Proposed Methodology

This study investigates the comparative performance of **Quantum Amplitude Estimation (QAE)** and **classical risk-estimation methods** for Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) across a broad spectrum of financial portfolios. The methodology is divided into five major components: **data preprocessing**, **portfolio generation**, **classical risk estimation**, **quantum risk estimation**, and **evaluation via benchmarking and backtesting**. Together, these components enable a comprehensive assessment of accuracy, convergence behavior, computational efficiency, and robustness under real market conditions.

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1. Data Collection and Preprocessing

1.1 Dataset

We use **10 years of daily OHLCV data** for **10 diversified Indian equities**, selected to represent multiple sectors (Technology, Banking, FMCG, Pharma, etc.). Data is sourced from Yahoo Finance/NSE and aligned across all trading dates.

1.2 Price Alignment

- Missing values are treated using forward-fill or removed to maintain consistent dates.
- Adjusted Close prices are used to capture corporate actions.

1.3 Return Computation

Daily **log returns** are computed as:

$$r_{t,i} = \ln\left(\frac{P_{t,i}}{P_{t-1,i}}\right)$$

for assets $i = 1, \ldots, 10$.

1.4 Statistical Inputs

From the return matrix $R \in \mathbb{R}^{T \times 10}$, we compute:

- Mean return vector: $\mu = \mathbb{E}[R]$
- Covariance matrix: $\Sigma = \mathrm{Cov}(R)$

These provide the classical distributional parameters and serve as the basis for both classical and quantum risk modeling.

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2. Portfolio Generation

To evaluate performance across a broad range of risk profiles, we generate **100,000 random portfolios**, varying both allocation patterns and concentration levels.

2.1 Portfolio Size Sampling

For each portfolio p:

- A random number of assets k_p is selected uniformly from $\{3, \ldots, 10\}$.
- k_p distinct assets are sampled without replacement.

2.2 Weight Generation

Weights are generated using a **Dirichlet distribution**:

$$w_p \sim \mathrm{Dirichlet}(\alpha \mathbf{1}_{k_p})$$

with $\alpha = 1$, ensuring:

- $w_{p,i} \geq 0$
- $\sum_i w_{p,i} = 1$

This approach produces fully invested, long-only portfolios with realistic variability.

2.3 Portfolio Return Model

For each portfolio:

$$\mu_p = w_p^\top \mu, \quad \sigma_p = \sqrt{w_p^\top \Sigma w_p}$$

These parameters are used for parametric VaR and serve as validation inputs for simulation-based methods.

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3. Classical Risk Estimation Framework

We implement three classical risk models to establish strong baselines against which quantum methods are evaluated:

3.1 Parametric (Variance–Covariance) VaR

Assuming multivariate normal returns, 1-day VaR at confidence c is:

$$\mathrm{VaR}_{p,c}^{\mathrm{param}} = V(z_c \sigma_p - \mu_p)$$

This method is extremely fast but relies on distributional assumptions.

3.2 Historical Simulation VaR

Using trailing return windows (e.g., 250 days):

- Compute empirical portfolio losses.
- VaR is taken as the empirical quantile:

$$\mathrm{VaR}_c^{\mathrm{hist}} = \mathrm{Quantile}_c(L_t)$$

This approach is non-parametric but limited by the size of available history.

3.3 Classical Monte Carlo VaR and CVaR

We simulate portfolio returns as:

$$R^{(i)} \sim \mathcal{N}(\mu, \Sigma)$$

$$L^{(i)} = -V(w_p^\top R^{(i)})$$

For each portfolio, 100,000 sampled losses estimate:

- VaR: empirical quantile of $L^{(i)}$
- CVaR: conditional mean of losses beyond VaR

To provide a **ground truth VaR**, we additionally generate **10 million Monte Carlo samples**. This yields an extremely accurate reference value:

$$\mathrm{VaR}_{p,c}^{\mathrm{true}} = \mathrm{Quantile}_c(L_{1\ldots 10M})$$

This serves as the benchmark for both classical and quantum estimation error.

4. Quantum Risk Estimation Framework

We apply **Quantum Amplitude Estimation (QAE)** and its variants to approximate tail probability distributions and risk measures.

4.1 Distribution Encoding

Portfolio loss distributions are encoded into quantum amplitudes via:

- **Amplitude Encoding**, or
- **Quantum State Preparation using log-normal approximations**, or
- **QGAN-generated distributions** for more realistic non-normal behavior.

The encoded distribution represents:

$$\Pr(L \leq x) \quad \text{in amplitude space}$$

4.2 Oracle Construction

We design a **loss oracle**:

$$f(x) = \mathbb{I}(L(x) > \mathrm{VaR}_{\text{threshold}})$$

for probability estimation, and an extended oracle for CVaR:

$$g(x) = \max(0, L(x) - \mathrm{VaR})$$

These oracles form the core of QAE circuits.

4.3 Amplitude Estimation Methods

We implement and benchmark:

1. **Standard Quantum Amplitude Estimation (QAE)**
2. **Iterative Amplitude Estimation (IQAE)**
3. **Maximum-Likelihood Amplitude Estimation (MLAE)**

For each portfolio:

- Compute estimated VaR/CVaR.
- Record quantum resource costs:
  - number of qubits
  - circuit depth
  - number of oracle calls
  - 1-qubit and 2-qubit gate counts

4.4 Quantum Runtime Environment

- Simulations performed on Qiskit Aer.
- Small-scale circuits validated on real quantum hardware (IBM Q) to assess noise sensitivity.
- Real-hardware results included as feasibility analysis.

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5. Evaluation Methodology

We evaluate classical vs quantum methods using two complementary approaches:

5.1 Benchmarking Against True VaR

For each of the 100,000 portfolios:

Accuracy Metrics

$$\text{MAE} = \frac{1}{N} \sum_p |\hat{\text{VaR}}_p - \text{VaR}_p^{\text{true}}|$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_p (\hat{\text{VaR}}_p - \text{VaR}_p^{\text{true}})^2}$$

$$\text{Relative Error} = \frac{|\hat{\text{VaR}}_p - \text{VaR}_p^{\text{true}}|}{\text{VaR}_p^{\text{true}}}$$

Efficiency Metrics

- Wall-clock runtime
- Error vs number of samples (MC)
- Error vs oracle calls (QAE)

Quantum Resource Metrics

- Qubit count
- Circuit depth
- Number of Grover iterations
- Gate count

Statistical Metrics

- Confidence interval width
- Kolmogorov–Smirnov distance from true distribution
- Tail stability at 95%, 99%, 99.5% VaR levels

This benchmarking reveals **quadratic convergence advantages** of QAE over classical Monte Carlo.

5.2 Real Market Backtesting

To assess real-world applicability, we conduct a **rolling VaR backtest**:

Procedure

1. For each day (t), estimate VaR using the previous 1-year window.
2. Observe the next-day realized profit/loss.
3. Record a **VaR exception** if:

$$L_{t+1} > \mathrm{VaR}_t$$

4. Compare exception frequency with theoretical expectation:
   - 95% VaR → 5% exceptions
   - 99% VaR → 1% exceptions

Backtesting Metrics

- Exception rate
- Kupiec likelihood ratio test
- Christoffersen independence test
- Duration-based test of conditional coverage

This component validates that the quantum-derived VaR behaves consistently with risk-management standards.

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6. Summary

The proposed methodology enables a **holistic, multi-layer comparison** between quantum and classical approaches:

- **Synthetic benchmarks** using true VaR demonstrate theoretical quantum advantage.
- **Real market backtests** evaluate practical applicability.
- **Resource analysis** establishes feasibility for near-term quantum hardware.
- **Large portfolio sampling** ensures statistically robust conclusions.