Lab Assignment – 04 – Spring 2020

Signal & Systems

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Summary:

- This assignment mainly dealt with finding the convolution of continuous signal.
- There are two ways to achieve the task, one of which uses the inbuilt function trapz and the other is based on the method used for discrete signals in lab assignment 3.

1. We are given

   $x(t) = \exp(-2t)(u(t) - u(t - 4))$

   and

$$h(t) = \begin{cases} 1 - t & 0 \le t < 1 \\ t - 1 & 1 \le t < 2 \\ 0 & \text{Elsewhere} \end{cases}$$

We have to find the convolution of these two signals. The basis of solving this problem is that we calculate the integral of x(t-tau) and h(t) to find y(t) which is the output of this function.

➔ <u>Here's the code:</u>

Here we have used function f9 to implement the action of x(t-tau).

atharva deshpande ▸ Documents ▸ MATLAB

Editor - C:\Users\atharva deshpande\Documents\MATLAB\f9.m

lab4one.m | f9.m | lab4two.m | f10.m

```
function [x] = f9(t)
x=zeros(size(t));
x(t<0 & t>=4)=0;
x(t>=0 & t<4)=exp(-2*t(t>=0 & t<4));
end
```

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lab4one.m | lab4two.m | f10.m | f9.m

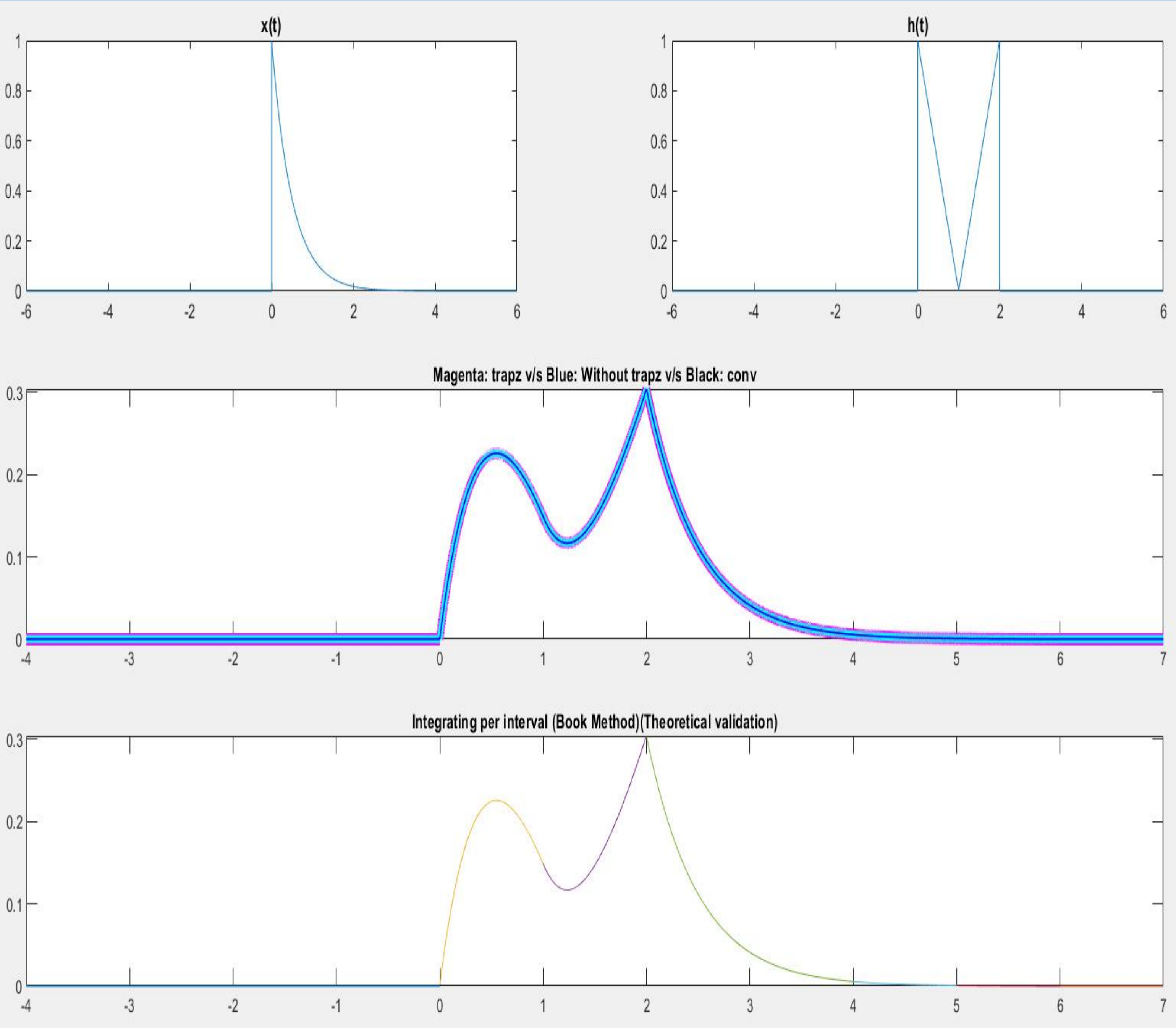
```

%
m=6;del=0.001;t=-m:del:m;
x=zeros(size(t));x(t<0 & t>=4)=0;x(t>=0 & t<4)=exp(-2*t(t>=0 & t<4));subplot(2,2,1);plot(t,x);title('x(t)');
h=zeros(size(t));h(t>=0 & t<1)=1-t(t>=0 & t<1);h(t>=1 & t<2)=t(t>=1 & t<2)-1;h(t<0 & t>=2)=0;subplot(2,2,2);plot(t,h);title('h(t)')
p=-2*m:del:2*m;
%First Method
y1=zeros(size(p));o=length(t);tau=t;
for i=1:o
    x1=f9(i*del-tau);
    z=x1.*h;
    y1(i)=trapz(tau,z);
end
y2=zeros(size(p));y2(1:m*2000)=0;y2(m*2000+1:24000)=y1(1:12000);
subplot(2,2,[3,4]);plot(p, y2,'LineWidth',6, 'Color', 'm'),xlim([-4,9]);hold('on');
%Second Method
a=length(x);b=length(h);l=a+b-1;
xe=zeros(1,l);he=zeros(1,l);y3=he;
xe(1:a)=x;xe(a+1:l)=0;he(1:b)=h;he(b+1:l)=0;
for i=1:l
    y3(i)=0;
    for k=1:i
        y3(i)=y3(i)+he(k)*xe(i-k+1);
    end
end
y3=y3*del;
subplot(2,2,[3,4]);plot(p, y3,'LineWidth',4, 'Color', 'c');title('Magenta: trapz v/s Blue: Without trapz v/s Black: conv');
%Third Method
y4=conv(x,h)*del;subplot(2,2,[3,4]);plot(p, y4,'LineWidth',1, 'Color', 'b');xlim([-4,7]);hold('off');
```

Editor - C:\Users\atharva deshpande\Documents\MATLAB\lab4one.m

lab4one.m | lab4two.m | f10.m | f9.m | +

```
%Fourth Method
p1=-2*m:del:0;p7=m:del:2*m;
y5=zeros(size(p1));y6=zeros(size(p7));
subplot(3,2,[5,6]);plot(p1,y5);hold('on');plot(p7,y6);
p2=0:del:1;
y7=zeros(size(p2));o=length(p2);tau=t;
for i=1:o
    x1=f9(i*del-tau);
    z=x1.*h;
    y7(i)=trapz(tau,z);
end
subplot(3,2,[5,6]);plot(p2,y7);xlim([-4,7]);
p3=1:del:2;
y8=zeros(size(p3));o=length(p3);tau=t;
for i=1:o
    x1=f9(i*del-tau+1);
    z=x1.*h;
    y8(i)=trapz(tau,z);
end
subplot(3,2,[5,6]);plot(p3,y8);
p4=2:del:4;
y9=zeros(size(p4));o=length(p4);tau=t;
for i=1:o
    x1=f9(i*del-tau+2);
    z=x1.*h;
    y9(i)=trapz(tau,z);
end
subplot(3,2,[5,6]);plot(p4,y9);
p5=4:del:5;
y10=zeros(size(p5));o=length(p5);tau=t;
for i=1:o
    x1=f9(i*del-tau+4);
    z=x1.*h;
    y10(i)=trapz(tau,z);
end
subplot(3,2,[5,6]);plot(p5,y10);
p6=5:del:6;
y11=zeros(size(p6));o=length(p6);tau=t;
for i=1:o
    x1=f9(i*del-tau+5);
    z=x1.*h;
    y11(i)=trapz(tau,z);
end
subplot(3,2,[5,6]);plot(p6,y11);title('Integrating per interval (Book Method)(Theoretical validation)');
```

➔ <u>Here's the output:</u>



➔ Here's the theoretical validation:

→ For $\boxed{t<0 \ \& \ t>6}$

$$\boxed{y(t)=0}$$

→ $\boxed{t>0 \ \& \ t<1}$

$$y(t) = \int_0^t e^{-2(t-\tau)}(1-\tau)\,d\tau$$

$$\boxed{y(t) = -\frac{3}{4}e^{-2t} - \frac{(2t-3)}{4}}$$

→ $\boxed{t>1 \ \& \ t<2}$

$$y(t) = \int_0^1 e^{-2(t-\tau)}(1-\tau)\,d\tau + \int_1^t e^{-2(t-\tau)}(\tau-1)\,d\tau$$

$$= \frac{(e^2-3)e^{-2t}}{4} + \frac{e^{2-2t}}{4} + \frac{1}{4}(2t-3)$$

$$\therefore \boxed{y(t) = \frac{(e^2-3)e^{-2t}}{4} + \frac{e^{2-2t}}{4} + \frac{1}{4}(2t-3)}$$

→ $\boxed{t>2 \ \& \ t<4}$

$$y(t) = \int_0^1 e^{-2(t-\tau)}(1-\tau)\,d\tau + \int_1^2 e^{-2(t-\tau)}(\tau-1)\,d\tau$$

$$= \frac{(e^2-3)e^{-2t}}{4} + \frac{(e^4+e^2)e^{-2t}}{4}$$

$$\therefore \boxed{y(t) = \frac{(e^2-3)e^{-2t}}{4} + \frac{(e^4+e^2)e^{-2t}}{4}}$$

Theoretical Validation: Part 1

→ $t > 4$ & $t < 5$

$$y(t) = \int_t^1 e^{-2(t-\tau)}(1-\tau)\,d\tau + \int_1^2 e^{-2(t-\tau)}(\tau-1)\,d\tau$$

$$\therefore \boxed{y(t) = \frac{e^{2-2t}}{4} + \frac{e^2}{4} + \frac{2t-3}{4} + \frac{1}{4}}$$

→ $t > 5$ & $t < 6$

$$y(t) = \int_t^2 e^{-2(t-\tau)}(\tau-1)\,d\tau$$

$$\boxed{y(t) = \frac{e^{4-2t}}{4} - \frac{(2t-6)}{4}}$$

	Observed	Calculated
$y(0.5)$	0.2245	0.2248
$y(1)$	0.1471	0.1483
$y(1.219)$	0.1168	0.1171
$y(2)$	0.3037	0.364
$y(4)$	0.00533	0.00539
$y(6)$	≈ 0	≈ 0

This ~~proves~~ states that calculated value ~~has~~ is more than observed value from the code

Theoretical Validation: Part 2

So, in the first question, the output signal was depicting positive values for t € [0,6]. The obtained values were more or less matching to those in on paper calculations. Also, it indicated the values calculated using integral are more than trapz.

2. We are given

$$x(t) = \ln(t)(u(t) - u(t - 3))$$

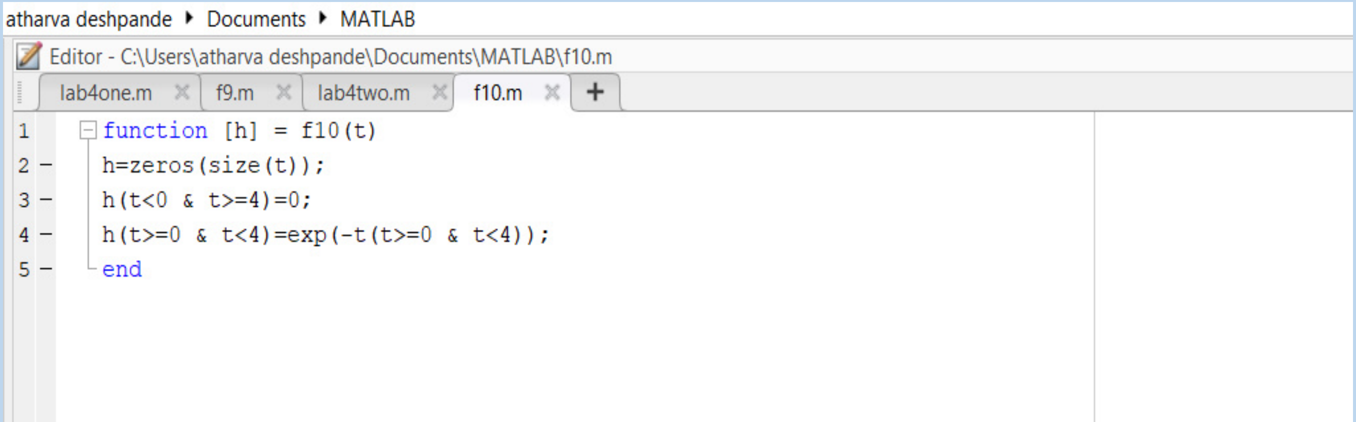
and

$$h(t) = \exp(-t)(u(t) - u(t - 4))$$

We have to find the convolution of these two signals. The basis of solving this problem is that we calculate the integral of x(t-tau) and h(t) to find y(t) which is the output of this function.

➔ <u>Here's the code:</u>

Here we have used function f10 to implement the action of x(t-tau).



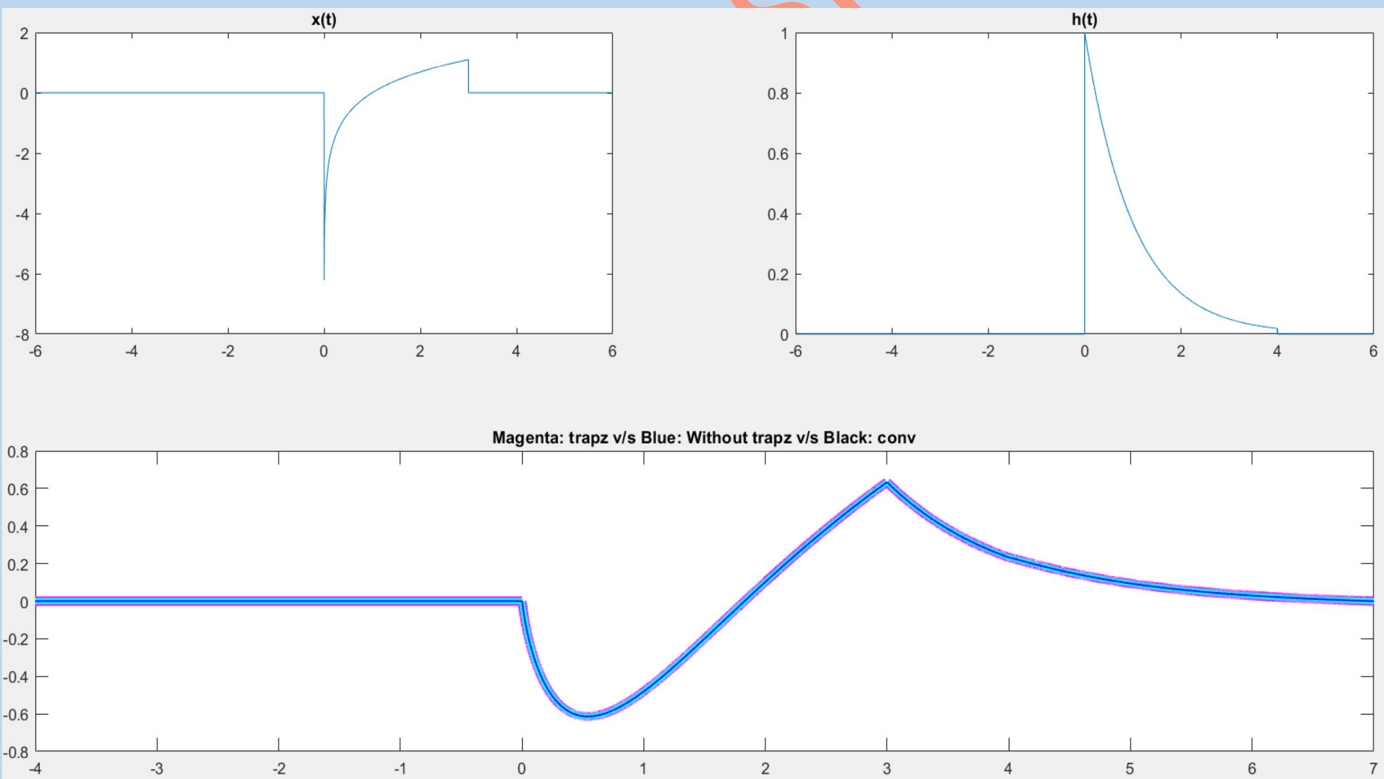

```
function [h] = f10(t)
h=zeros(size(t));
h(t<0 & t>=4)=0;
h(t>=0 & t<4)=exp(-t(t>=0 & t<4));
end
```

Editor - C:\Users\atharva deshpande\Documents\MATLAB\lab4two.m

lab4one.m | lab4two.m | f10.m | f9.m | +

```
m=6;del=0.001;t=-m:del:m;
x=zeros(size(t));x(t<del & t>=3)=0;x(t>=del & t<3)=log(t(t>=del & t<3));subplot(2,2,1);plot(t,x);title('x(t)');
%As del-->>0 we can take 0 as del.
%Also we eliminate the problem of no value for a particular range:)
h=zeros(size(t));h(t<0 & t>=4)=0;h(t>=0 & t<4)=exp(-t(t>=0 & t<4));subplot(2,2,2);plot(t,h);title('h(t)');
p=-2*m:del:2*m;
%Fisrt Method
y1=zeros(size(p));o=length(t);tau=t;
for i=1:o
    h1=f10(i*del-tau);
    x1=x;
    z=x1.*h1;
    y1(i)=trapz(tau,z);
end
y2=zeros(size(p));y2(1:m*2000)=0;y2(m*2000+1:24000)=y1(1:12000);
subplot(2,2,[3,4]);plot(p, y2,'LineWidth',6, 'Color', 'm'),xlim([-4,9]);hold('on');
%Second Method
a=length(x);b=length(h);l=a+b-1;
xe=zeros(1,l);he=zeros(1,l);y3=he;
xe(1:a)=x;xe(a+1:l)=0;he(1:b)=h;he(b+1:l)=0;
for i=1:l
    y3(i)=0;
    for k=1:i
        y3(i)=y3(i)+he(k)*xe(i-k+1);
    end
end
y3=y3*del;subplot(2,2,[3,4]);plot(p, y3,'LineWidth',4, 'Color', 'c');title('Magenta: trapz v/s Blue: Without trapz v/s Black: conv'
%Third Method
y4=conv(x,h)*del;subplot(2,2,[3,4]);plot(p, y4,'LineWidth',1, 'Color', 'b');xlim([-4,7]);hold('off');
```

➔ <u>Here’s the output:</u>



Since theoretical validation not necessary in the question, it has not been uploaded.