



Probability

Meaning and Concepts

- **Probability** refers to chance or likelihood of a particular event taking place.
- An **event** is an outcome of an experiment.
- An **experiment** is a process that is performed to understand and observe possible outcomes.
- The outcomes of an experiment are said to be **equally likely** if any one of them cannot be expected to occur in preference to another.
- Set of all outcomes of an experiment is known as **Sample Space**.

1. Classical OR '*a priori*' Probability:

If a trial results in n exhaustive cases which are mutually exclusive and equally likely and out of which m are favorable to the happening of event A, then the probability P of the happening of event A i.e. $P(A)$ is given by,

$$P(A) = \frac{m}{n}$$

2. Statistical OR Empirical Probability:

If a trial be repeated for a large number of times say n under the same conditions and a certain event A occurs on $p \times n$ occasions, then, the probability of happening of event A is given by,

$$P(A) = \lim_{n \rightarrow \infty} \frac{p \times q}{n}$$

3. Subjective Probability

Subjective probability is a type of probability that represents an individual's personal degree of belief or confidence in the occurrence of an event, rather than the objective frequency of the event. It is based on an individual's own experience, knowledge, and intuition, rather than on the laws of probability.

Algebra of sets - Wikipedia

In mathematics, the algebra of sets, not to be confused with the mathematical structure of an algebra of sets, defines the properties and laws of sets, the set-theoretic operations of union, intersection, and complementation and the relations of set equality and set inclusion.

W https://en.wikipedia.org/wiki/Algebra_of_sets

Joint probability is a statistical measure that calculates the likelihood of two events occurring together and at the same point in time $P(A \cap B)$.

Mutually exclusive is a statistical term describing two or more events that cannot happen simultaneously.

Independent events are those events whose occurrence is not dependent on any other event.

Theorems on Probability

1. Addition Theorem of Probability

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$$

If events A and B are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B)$$

2. Multiplication Theorem of Probability

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B|A)$$

If events A and B are independent,

$$P(A \text{ and } B) = P(A) \times P(B)$$

3. Total Theorem of Probability

If $A_1, A_2, A_3, \dots, A_n$ are mutually exclusive and exhaustive events in the sample space S and each has non-zero probability of occurrence. Let B be any event associated with S , then

$$\begin{aligned} P(B) &= P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + \dots + \\ P(A_n)P(B/A_n) &= \sum_{i=1}^n P(A_i) \times P(B/A_i) \end{aligned}$$

4. Bayes Theorem

Bayes Theorem,

$$P(A|B) = P(A) \times \frac{P(B|A)}{P(B)}$$

posterior prior likelihood marginal

Posterior probability is the revised probability of an event occurring after taking into consideration new information.

Prior probability, in Bayesian statistical inference, is the probability of an event based on established knowledge, before empirical data is collected.

Likelihood is the probability of the evidence, given the belief is true

Marginal probability represents the probability of the evidence, under any circumstance.
