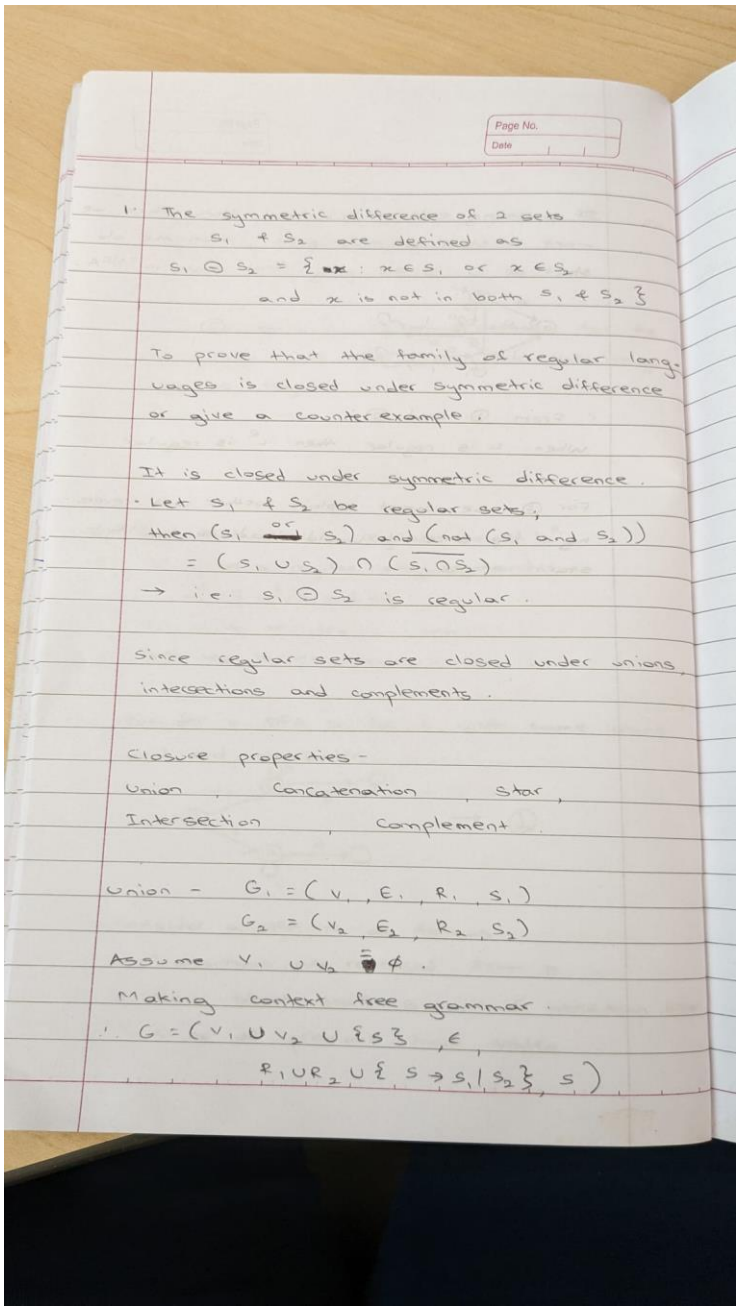
1. The symmetric difference of two sets is a new set that contains every element that is in either set except the elements that are in both sets. For example, the symmetric difference of {3, 7, 2, 12, 9} and {8, 12, 4, 16, 7, 5} would be {3, 2, 9, 8, 4, 16, 5}. Prove that the family of regular languages is closed under symmetric difference.



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1. The symmetric difference of 2 sets $S_1$ & $S_2$ are defined as

$S_1 \ominus S_2 = \{ x : x \in S_1$ or $x \in S_2$ and $x$ is not in both $S_1$ & $S_2 \}$

To prove that the family of regular languages is closed under symmetric difference or give a counter example.

It is closed under symmetric difference.

- Let $S_1$ & $S_2$ be regular sets,

then $(S_1$ or $S_2)$ and (not ($S_1$ and $S_2$))

$= (S_1 \cup S_2) \cap (\overline{S_1 \cap S_2})$

→ i.e. $S_1 \ominus S_2$ is regular.

Since regular sets are closed under unions, intersections and complements.

Closure properties -

Union, Concatenation, Star,

Intersection, Complement

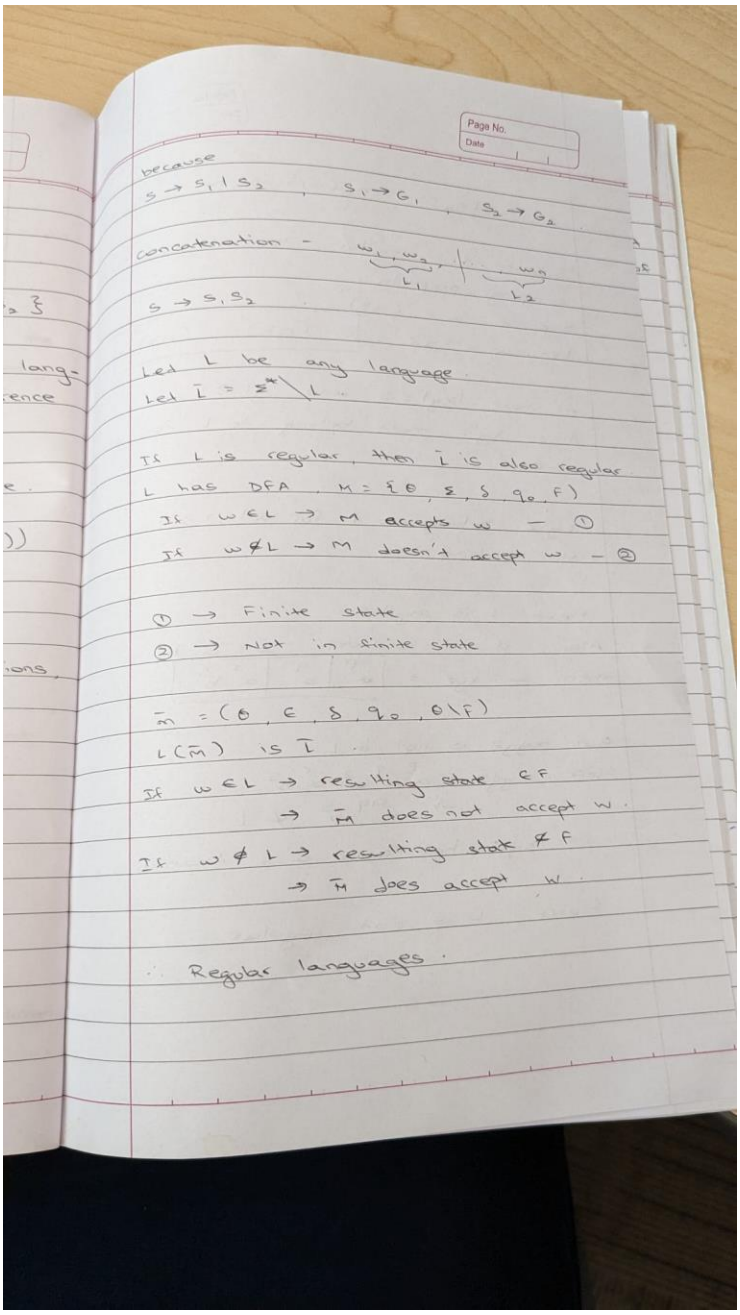
Union - $G_1 = (V_1, E_1, R_1, S_1)$

$G_2 = (V_2, E_2, R_2, S_2)$

Assume $V_1 \cup V_2 = \phi$.

Making context free grammar.

$G = (V_1 \cup V_2 \cup \{S\}, \epsilon, R_1 \cup R_2 \cup \{S \to S_1 | S_2\}, S)$



because

$S \to S_1 \mid S_2$ , $S_1 \to G_1$ , $S_2 \to G_2$

concatenation - $w_1, w_2, \ldots$ ($L_1$) | $\ldots w_n$ ($L_2$)

$S \to S_1 S_2$

Let L be any language

Let $\bar{L} = \Sigma^* \setminus L$

If L is regular, then $\bar{L}$ is also regular

L has DFA, $M = (Q, \Sigma, \delta, q_0, F)$

If $w \in L \to M$ accepts w — ①

If $w \notin L \to M$ doesn't accept w — ②

① → Finite state

② → Not in finite state

$\bar{M} = (Q, \Sigma, \delta, q_0, Q \setminus F)$

$L(\bar{M})$ is $\bar{L}$

If $w \in L \to$ resulting state $\in F$

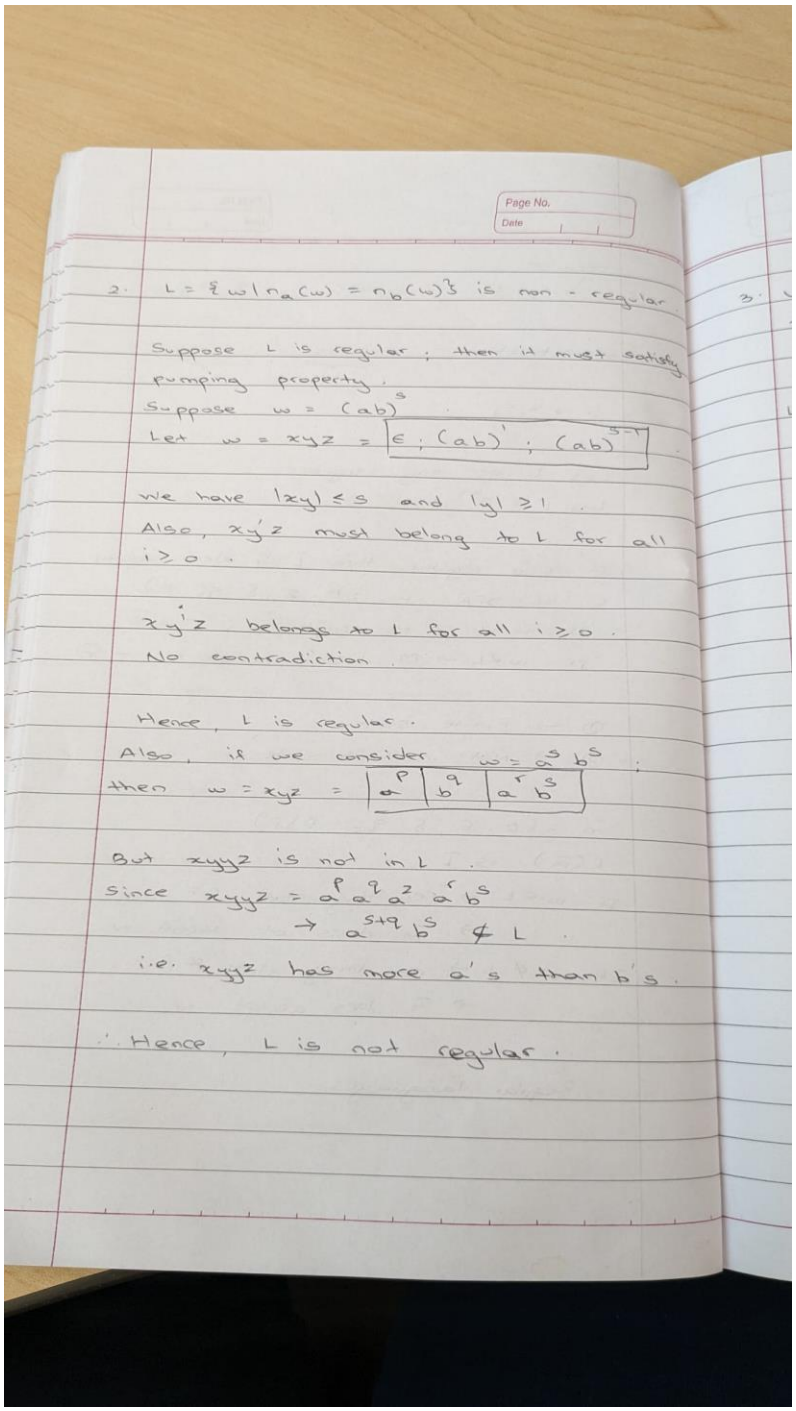
$\to \bar{M}$ does not accept w.

If $w \notin L \to$ resulting state $\notin F$

$\to \bar{M}$ does accept w.

∴ Regular languages.

2. Prove that the language L = {w: $n_a(w) = n_b(w)$} is not regular.



2. $L = \{w \mid n_a(w) = n_b(w)\}$ is non-regular

Suppose L is regular; then it must satisfy pumping property.

Suppose $w = (ab)^s$

Let $w = xyz = \boxed{\epsilon \,;\, (ab)^1 \,;\, (ab)^{s-1}}$

We have $|xy| \leq s$ and $|y| \geq 1$.

Also, $xy^iz$ must belong to L for all $i \geq 0$.

$xy^iz$ belongs to L for all $i \geq 0$.

No contradiction

Hence, L is regular.

Also, if we consider $w = a^s b^s$

then $w = xyz = \boxed{a^p \mid b^q \mid a^r b^s}$

But $xyyz$ is not in L.

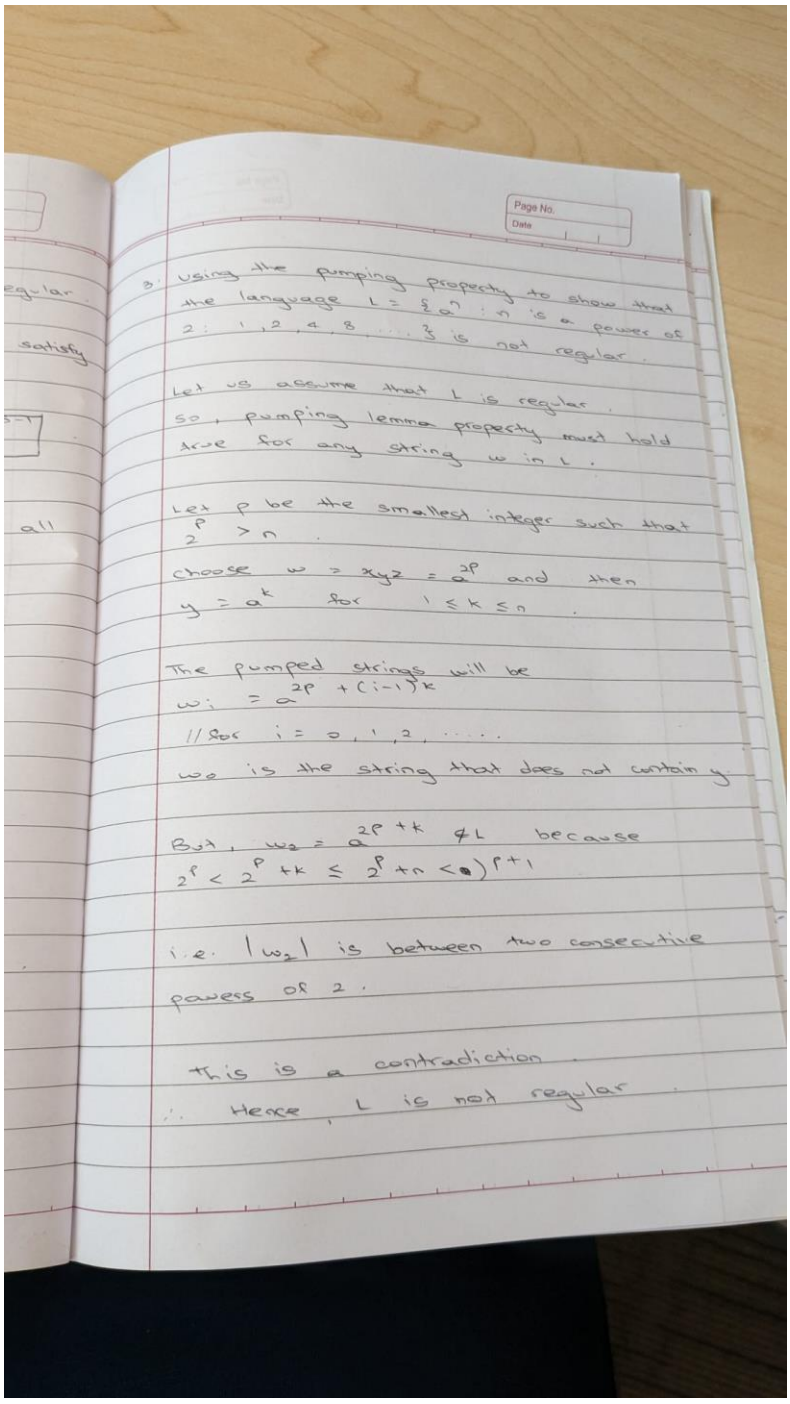
Since $xyyz = a^p a^q a^z a^r b^s$

$\rightarrow a^{s+q} b^s \notin L$

i.e. $xyyz$ has more a's than b's.

∴ Hence, L is not regular.

3. Prove that the language L with Σ = {a}, where L = {$a^n$: n is a power of 2} is not regular.



3. Using the pumping property to show that the language $L = \{a^n : n \text{ is a power of } 2 : 1, 2, 4, 8, \ldots\}$ is not regular.

Let us assume that L is regular.
So, pumping lemma property must hold true for any string w in L.

Let p be the smallest integer such that $2^p > n$.

Choose $w = xyz = a^{2^p}$ and then $y = a^k$ for $1 \leq k \leq n$.

The pumped strings will be
$w_i = a^{2^p + (i-1)k}$
// for $i = 0, 1, 2, \ldots$.

$w_0$ is the string that does not contain y.

But, $w_2 = a^{2^p + k} \notin L$ because
$2^p < 2^p + k \leq 2^p + n < 2^{p+1}$

i.e. $|w_2|$ is between two consecutive powers of 2.

This is a contradiction.
∴ Hence, L is not regular.