1. Prove by induction that if S is a finite set, then $|2^S| = 2^{|S|}$.

To prove $|2^S| = 2^{|S|}$.

→ Basis :

If $S = \{\}$, then $|S| = 0$.

$2^{\{\}}$ consists of 1 element i.e. $\{\}$

$\therefore 2^0 = 1$.

$\therefore |2^{\{\}}| = 1$ — ①

and $2^{|S|} = \{\{\}\} = 1$ — ②

∴ frome above, ① = ②.

Inductive step :

Let us assume that $|2^{\{1,2,\dots,n\}}| = 2^{|\{1,2,\dots,n\}|}$ is true.

So, we need to show that for $\{1,\dots,n+1\}$

We have all subsets of $2^{\{1,2,\dots,n\}}$ in $\{1,\dots,n+1\}$

Hence, $|2^{\{1,2,\dots,n\}}| \subseteq \{1,2,\dots,n+1\}$ that does not have $n+1$.

$\therefore 2|2^{\{1,2,\dots,n\}}| = 2(2^{|\{1,2,\dots,n\}|})$

So, we can say that,

$|2^{(1,2,\dots,n+1)}| = 2^{n+1}$,

which is $2^{|\{1,2,\dots,n+1\}|}$.

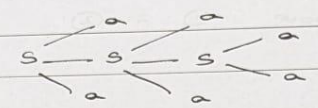
Hence Proved.

2. Give a simple English (not math) description of the language generated by the grammar with the productions: S -> aSa | aa.

2. $S \rightarrow aSa \mid aa$.

→ Here the given grammar $S \rightarrow aSa \mid aa$ creates all even strings of length of 'a' such as aa, aaaa, aaaaaa & so on. We can see that there is a recursive pattern.

eg -



i.e. aSa

aaSaa

aaaaaa

P → $S \rightarrow aSa$ — ①

$S \rightarrow aa$ — ②

So, by using ② ;

$S \rightarrow aSa \rightarrow aaaa$

↓

aaSaa ... using ①

↓

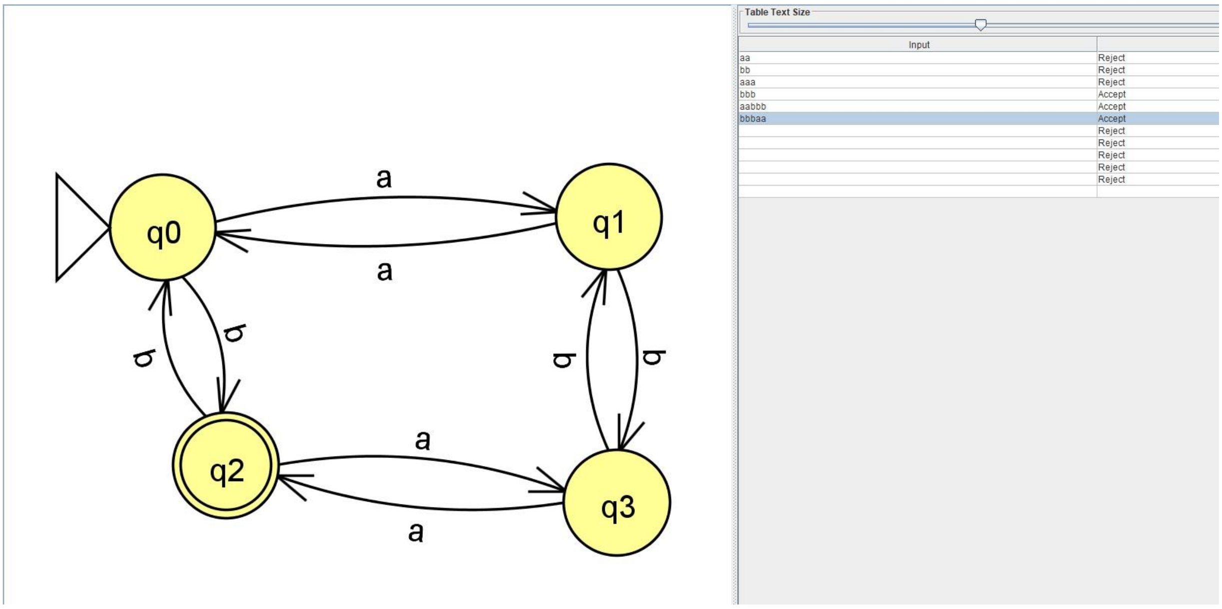
aaaaaa ... using ②

i.e. All even strings of 'a' .

∴ $L(G) = \{aa\}^*$.

3. Create a dfa for Σ = {a, b} that accepts the set of all strings with an even number of 'a's and an odd number of 'b's. Zero is even, so your dfa should accept 'b', but should not accept 'aa'.

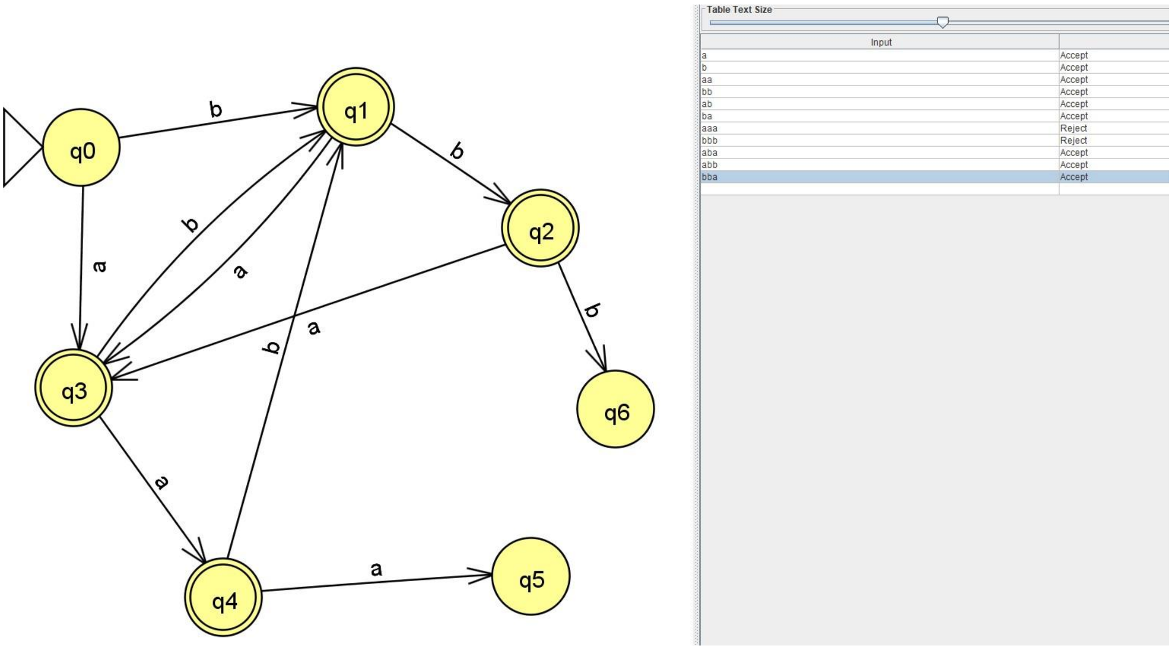
For the given inputs the results shows the following:



Input	
aa	Reject
bb	Reject
aaa	Reject
bbb	Accept
aabbb	Accept
bbbaa	Accept
	Reject
	Reject
	Reject
	Reject
	Reject

4. Create a dfa for Σ = {a, b} that accepts the set of all strings in which the same symbol does not occur three or more times in a row. For example it should not accept "aaa" or "aabbbbab".

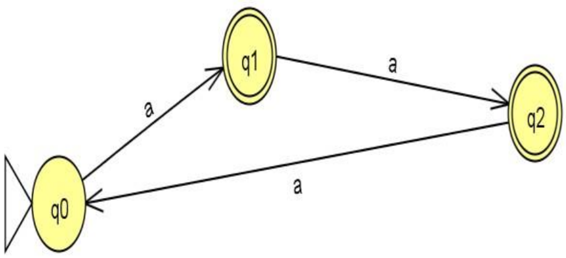
For the given inputs the results shows the following:

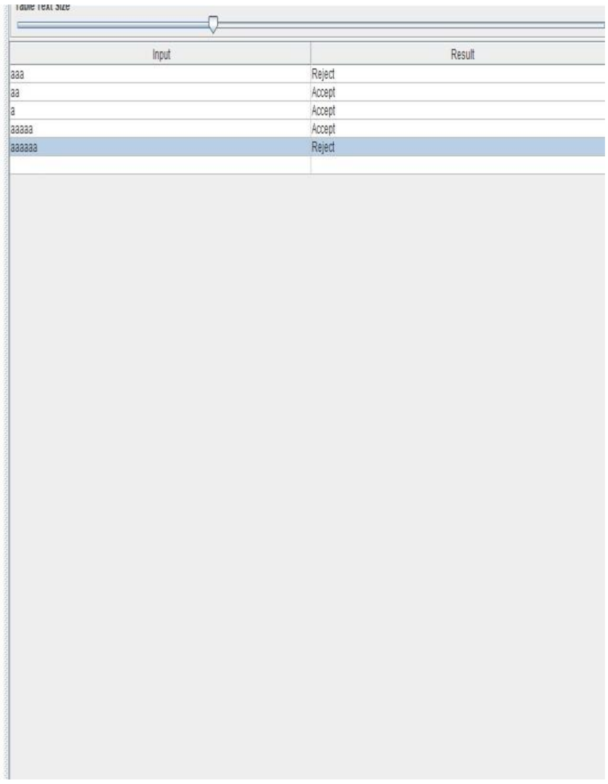


Input	
a	Accept
b	Accept
aa	Accept
bb	Accept
ab	Accept
ba	Accept
aaa	Reject
bbb	Reject
aba	Accept
abb	Accept
bba	Accept

5. Show that the language L = $\{a^n : n \text{ is not a multiple of } 3\}$ is regular.

For the given inputs the results shows the following:





Input	Result
aaa	Reject
aa	Accept
a	Accept
aaaaa	Accept
aaaaaa	Reject