Programme: B.Tech in Computer Science and Engineering (Data Science)

Year: II/Semester III (Exam Year: 2024-2025)

Subject: Mathematics for Intelligent Systems   **Time:** 02:30 pm - 04:30 pm (02:00 Hrs.)

Date: 21 Mar 2025

Max Marks: 60   **RE END SEMESTER EXAMINATION SEM-III (MARCH 2025)**

Instructions: 1. This question paper contains 3 pages

2. Answer to each new question to be started on a fresh page..

3. Figure in right hand side indicates full marks.

4. All questions are compulsory.

5. Assume suitable data, wherever necessary.

6. Draw neat, labelled diagrams, if required.

1. Neeraja belongs to an athletics club. In javelin throwing competitions, her throws are normally distributed with mean 39.23 meters and standard deviation of 3.35 meters.   **4**
   a. What is the probability of her throwing between 36 and 42 meters?
   b. To get chosen to compete, Neeraja needs to throw beyond 44 meters. What is the probability of her qualifying?

2. Let X and Y be two jointly continuous random variables with joint PDF. Calculate the covariance between X and Y. Are X and Y independent?   **5**

$$f(x,y) = \frac{6}{5}(x + y^2); \qquad 0 \le x \le 1, 0 \le y \le 1$$

3. Solve either (a) or (b):-   **6**

   a. A machine which is known to produce 1% defective components is used for a production run of 40 components. Using both Binomial and Poisson Distributions, calculate the probability that no defective components, one defective component and at most 2 defective components will be found.   **6**

   ------------ OR ------------

   b. i. The mileage which car owners get with a certain kind of radial tyre is an exponential random variable having a mean of 40,000 km. Compute the probabilities that one of these tires will last more than 20,000 km but at most 30,000 km.   **6**

   ii. An aircraft emergency locator transmitter (ELT) is a device designed to transmit a signal in the case of a crash. The Altigauge Manufacturing Company makes 80% of the ELTs, the Bryant Company makes 15% of them, and the Chartair Company makes the other 5%. The ELTs made by Altigauge have a 4% rate of defects, the Bryant ELTs have a 6% rate of defects, and the Chartair ELTs have a 9% rate of defects (which helps to explain why Chartair has the lowest market share). If a randomly selected ELT is then tested and is found to be defective, evaluate the probability that it was made by the Altigauge Manufacturing Company.

4. Compute the basis and dimension of the orthogonal complement $W^\perp$ of
$$W = \{(1,4,5,6,9), (3,12,1,4,-1), (1,4,1,2,1), (2,8,5,7,8)\}$$ 5

5. Given $T(1,2,-2) = (7,10), T(3,0,1) = (9,-3), T(1,1,1) = (5,-1)$. Compute $T(a,b,c)$.   4

6. Compute $[v]_S$ given $v = (3,-5,4); S = \{(1,0,3),(2,1,8),(1,-1,2)\}$ 3

7. Compute the eigenvalues and eigenvectors of the matrix A. Is A diagonalizable? If yes, write the diagonal matrix D and the diagonalizing matrix P.   6

$$A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$$

8. Solve either (a) or (b):-   5

    a. Use Caley Hamilton Theorem to compute   5

$$A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I, \quad \text{Where} \quad A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

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    b. Check whether the Matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is derogatory, also compute the minimal   5

    polynomial.

9. Find the directional derivatives of $\emptyset = xy^2 + yz^3$ at the point $(1,-1,1)$ in the direction of the vector $(1,2,2)$.   6

10. Solve either (a) or (b):-   8

    a. Solve the following LLP using simplex method   8

    Maximize $Z = 4x_1 + 6x_2$

    Subject to $\quad 2x_1 + 3x_2 \leq 6$

$$x_1 - x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

------------ OR ------------

    b Solve by Big-M method   8

    Minimize $\quad Z = 12x_1 + 20x_2$

    Subject to $\quad 6x_1 + 8x_2 \geq 100$

$$7x_1 + 12x_2 \geq 120$$
$$x_1, x_2 \geq 0$$

11. Solve either (a) or (b):- **8**

a. ## Solve NLPP by Kuhn-Tucker method: **8**

Minimize $\quad Z = x_1^2 + x_2^2 + 60x_1$

Subject to $\quad x_1 \geq 80$

$\qquad\qquad x_1 + x_2 \geq 120$

$\qquad\qquad x_1, x_2 \geq 0$

------------ OR ------------

b. i) Solve NLPP by Lagrange's method: **8**

Maximize $\quad Z = 6x_1 x_2$

Subject to $\quad 2x_1 + x_2 = 10$

$\qquad\qquad x_1, x_2 \geq 0$

ii) Minimize $f(x_1, x_2) = 8x_1 + x_1^2 - 12x_2 + 2x_2^2 + 2x_1 x_2$ starting with $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ using Gradient Descent Technique