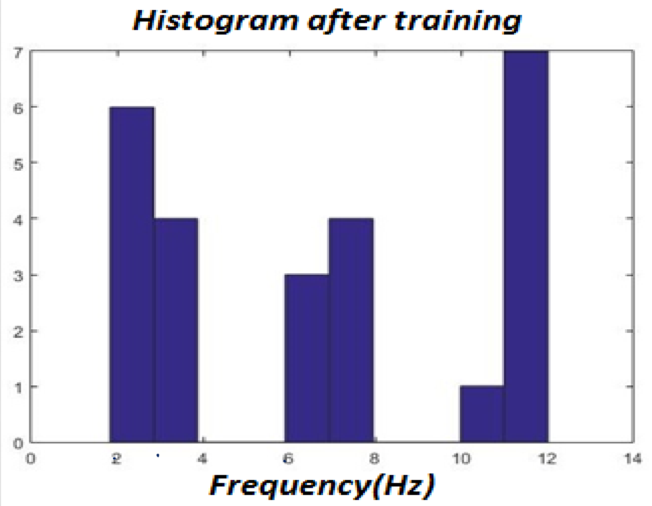
$$n = 25$$

$$\eta_\omega = 0.001$$

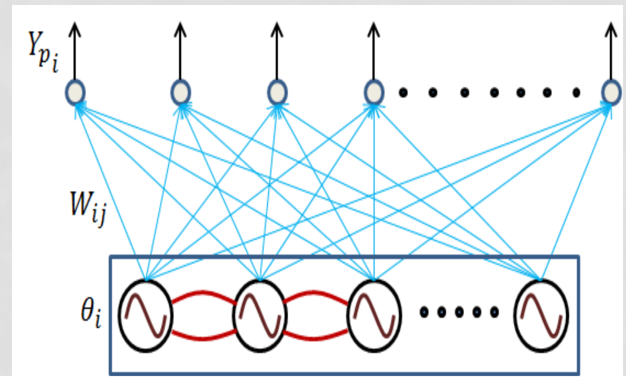
$$\alpha_j = 0.2$$







GENERALIZED NETWORK#2
FROM LEARNING FREQUENCY SPECTRUM REPRESENTATION TO MODELLING ARBITRARY M NO OF TIME SERIES SIGNALS HAVING SAME FREQUENCY COMPONENTS PRESENT IN THE SIGNAL USED TO TRAIN NETWORK#1



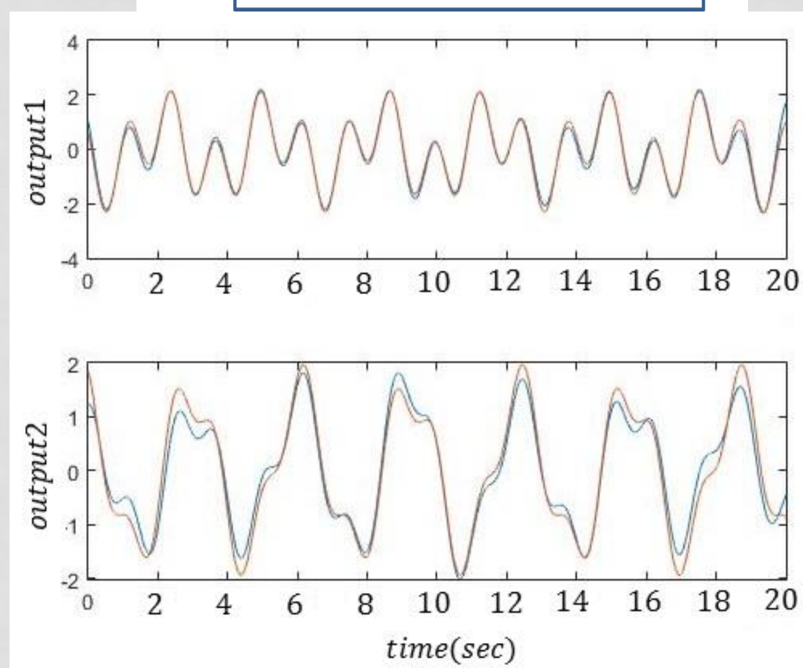
$$L = \frac{1}{2}\sum_{i=1}^{m}\sum_{t}\left\|Y_{d_i}(t) - Y_{p_i}(t)\right\|^2$$

$$Y_{p_i}(t) = real\left(\sum_{j=1}^{n} W_{ij}e^{i\theta_j}\right)$$

$$W_{ij} = K_{ij}e^{i\emptyset_{ij}}$$

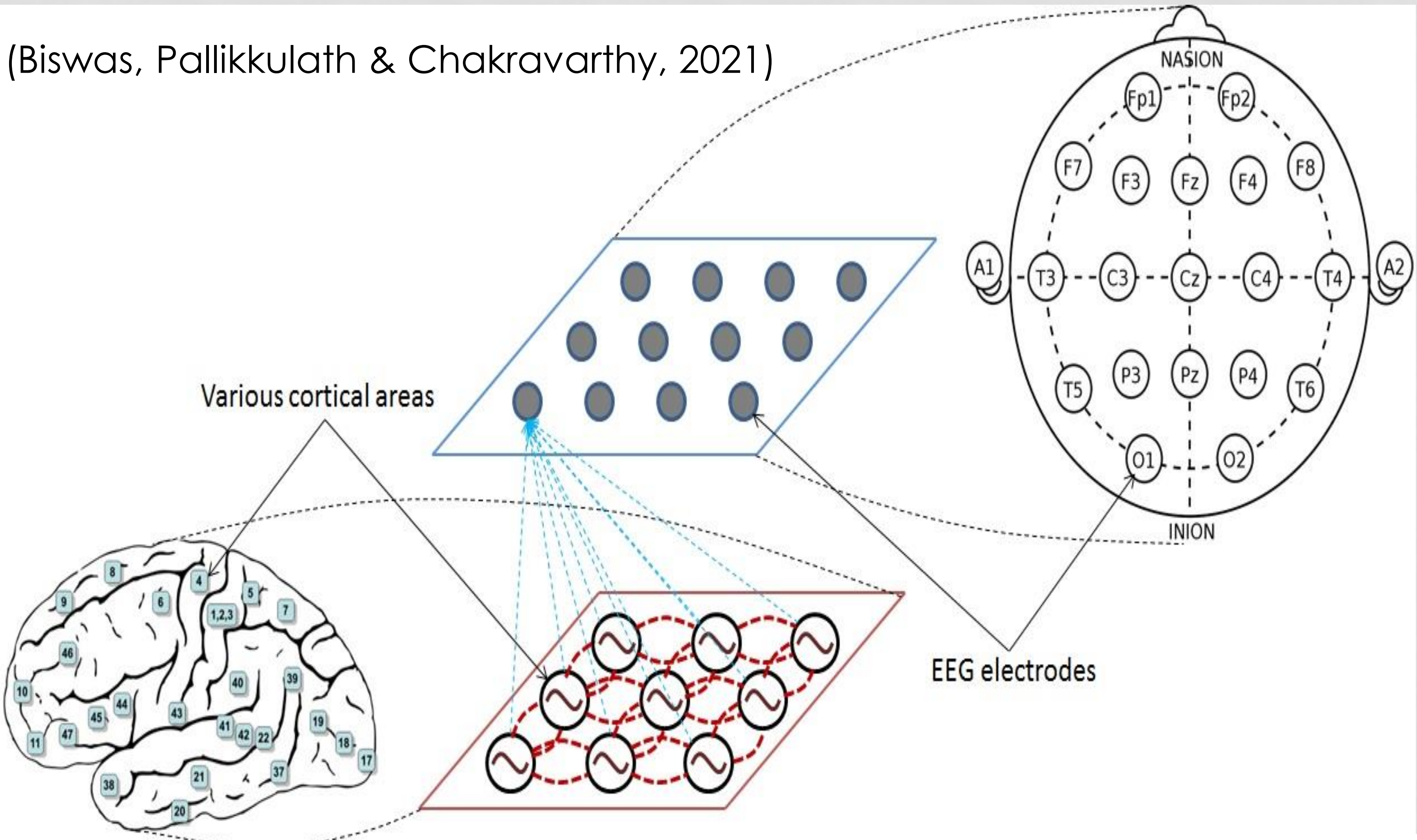
$$\Delta K_{ij} = (-1)\eta_{K_{ij}}\sum_{t}\left(Y_{d_i}(t) - Y_{p_i}(t)\right)\cos\left(\theta_j(t) + \emptyset_{ij}\right)$$

$$\Delta\emptyset_{ij} = (-1)\eta_{\emptyset_{ij}}\sum_{t}\left(Y_{d_i}(t) - Y_{p_i}(t)\right)\left(-K_{ij}\sin\left(\theta_j(t) + \emptyset_{ij}\right)\right)$$



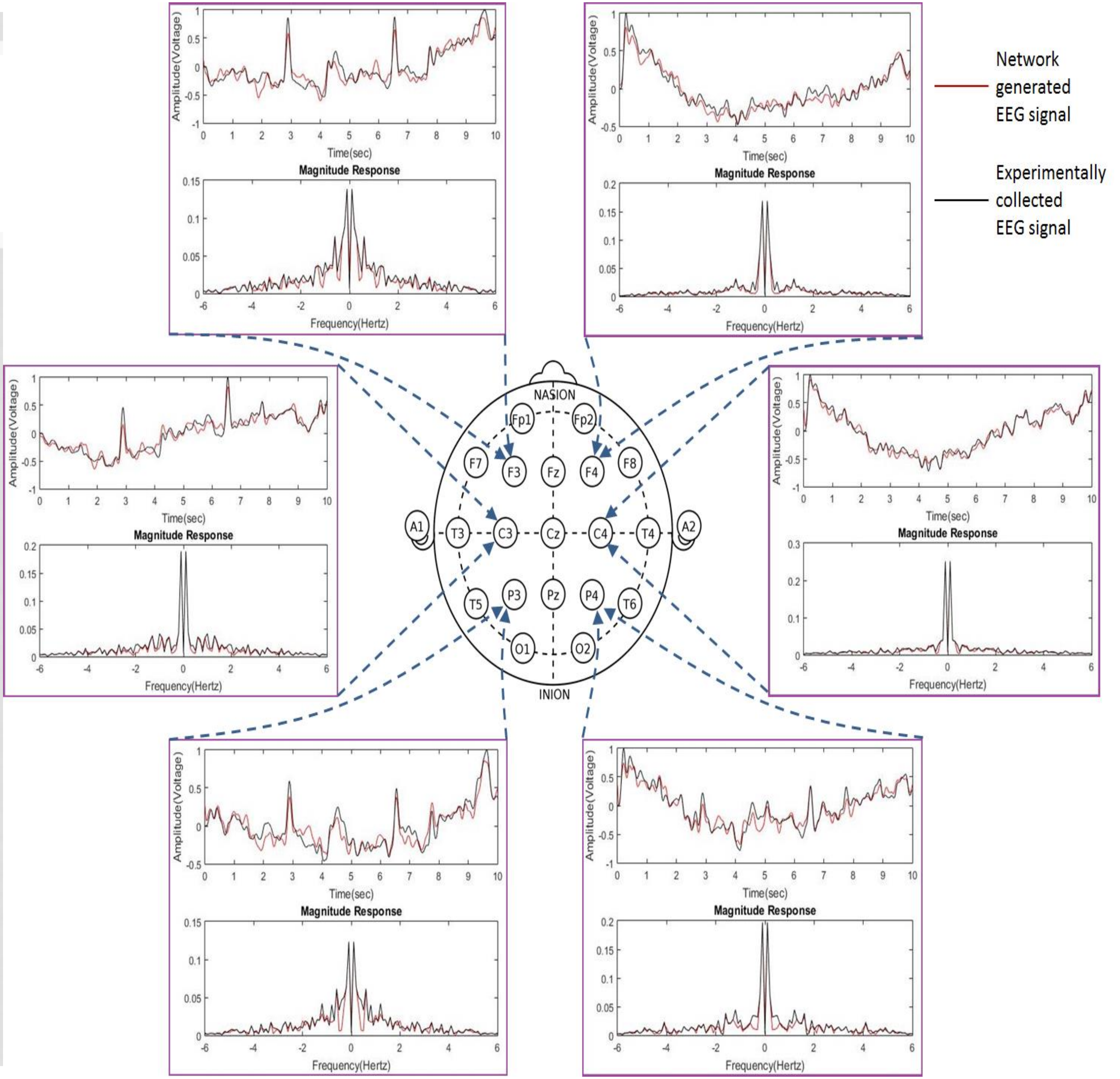
MODELLING ELECTROENCEPHALOGRAPHIC SIGNAL USING NETWORK#2

(Biswas, Pallikkulath & Chakravarthy, 2021)

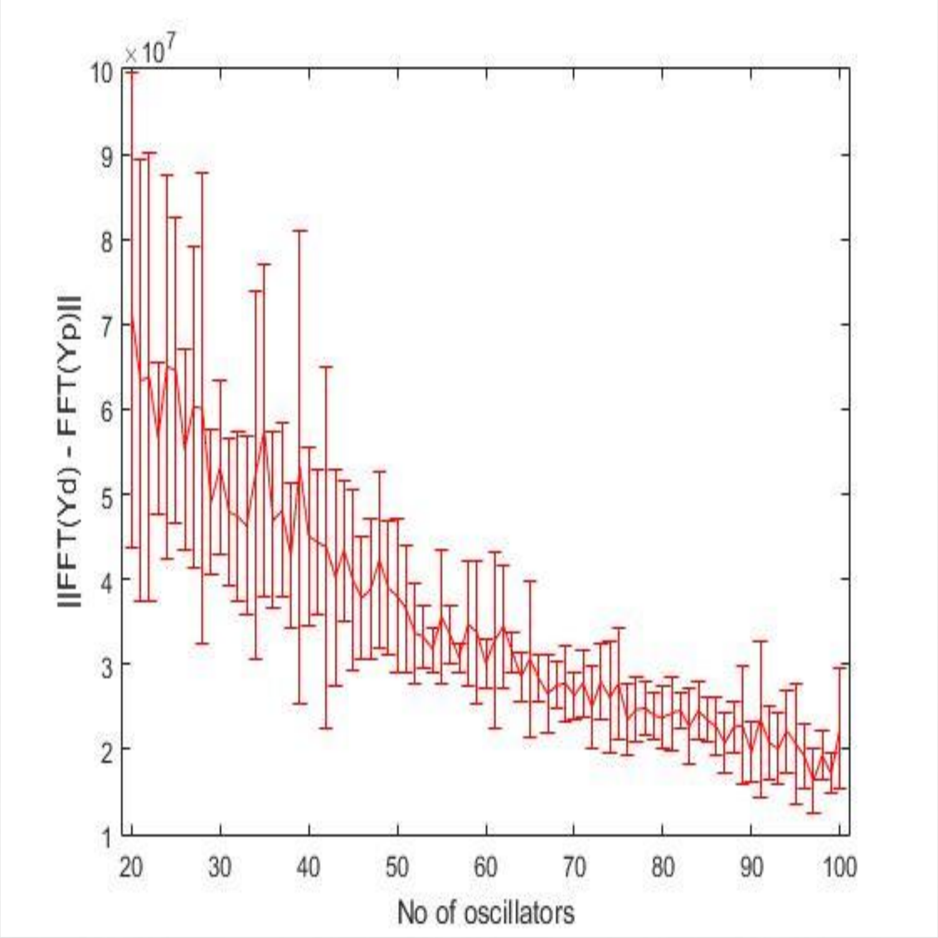


PHASE #1 NETWORK LEARNS THE FREQUENCY SPECTRUM OF A EEG SIGNAL COLLECTED FROM ONE ELECTRODE AND IN THE

PHASE #2: NETWORK LEARNS TO RECONSTRUCT N-EEG CHANNELS



RECONSTRUCTION ACCURACY INCREASES WITH #OSCILLATORS IN THE NETWORK



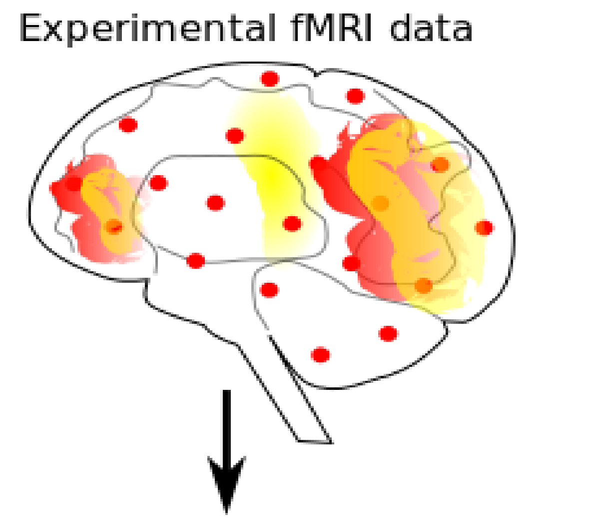
FUTURE STEPS

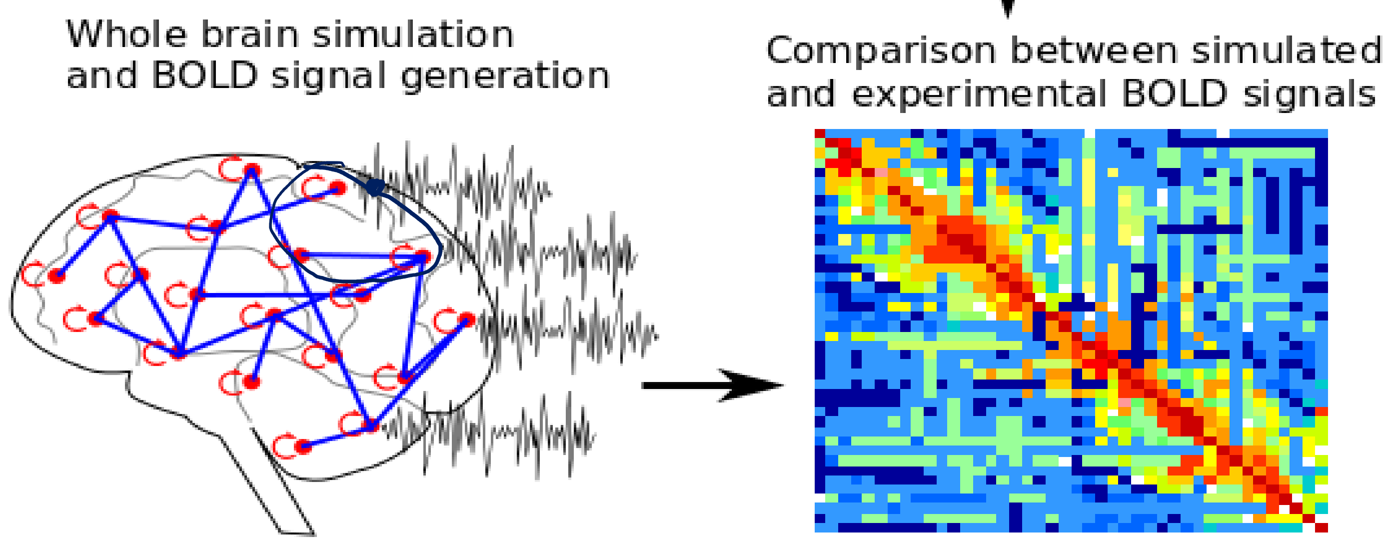
- Large scale Brain models using Oscillatory Deep Neural Networks
- Modeling the Auditory system using Oscillatory Neural Networks
- Generalized Deep Oscillatory Neural Networks

1. LARGE SCALE BRAIN MODELS

The Virtual Brain (TVB)

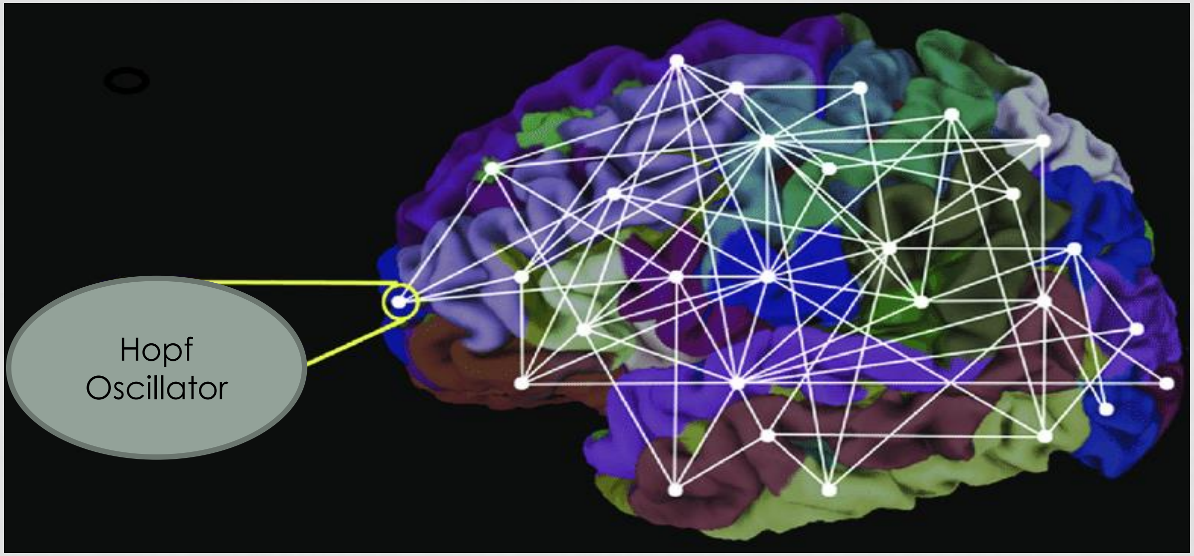
- Network of Wilson-Cowan Oscillators
- Connections computed by General optimization

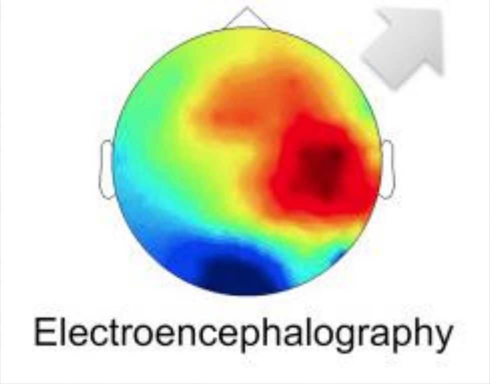


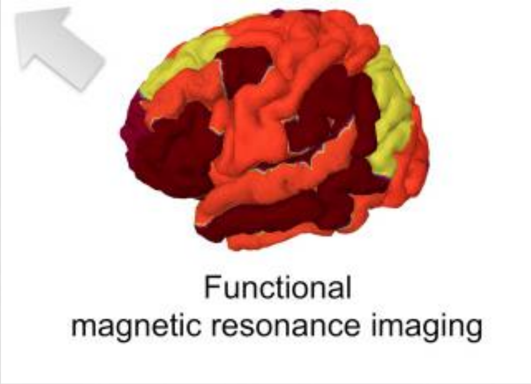


https://www.thevirtualbrain.org/tvb/zwei

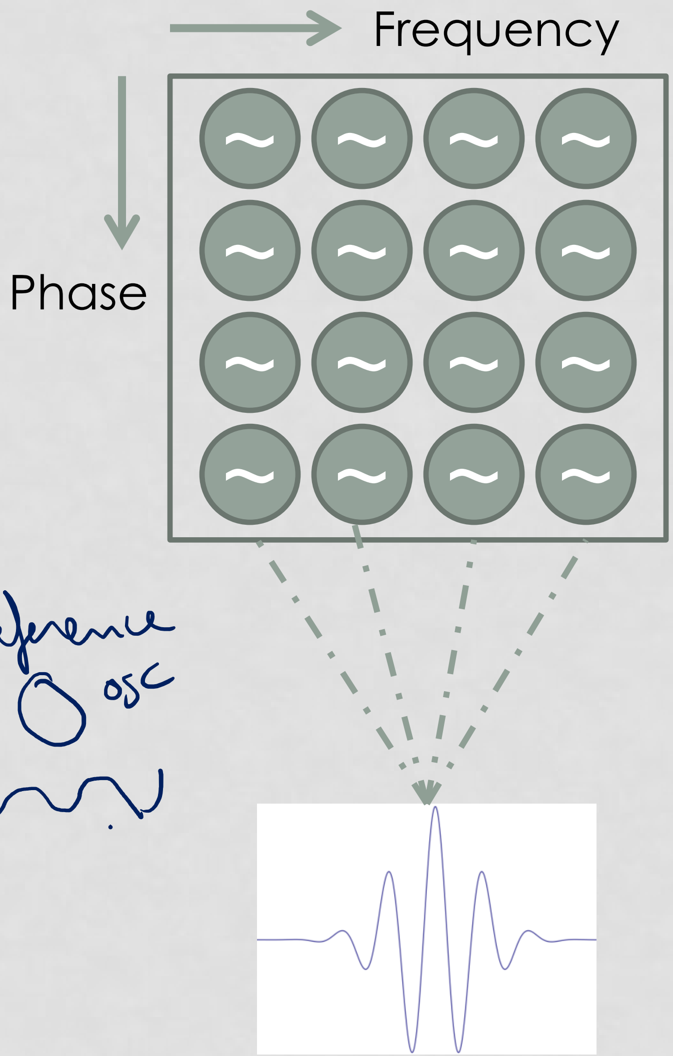
LARGE SCALE BRAIN MODELING USING OSCILLATORY NEURAL NETWORK







2. TONOTOPIC MAP
MAP OF SOUNDS OR FREQUENCIES



Reference osc

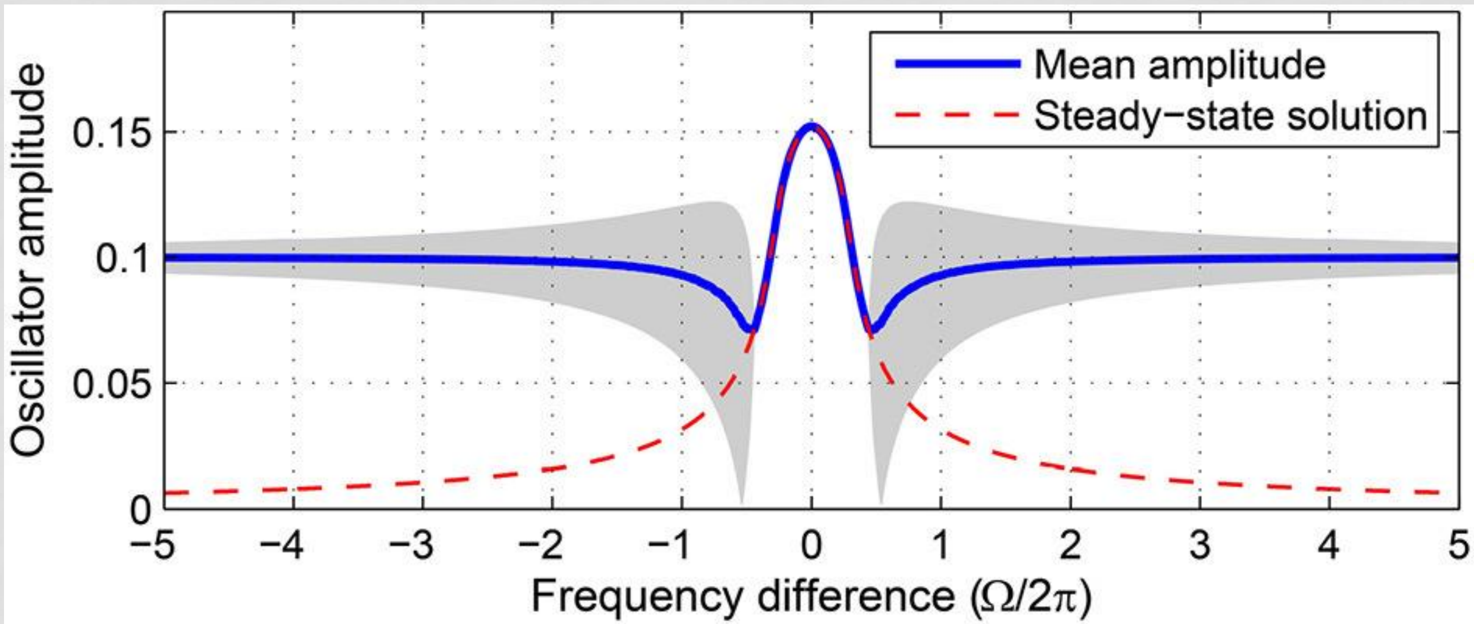
Fourier Series

$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

HOPF OSCILLATORS EXHIBITS RESONANCE

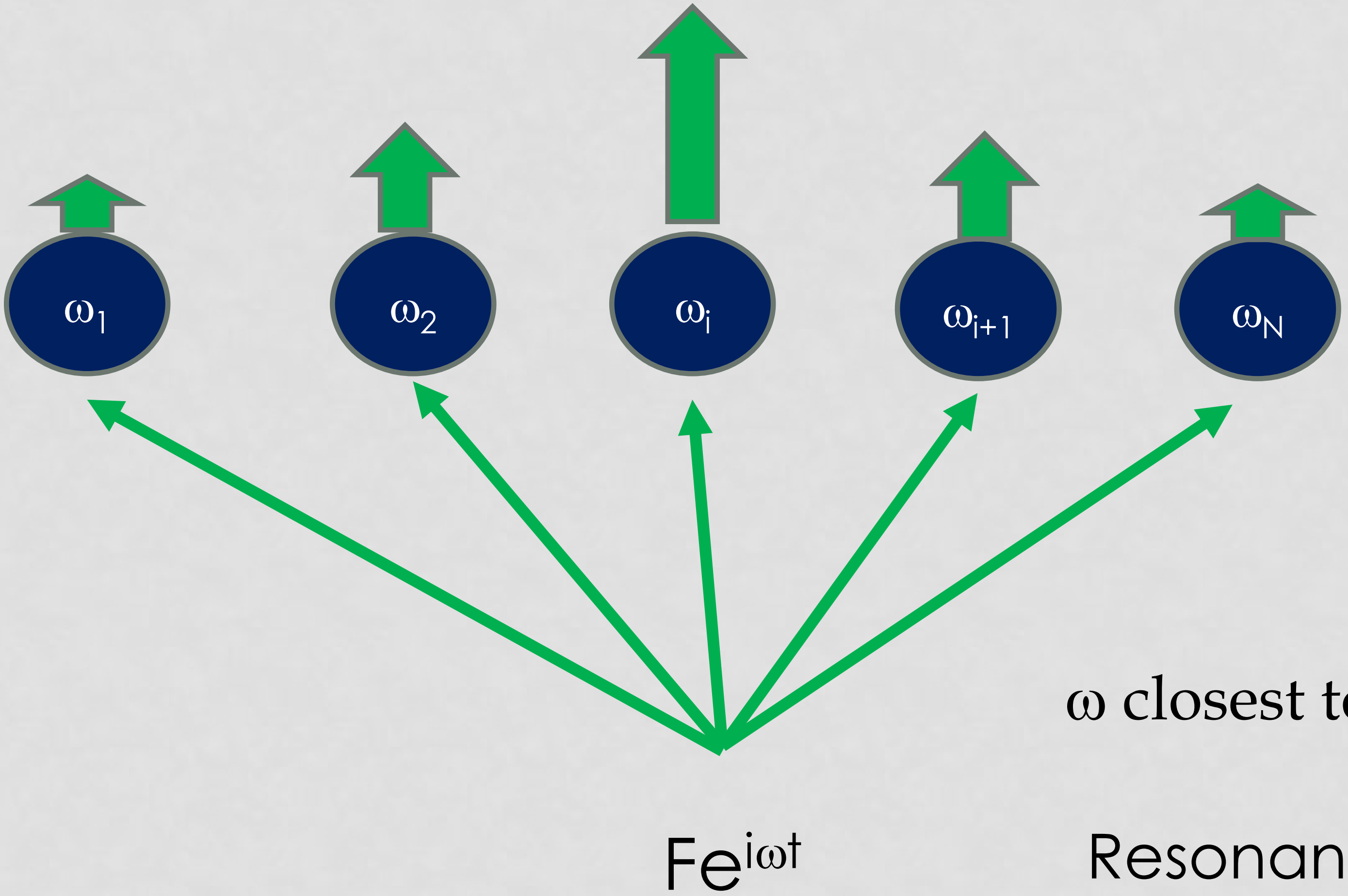
$ze^{i\omega t}$



$Fe^{i\omega t}$

$\Omega=\omega-\omega_0$

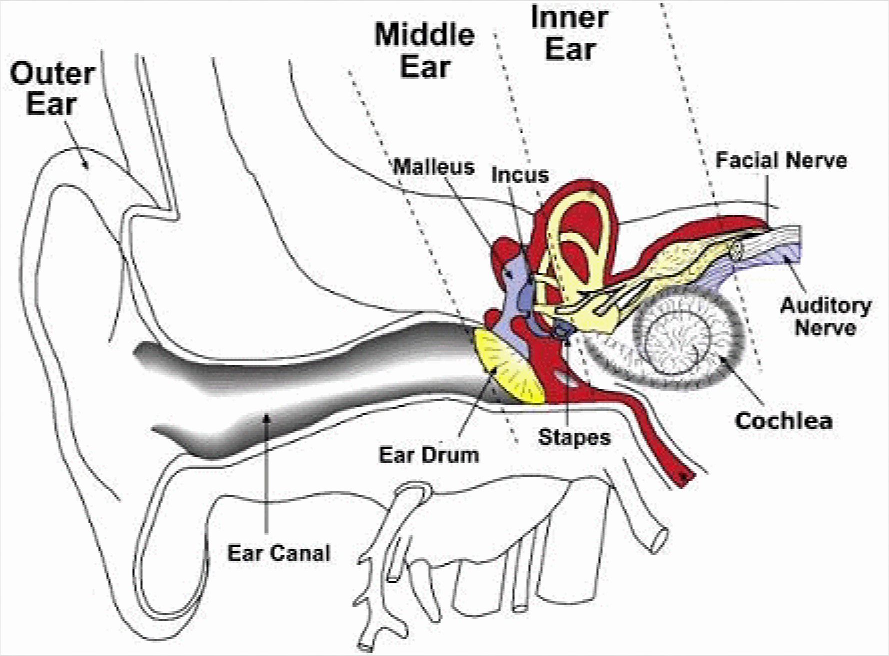
(kim & Large,)

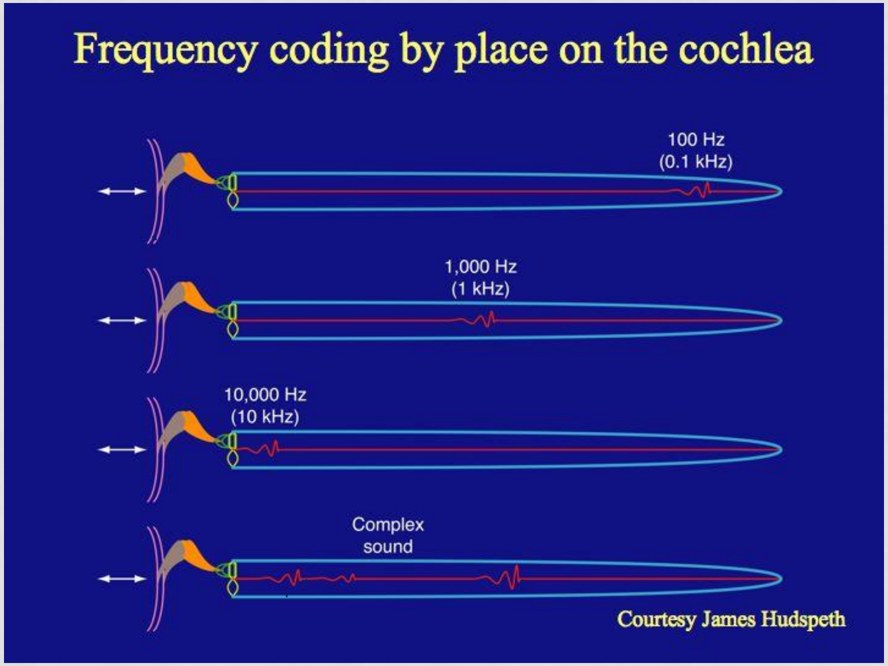


ω closest to ω_i

Resonance!

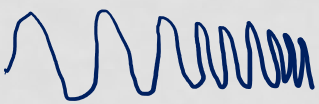
FOURIER DECOMPOSITION IN COCHLEA

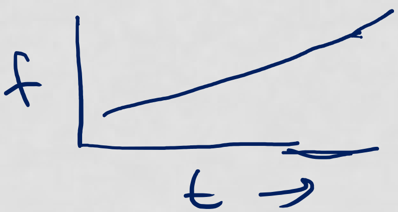


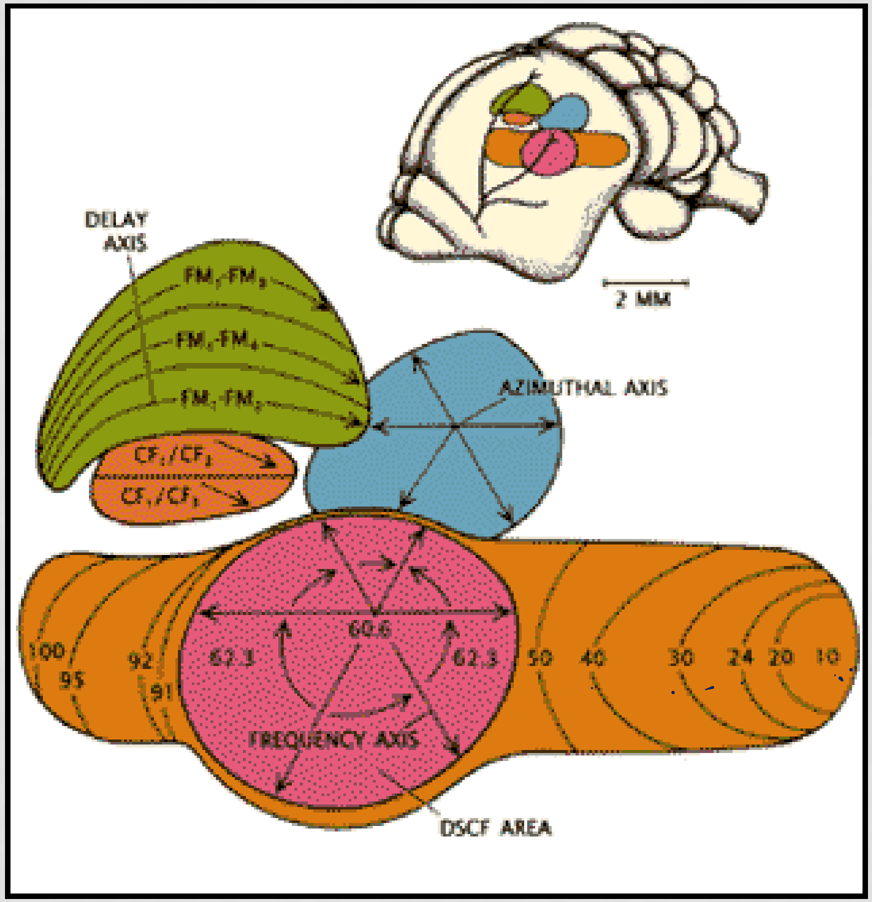


AUDITORY CORTEX OF BATS

chirp signals.



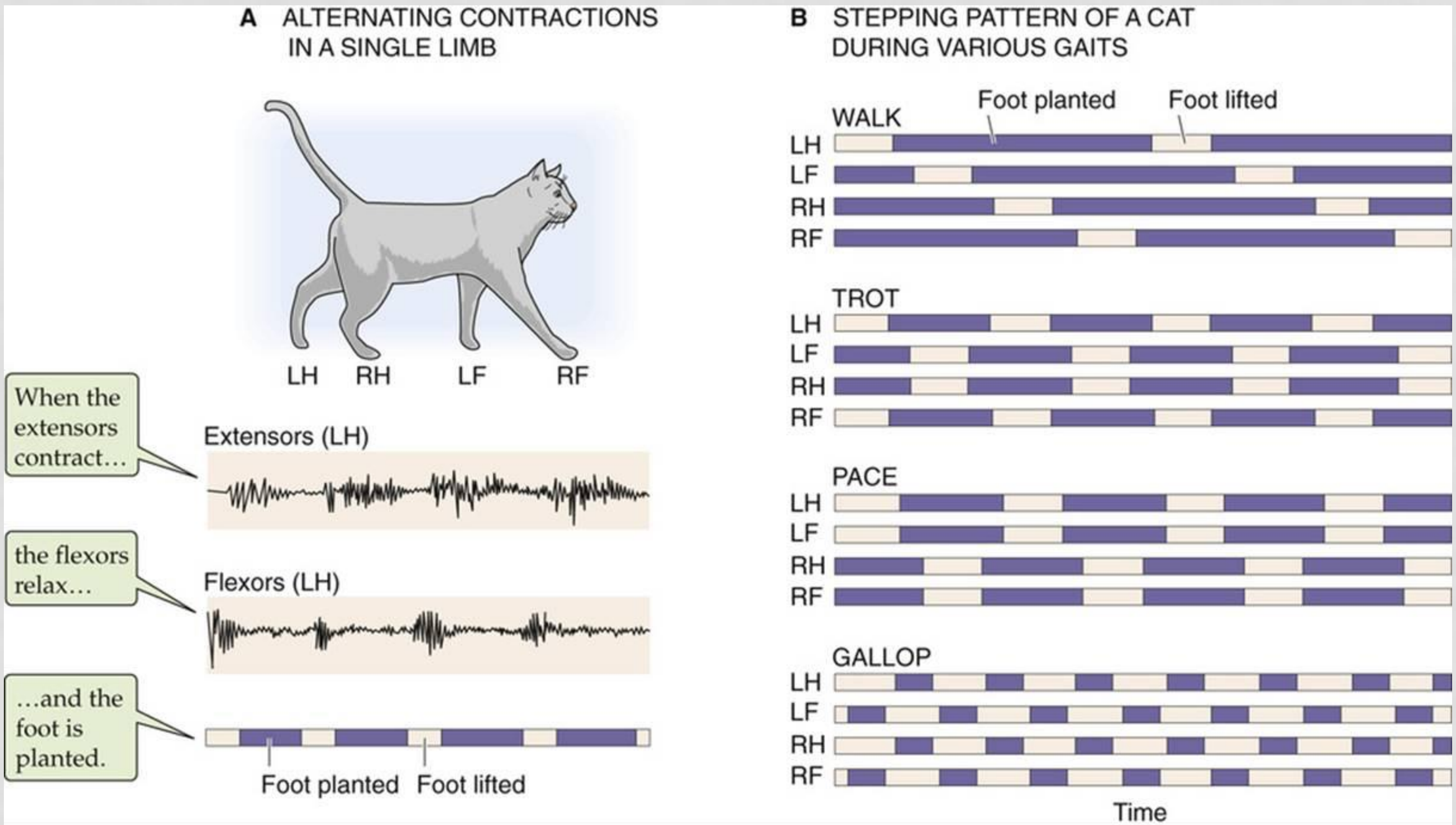




Nobuo Suga

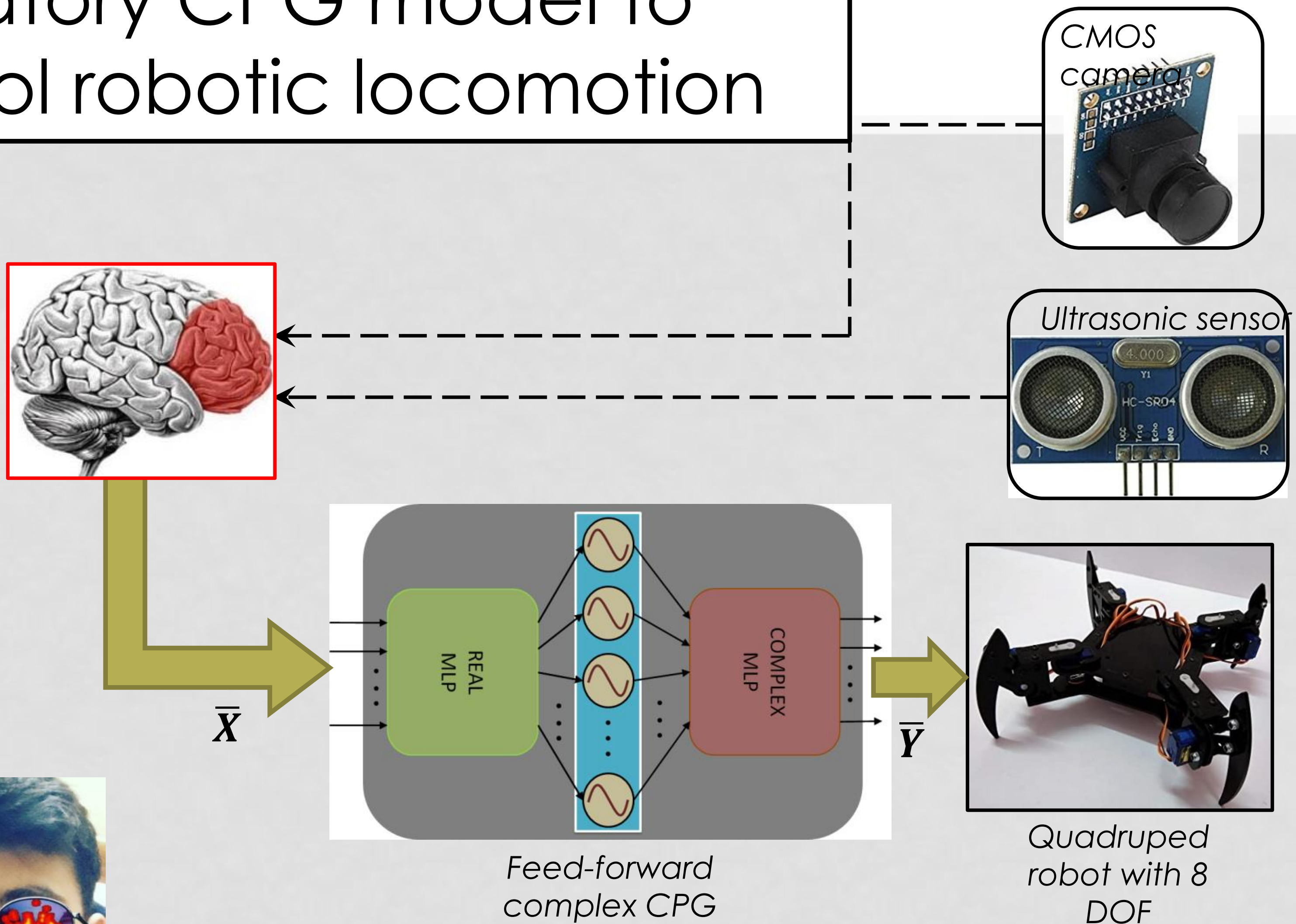
http://nelson.beckman.illinois.edu/courses/neuroethol/models/bat_echolocation/bat_echolocation.html

3. MODELING LOCOMOTOR RHYTHMS

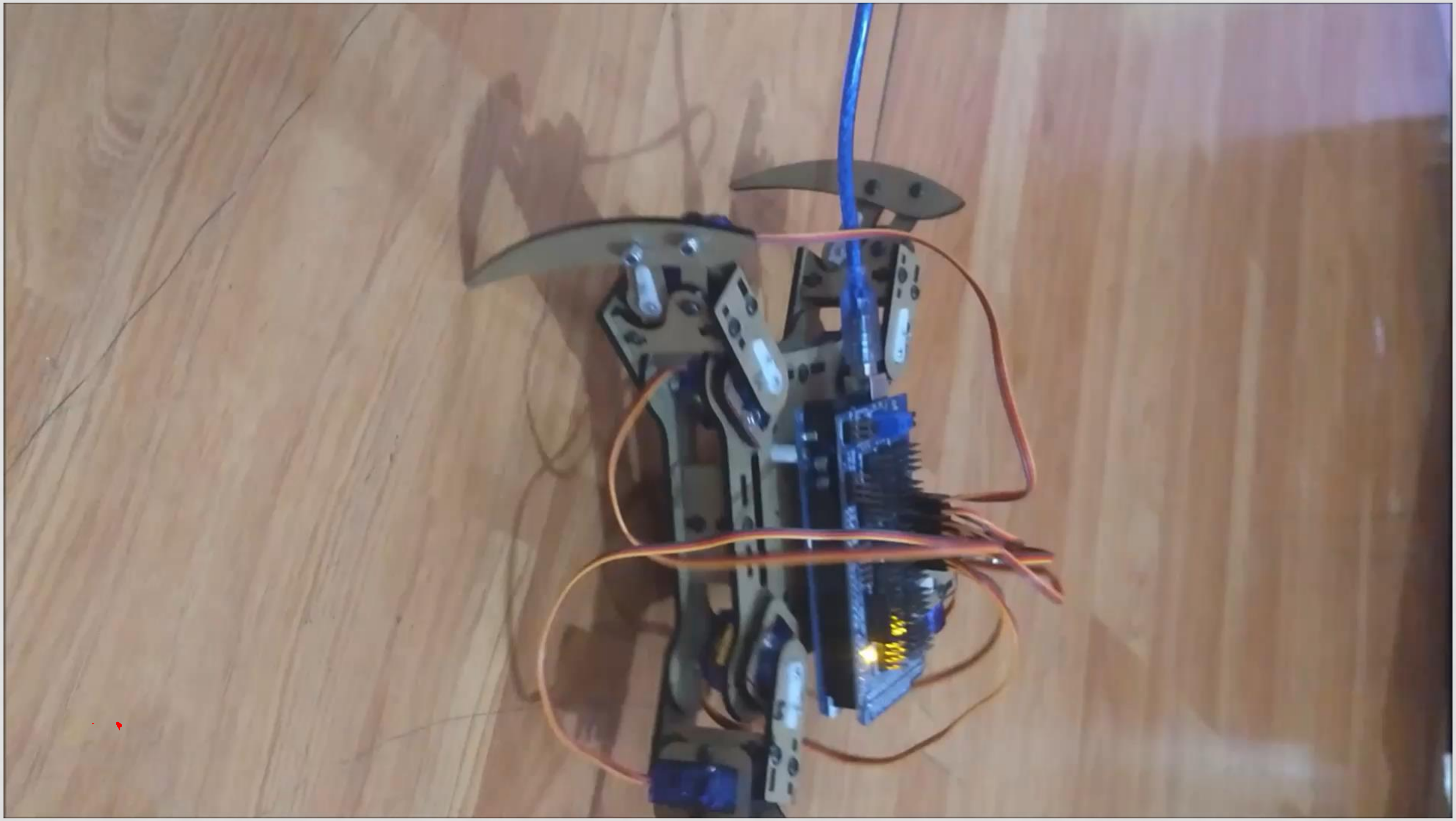


https://doctorlib.info/physiology/medical/88.html

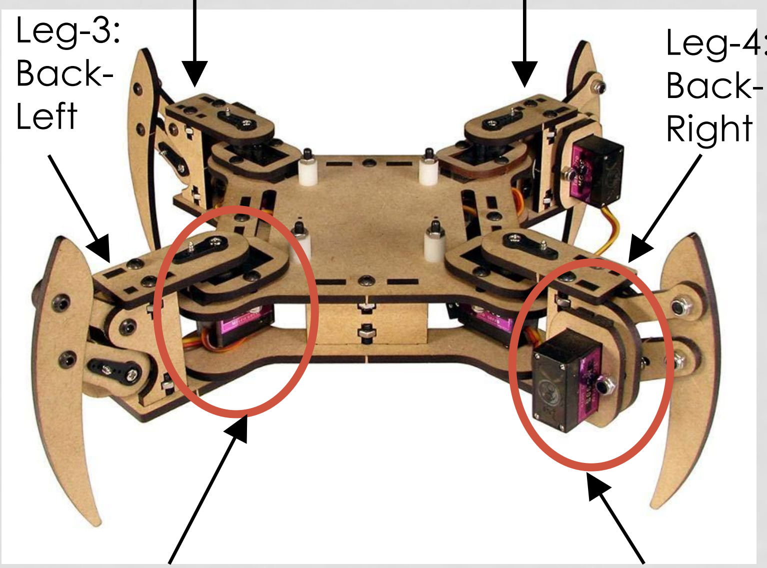
A feed-forward supervised oscillatory CPG model to control robotic locomotion

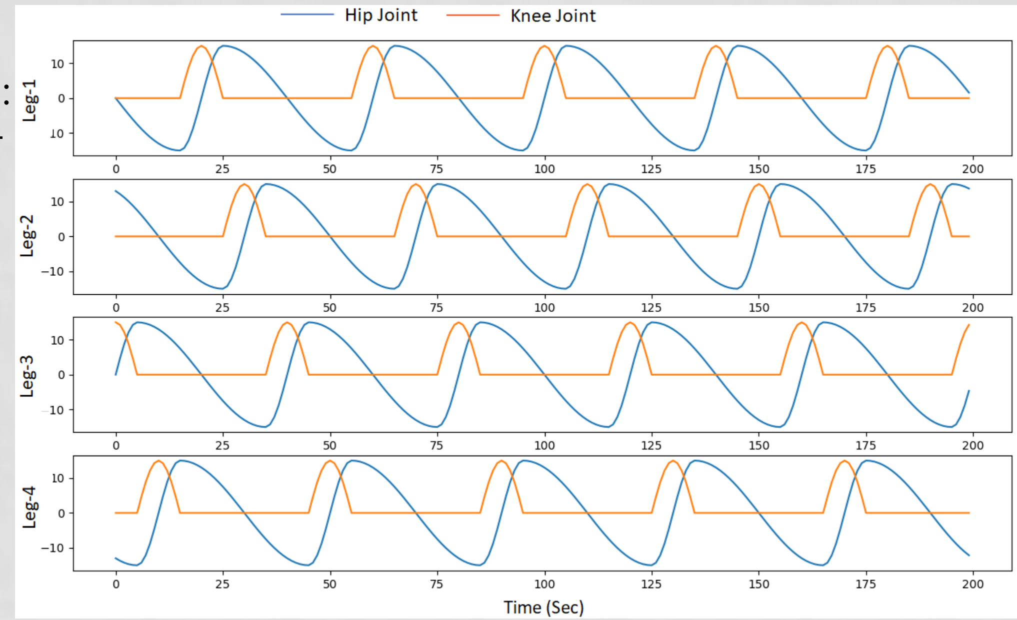


CURRENT IMPLEMENTATION



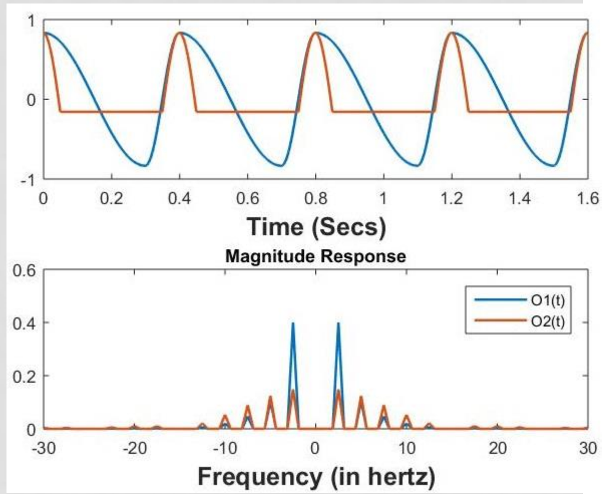
MEPED V2 QUADRUPED ROBOT:

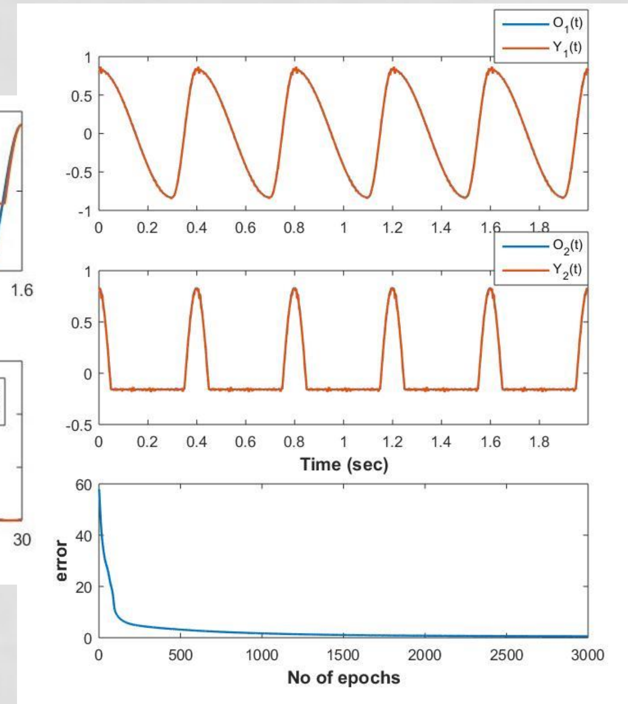




SUPERVISED CPG NETWORK TO PRODUCE GAIT RHYTHMS:

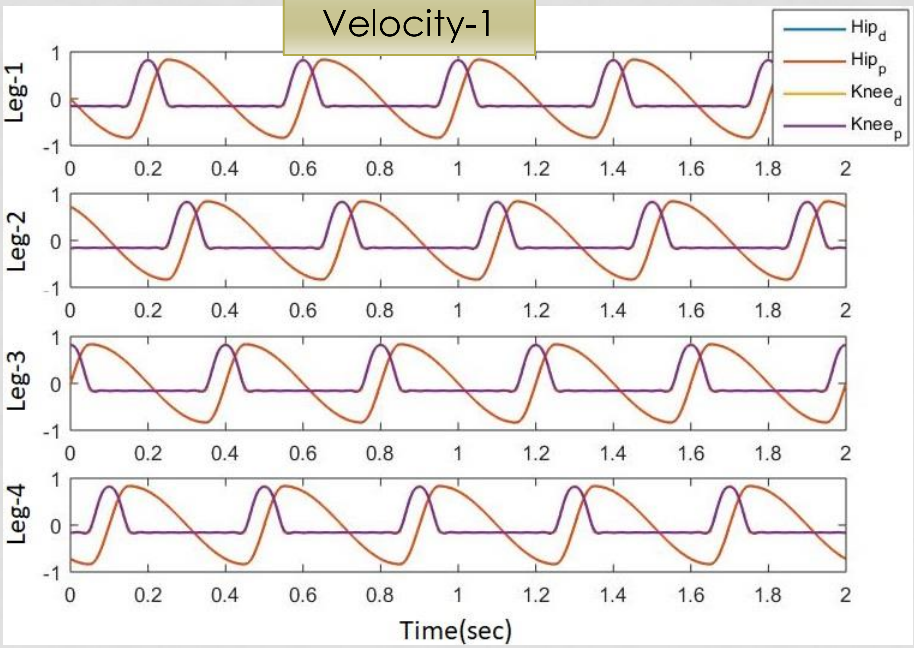
- As the gait rhythms are periodic in nature the constitutive frequency components are harmonics of the fundamental frequency (f_d) of the gait rhythm.
- A perceptron with complex weights projecting the activity of reservoir of oscillators with natural frequencies as harmonics of f_d can learn to produce desired gait pattern with reasonable accuracy.



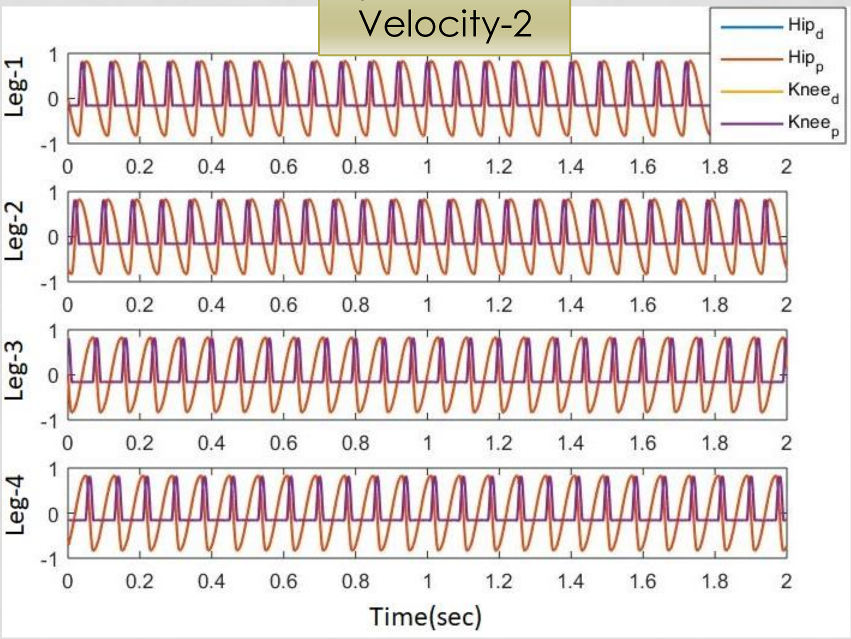


THE FOUR DIFFERENT GAIT PATTERNS LEARNT BY THE NETWORK:

Gait pattern-1: Velocity-1

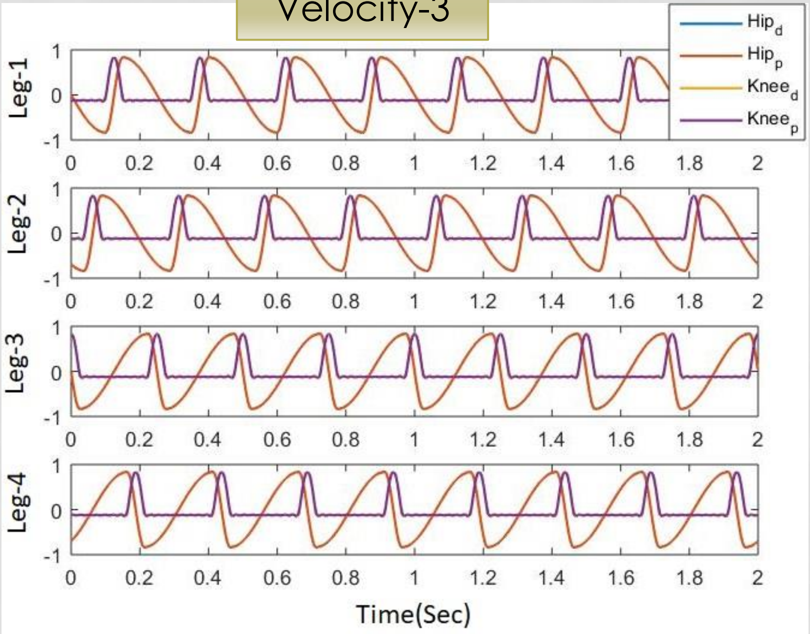


Gait pattern-2: Velocity-2

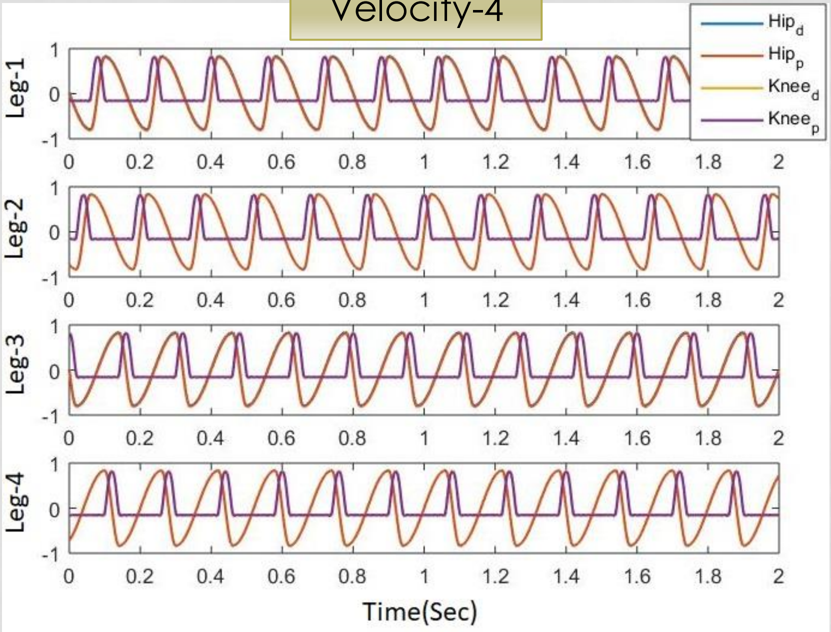


DESIRED AND THE NETWORK LEARNT GAIT PATTERNS:

Gait pattern-3: Velocity-3



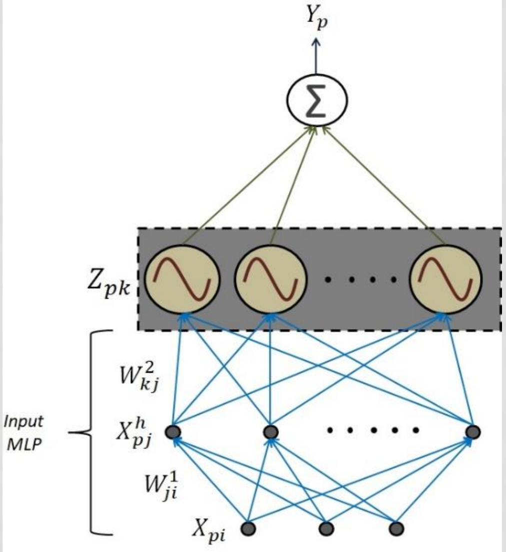
Gait pattern-4: Velocity-4

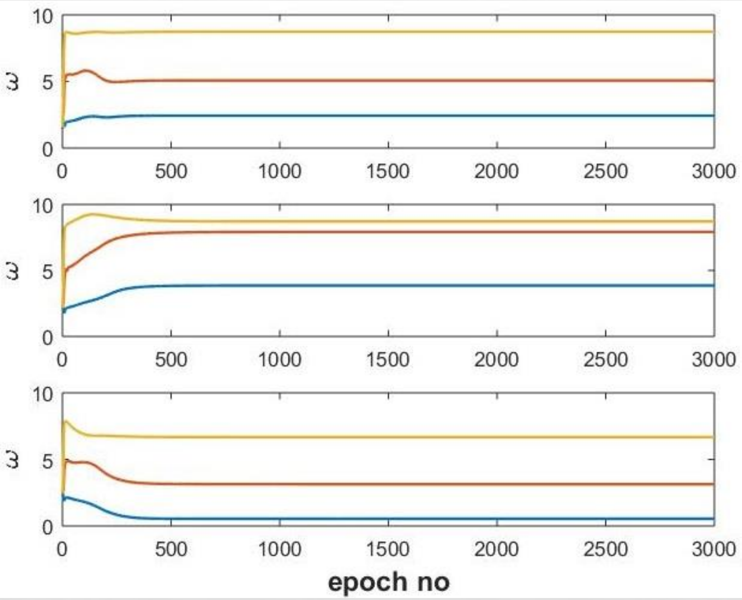


A FEED-FORWARD HYBRID NETWORK WITH HOPF OSCILLATORS TO LEARN FOURIER DECOMPOSITION OF MULTIPLE SIGNALS

Network-1:

- The output of the "input MLP" is the natural frequencies of the oscillators.
- The parameters of the MLP are supposed to learn the frequency components in the desired output signal.

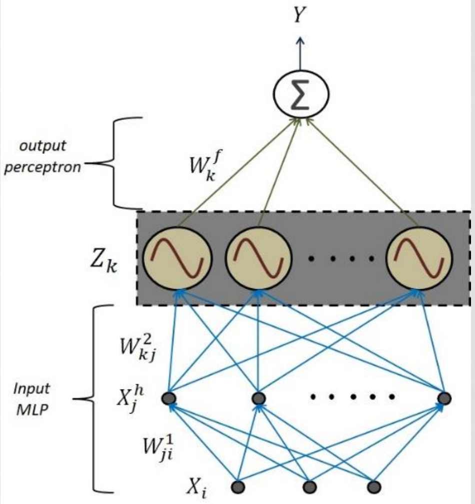


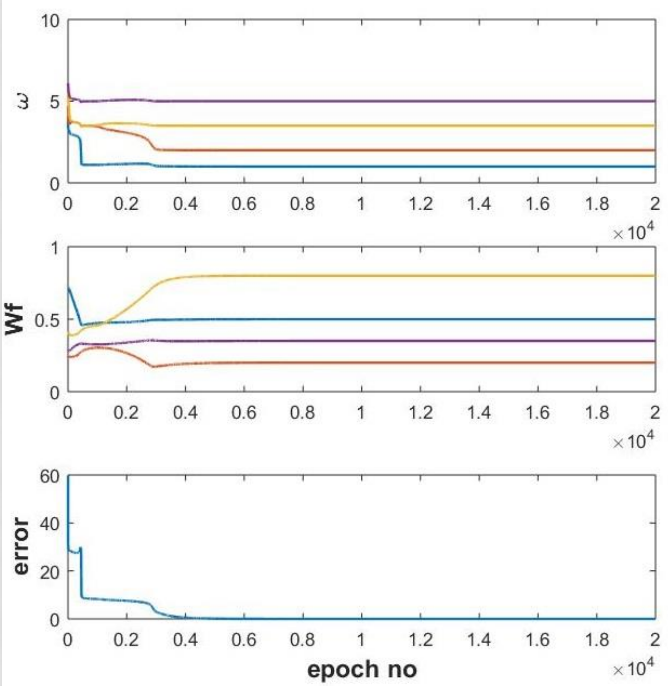


	Desired input			Desired output
1	0.7544	0.8892	0.7104	$\cos(8.7403t) + \cos(5.0725t) + \cos(2.4314t)$
2	0.2465	0.4514	0.9312	$\cos(3.8588t) + \cos(8.7079t) + \cos(7.9213t)$
3	0.6157	0.9312	0.6311	$\cos(6.6784t) + \cos(0.5457t) + \cos(3.1394t)$

NETWORK-2: CAN LEARN FREQUENCY AS WELL AS THE MAGNITUDE OF A OUTPUT SIGNAL

- The real weights of the “output” perceptron learn the magnitude of the various frequency components learnt by the oscillators.

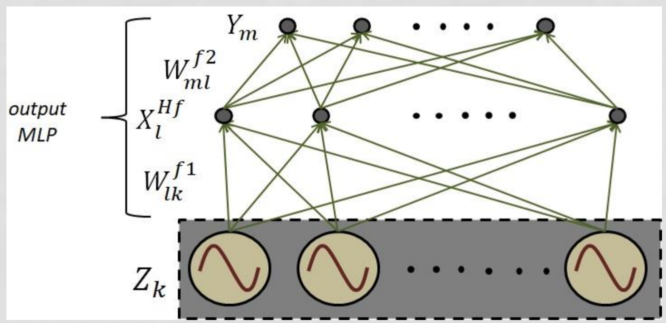


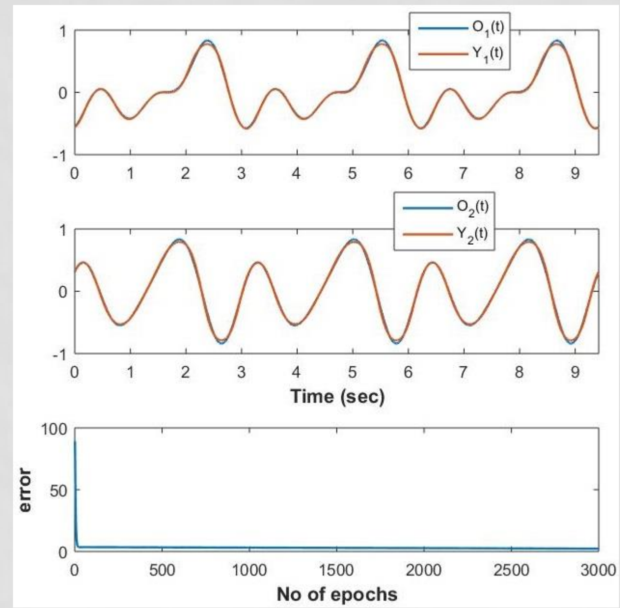


	Desired input			Desired output
1	0.4108	0.7703	0.3400	$0.2\cos(2t) + 0.8\cos(3.5t) + 0.5\cos(t) + 0.5\cos(5t)$

NETWORK-3: CAN LEARN THE MAGNITUDE AND THE PHASE OFFSET OF THE DESIRED OUTPUT SIGNAL

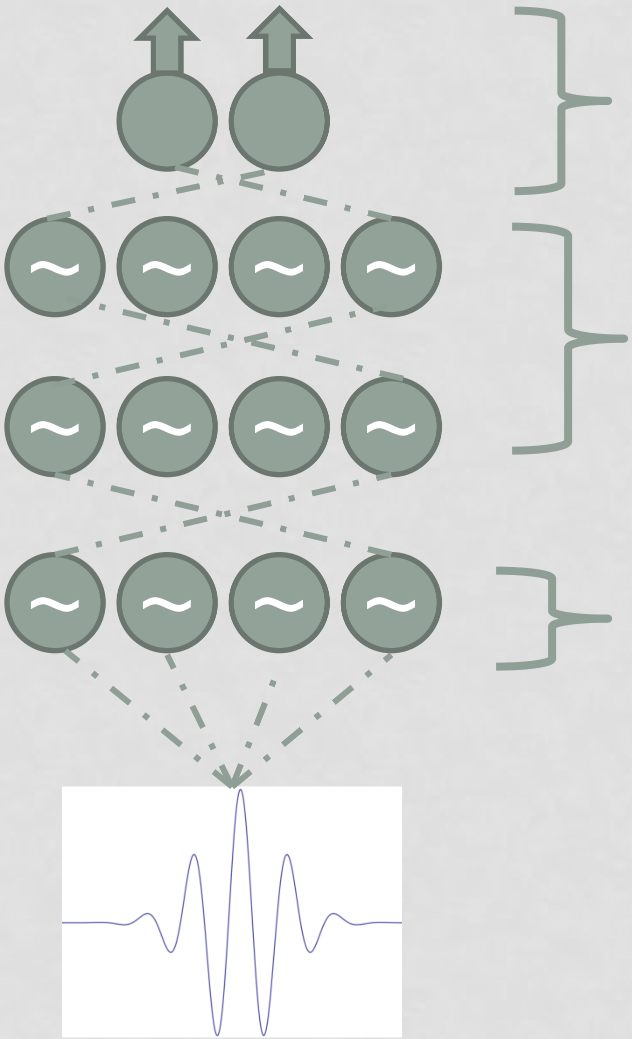
- Given that the constitutive frequency components of the desired output signals at the output nodes are same as the natural frequencies of the Hopf oscillators the network can learn to predict the output by tuning the parameters of the feed-forward complex MLP.





Desired input	Desired output 1	Desired output 2
$\overline{Z(t)}$ $(\omega = [2; 4;\ 6])$	$0.9963\cos(2t + 4.3585)$ $+ 0.7657\cos(4t + 5.9176)$ $+ 0.6521\cos(6t + 3.9679)$	$0.5754\cos(2t + 3.8775)$ $+ 1.2634\cos(4t + 4.0434)$ $+ 0.3763\cos(6t + 0.9521)$

4. GENERALIZED OSCILLATORY DEEP NEURAL NETWORKS



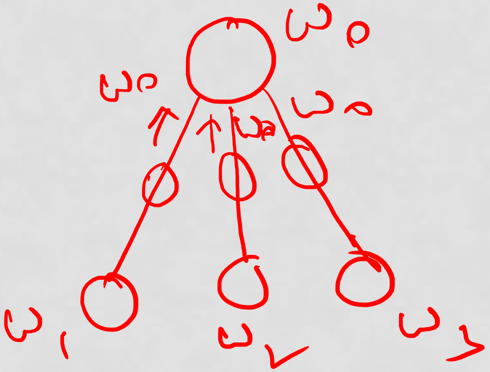
Final outputs: non-sinusoidal signals

Hidden neuron outputs: sinusoidal signals

$$Ae^{i(\omega t+\varphi)}$$

Fourier Decomposition stage

Training: complex-valued
Backpropagation algorithm



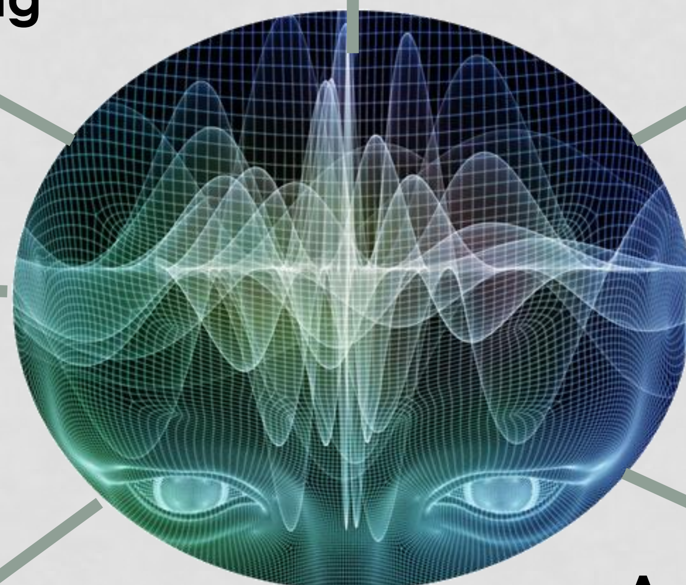
BASIC SCIENCE

AN OSCILLATOR THEORY OF BRAIN FUNCTION

Oscillations in
Spatial Navigation

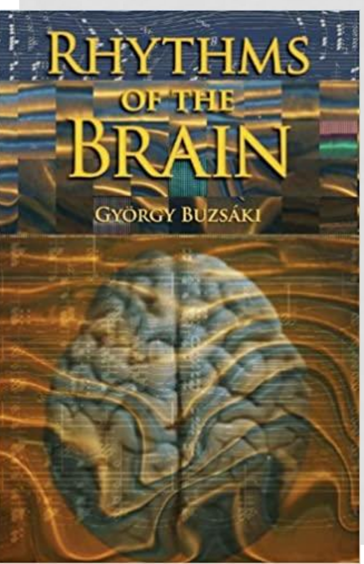
Oscillations in
Sensory Processing

Oscillations in
Attention



Oscillations in
Memory

Oscillations in
Motor Function



Oscillations in
Autonomic control

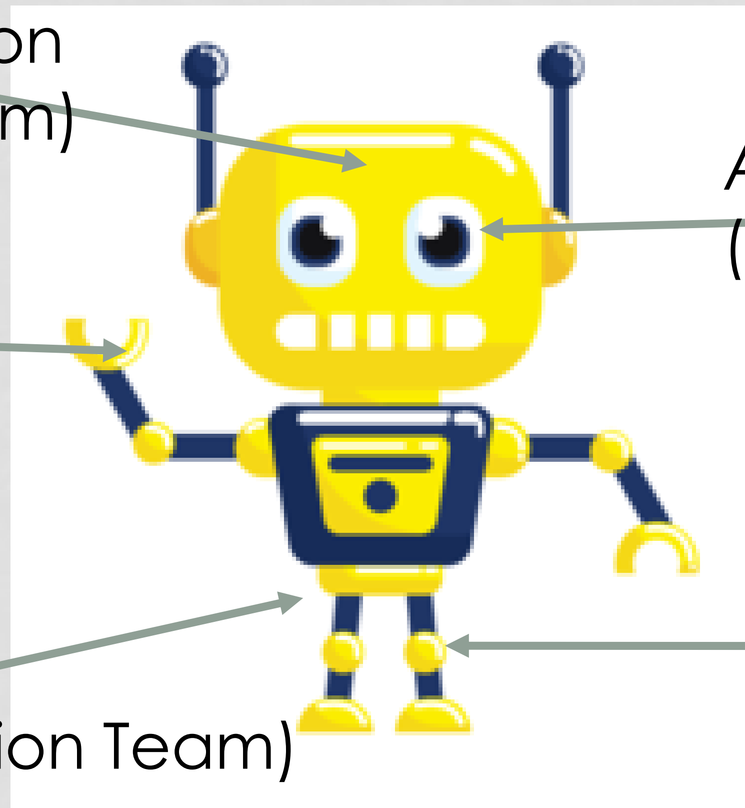
Oscillations in
Mood

Oscillations in
Brain disorders

APPLICATION

HANU: HIERARCHICAL ADAPTIVE NAVIGATING UNIT

A SEARCH-AND-LOCATE ROBOT WITH A BRAIN-INSPIRED NAVIGATIONAL SYSTEM



Verbal Instruction
(Language team)

Attentional Search
(Deep Vision Team)

Reaching &
Grasping
(Stroke Team)

Navigation
(Spatial Navigation Team)

Locomotion
(Oscillator Team)

ACKNOWLEDGEMENTS

- Dipayan Biswas
- Aasit
- Surya Kiran
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