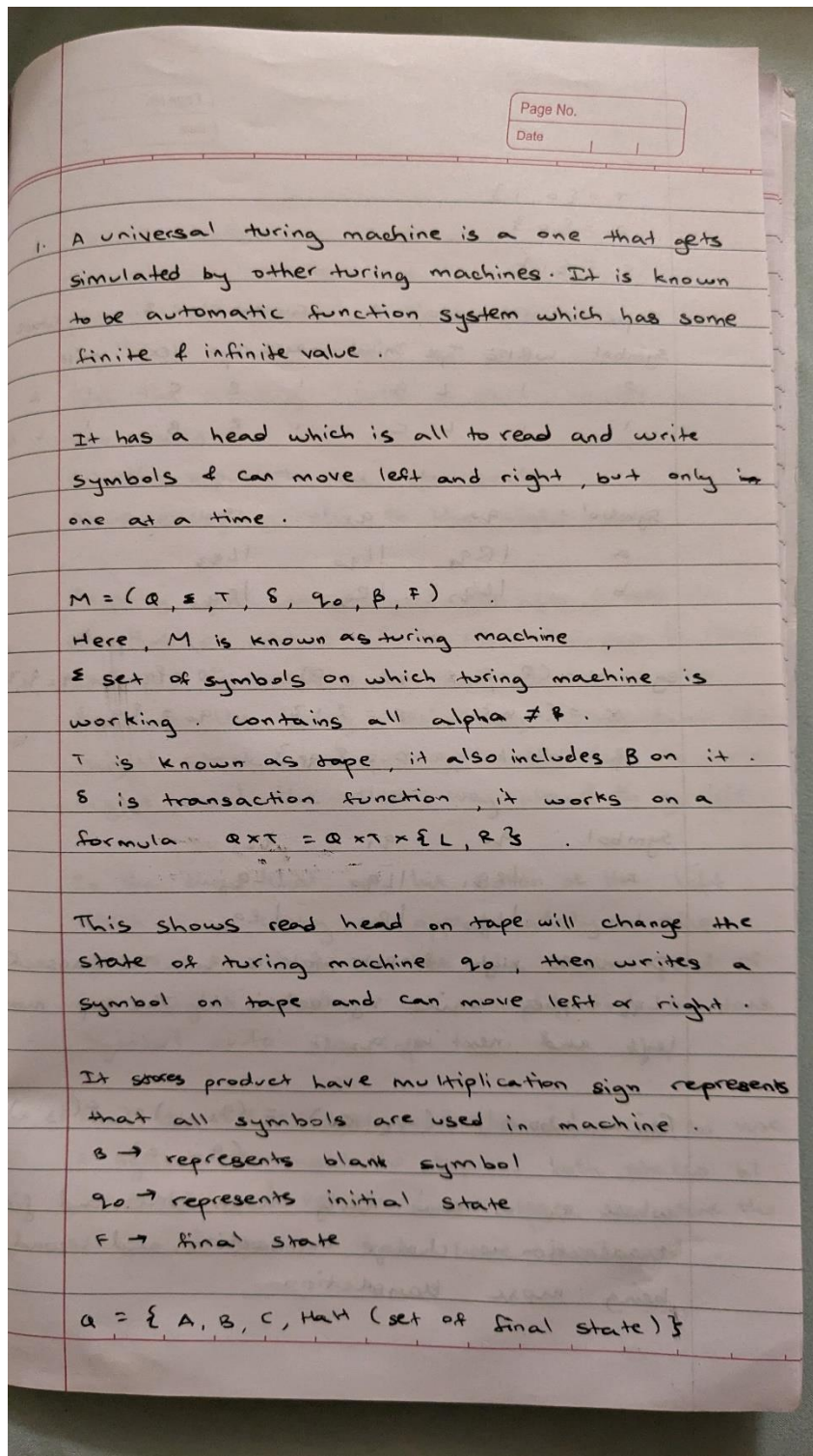


1. Give a formal definition of a Turing machine that on each move can either change the tape symbol or move the read/write head, but not both. Then describe how it can be simulated by a standard Turing machine.



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$$\Sigma = \{0, 1\}$$

$$\Gamma = \{0\}$$

$$\epsilon = \{1\}$$

Tape	Current state A	Current state B	State C
Symbol	WRITE	WRITE	Write
0	L B	R C	R B
1	L C	R B	L Right

Symbol	q_0	q_1	q_2
a	$ Rq_1$	$ Lq_0$	$ Lq_3$
b	$ Lq_2$	$ Rq_1$	$ Rq_4$

eg- $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$, $Q = \{q_0, q_1, q_2, q_3, q_4\}$
 $\Sigma = \{a, b\}$, $\Gamma = \{1\}$, $q_0 = \{q_0\}$
 $B = \text{blank}$, $F = \{q_4\}$

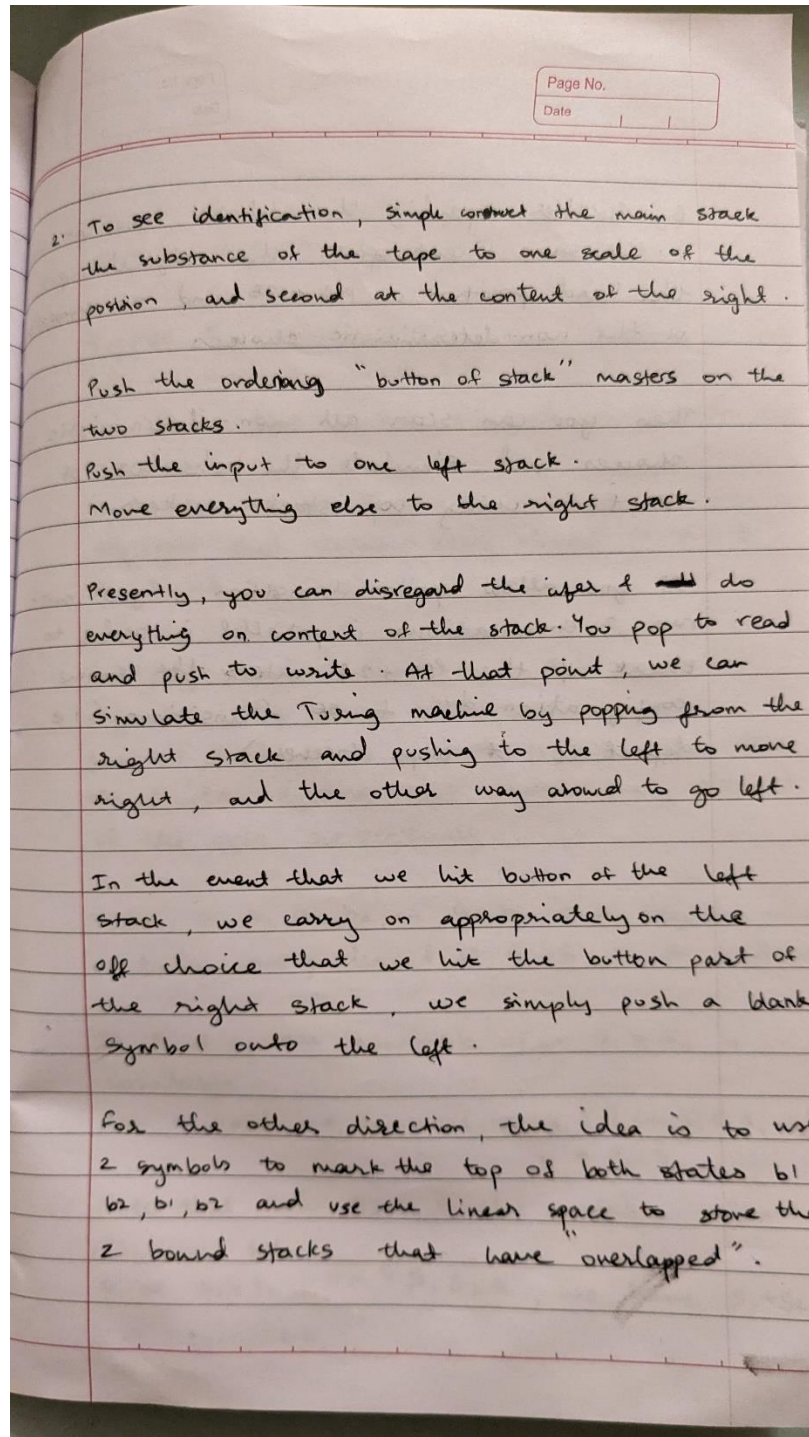
Symbol	q_0	q_1	q_2
a	$ Rq_1$	$ Lq_0$	$ Lq_3$
b	$ Lq_2$	$ Rq_1$	$ Rq_4$

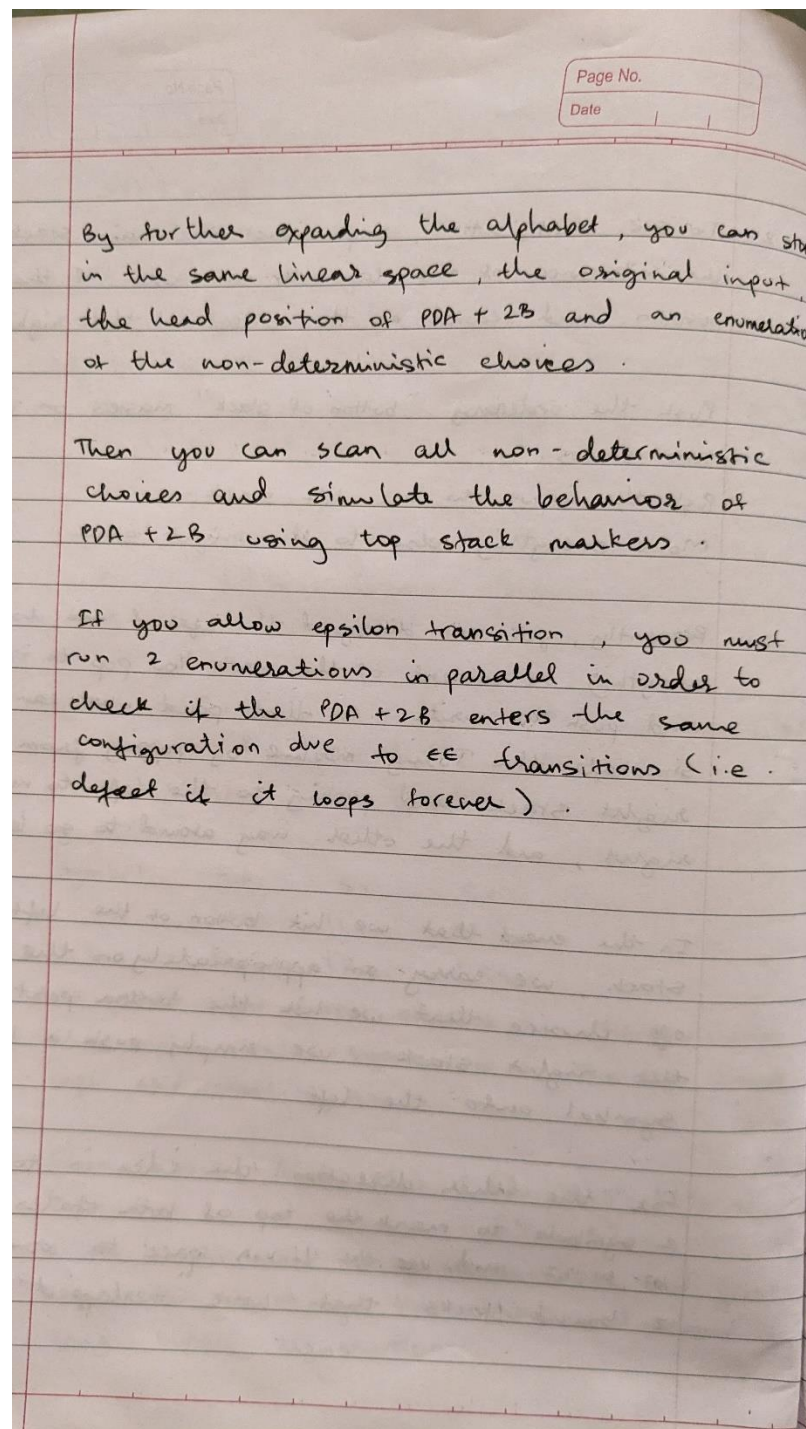
It moves right & next state is q_1 . The transaction $|Lq_2$ implies write symbol is 1. The tap moves left and next is q_2 .

From above, $\delta(q_k, a) = (q_2, b)$ or $\delta(q_3, a) = (q_4, \epsilon)$

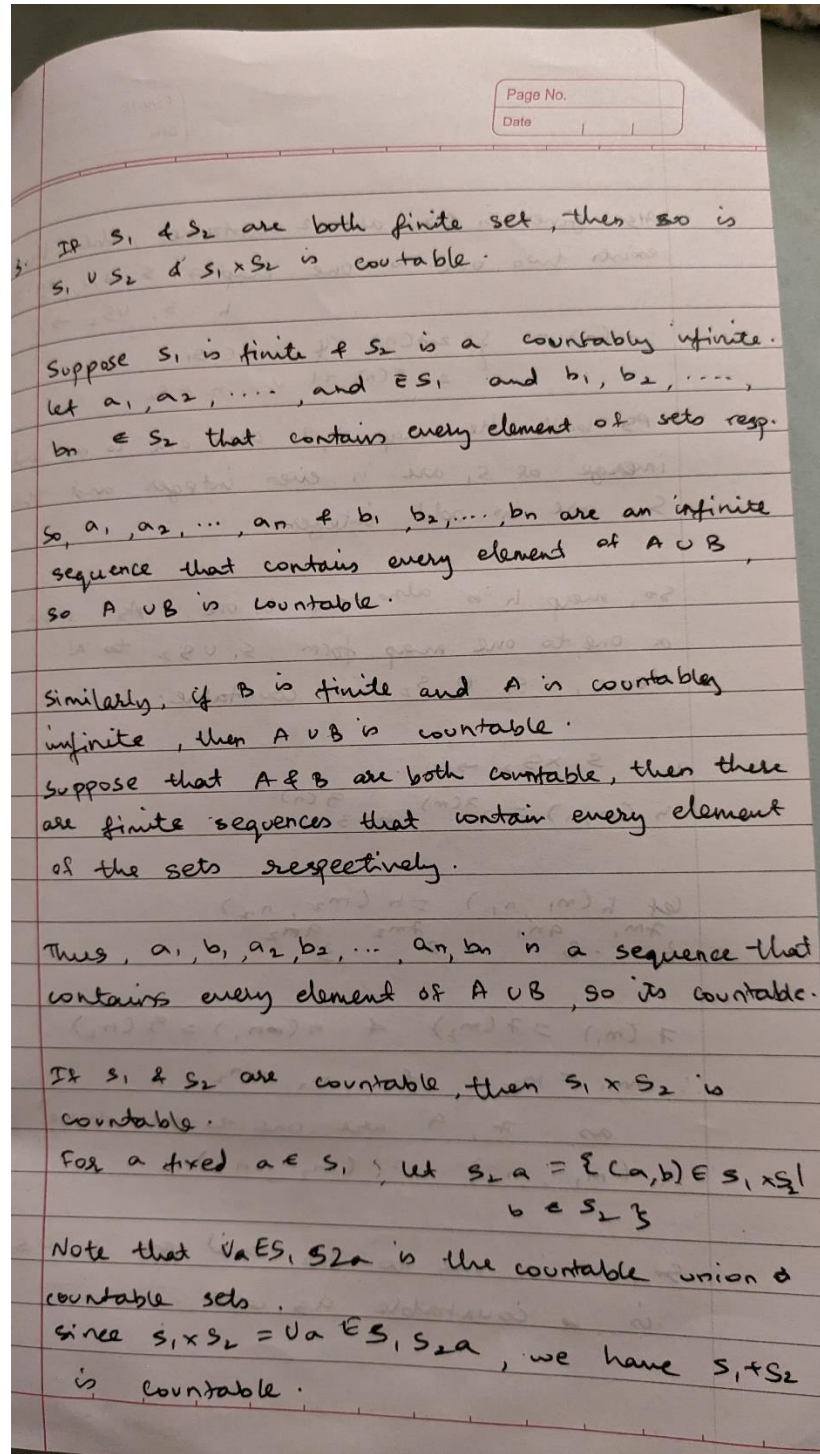
where q_k is some newly chosen state and first transaction is change transaction and second being move transaction.

2. A two-stack npda is an npda with two independent stacks. A move depends on the tops of the two stacks and results in new values being pushed on these two stacks. Show that two-stack npdas are equivalent to standard Turing machines. As usual, showing equivalency requires showing both directions.





3. Assume that S_1 and S_2 are countable sets. Prove that $S_1 \cup S_2$ and $S_1 \times S_2$ are also countable.



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Also, given S_1 & S_2 are 2 countables, then there exists two one-to-one map $f: S_1 \rightarrow \mathbb{N}$, $g: S_2 \rightarrow \mathbb{N}$
 $h: S_1 \cup S_2 \rightarrow \mathbb{N}$

$$h(n) = \begin{cases} 2f(n) & \text{if } n \in S_1 \\ 2g(n) + 1 & \text{if } n \in S_2 \end{cases}$$

As both the map f, g are one to one and image of S_1 are in even integer and that of S_2 are in odd integer.

So, map h is also one to one. As there exists a one to one map from $S_1 \cup S_2$ to \mathbb{N} .
Hence, $S_1 \cup S_2$ is countable.

$$h: S_1 \times S_2 \rightarrow \mathbb{N}$$

$$h(m, n) = 2^{f(m)} \cdot 3^{g(n)}$$

$$\text{Let } h(m_1, n_1) = h(m_2, n_2) \\ 2^{f(m_1)} \cdot 3^{g(n_1)} = 2^{f(m_2)} \cdot 3^{g(n_2)}$$

$$f(m_1) = f(m_2) \quad \& \quad g(n_1) = g(n_2)$$

$$\therefore m_1 = m_2 \quad \& \quad n_1 = n_2$$

as f, g are one to one funcⁿ.

$$\therefore (m_1, n_1) = (m_2, n_2)$$

So, h is one to one and so $S_1 \times S_2$ is countable as well.