

## Graph coloring problem

Let  $G$  be a graph and  $m$  be a positive integer. We need to find whether the nodes of  $G$  can be colored in such a way that no two adjacent nodes have the same color yet only  $m$  colors are used.

### Imp note

If  $d$  is the degree of given graph  $G$ , then, it can be colored with  $(d+1)$  colors.

Time complexity -  $O(nm^n)$

### Approach -

①  $f^n$  ①,  $mColoring(k)$

in an  $\infty$  loop  
 if  $(k+1)$  node does not have valid color use check  $nextvalue(k)$  for  $k$  for valid color for  $(k+1)$  node

1) call  $nextvalue(k)$  to get valid color

2) return if no color found

3) if node is last, write array  $x$

else go for next node by  $mcoloring(k+1)$

②  $f^n$  ②  $nextvalue(k)$

in an  $\infty$  loop  
 to check validity of each new color if previous color found to be same as adjacent

1) choose next color by using 'mod'  $0 \rightarrow 1 \rightarrow 2 \rightarrow \dots$

2) if no color chosen, return

3) use loop to travel for each node (other)  $j$  in range  $(\pm kn)$

i) check if adjacent if both true, break, this will go to check for next color

ii) color same

4) if step 3, does not break

checked by  $k, j = (n+1)$  then return

no. of nodes

$m$  - no. of colors (1,2,3)  
 $x$  - global array, solution in  $n$ -tuple, index  $\rightarrow$  node  
 $K$  - current node (0,1,2,3)  
 $n$  - no. of nodes.

$x = [0]^n$

mColoring(K)

{

  repeat

    { nextValue(K)

      if ( $x[K] == 0$ ) then return

      if ( $K == n-1$ ) then write  $x[1:n]$

      else

        mColoring(K+1)

    } until (false)

}

nextValue(K)

{

  repeat

    {

$x[K] = (x[K] + 1) \bmod (m+1)$

      if ( $x[K] == 0$ ) then return

      for  $j \leftarrow 0$  to  $n-1$

        do if ( $G[j][K] \neq 0$  and

$x[j] == x[K]$ )

        then break

      if ( $j == n-1$ ) then return

    } until (false)

}

# next color, stored in  $x[K]$

# no valid color,  $x[K] == 0$

#  $K$  is 0 indexed so, if  $K == n-1$

it means  $K$  is last node

# move to next node

#  $x[K] == 0$ , terminates this

loop

# for each loop,  $x[K]$  is tried

# for valid color

# no valid color found

# for each available node

# if adjacent

# same color

# if for loop executes without

break then the color is valid

here return.