Black-Litterman Model Explanation

1 PART 1 — What is the Black-Litterman Model? (Zero-Knowledge Start)

Imagine you want to invest in multiple stocks.
You want to find the **best weights** (allocation
Traditional portfolio optimization (Markowitz Mean-Variance) has two big problems:

1.0.1 1. If you use historical returns, the output is unstable

Small changes in input → large, crazy changes in weights
(e.g., 90

1.0.2 2. If you add your own views, there is no clear way to blend them

Example: "I think INFY will outperform TCS by 2
Markowitz cannot incorporate such views smoothly.

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2 Black-Litterman fixes both problems

It **combines two things**:

2.0.1 1. Market equilibrium returns (neutral, objective)

These are returns implied by the market's current prices
— what an "average" investor believes.

2.0.2 2. Your own views (subjective)

Such as:

- "I think stock A will return 5
- "I think Reliance will outperform HDFC Bank by 1
- "I am 60

Then it blends (1) and (2) using Bayesian statistics.
Result → stable, realistic portfolio weights.

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3 Intuition in 1 sentence

> **Black-Litterman = Market Consensus + Your Views (with confidence weighting)**
>
> → producing **posterior/expert-adjusted expected returns**, which then go into classical mean-variance optimization to get portfolio weights.

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4 PART 2 — Core Idea Using a Simple Example

Suppose you invest in 2 stocks:

- TCS
- INFOSYS

The market suggests:

- TCS expected return: 8
- INFY expected return: 10

But **your view**:

> "I think TCS will outperform INFY by 1

BL will **mathematically blend** these two.
If you are **very confident**, it will trust your view strongly.
If you are **not confident**, it will mostly trust the market.
This is the entire model.

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5 PART 3 — Mathematical Structure

Black-Litterman steps (in correct sequence):

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6 Step 1 — Compute Prior (Equilibrium) Returns

$$\pi = \lambda \Sigma w_{mkt}$$

Where:

- π = implied (equilibrium) returns
- λ = risk aversion (scalar)
- Σ = covariance matrix
- w_{mkt} = market-cap weights of assets

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7 Step 2 — Encode Your Views

Use a matrix **P** and vector **Q**

7.0.1 Example view:

“TCS - INFY = 1

Then

$$P = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.01 \end{bmatrix}$$

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8 Step 3 — Confidence in each view

A diagonal matrix

$$\Omega = \text{diag}(\text{uncertainty})$$

Lower uncertainty   more confidence.

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9 Step 4 — Compute Posterior Returns

This is the heart of Black-Litterman:

$$\mu = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1} \left[(\tau\Sigma)^{-1}\pi + P^T\Omega^{-1}Q\right]$$

Where:

- τ = scaling parameter (small number like 0.025)
- μ = posterior returns (use these for optimization)

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10 Step 5 — Run Mean-Variance Optimization

$$w = \frac{1}{\lambda}\Sigma^{-1}\mu$$

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11 PART 4 — Fully Solved Numerical Example

We use a **2-asset example** for clarity.

11.0.1 Assets:

1. TCS
2. INFOSYS

11.0.2 Market weights:

- TCS = 60
- INFY = 40

$$w_{mkt} = \begin{bmatrix} 0.6 \ 0.4 \end{bmatrix}$$

11.0.3 Covariance matrix:

$$\Sigma = \begin{bmatrix} 0.04 & 0.01 \ 0.01 & 0.09 \end{bmatrix}$$

Risk aversion: ($\lambda = 2.5$)
Tau: ($\tau = 0.025$)

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12 Step 1 — Compute Prior Returns ( )

$$\pi = \lambda \Sigma w_{mkt}$$

Compute:

$$\Sigma w_{mkt} = \begin{bmatrix} 0.04(0.6) + 0.01(0.4) \ 0.01(0.6) + 0.09(0.4) \end{bmatrix} =============$$

$\begin{bmatrix} 0.028 \ 0.042 \end{bmatrix}$

Now multiply by :

$$\pi = 2.5 \begin{bmatrix} 0.028 \ 0.042 \end{bmatrix} =============$$

$\begin{bmatrix} 0.07 \ 0.105 \end{bmatrix}$

So market-implied returns:

- TCS = 7
- INFY = 10.5

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13 Step 2 — Your View

You think:

TCS will outperform INFY by 1

So (Q = 0.01)

$$P = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

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14 Step 3 — Confidence

Assume:
Variance of view = 0.0025 (fairly confident)

$$\Omega = \begin{bmatrix} 0.0025 \end{bmatrix}$$

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15 Step 4 — Posterior Return Calculation

We compute:

$$\mu = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\pi + P^T\Omega^{-1}Q]$$

Let's do it cleanly:

15.0.1 First compute Σ:

$$\tau\Sigma = 0.025 \begin{bmatrix} 0.04 & 0.01\, 0.01 & 0.09 \end{bmatrix} =============$$

$\begin{bmatrix} 0.001 & 0.00025\, 0.00025 & 0.00225 \end{bmatrix}$

Compute its inverse:

$$(\tau\Sigma)^{-1} = \begin{bmatrix} 1066.7 & -118.5 & -118.5 & 473.5 \end{bmatrix}$$

15.0.2 Compute part A:

$$A = (\tau\Sigma)^{-1}\pi$$

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$\begin{bmatrix} 1066.7 & -118.5 & -118.5 & 473.5 \end{bmatrix}$
$\begin{bmatrix} 0.07\, 0.105 \end{bmatrix}$
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$\begin{bmatrix} 63.0\, 44.8 \end{bmatrix}$
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