MATH0103 Discrete Mathematics

Year:	2023–2024
Code:	MATH0103
Level:	5 (UG)
Normal student group(s):	UG Year 2 BASc and NatSci students
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Assessment:	90% examination with standard calculators allowed 10% weekly coursework
Normal Pre-requisites:	One of A level Further Mathematics or MATH0012 or MATH0041, or MATH0045 and MATH0047
Lecturer:	Dr R Reynolds

Course Description and Objectives

One of the most powerful ideas in mathematics is abstraction. In this course you will see the importance of the ability to understand and work with abstract concepts and constructions. You will be introduced to the basic language of mathematics, some important algebraic structures, and the idea of a proof. After taking this course, a successful student should be able to take the following options in their third year:

1. Algebra 4: Groups and Rings;
2. Number Theory;
3. Graph Theory and Combinatorics.

The course will be divided into four sections with a balance between abstract ideas and more concrete methods, particularly in number theory.

In section 1 we will cover some ideas from set theory and we will introduce the concept of mathematical proof. In section 2 we will study an important algebraic structure called a group. We aim to give a thorough grounding in the basics of group theory and to build confidence working with abstract definitions and concepts. In section 3 we will focus on number theory. In particular, we will introduce $\mathbb{Z}/(n)$, the integers modulo n, and we will learn methods to solve equations in this new setting. Finally, in section 4 we will study another important algebraic structure called a field which is a generalisation of the real numbers. We will learn some new examples of fields and some general theory which generalises results learnt from linear algebra. In particular, sections 2 and 4 will provide a good basis for studying Algebra 4.

Recommended Texts

Martin Liebeck, “A concise introduction to pure mathematics”, Chapman–Hall

Charles Pinter, “A book of abstract algebra”, Dover

Kenneth H Rosen, “Discrete Mathematics and its Applications”, McGraw–Hill

Detailed Syllabus

- **Set theory and methods of proof.** Union, intersection, complement, cartesian product, functions, injections, surjections, bijections, inverses, equivalence relations. Some methods of proof: Proving results in set theory by truth tables, or by a logical argument. Proof by contradiction (e.g. existence of infinitely many prime numbers). Proof by induction.
- **Group theory.** Permutations and the symmetric group. Disjoint cycle notation. Order of a permutation. Parity of a permutation. Abstract group operations. Finite examples from geometry. Subgroups, Lagrange's theorem, cyclic subgroups, order of an element. Examples of finding all the subgroups of a given finite group. Group homomorphisms/isomorphisms.
- **Number theory.** Congruences. Euclid's algorithm for hcf. Finding multiplicative inverses and solving linear congruences. Chinese remainder theorem. Uniqueness of factorization. The multiplicative group. Fermat's little theorem. RSA cryptography.
- **The notion of a field.** Examples: $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, finite fields of prime order. Gaussian elimination over an arbitrary field. Euclid's algorithm for polynomials, and uniqueness of factorization in $\mathbb{F}[X]$.