DSGT-4 H·E Suryavanshi

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Nimzari Naka, Shirpur, Dist - Dhule (MS)

Ph No. : (02563) 259600,259801 Telefax : (02563) 259801

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TEST (I / II) / PRELIMINARY EXAMINATION

Name of Candidate : ____________________

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Year : FE / SE / TE / BE Branch : ________ Division : ________ Roll No. : ________

Semester : I / II Name of Subject : ____________________

Total Supplements : 1 + ________ = ________

Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
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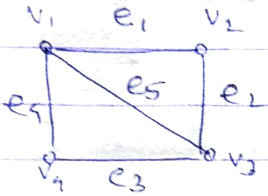
* Basic terminology of Graph-

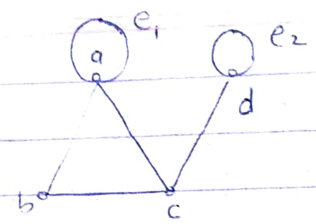
A graph is collection of points (vertices) and collection of lines (edges).

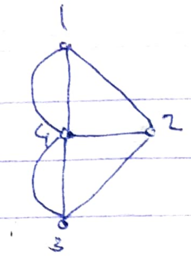
mathematically, graph G is an ordered pair (V, E) where V- set of vertices & E- set of edges.

Let e_{ij} is associated with (v_i, v_j). Then v_i and v_j called end vertices or terminal vertices.

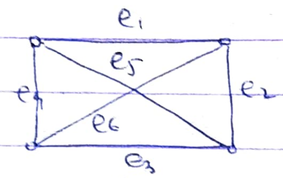
Vertex is also refered to as node, junction or point and edge is also called as line, element or arc.



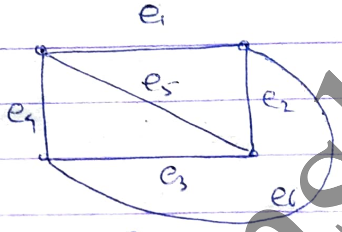




Representation of graph is not unique. Following graphs G_1 & G_2 represents the same path.



G_1

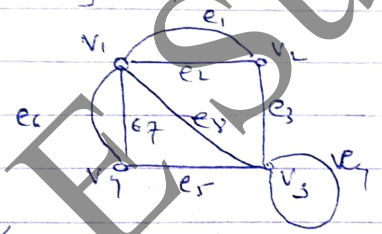


G_2

* **Self-loops and parallel edges**

- For any edge e_{ij}, if the end-points v_i & v_j are same, then e_{ij} is called as self loop or loop
- If there are more than one edge associated with given pair of vertices then those edges are called parallel / multiple edges.

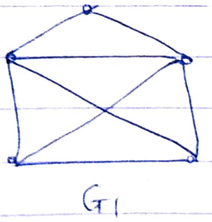
eg $e_1 = (v_1, v_2)$ & $e_2 = (v_1, v_2)$

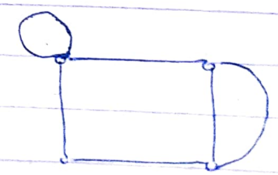


Similarly, e_6 & e_7 are parallel edges

* **Simple and Multiple Graphs**

A graph that has neither self-loops nor parallel edges is called simple graph otherwise it is called multiple graph.


G_1


G_2

G_1 — Simple Graph
G_2 — Multiple Graph

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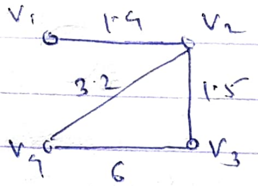
* Weighted Graph

Let G be a graph with vertex set V & edge set E. If each edge or each vertex or both are associated with some positive real no. then the graph is called a weighted graph.



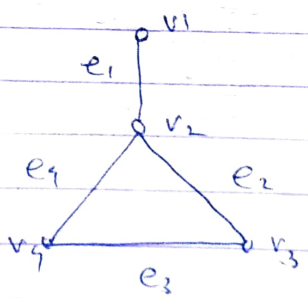
* Finite and Infinite Graphs

A graph with finite no. of vertices as well as finite no. of edges is called a finite graph, otherwise it is infinite graph.

A graph shown above is finite graph.

* Adjacency and Incidence

- If two vertices are joined directly by at least one edge then these vertices are called adjacent vertices. eg. v_1 & v_2 - adjacent vertices but v_1 & v_4 are not adjacent
- Two non-parallel edges are said to be adjacent if they are incident on a common vertex.
  eg. e_1 & e_2 are adjacent
  e_3 & e_4 also adjacent
- For incidence, if the vertex v_i is the end vertex of edge $e_{ij} = (v_i, v_j)$ then the edge e_{ij} is said to be incident on v_i. Similarlly, e_{ij} is said to be incident on v_j. Eg. e_1 incident on v_1 & v_2



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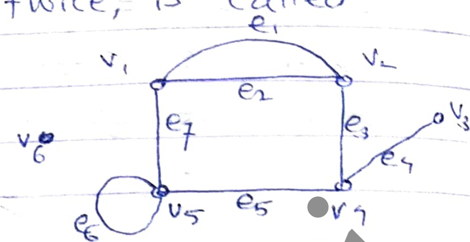
* **Degree of vertex**

- The no. of edges incident on a vertex v_i with self-loop counted as twice, is called the degree of vertex v_i.
  - denoted by $d(v_i)$



$d(v_1) = 3$, $d(v_2) = 3$

$d(v_3) = 1$, $d(v_4) = 3$

$d(v_5) = 4$ $d(v_6) = 0$

* **Isolated vertex and Pendant vertex**

- vertex with degree zero called Isolated
- vertex of degree 1 is called pendant

eg. In above fig.

v_6 i.e $d(v_6) = 0$ → Isolated vertex

v_3 i.e $d(v_3) = 1$ → pendant vertex

* **Handshaking Lemma**

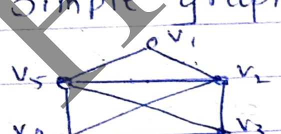
Let G be the graph with e - no. of edges & n - no. of vertices.

Since each edge contributes two degree the sum of the degrees of all vertices in G is twice the no. of edges in G.

i.e $$\sum_{i=1}^{n} d(v_i) = 2e$$

① Show that maximum degree of any vertex in simple graph with n-vertices is $\boxed{(n-1)}$

⇒



Simple Graph

$G = (E, V) = (8, 5)$

Now $n = 5$

∴ maximum degree $d(v_i) = (n-1) = 4$

∴ eg $d(v_2) = d(v_5) = 4$

② Show that maximum no. of edges in simple graph with n-vertices is $\boxed{\frac{n(n-1)}{2}}$

⇒

By handshaking Lemma

$$\sum_{i=1}^{n} d(v_i) = 2e \quad (e = \text{no. of edges in } G)$$

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$\therefore \; d(v_1) + d(v_2) + \cdots + d(v_n) = 2e$

Since maximum degree of each vertex $= (n-1)$

$\therefore \; \underbrace{(n-1) + (n-1) + \cdots + (n-1)}_{n\text{-time}} = 2e$

$\therefore \; n(n-1) = 2e$

$\therefore \; \boxed{e = \frac{n(n-1)}{2}}$

(3) How many nodes are necessary to construct a graph with 6-eadges in which each node is of degree 2

$\Rightarrow$

$\therefore \; \sum_{i=1}^{n} d(v_i) = 2e = 2 \times 6 = 12$

$\therefore \; d(v_1) + d(v_2) + \cdots + d(v_n) = 12$

$\underbrace{2 + 2 + \cdots + 2}_{n \text{ times}} = 12$

$2n = 12$

$\boxed{n = 6}$ $\therefore$ 6 nodes Required

(4) Determine no. of edges in graph with 6-nodes, 2 of degree 4 and 4 of degree 2. Draw graph

$\Rightarrow$

$n = 6$ $\therefore$ by handshaking lemma $\sum_{i=1}^{6} d(v_i) = 2e$

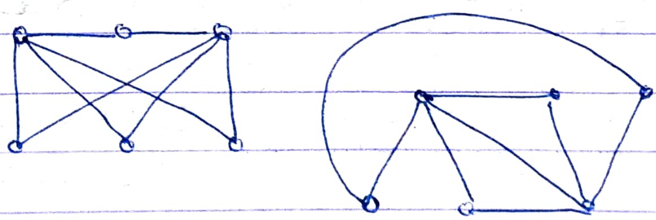
$\therefore \; d(v_1) + d(v_2) + \cdots + d(v_6) = 2e$

Now 2V of degree 4 & 4V of degree 2

$\therefore \; (4+4) + (2+2+2+2) = 2e$

$16 = 2e$

$\boxed{e = 8}$ $\therefore$ 8 edges Required



(5) Is it possible to construct graph with 12 nodes such that 2 nodes have degree 3 & Remaining nodes have degree 4

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$\Rightarrow$ $n = 12$ (nodes / vertices)

$\therefore$ by handshaking lemma

$$\sum_{i=1}^{12} d(v_i) = 2e$$

$$\therefore (2 \times 3) + (10 \times 4) = 2e$$

$$6 + 40 = 2e$$

$$46 = 2e$$

$$\boxed{e = 23}$$

$\therefore$ Yes it is possible to construct such graph.

⑥ Is it possible to draw a simple graph with 4 vertices & 7 edges? Justify

$\Rightarrow$ In simple graph

maximum ~~degree~~ edges $= \dfrac{n(n-1)}{2}$ (n = vertices)

$\therefore$ Graph with 4 vertices $= \dfrac{4 \times 3}{2} = 6$ edges

Therefore, graph with 4 vertices & cannot have 7 edges. Hence the graph doesn't exist

=x=

* Some Important and Useful Graphs

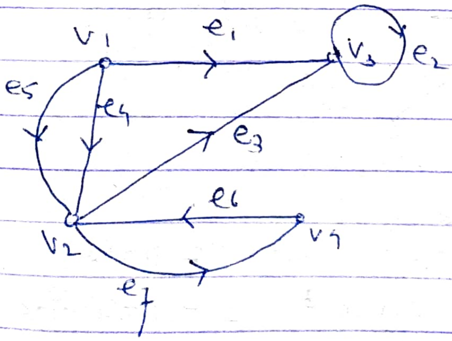
① Directed Graph (Diagraph)

– If each edge of graph has direction, then the graph is called directed graph.

$D = (V, A)$ D – Graph

V – set of vertices

A – directed edges / Arcs



$e_1 = (v_1, v_3)$	$e_6 = (v_4, v_2)$
$e_2 = (v_3, v_3)$	$e_7 = (v_2, v_4)$
$e_3 = (v_2, v_3)$	
$e_4 = (v_1, v_2)$	
$e_5 = (v_1, v_2)$	

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v_1 & v_2 - joined by more than one arce with same direction. Such arcs are called multiple arcs.

Arcs e_6 & e_7 — Not multiple arc because direction different.

Diagraph without loops & multiple arcs are known as simple digraphs.

Incidence

- Indegree and Outdegree
  - indegree of vertex u of digraph D is defined as no. of arcs which are incident into u.

    denoted by $\overrightarrow{d}(u)$
  - Similarly, outdegree = no. of arcs which are incident of out u & denoted by $\overleftarrow{d}(u)$

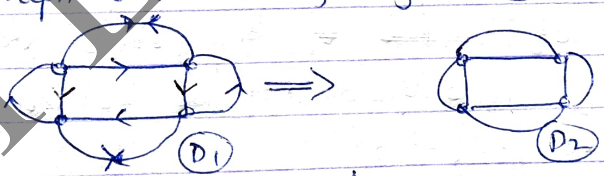
*// Self-loop considered in both indegree & outdegree

Indegree : $\overrightarrow{d(v_1)} = 0$, $\overrightarrow{d(v_2)} = 3$, $\overrightarrow{d(v_3)} = 3$, $\overrightarrow{d(v_4)} = 1$

outdegree : $\overleftarrow{d(v_1)} = 3$, $\overleftarrow{d(v_2)} = 2$, $\overleftarrow{d(v_3)} = 1$, $\overleftarrow{d(v_4)} = 1$

Underlying Graph of Digraph

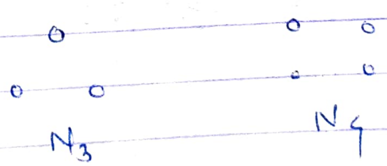
Graph obtained by neglecting directions of arcs



D_1 - simple graph but D_2 is not Simple graph & contains parallel edges

② Null Graph (N_n)

If Edge set of graph with n-vertices is empty set then graph is null graph.



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③ Complete Graph (K_n)

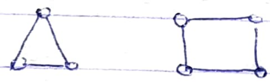
If in a graph G, degree of each vertex is $(n-1)$ then G is called complete graph.

K_2 K_3 K_4

In complete graph K_n, no. of edges $= \frac{n(n-1)}{2}$

④ Regular Graph

Degree of each vertex is same



Complete graph is also a regular graph but regular graph need not be a complete graph.

⑤ Bipartite Graph -

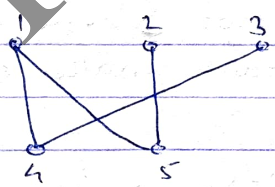
$G = (V, E)$ V - vertices, E - edges & G - Graph

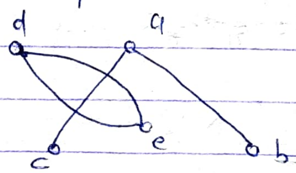
G is bipartite if its vertex set V can be partitioned into two disjoint subsets V_1 & V_2

where $V_1 \cup V_2 = V$ &

$V_1 \cap V_2 = \phi$

- Each edge of ~~vertex~~ V_1 G joins vertex V_1 to V_2
- Vertices of V_1 should not be joined, similarly, V_2
- Do not have self-loop





$V_1 = \{1, 2, 3\}$ $V_1 = \{a, d\}$

$V_2 = \{4, 5\}$ $V_2 = \{b, c, e\}$

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⑥ Complete Bipartite Graph ($K_{m,n}$)

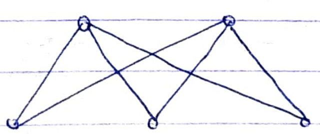
A Bipartite Graph is called complete bipartite graph if each vertex of V_1 is joined to every vertex of V_2 by an unique edge.

- denoted by

$K_{m,n}$ m = no. of vertices in V_1 &

n = no. of vertices in V_2

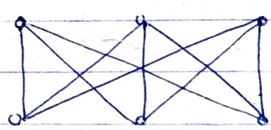
∴ total no. of edges in complete Bip. Graph $m * n$



$K_{2,3}$

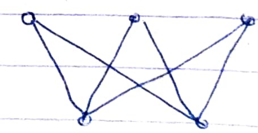
$m=2$, $n=3$

$m<n$



$K_{3,3}$

$m=n=3$

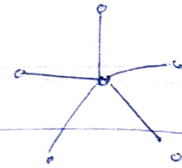


$K_{3,2}$

$m=3$, $n=2$

$m>n$

Graph $K_{1,n}$ known as star eg $K_{1,5}$



- Every complete bipartite graph is regular if $m=n$
  $\therefore$ $K_{3,3}$ — regular

Ex. ① How many edges has each of the following graph
i> K_{10} ii> $K_{5,7}$

— (i) K_{10} — bipartite graph edges $= \frac{n(n-1)}{2} = \frac{10\times 9}{2} = 45$

— (ii) $K_{5,7}$ — complete bi-graph edges $= m \times n = 5 \times 7 = 35$

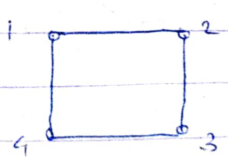
* **Isomorphism** -

Two graphs are thought of as equivalent (called isomorphic) if they have identical behaviour interms of graph-theoritic properties.

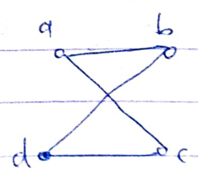
Two graphs $G_1(V_1 E_1)$ & $G'(V' E')$ are said to be isomorphic to each other if there is one-one correspondence betn vertices and edges such that incidence relationship is preserved.

- eg. e - incident edge on vertices V_1 & V_2 of G then e' must be incident on v_1' & v_2' of G'
- Adjacency betn vertices is preserved
- denoted by $G_1 \cong G_2$

Isomorphic graphs must have
① Same no. of vertices
② Same no. of edges
③ An equal no. of vertices with given degree



G1



G2

G_1 is isomorphic to G_2
$\therefore$ one-one correspondence betn vertices
1–a, 2–b, 3–d, 4–c

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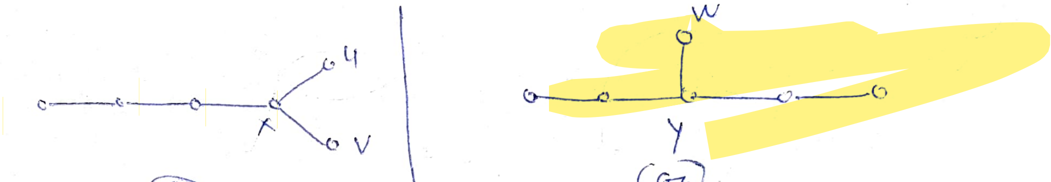


(G_3) (G_4)

$G_3 \not\cong G_4$ — Not isomorphic

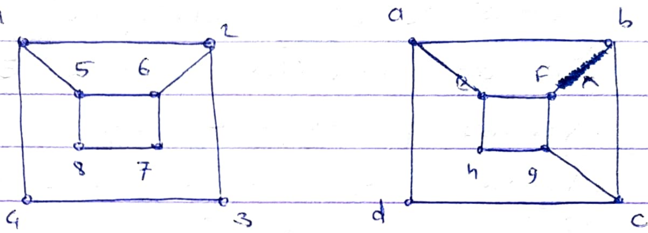
Since

G_3 has vertex x (degree-3) connected to two pendent vertices

G_4 has vertex y (degree-3) but not connected to pendent vertices

$\therefore$ Adjacency not preserved.

Ex. ① Determine whether following graphs are isomorphic or not.



$\Rightarrow$ G1 G2

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- Both G1 & G2 contains 8-vertices & 10-edges.
- No. of vertices with degree 2 in G1 & G2 $\Rightarrow$ 4
- No. of vertices with degree 3 in G1 & G2 $\Rightarrow$ 4
- For Adjacency,

vertex 1 in G_1 (degree = 3) connected / adjacent to two vertices with degree 3 (i.e 2, 5) and one vertex with degree 2 (i.e 4)

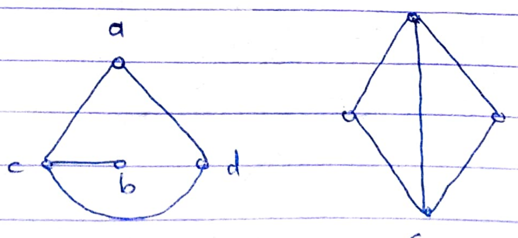
There is no such a vertex in G_2

Hence, adjacency is not presered

So G_1 & G_2 are not isomorphic ($G_1 \not\cong G_2$)

Ex. ② Find whether following pairs of graph are isomorphic or not.

Ⓐ



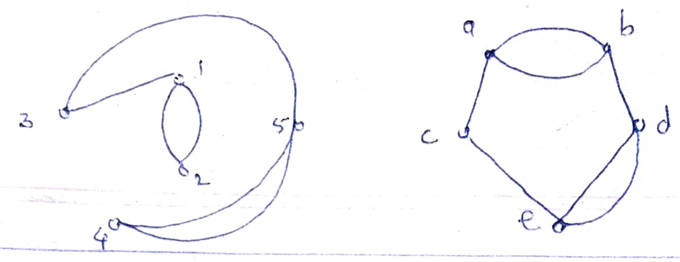
G_1 G_2

$\Rightarrow$ $G_1 \not\cong G_2$

Not isomorphic

Since, No. of edges in $G_1 = 4$ while in $G_2 = 5$

(B)

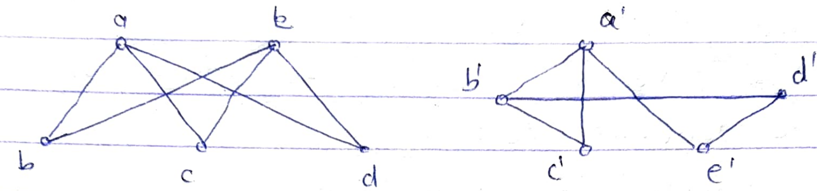


G_3 G_4

$\Rightarrow$

$\therefore$ $G_3 \not\cong G_4$ No. of edges in $G_3 = 6$ while G_4 has 7

(C)



G_5 G_6

$\Rightarrow$

$\therefore$ $G_5 \not\cong G_6$ (Not isomorphic)

In G_5, a - vertex of degree 3 is adjacent to 3 vertices of degree 2. But in G_6, both the vertices a' & b' of degree 3 are not adjacent to 3 vertices of degree 2

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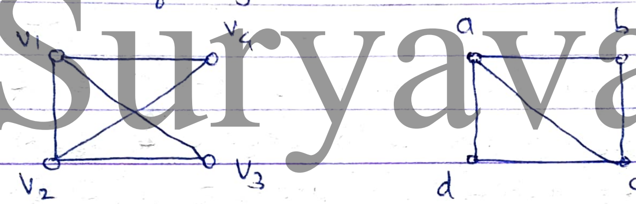
(D)

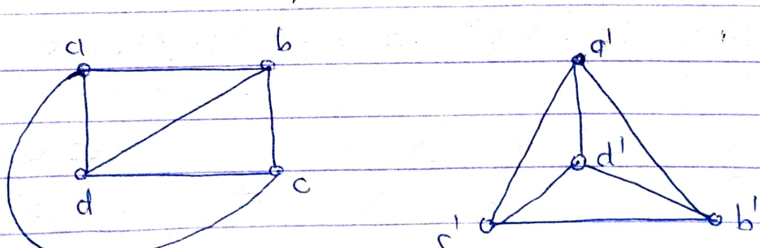


G_7 G_8

$\Rightarrow$

- Yes $G_7 \cong G_8$ (isomorphic)
- In G_7 & G_8, there are 2 vertices of degree 2 and 2 vertices of degree 3
- Also adjacency is preserved
- $\therefore$ one-one correspondence betn vertices is given by $v_1 \to a$, $v_2 \to c$, $v_3 \to d$ & $v_4 \to b$

(E)



G_9 G_{10}

$\Rightarrow$ Both contains same no. of vertices (4) & same no. of edges (6). Also adjacency is preserved $G_9 \cong G_{10}$

(i.e. isomorphic)

Ex. (3) Find whether K_6 and $K_{3,3}$ are isomorphic or not?

$\Rightarrow$

Both contains 6 - vertices

But, no. of edges in $K_6 \Rightarrow \frac{6\times5}{2} = 15$ $\left(\frac{n(n-1)}{2}\right)$

while no. of edges in $K_{3,3} \Rightarrow 3\times3 = 9$ $(m*n)$

$15 \neq 9$

Hence K_6 & $K_{3,3}$ are not isomorphic

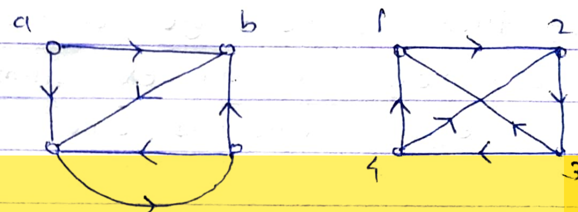
Ex. (4) Determine whether the following graphs $G = (V, E)$ and $G^* = (V^*, E^*)$ are isomorphic or not

$G = (\{a,b,c,d\}, \{(a,b)(a,d)(b,d)(c,d)(c,b)(d,c)\})$

$G^* = (\{1,2,3,4\}, \{(1,2)(2,3)(3,1)(3,4)(4,1)(4,2)\})$

$\Rightarrow$

Graphs are



G G^*

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$G \not\cong G^*$

Not isomorphic

- No. of vertices & edges are same in G & G^* but degree of vertices do not match.

G contains 1-vertex of degree 2 & 3 of degree 3 while G^* contains 4-vertex & all of degree 3

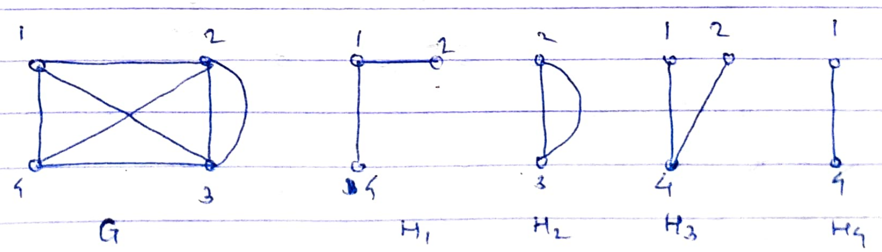
=x=

* New Graphs from old ones

(1) Subgraph

Let $G = (V, E)$ be any given graph.

Then $G' = (V', E')$ is called subgraph of G if $V' \subseteq V$ and $E' \subseteq E$



Here H_1, H_2, H_3 and H_4 are subgraphs of G

Properties

① Each graph is subgraph of itself.
② A single vertex of a graph G is a subgraph of G
③ A single edge together with its end vertices is also a subgraph of a graph G
④ A subgraph of a subgraph of a graph G is a subgraph of G.

—x—

② Edge Disjoint Subgraphs

Two subgraphs H_1 and H_2 of graph G are said to be edge disjoint subgraphs of G if there is no edge common betn H_1 and H_2 (but may have vertex common)

e.g. H_1 and H_2 are disjoint subgraphs of G

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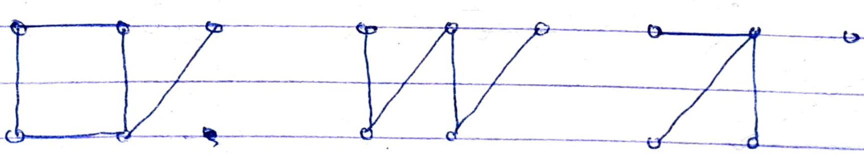
③ Vertex Disjoint Subgraphs

Two subgraphs H_1 and H_2 of graph G are said to be vertex disjoint subgraphs of a graph G if there is no vertex common between them (i.e they do not have ~~ver~~ common edge also)

eg. H_2 and H_4 – vertex disjoint subgraphs

④ Spanning Subgraphs

Let $G = (V, E)$ be any graph.
Then G' is said to be the spanning subgraph of G if its vertex set V' is equal to the vertex set V of G

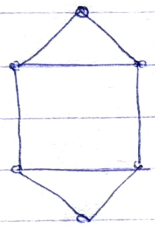


G G' G''

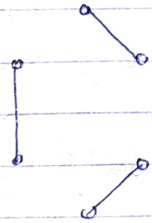
In fig. G' and G'' are spanning subgraphs of the graph G.

(5) Factors of Graph

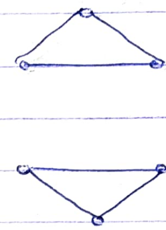
A K-Factor of a graph is defined to be a spanning subgraph of the graph with the degree of each of its vertex is being K.



G



1-Factor graph



2-Factor graph

- A graph might have many different K-Factors or might not have any K-Factor at all for some K.
- Fig. shown below depicts a graph which does not have any 1-Factor graph.

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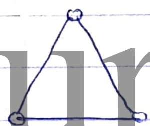
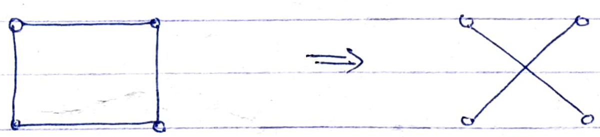


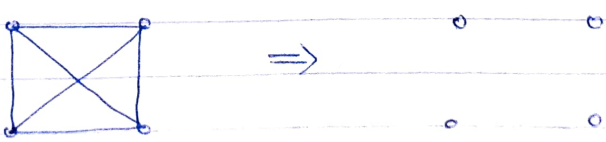
(6) Complement of a graph

Let G be a simple graph.
Then the complement of G denoted by $\bar{G}$ is the graph whose vertex set is the same as the vertex set of G and in which two vertices are adjacent if and only if they are not adjacent in G.



G $\qquad\qquad$ $\bar{G}$

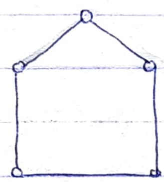
Complement of a complete graph is a null graph and vice-versa.



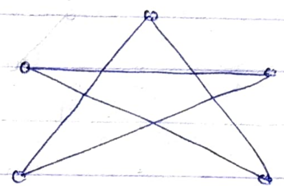
G $\qquad\qquad$ $\bar{G}$

A graph is said to be self complementory if it is isomorphic to its complement.

A graph G and its self complementary graph are shown in fig. below.



graph G.



G'

Self complementory graph of G

$G \cong G'$

Ex ① Find complement of graph shown below. Is it self-complementory?

⇒

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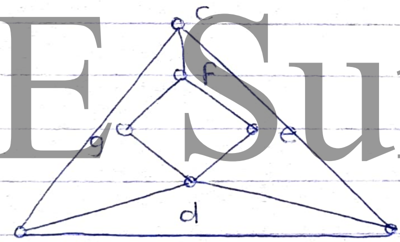
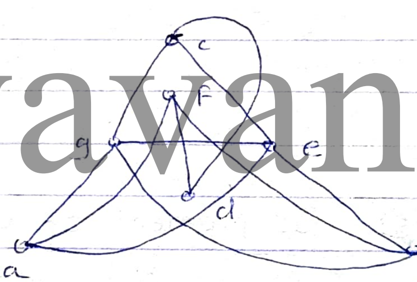


G

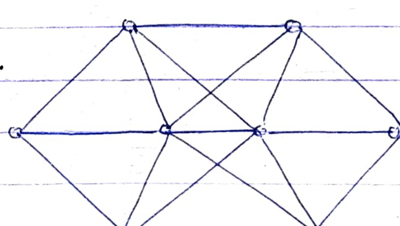
⇒



$\overline{G}$

Graph G is not self-complementory because it is not isomorphic to its complement $\overline{G}$.

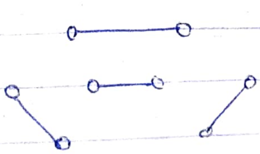
Ex.② Find 1-factor and 2-factor graphs of G graph shown below.

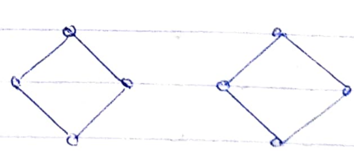


G

⇒

1-factor graph | 2-factor graph





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Semester : I / II Name of Subject : ____________________

Total Supplements : 1 + ______ = ______

Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

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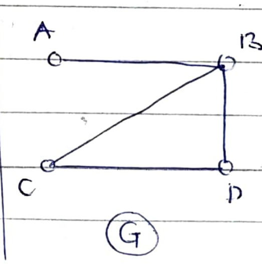
(Start From here only)

Ex. ③ For the graph G show in fig determine whether $H = (V', E')$ is subgraph of G, where

i> $V' = \{A, B, F\}$ & $E' = \{\{A,B\}, \{A,F\}\}$

ii> $V' = \{B, C, D\}$ & $E' = \{(B,C)\ (B,D)\}$



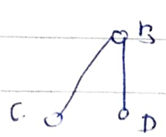
⇒

i> Given $V' = \{A, B, F\}$ & $E' = \{\{A,B\}\ \{A,F\}\}$

No, H is not subgraph of G. because it contains vertex 'F' which doesn't belongs to G

ii> Given $V' = \{B, C, D\}$ & $E' = \{\{B,C\}\ \{B,D\}\}$

Yes, All the vertices & edges of H belongs to G



* Operations on Graphs

1. Union of two graphs
2. Intersection of two graphs
3. Ring Sum of two graphs
4. Removal of an Edge
5. Removal of a Vertex

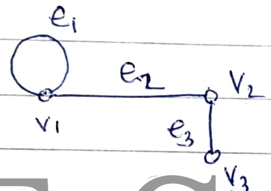
⇒ 1. **Union of two graphs**

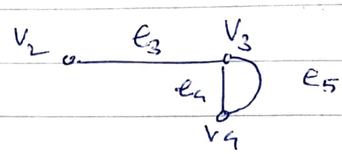
Let $G_1 = (V_1\ E_1)$ and $G_2 = (V_2\ E_2)$.

The union of two graphs is denoted by

$\boxed{G_1 \cup G_2}$

where, $V_1 \cup V_2$ and $E_1 \cup E_2$.





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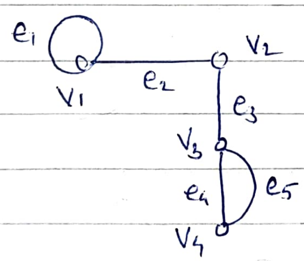
G_1 : $V_1 = \{V_1, V_2, V_3\}$; $E_1 = \{e_1, e_2, e_3\}$

G_2 : $V_2 = \{V_2, V_3, V_4\}$; $E_2 = \{e_3, e_4, e_5\}$

Then,

$G_1 \cup G_2$ is shown below



$G_1 \cup G_2 \Rightarrow V_1 \cup V_2 = \{V_1, V_2, V_3, V_4\}$

$E_1 \cup E_2 = \{e_1, e_2, e_3, e_4, e_5\}$

⇒ 2. **Intersection of Two graphs**

Let $G_1 = (V_1\ E_1)$ and $G_2 = (V_2\ E_2)$

$\boxed{G_1 \cap G_2}$

where $V_1 \cap V_2$ and $E_1 \cap E_2$

In Graph G_1 & G_2 shown above

$V_1 \cap V_2 = \{V_2\ V_3\}$

$G_1 \cap G_2 \Rightarrow$ 

$E_1 \cap E_2 = \{e_3\}$

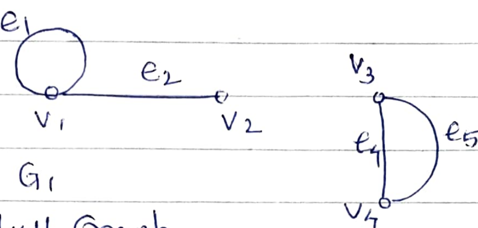
- If G_1 & G_2 are Edge disjoint graph then, $G_1 \cap G_2 \Rightarrow$ Null Graph
- If G_1 & G_2 are Vertex disjoint graph then, $G_1 \cap G_2 \Rightarrow$ Empty set

⇒ ③ **Ring Sum of Two Graphs**

Let $G_1 = (V_1 E_1)$ and $G_2 = (V_2 E_2)$

Ring sum of two graphs consisting of the vertex set $V_1 \cup V_2$ and edges that in either G_1 or G_2 but not in both.

$G_1 \oplus G_2$



For any graph G_1

$G \oplus G$ = Null Graph

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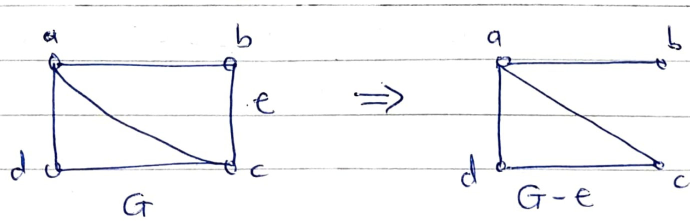
⇒ ④ **Removal of an Edge**

Let $G = (V, E)$ be any graph.

Let $e \in E$. Then the graph $(G - e)$ can be obtained by removing the edge 'e' from the graph.

— Removal of any edge 'e' from graph G doesn't mean the removal of its end vertices

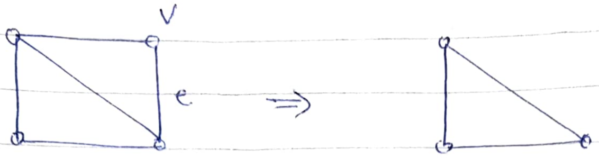


⇒ ⑤ **Removal of vertex**

Let $G = (V, E)$ be any graph.

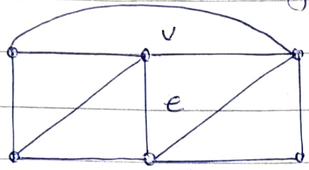
Let $v \in V$. The graph $(G - v)$ can be obtained by removing vertex 'v' from graph G.

— Removal of v means, removal of all these edges also which are incident on v



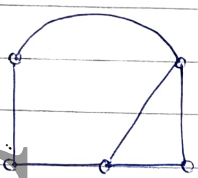
G (G-v)

Ex ① Draw the graphs ① G-v ② G-e, where the graph G is shown in fig.

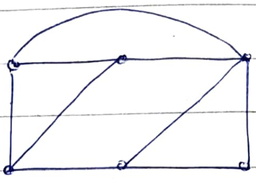


G

⇒ ① G-v



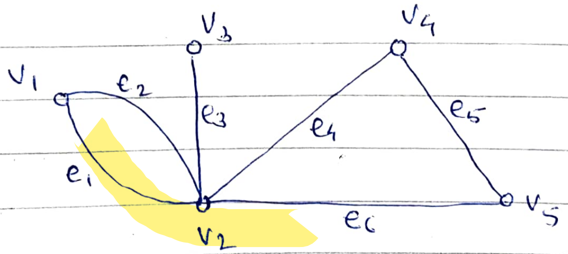
② G-e



E Suryansh

* Paths and Circuits

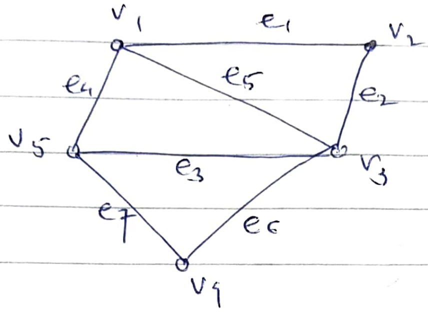
Let $G = (V, E)$ be any graph.
Let V_0 and V_n be any two vertices in V.
A path 'p' of length 'n' from V_0 to V_n is a sequence of vertices and edges of the form $V_0\, e_1\, V_1\, e_2 \cdots e_n\, V_n$. where each edge e_j is an edge betn V_{j-1} and V_j.
The vertices V_0 and V_n called end-points of path and other vertices $V_1 V_2 \cdots V_{n-1}$ called interior vertices



path I = $V_1\, e_2\, V_2\, e_4\, V_4$
path II = $V_4\, e_4\, V_2\, e_2\, V_1\, e_1\, V_2\, e_3\, V_3$
path III = $V_3\, e_3\, V_2\, e_2\, V_1\, e_1\, V_2\, e_3\, V_3$

In simple graph (no loops & parallel edges), a path may be described by giving only the sequence of vertices traversed in the path.

- For ex. the path $(v_5\ e_4\ v_1\ e_1\ v_2\ e_2\ v_3)$ can be written as $(v_5\ v_1\ v_2\ v_3)$



① <u>Simple Path</u>

- Path is called simple if <u>edges do not repeat</u> in the path

ex. path I and II — Simple path
path III - Not simple bcoz e_3 repeated twice

② <u>Elementary Path</u>

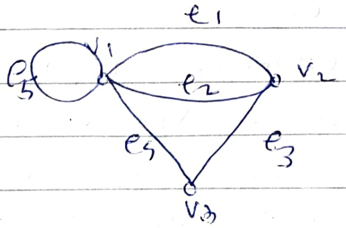
- Vertices do <u>not repeat</u> in the path

ex. path I - elementary

③ <u>Circuit</u>

- If the end <u>vertices</u> of the path <u>are same</u> then it is called circuit.

ex. path - III — circuit



$C_1 = (v_1\ e_1\ v_2\ e_2\ v_1)$

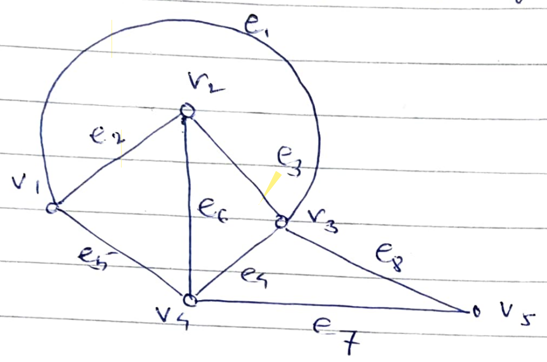
$C_2 = (v_3\ e_4\ v_1\ e_1\ v_2\ e_3\ v_3)$

④ <u>Simple and Elementary Circuit</u>

A circuit in a graph G is called simple circuit if it does not include the same edge twice

- A circuit is called elementary circuit if it does not meet the same vertex twice (except for first and last vertex)

- Length of circuit = No. of edges



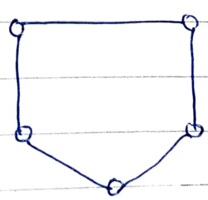
G

$C_1 = (v_1 e_1 v_3 e_3 v_2 e_2 v_1)$ — Simple & elementary)
$C_2 = (v_1 e_2 v_2 e_6 v_4 e_7 v_5 e_8 v_3 e_4 v_4 e_5 v_1)$ — elementary

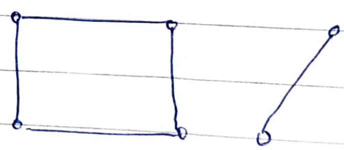
* Connected and Disconnected Graphs

- A graph is connected graph if there exists a path between every pair of vertices, otherwise the graph is disconnected.

- Disconnected graph consists of two or more parts called components
- Each component is connected graph but there is no path between two vertices if they belongs to two different components.
- Connected graph has only one component.



G_1
Connected Graph



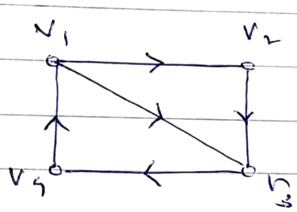
G_2
Disconnected Graph

Diagraph

- A directed graph (digraph) is called <u>strongly connected</u> if for every pair of vertices 'a' & 'b' in the diagraph, there is a path from 'a' to 'b' as well as path from 'b' to 'a'.

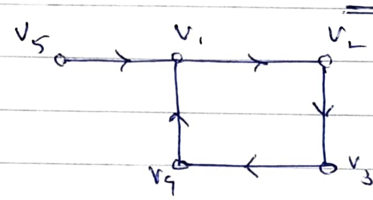
- A diagraph is <u>weakly connected</u> if it is not strongly connected and its underlying graph is connected.

- A diagraph which is neither strongly connected nor weakly connected is known as <u>disconnected diagraph</u>.



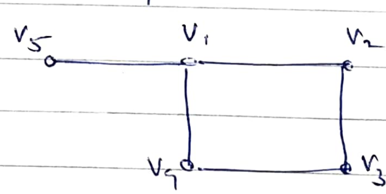
D_1

(Strongly connected)



D_2

(Weakly connected)



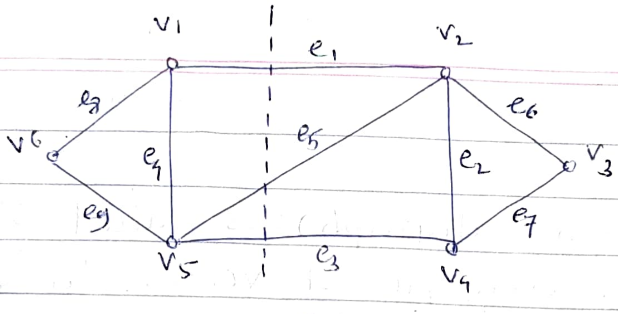
(Underlying graph of D_2)

* Edge and vertex Connectivity

① Edge Connectivity

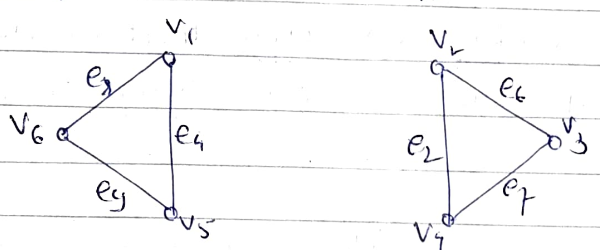
- In connected graph, <u>cut-set</u> is minimal set of edges whose removal disconnects the graph and increases the components of graph by one

- i.e cut-set in connected graph G is set of edges whose removal from G leaves G disconnected



G

For cut-set { e_1, e_3, e_5 } graph G will have 2 components G_1 and G_2

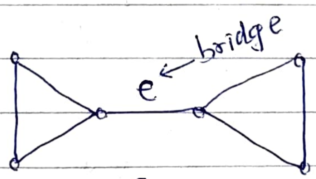


G_1 G_2

Hence, if 'S' is cut-set then (G-S) has exactly two components

Bridge or Isthmus

- If cut-set contains only one edge then that edge is called an isthmus or bridge



$\lambda(G_3) = 1$

G_3

Edge Connectivity $\lambda(G)$

- No. of edges in smallest cut-set of a connected simple graph is called edge connectivity
- i.e smallest no. of edges whose removal disconnects the graph G.
- In above graph G_3, $\lambda(G_3) = 1$

② Vertex Connectivity $K(G)$

- Smallest no. of vertices whose removal disconnects the graph

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Semester : I / II Name of Subject : ____________________

Total Supplements : 1 + ______ = ______

Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
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Signature of Examiner : Signature of Moderator :

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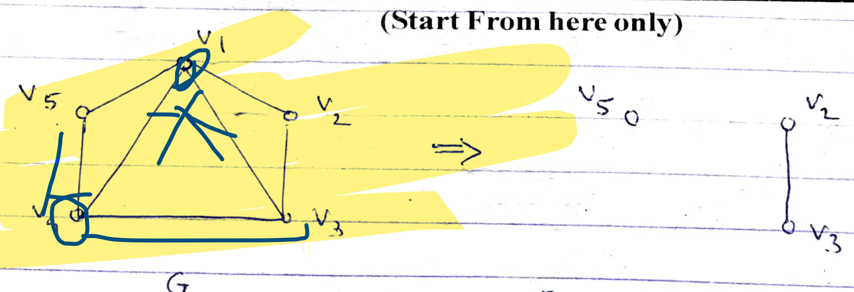


Removal of $\{v_1 v_4\}$ disconnects G
$K(G) = 2$ i.e $\{v_1 v_4\}$

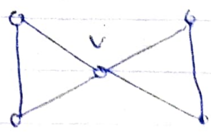
<u>K-connected graph</u>

- graph whose vertex connectivity is 'K'

<u>Seperable Graph</u>

- vertex connectivity is one i.e $K(G)=1$
- cut-vertex or cut-point

e.g. 'v' is cut-point



- Edge and vertex connectivity are related to minimum degree of vertex in graph.

i.e $K(G) \le \lambda(G) \le \delta$

where,

$K(G)$ – Vertex connectivity

$\lambda(G)$ – Edge connectivity

δ – minimum degree of vertex in G

- Edge connectivity is also related to no. of edges & vertices

$$\therefore \quad \lambda(G) \le \left\lfloor \frac{2e}{n} \right\rfloor$$

Ex ① Find edge connectivity for complete graph K_5

$\Rightarrow$ Here $n=5$ and $e = \frac{n(n-1)}{2} = \frac{5 \times 4}{2} = 10$

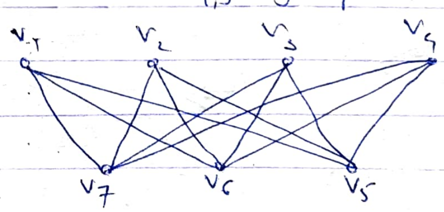
$\therefore$ edge connectivity $\le \frac{2e}{n}$

$$\frac{2 \times 10}{5} = 4$$

$\therefore \quad \lambda(G) = 4$

Ex. ② Find $\lambda(G)$, $K(G)$ for $K_{4,3}$ graph

$\Rightarrow$ $K_{4,3}$ $\therefore$



Here $n = 7$

$e = 12$

$\therefore$ edge connectivity $(\lambda) = \frac{2e}{n} = \frac{2 \times 12}{7}$

$$= \frac{24}{7}$$

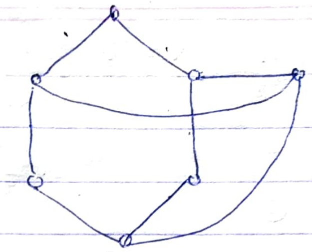
$$\lambda(G) = 3$$

Now,

if vertex (v_5, v_6 & v_7) removed then graph will be disconnected

$\therefore \quad K(G) = 3$ i.e vertex connectivity

Ex. ③ Find edge connectivity of Graph given in fig.



$\Rightarrow$ Here $n = 7$
and $e = 9$

$\therefore$ Edge connectivity $\lambda(G) \leq \frac{2e}{n}$

$$= \frac{2 \times 9}{7}$$

$\therefore \lambda(G) = 2$ i.e. edge connectivity

* Shortest Path Algorithm

The algorithm was found by Dijkstra in 1959. and is known as Dijkstra's shortest path algo.

This algorithm gives the shortest length of the path from the vertex 'a' to vertex 'z' but it does not give the actual path for the shortest distance from vertex 'a' to vertex 'z'.

Algorithm -

$G = (V, E) \Rightarrow$ be a simple graph
a & $z \Rightarrow$ any two vertices of the graph.
$L(x) \Rightarrow$ Label - length of shortest path from 'a' to 'x'
$W_{ij} \Rightarrow$ Weight of edge $e_{ij} = (v_i, v_j)$

Step ① $P = \{\phi\}$
$T = \{$ All vertices of Graph G $\}$

where
P = Set of vertices having permanent label.

Set $L(a) = 0$
$L(x) = \infty \ \forall x \in T$ and $x \neq a$

Step ②
Select vertex 'v' from T which has smallest label.
This label is called permanent label of 'v'.
$P = P \cup \{v\}$ & $T = T - \{v\}$

IF $v = z$ then $L(z)$ is the length of Shortest path from a to z & stop.

Step ③

IF $v \neq z$ revise the labels of vertices of T i.e vertices that do not have permanent label

$L(x) = \min \{ \text{old-}L(x),\ L(v) + w(v, x) \}$

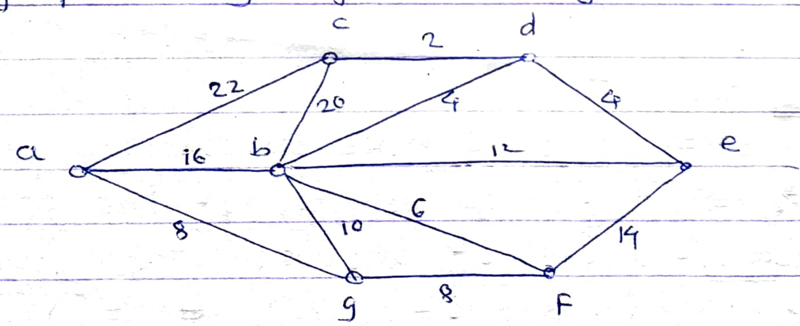
where

$w(v, x)$ – weight of edge joining 'v' to 'x'

IF there is no direct edge from 'v' to 'x' then $w(v, x) = \infty$

Step ④ Repeate step ② and ③ until 'z' gets permanent label.

Ex ① Find the shortest path for the following graph using Dijkstra's Algorithm. (from a to e)



⇒ According to the Dijkstras shortest path algorithm

① $P = \{\phi\}$ $T = \{a, b, c, d, e, f, g\}$

$L(a) = 0$ $L(x) = \infty$ $\forall x \in T$, $x \neq a$

② $v = a$, the permanent label of $a = 0$ i.e $L(a) = 0$

$P = \{a\}$ $T = \{b, c, d, e, f, g\}$

$L(b) = \min(\text{old-}L(b),\ L(a) + w(a, b))$

$= \min(\infty,\ 0 + 16) = \min(\infty, 16)$

$\therefore L(b) = 16$

$L(c) = \min\left(\text{old\_}L(c),\ L(a) + w(a,c)\right)$

$= \min(\infty,\ 0 + 22) = \min(\infty, 22)$

$L(c) = 22$

Similarly,

$L(d) = \min(\infty,\ 0+\infty) = \infty$

$L(e) = \min(\infty,\ 0+\infty) = \infty$

$L(f) = \min(\infty,\ 0+\infty) = \infty$

$L(g) = \min(\infty,\ 0+8) = 8$

③ $v = g$, permanent label of $g = 8$ i.e $L(g) = 8$

$P = \{a, g\}$ $T = \{b, c, d, e, f\}$

$L(b) = \min\left(\text{old\_}L(b),\ L(g) + w(g,b)\right)$

$= \min(16,\ 8+10) = \min(16, 18)$

$\therefore L(b) = 16$

Similarly,

$L(c) = \min\left(\text{old\_}L(c),\ L(g) + w(g,c)\right)$

$= \min(22,\ 8+\infty) = \min(22, \infty)$

$\therefore L(c) = 22$

$L(d) = \min(\infty,\ 8+\infty) = \infty$

$L(e) = \min(\infty,\ 8+\infty) = \infty$

$L(f) = \min(\infty,\ 8+8) = 16$

④ $v = b$, permanent label of $b = 16$ i.e $L(b) = 16$

$P = \{a, g, b\}$ $T = \{c, d, e, f\}$

$L(c) = \min\left(\text{old\_}L(c),\ L(b) + w(b,c)\right)$

$= \min(22,\ 16+20)$

$\therefore L(c) = 22$

$L(d) = \min(\infty,\ 16+4) = 20$

$L(e) = \min(\infty,\ 16+12) = 28$

$L(f) = \min(\cancel{\infty}\,16,\ 16+6) = \cancel{22}\ 16$

⑤ $v = f$, the permanent label of $f = 16$ i.e $L(f) = 16$

$P = \{a, g, b, f\}$ $T = \{c, d, e\}$

$L(c) = \min(\text{old\_}L(c),\ L(F) + w(F, c))$

$= \min(22,\ 16 + \infty)$

$\therefore L(c) = 22$

$L(d) = \min(20,\ 16 + \infty) = 20$

$L(e) = \min(28,\ 16 + 14) = 28$

⑥ $v = d$, the permanent label of d is 20 i.e $L(d) = 20$

$P = \{a, g, b, F, d\}$ $T = \{c, e\}$

$L(c) = \min(\text{old\_}L(c),\ L(d) + w(d, c))$

$= \min(22,\ 20 + 2)$

$\therefore L(c) = 22$

$L(e) = \min(28,\ 20 + 4) = 24$

⑦ $v = c$, the permanent label of $c = 22$ i.e $L(c) = 22$

$P = \{a, g, b, F, d, c\}$ $T = \{e\}$

$L(e) = \min(\text{old\_}L(e),\ L(c) + w(c, e))$

$= \min(24,\ 22 + \infty)$

$L(e) = 24$

⑧ $v = e$, the permanent label of $e = 24$ i.e $L(e) = 24$

$P = \{a, g, b, F, d, c\}$ $T = \{\phi\}$

Hence,

the length of shortest path from 'a' to 'e' is 24

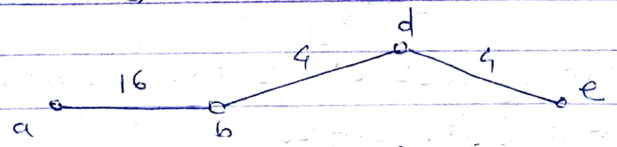
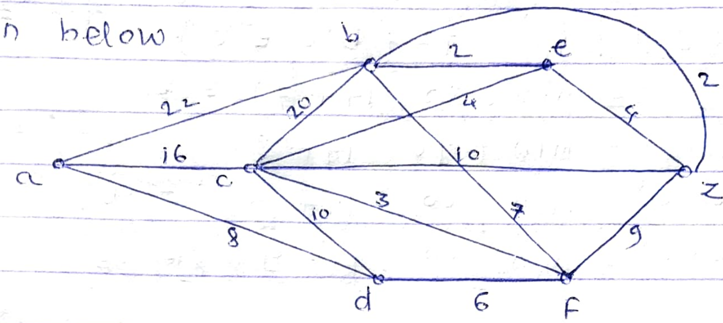


Fig. shortest path from a to e i.e (a-b-d-e)

Ex. (2) Determine a shortest path from 'a' to 'z' in the graph shown below



$\Rightarrow$

Dijkstras Algorithm to find shortest path from 'a' to 'z'

(1) $P = \{\phi\}$ $T = \{a, b, c, d, e, f, z\}$

$L(a) = 0$, $L(x) = \infty$, $\forall x \in T$, $x \neq a$

(2) $v = a$, permanent label of a is 0 i.e $L(a) = 0$

$P = \{a\}$ $T = \{b, c, d, e, f, z\}$

$L(b) = \min(\text{old\_}L(b) + L(a) + w(a,b))$

$= \min(\infty, 0 + 22)$

$\therefore L(b) = 22$

$L(c) = \min(\infty, 0 + 16) = 16$

$L(d) = \min(\infty, 0 + 8) = 8$

$L(e) = \min(\infty, 0 + \infty) = \infty$

$L(f) = \min(\infty, 0 + \infty) = \infty$

$L(z) = \min(\infty, 0 + \infty) = \infty$.

(3) $v = d$, permanent label of d is 8 i.e $L(d) = 8$

$P = \{a, d\}$ $T = \{b, c, e, f, z\}$

$L(b) = \min(\text{old\_}L(b), L(d) + w(d,b))$

$= \min(22, 8 + \infty)$

$L(b) = 22$

$L(c) = \min(16, 8 + 10) = 16$

$L(e) = \min(\infty, 8 + \infty) = \infty$

$L(f) = \min(\infty, 8 + 6) = 14$

$L(z) = \min(\infty, 8 + \infty) = \infty$

$v = f$, - the permanent label of f is 14 i.e $L(f) = 14$

$P = \{a, d, f\}$ $T = \{b, c, e, z\}$

$L(b) = \min(22, 14+7) = 21$

$L(c) = \min(16, 14+3) = 16$

$L(e) = \min(\infty, 14+\infty) = \infty$

$L(z) = \min(\infty, 14+9) = 23$

(4) $v = c$, permanent label of c is 16 i.e $L(c) = 16$

$P = \{a, d, f, c\}$ $T = \{b, e, z\}$

$L(b) = \min(21, 16+20) = 21$

$L(e) = \min(\infty, 16+4) = 20$

$L(z) = \min(23, 16+10) = 23$

(5) $v = e$, permanent label of e is 20 i.e $L(e) = 20$

$P = \{a, d, f, c, e\}$ $T = \{b, z\}$

$L(b) = \min(21, 20+2) = 21$

$L(z) = \min(23, 20+4) = 23$

(6) $v = b$, permanent label of b is 21 i.e $L(b) = 21$

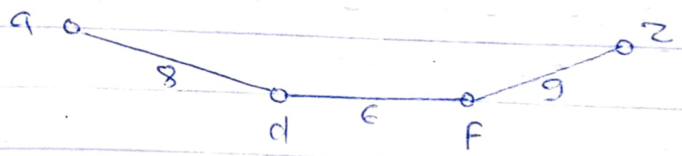
$P = \{a, d, f, c, e, b\}$ $T = \{z\}$

$L(z) = \min(23, 21+2) = 23$

(7) $v = z$, permanent label of z is 23 i.e $L(z) = 23$

$P = \{a, d, f, c, e, b, z\}$ $T = \{\phi\}$

Hence, the length of shortest path from a to z is 23 & path is $a - d - f - z$



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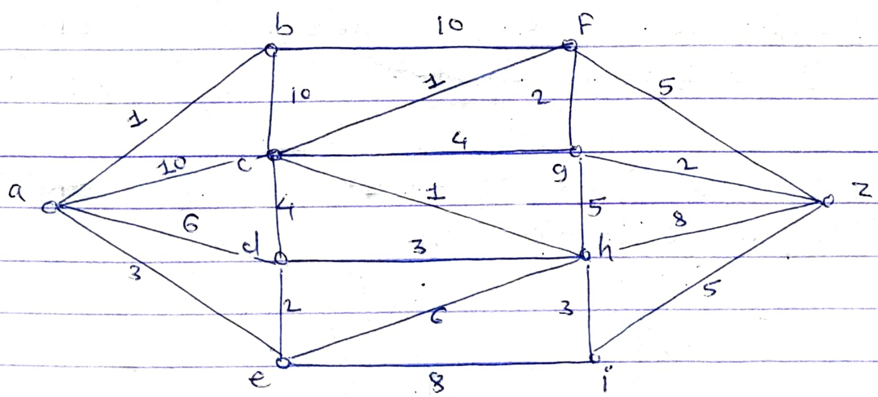
Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

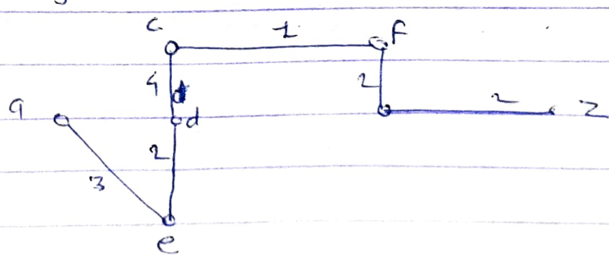
(Start From here only)

H.W.

Ex. ① Find shortest path from a to z using Dijkstra's algo.

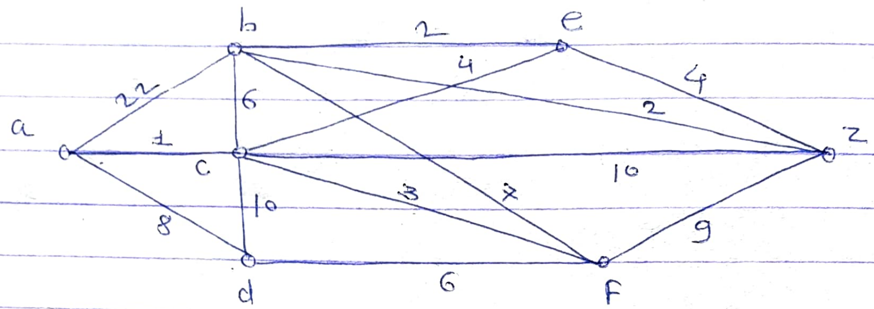


Ans Length of shortest path from a to z is 15 (a-e-d-c-f-g-z)



H.W.

Ex ② Apply Dijkstra's shortest path algorithm to find the shortest path between vertices a and z.



Ans. – length of shortest path is 9

path = a – c – e – z

=x=

* Eulerian Path and Eulerian Circuits

– Swiss Mathematician Leonhard Euler

① Eulerian Path

– Every edge of the graph G appears exactly once in the path

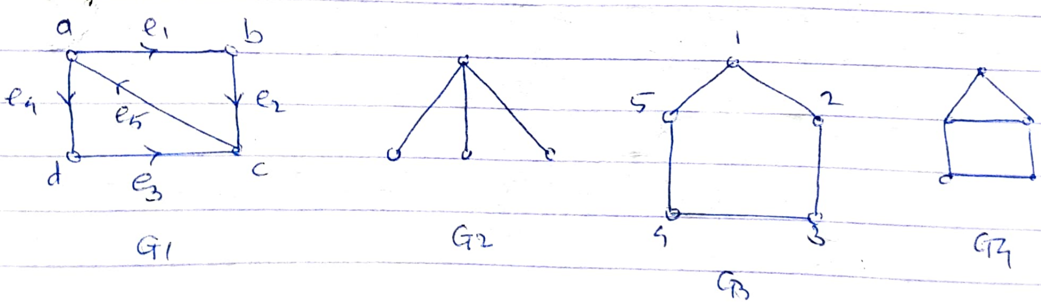
② Eulerian Circuit

– Circuit which contains every edge of the graph G exactly once

③ Eulerian Graph

– A graph which has an Eulerian circuit called Eulerian Graph.

– A graph which has an eulerian path may not have an eulerian circuit.



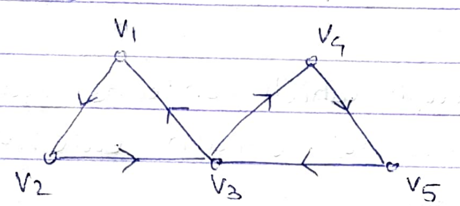
G_1 has eulerian path $(e_1 e_2 e_3 e_4 e_5)$ but G_2 not

G_3 contains eulerian ckt (123451) but G_4 not

- Existence of eulerian paths/ckts in a graph is related to the degree of vertices as follows

Theorems

1. Graph possesses eulerian path iff it is connected & has '0' (zero) or two vertices of degree odd (i.e odd degree vertices)
2. Graph possesses eulerian circuit iff it is connected & its vertices are all of even degree.
3. Directed graph possesses eulerian circuit if it is connected & incoming & outgoing degree of every vertex is same.



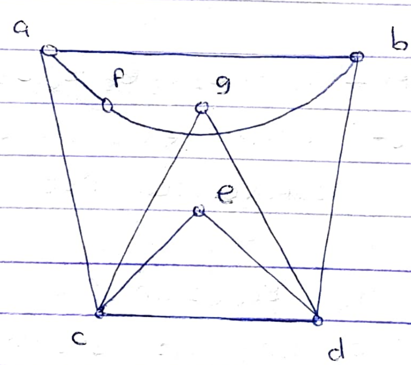
$\overrightarrow{d}(V_1) = \overleftarrow{d}(V_1) = 1$

$\overrightarrow{d}(V_2) = \overleftarrow{d}(V_2) = 1$

$\overrightarrow{d}(V_3) = \overleftarrow{d}(V_3) = 2$

Hence, diagraph has eulerian ckt – $V_1 V_2 V_3 V_4 V_5 V_3 V_1$

Ex ① Find out the eulerian ckt in the following graph. Also find an euler path from vertex 'a' to 'b'.



⟹ Eulerian circuit –

According to the theorem, graph doesnt contains all vertices of even degree Hence, graph doesnt have eulerian circuit

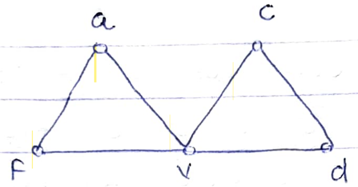
Eulerian Path –

Also, graph has two vertices of odd degree So, graph has eulerian path given as below

⟹ a – c – e – d – c – g – d – b – f – a – b

Ex ② Draw the graph which has eulerian ckt. and cut-vertex also.

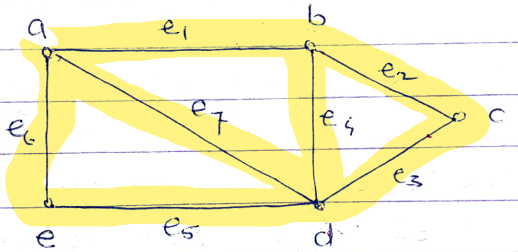
⇒



- Degree of each vertex is even
Hence, graph has eulerian ckt.
- Cut vertex is 'v'. Its removal forms disconnected graph with two components
- Also, Eulerian path is a-v-c-d-v-f-a

Ex ③ Draw the graph which contains eulerian path but does not contain eulerian ckt.

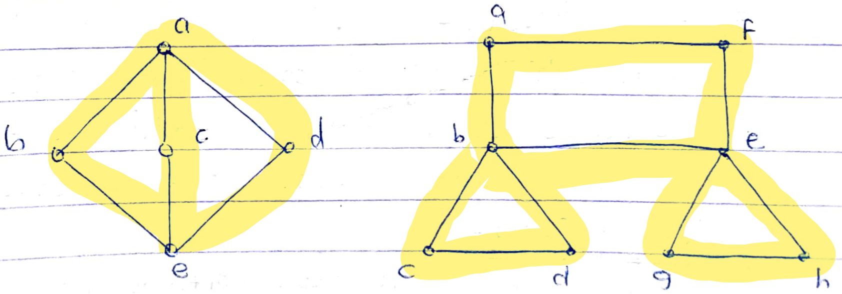
⇒



Eulerian Path - a $e_1 e_2 e_3 e_5 e_6 e_7 e_4$ b

Eulerian ckt- doesnt exist. bcoz degree of all vertices not even.

Ex ④ Determine whether Eulerian path & ckt. exits in the graph G1 and G2 shown below.



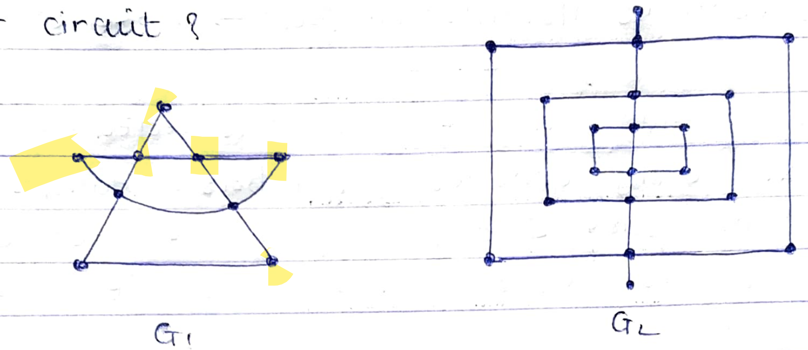
G1 G2

⇒ G1 contains Eulerian path bcoz exactly two vertices have odd degree (a, e) ⇒ a-d-e-b-a-c-e

G_1 doesnt contains eulerian ckt. bcoz degree of all vertices is not even

In G_2, degree of each vertex is even. Hence G_2 has both eulerian ckt. & path ⇒ a-f-e-h-g-e-b-d-c-b-a

Ex. ⑤ Which of the following graph possess Euler's path or circuit?



⇒ - In G_1, each vertex is of even degree. Hence it possess eulerian circuit

- In G_2, graph is connected and has exactly two vertices of odd ~~gr~~ degree. Hence, there exist eulerian path.

* Hamiltonian Path and Hamiltonian Circuit

- Sir William Hamiltonian (1859)

Hamiltonian Circuit

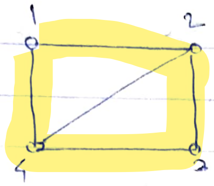
- A circuit in connected graph G is called Hamiltonian Circuit if it contains every vertex of G exactly once (except first & last vertex).

Hamiltonian Path

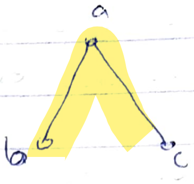
- A path in connected graph G is a Hamiltonian path if it contains every vertex of G exactly once.

A graph which has Hamiltonian circuit is called a Hamiltonian Graph

A graph that contains the Hamiltonian path may not contain Hamiltonian circuit



G_1



G_2

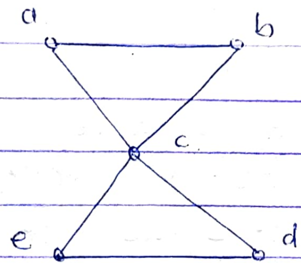
G_1 contains Hamiltonian ckt → (1-2-3-4-1)
G_2 contains Hamiltonian path → (b-a-c)

- **Hamiltonian path & ckt's in connected graph**
  - Not every connected graph has a Hamiltonian ~~graph~~ path
  - A necessary & sufficient condition for a connected graph to have a Hamiltonian circuit is still unknown.

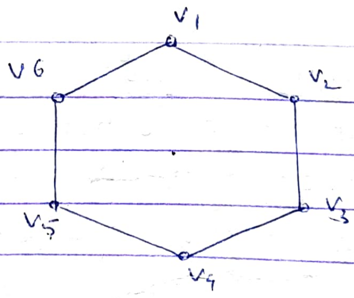
Theorem ①

Let G be a simple graph with n-vertices. If the sum of degree for each pair of vertices is (n-1) or large, then there exist a Hamiltonian path.

- It is sufficient condition but not necessary condition for the existence of Hamiltonian path.



G_1



G_2

In G_1, $n = 5$
∴ Sum of degree for each pair must be $>= (n-1)$ i.e. (4)
∴ There exist Hamiltonian path (a-b-c-e-d)

In G_2 $n = 6$
∴ sum of degree of pair must be $>= (n-1)$ i.e 5 which is not true
But the graph G_2 has Hamiltonian path - $v_1 v_2 v_3 v_4 v_5 v_6$

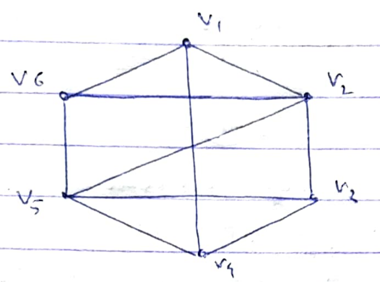
$$\boxed{\text{Total no. of Hamiltonion Ckt in a graph} = \frac{(n-1)!}{2}}$$

Theorem ②

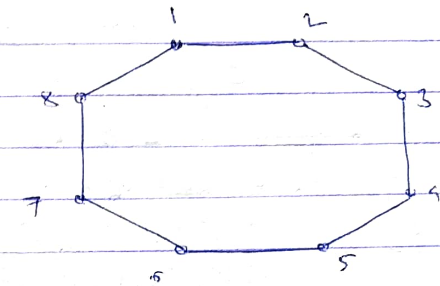
Let $G = (V, E)$ be a simple connected graph. If the degree of each vertex 'v' $\geq n/2$

i.e $d(v) \geq (n/2)$ $\forall v \in V$.

Then Graph will contain Hamiltonian Circuit.



G_1



G_2

According to theorem ② degree of each vertex is greater than or equal to $n/2$ (i.e 3)

$\therefore$ G_1 contains Hamiltonian circuit - $(v_1 v_2 v_3 v_4 v_5 v_6 v_1)$

In G_2, $n = 8$

It contains Hamiltonian ckt - (1 2 3 4 5 6 7 8 1)

But $d(v) = 2 \neq 8/2 = 4$

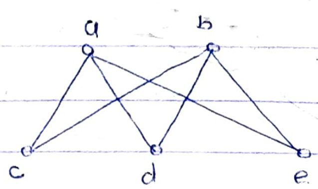
according to theorem ②

Theorem ③

Let G be a connected simple graph. If G has a Hamiltonian circuit then for every proper non-empty subset S of V, the components in graph (G−S) is less than or equal to the number of vertices in S

i.e. $(G - S) \leq |S|$ where $S \subseteq V$

Ex ① consider complete Biportite graph $K_{2,3}$

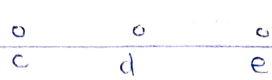


$S = V_1 = \{a, b\}$ $|V_1| = m$

$V_2 = \{c, d, e\}$ $|V_2| = n$

i.e $m < n$

Now $(K_{2,3} - V_1)$ — is null graph with 3-component



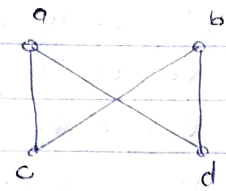
let x = components

$\therefore x = 3$

According to theorem ③ No. of components are less than or equal to total vertices in S.

but here $x \neq m$ $\therefore$ It does not contain Hamiltonian Circuit

Ex (2) Let $K_{2,2}$ - complete bipertite graph



Total vertices = 4 each vertex v is $d(v)=2$

& $d(v) \geq n/2$

$2 \geq 4/2$ by theorem (2)

Hence, graph has Hamiltonian circuit

<u>Note</u> : In complete Bipertite graph $(K_{m,n})$, there exist Hamiltonian circuit & path also if <u>$m=n$</u>

* <u>Travelling Salesman Problem</u>

- Problem related to Hamiltonian circuit

<u>Problem</u> -

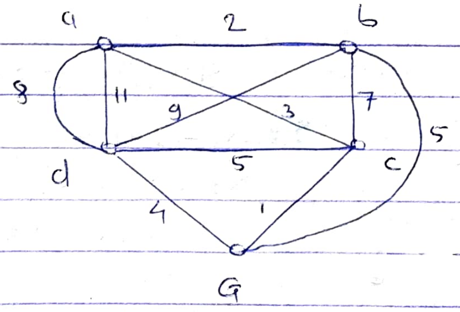
A salesman is required to visit a no. of cities during a trip. Given the distance between the cities, in what order should he travel so as to visit every city precisely once and return home with minimum distance travelled ?

Let $G=(V,E)$ be weighted graph shown in fig.

V = represents the cities

E = represents the road

& W(E) = represents the distance betn two cities



Hamiltonian ckts starts from 'a'

(1) abceda with weight = 22

(2) abecda ⇒ wt = 21

(3) acebda ⇒ wt = 26 &

so on

If G is K_n with n-nodes, then there are $\frac{(n-1)!}{2}$ possible Hamiltonian ckts. However for large value of n, it is highly inefficient algorithm

• Nearest - Neighbour Method

- provides good results for salesman problem.

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Semester : I / II Name of Subject : ____________
Total Supplements : 1 + ______ = ______

Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

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(Start From here only)

* Nearest-Neighbour Method

① Start with any vertex (v_1) & choose the vertex closest to v_1 to form initial path of edge.

② Let v_n denotes latest vertex added to path. Select v_{n+1} closest to v_n & not in path. Then add it to path

③ Repeate Step ② untill all vertices of G are included in path.

④ At last, complete the ckt by adding the edge which connects starting vertex to the last vertex.

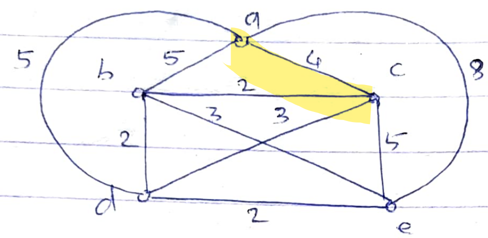
The circuit obtained by this method will be the required Hamiltonian circuit.

Ex.① Use nearest-neighbour method to find out Hamiltonian ckt.

① starting at vertex 'a'

② Starting at vertex 'd'

③ Determine the minimum Hamiltonian ckt.

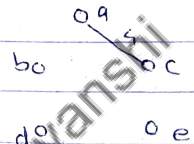


$\Rightarrow$

① start with 'a'

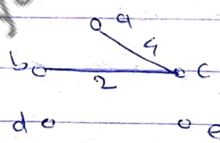
Select vertex with minimum cost.

i> Path = { a, c }



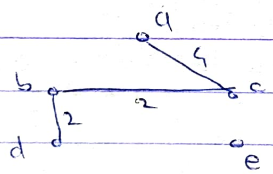
ii> Select b wt (2)

Path = { a, c, b }

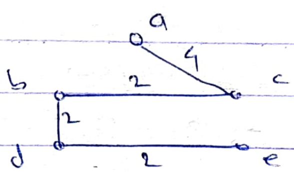


iii> select d wt (2)

path = { a, c, b, d }



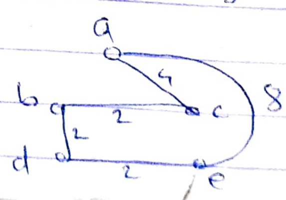
iv> path = { a, c, b, d, e }



v> Since all vertices are traversed, to complete Hamiltonian ckt, connect a to e

∴ Hamiltonian ckt = { a, c, b, d, e, a }

Total wt. of ckt = 18



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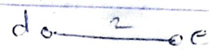
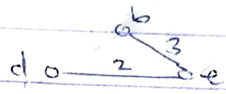
(2) Start from 'd'

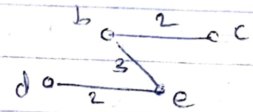
i) path = {d, e}



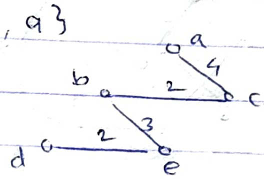
ii) path = {d, e, b}



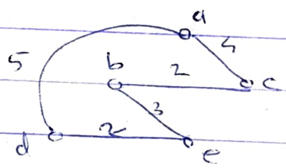
iii) Path = {d, e, b, c}



iv) Path = {d, e, b, c, a}



v) Hamiltonian
ckt = {d, e, b, c, a, d} & weight = 16



(3) The minimum Hamiltonian ckt is "debcad" with weight = 16

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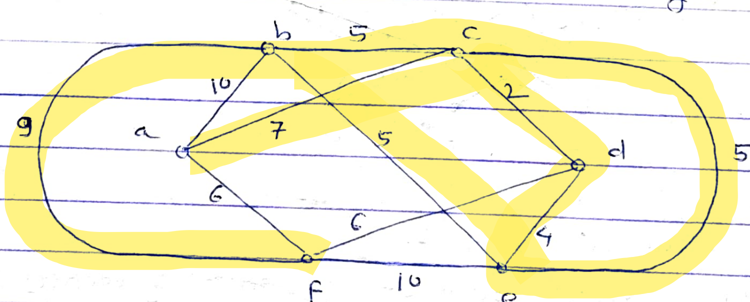
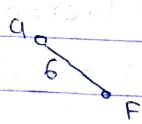
Ex. (2) Use nearest-neighbour method to find the Hamiltonian ckt. starting from 'a' in the following graph. Find its weight

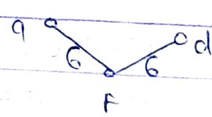


⇒ starting from 'a'

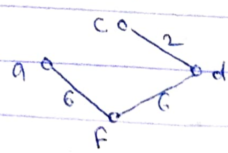
i) path = {a, f}



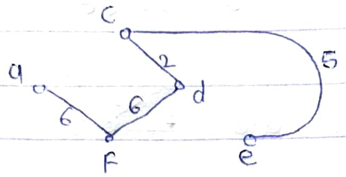
ii) path = {a, f, d}



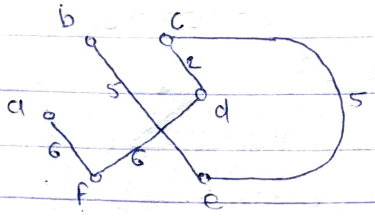
iii) path = {a, f, d, c}



iv> path = $\{a, f, d, c, e\}$

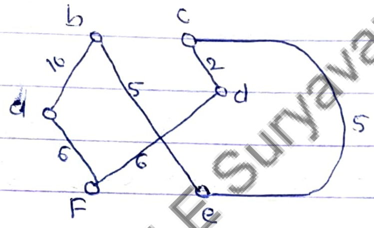


v> path = $\{a, f, d, c, e, b\}$



vi> Hamiltonion ckt = $\{a, f, d, c, e, b, a\}$

weight = 34



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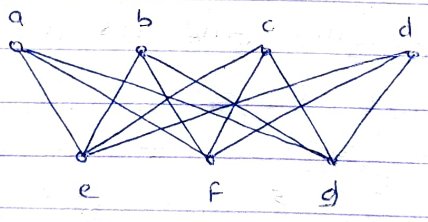
Ex. (3) Find Hamiltonian path and ckt. in $K_{4,3}$

$\Rightarrow$

The complete Bipartite Graph $K_{4,3}$ given by



Here,

Total vertices = 7

also, degree sum of any pair of vertices $\geq (7-1)$

Hence, by theorem (1) graph $K_{4,3}$ has Hamiltonian path & it is $\Rightarrow$ a-g-b-f-c-e-d

Also, $K_{4,3}$ doesn't contain a Hamiltonion circuit because $m \neq n$ i.e. $(4 \neq 3)$

Ex (4) Is there a Hamiltonion path in complete Biportite graph $K_{4,4}$ and $K_{4,5}$?

⇒

(1) In $K_{4,4}$

Total vertices = 8

Degree of each vertex = 4

Hence, degree sum of any pair is 8 (i.e 4+4)

By theorem (1)

degree sum $\geq (n-1)$

$8 \geq (8-1)$

Hence, $K_{4,4}$ contains Hamiltonion path

(2) In $K_{4,5}$

Total vertices = 9

4-vertices with degree = 5

& 5-vertices with degree = 4

therefore, degree sum of any pair

$(4+4) = 8 \geq (9-1)$ i.e $(n-1)$ & $n=9$ total vertices

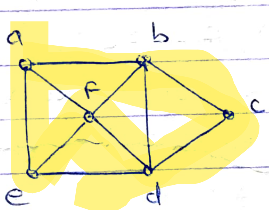
$(5+5) = 10 \geq (9-1)$

$(4+5) = 9 \geq (9-1)$

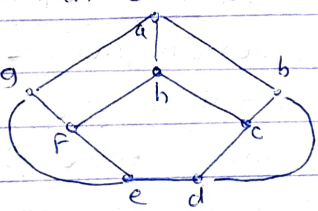
Hence, by theorem (1),

- $K_{4,5}$ contains Hamiltonian path.

Ex. (5) Is there a Hamiltonian circuit in the graphs G_1 & G_2



G_1



G_2

⇒ In G_1,

Hamiltonian path = a-b-c-d-f-e

Hamiltonian ckt = a-b-c-d-f-e-a

In G_2, Hamiltonian path = a-b-d-c-h-f-e-g

Hamiltonian ckt = a-b-d-c-h-f-e-g-a

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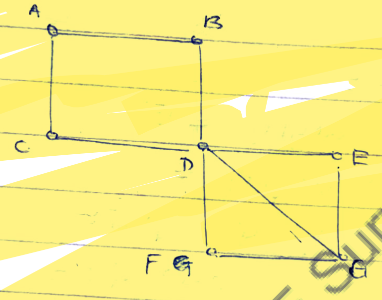
Ex ⑥ Is there a Hamiltonian ckt in a complete Bipartite Graph $K_{4,4}$, $K_{4,5}$ & $K_{4,6}$?

⇒

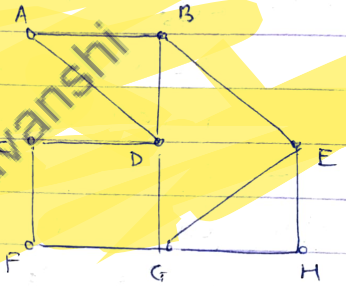
In complete Bipartite graph $K_{m,n}$
Hamiltonian ckt exist iff $m=n$

In $K_{4,4}$, Hamiltonian ckt exist But
In $K_{4,5}$ & $K_{4,6}$ (i.e $m \neq n$) Hamiltonian ckt. doesn't exist.

Ex. ⑦ Determine which of the following graphs G_1 & G_2 represents Eulerian path, Eulerian ckt, Hamiltonian path & Hamiltonian ckt.



G_1



G_2

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⇒ In G_1

① No Eulerian ckt. bcoz degree of each vertex is not even.

② Eulerian path exist bcoz exactly two vertices of odd degree (D & F) are present in G_1
∴ Eulerian path ~~G~~ GEDBACDFGD

③ No Hamiltonian ckt. bcoz to cover all vertices exactly once, vertex D has to be traversed twice

④ Hamiltonian path = EGFDBAC

In G_2

① No eulerian ckt. (degree not even)

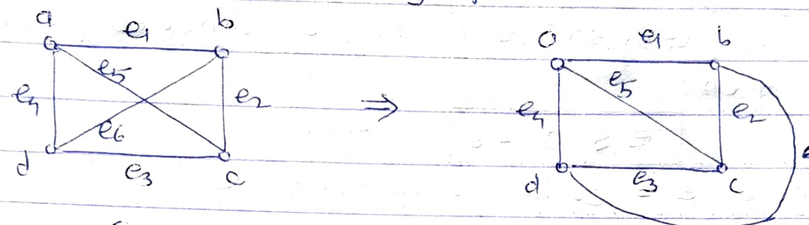
② Eulerian path = BADCFGHEGDBE

③ It has Hamiltonian path & ckt. Both
It is ABEHGFCDA

* ## Planner Graph

A graph is a planner graph if it can be drawn on the plane with no intersecting edges i.e no edge can cross each other.

A graph which is non-planner in one representation may become a planner graph after redrawing it



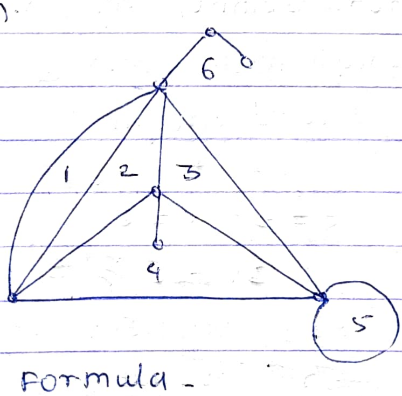
G1 (Non-planner graph) $\Rightarrow$ G2 (Planner Graph)

Regions -

A planner representation of graph divides the plane into regions (also called windows, faces & meshes). A region is characterized by the set of edges forming its boundary.

A region is finite, if its area is finite & it is said to be infinite if its area is infinite.

A planner ~~has~~ graph has exactly one infinite region.



Regions - 1, 2, 3, 4 & 5 finite

Region - 6 infinite

Eulers Formula -

A planner graph may have no. of regions. The no. of regions in graph depends on no. of vertices & no. of edges.

The relation betn vertices, edges & regions is given by Eulers Formula

$$\boxed{v - e + r = 2}$$

v - vertices, e - edges

r - regions

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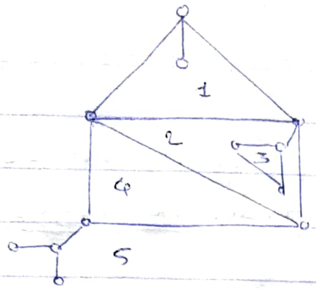


G

Here $v = 12$, $e = 15$

$\therefore r = \cancel{v - e + 2} = 2 - v + e$

$= \cancel{12 - 15 +} = 2 - 12 + 15$

$\boxed{r = 5}$

Corollary

If $G(V, E)$ is a simple connected planner graph then

$$\boxed{e \leq 3v - 6} \quad \text{—— ①}$$

where e = total no. of edges &

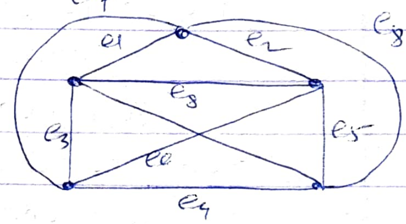
v = total no. of vertices in graph G.

- The most important application of this corollary is to show that the complete graph K_5 on 5 vertices are non-planner.

K_5 is known as Kuratowski's First Graph.

In K_5 $\quad v = \cancel{10}\ 5$

$e = \frac{n(n-1)}{2} = \frac{5(4)}{2} = 10$



$\therefore e \leq 3v - 6$

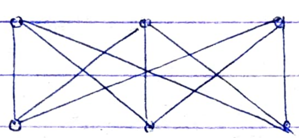
$10 \leq 3(5) - 6$

$10 \nleq 9$

Hence K_5 is non-planner graph $\quad G(K_5)$

- Every simple planner graph must satisfy ① but converse need not be true

- $K_{3,3}$ Kuratowski's second Graph



$K_{3,3}$

$e = 9, \ v = 6$

$\therefore 3v - 6 \geq e$

$K_{3,3}$ satisfys eq^n ① but still its not planner graph.

- Kuratowski enables us to determine the planarity of a graph.

The planarity of graph is clearly not affected if an edge is divided into two edges by the insertion of a new vertex of degree 2 or if two edges are incident with vertex of degree 2 are combined as single edge by the removal of vertex.

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DSCIT - 4 H.E. Suryawanshi (7)

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Semester : I / II Name of Subject : ______________________

Total Supplements : 1 + ______ = ______

Signature of Student

Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner :

Signature of Moderator :

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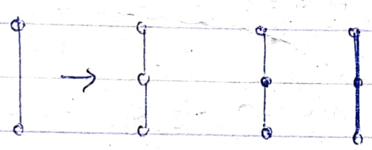
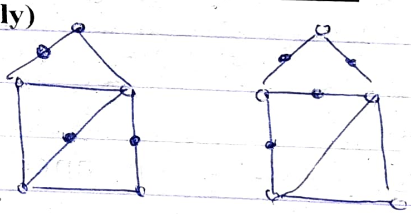
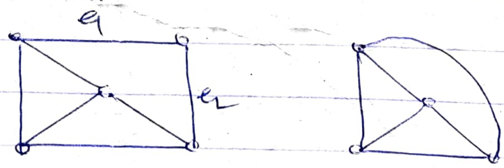


(a)

(b)



(c)

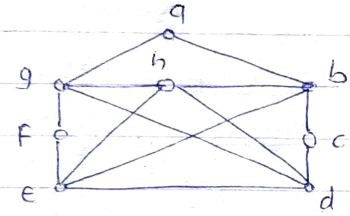


(D)

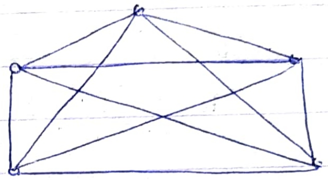
* Kuratowski Theorem

A graph is planner graph iff it does not contain any subgraph that is isomorphic to within vertices of degree 2 to either K_5 or $K_{3,3}$

consider the graph G



G



K_5 (Kuratowski's First Graph)

- Merge the edges which are incident on the vertices a, f, c.
- After merging the graph G becomes isomorphic to Graph K_5

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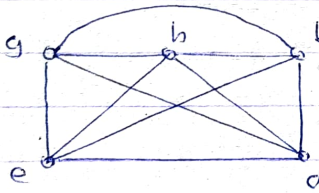
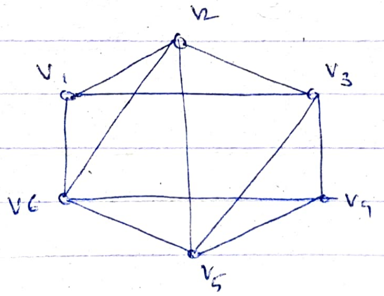
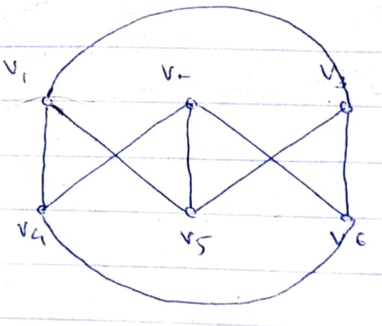


G (After merging edges of a, f & c)

Ex. (1) Draw the planner representation of graph shown in Fig. below.

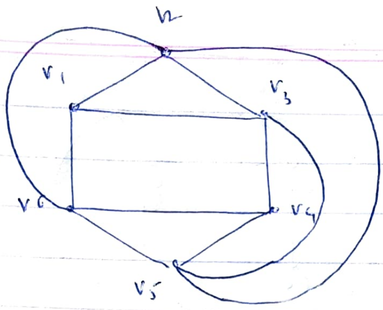


G_1

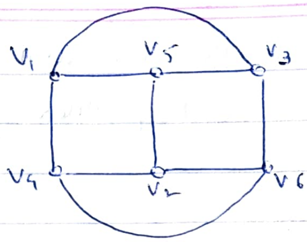


G_2

⇒

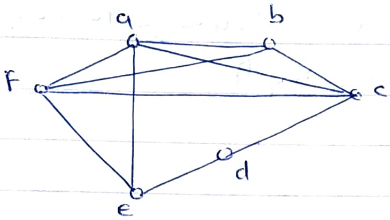


G1

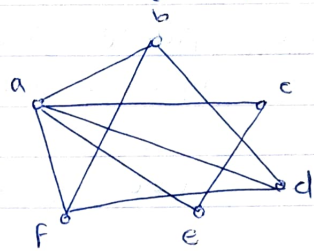


G2

Ex. ② Draw the planner representation of Graph G3 & G4

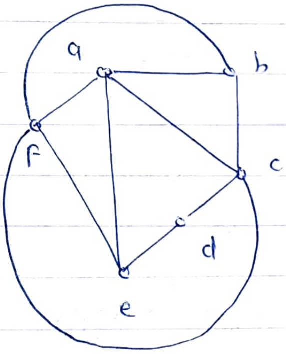


G3

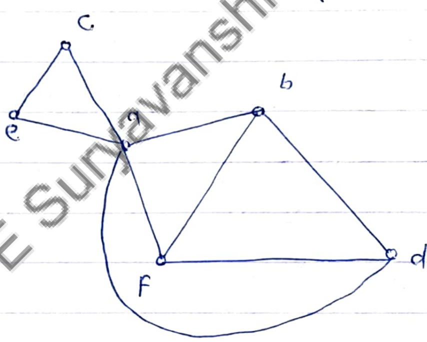


G4

⇒

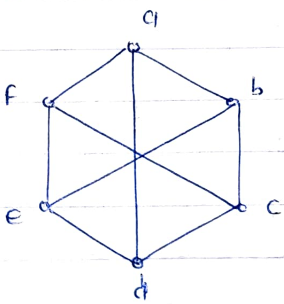


G3

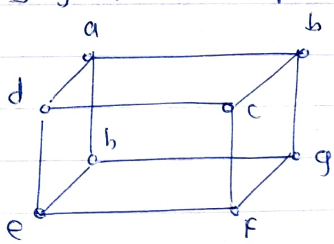


G4

Ex. ③ Which of the following graphs are planner?



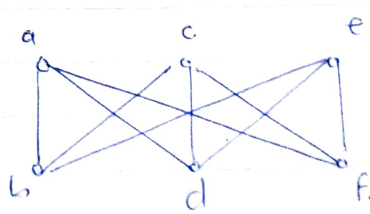
G5



G6

⇒

G5 is a kuratowski's second graph $K_{3,3}$ & it can be drawn as →



Hence it is non-planner graph

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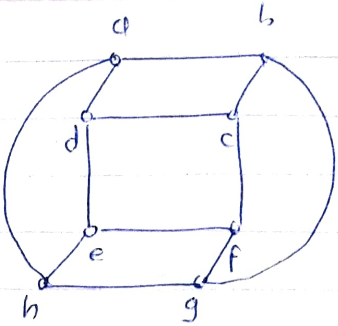
G_6 can be re-drawn as →
Hence G_6 is planner graph



G_6

• Ex ④ Determine the no. of regions defined by a connected planner graph with 6-nodes & 10 edges. Draw simple & non-simple graph.

⇒

Given $v = 6$ & $e = 10$

$\therefore$ $v - e + r = 2$

$6 - 10 + r = 2$

$\therefore$ $r = 6$

Hence graph should have 6-regions

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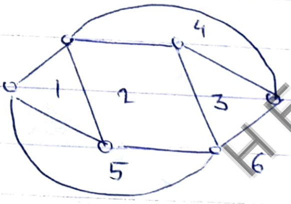
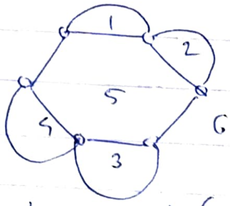


Simple graph



Non-simple (Multiple) Graph

• Ex ⑤ How many edges must a planner graph have if it has 7 regions & 5 nodes. Draw one such graph.

⇒

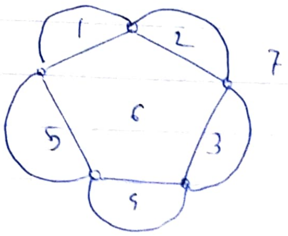
Given $v = 5$, $r = 7$

$v - e + r = 2$

$5 - e + 7 = 2$

$\therefore$ $e = 10$

Hence, graph have 10 edges



Ex ⑥ A connected planner graph has nine vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4, & 5. How many edges are there? How many faces (regions) are there?

$\Rightarrow$ By Handshaking lemma

$$\sum_{i=1}^{n} d(v_i) = 2e$$

$$2+2+2+3+3+3+4+4+5 = 2e$$

$$28 = 2e$$

$$\therefore \boxed{e = 14}$$

$\therefore$ Total edges $= 14$

By Eulers Formula

$$v - e + r = 2$$

Here $v = 9$, $e = 14$

$$\therefore 9 - 14 + r = 2$$

$$\boxed{r = 7}$$

$\therefore$ Total regions (faces) $= 7$

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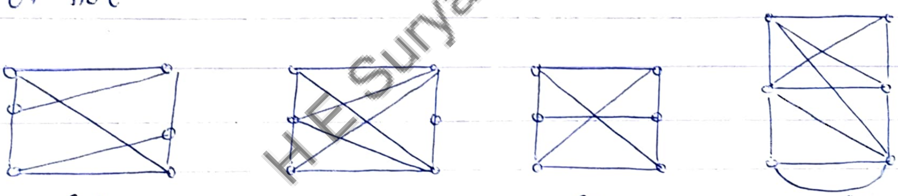
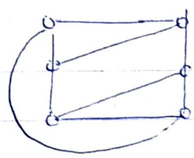
Ex (7) Identify whether the graphs given are planner or not



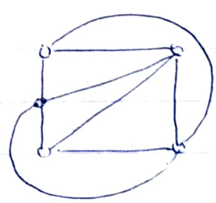
G1 G2 G3 G4

$\Rightarrow$

G1 planner Graph $\Rightarrow$

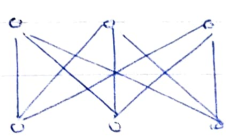


G2 planner Graph $\Rightarrow$

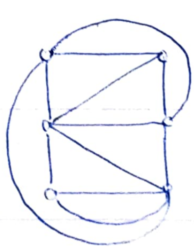


G3 Non-planner Graph

It's kuratowskis graph $K_{3,3}$ $\Rightarrow$



G4 planner Graph $\Rightarrow$



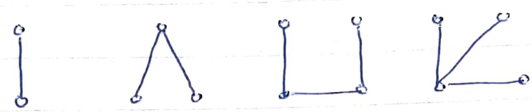
TREES

Trees were discovered by Kirchoff in 1847 while investigating the electrical networks.

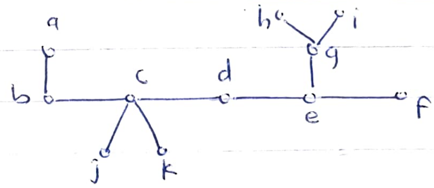
In computer science, trees are useful in organizing and relating data in data base and analysis of algorithm.

* Definition & properties of trees

- A tree is simple & connected graph without any circuits.
- Ex.



- A collection of disjoint trees is known as **forest**.
- A vertex of degree 1 in a tree is called a leaf or terminal node.
- A vertex of degree greater than one is called a branch node or internal node.



b, c, d, e ⇒ branch nodes
a, h, i etc ⇒ leaf nodes

- A tree which is defined as non-cyclic connected graph, can be defined in terms of no. of edges & vertices in the given graph.

Theorems:

1. G is a tree iff there exists a unique path between every pair of vertices of G
2. G is a tree iff if G is connected & has exactly (n-1) edges, where n is the no. of vertices in G.

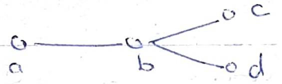
Summary -
1. G is connected & circuitless graph
2. G is connected & has (n-1) edges.
3. G is circuitless & has (n-1) edges
4. There is exactly one path betn every pair of vertices in G.

- In any tree, there are at least two pendant vertices.
- No vertex can be zero degree, at least two vertices of one degree in a tree.

* <u>Eccentricity of a vertex</u> $E(V)$

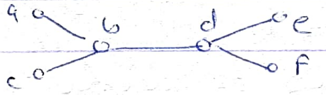
The eccentricity $E(V)$ of a vertex V in a graph G is the distance from V to the farthest from V in G

i.e. $E(V) = \max\_\text{dist}(V_i, V)$



$E(a) = 2$, $E(b) = 1$, $E(c) = 2$, $E(d) = 2$

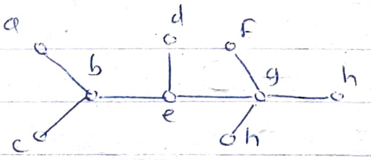
- A vertex with minimum eccentricity is called a center of G. (Ex. vertex b is center)
- Every tree has either one or two centers.



Here $E(a) = E(c) = E(e) = E(f) = 3$
& $E(b) = E(d) = 2$
∴ b & d - centers

* <u>Cut-points</u>

In any tree, all the vertices except pendant vertices (degree-1) are cut-points/vertices.



b, e, g, = cut-points.

Ex. ① Is it possible to draw a tree with five vertices having degree 1, 1, 2, 2, 4 ?

⇒ Given $n = 5$ ∴ $e = (n-1) = 4$ edges $\boxed{e = 4}$ → Tree

Now, By handshaking lemma

$$\sum_{i=1}^{n} d(v_i) = 2e$$

$1+1+2+2+4 = 2 \times e$ ⇒ $\boxed{e = 5}$ Graph

∴ It is not possible to draw such a tree.

Ex. 2 Show that it is possible to draw a tree with 10 vertices which has vertices either of degree 1 or 3. Draw the tree.

Is it possible to draw the same type of tree with 11 vertices.

$\Rightarrow$ Given

$n = 10$ $\therefore$ $e = (n-1) = 9$ edges

Let x be no. of vertices of degree 1 &

y be no. of vertices of degree 3

$\therefore$ $x + y = 10$ —— (1)

Now,

By handshaking lemma

$$\sum_{i=1}^{n} d(v_i) = 2e$$

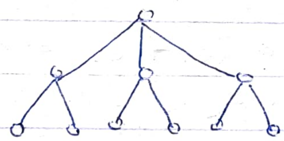
$$1x + 3y = 2 \times 9$$

$$x + 3y = 18 \quad \text{—— (2)}$$

on solving (1) & (2)

$x = 6$ & $y = 4$

$\therefore$ there are 6-vertices of degree 1 & 4-vertices of degree 3 in a tree with 10 vertices



Now, $x + y = 11$ —— (3) $e = (n-1) = 10$

& By handshaking lemma

$x + 3y = 20$ —— (4)

on solving (3) & (4)

$x = \frac{13}{2}$ & $y = \frac{9}{2}$ which is impossible

$\therefore$ There is no tree with 11 vertices which has vertices of degree 1 or 3

DSGT-4 H. E. Suryavanshi (8)

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Signature of Student Signature of Supervisor

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Marks Obtained											
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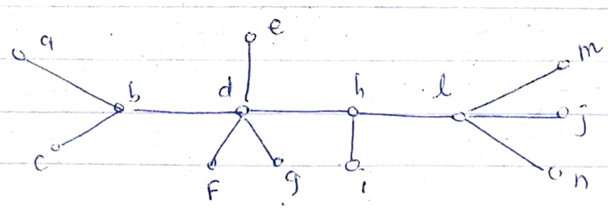
Ex. (6) Find the center of the following tree.



⇒ Center of tree = vertices with minimum eccentricity

$E(a) = 5$, $E(b) = 4$, $E(c) = 5$, $E(d) = 3$

$E(e) = 4$, $E(f) = 4$, $E(g) = 4$, $E(h) = 3$

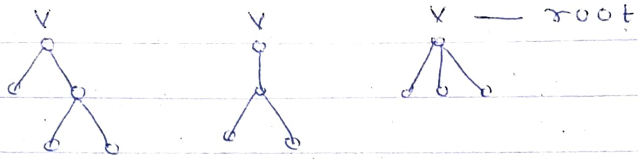
$E(l) = 4$, $E(m) = E(n) = E(j) = 5$

Here, $E(d) = E(h) = 3$ (minimum eccentricity)

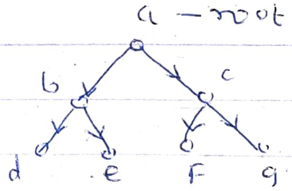
Hence d & h are the centers of the tree

* Rooted & Binary Trees

- **Rooted Trees**
  - A tree in which one vertex (called root) is distinguished from all other vertices is known as rooted tree.
  - Trees without any root are called Free trees or simply trees.
  - A vertex of degree-1 is called a leaf &
  - All the vertices (including roots) that are not leaves are called interior nodes.



- A directed tree (a tree with directions) is called a rooted tree if there is exactly one vertex whose incoming degree is zero & all other vertices having incoming degree one.
  - The vertex with incoming degree-~~0~~ zero is known as root of the tree.



a – root
b, c – branch node
d, e, f, g – leaf nodes

- In directed tree, vertex with outgoing degree 0 (zero) is called leaf (pendant) and vertex whose outgoing degree is non-zero is called branch node or an internal node.

(1) Vertex - x in rooted tree is said to be at level - n if there is path of length - n from root to vertex - x

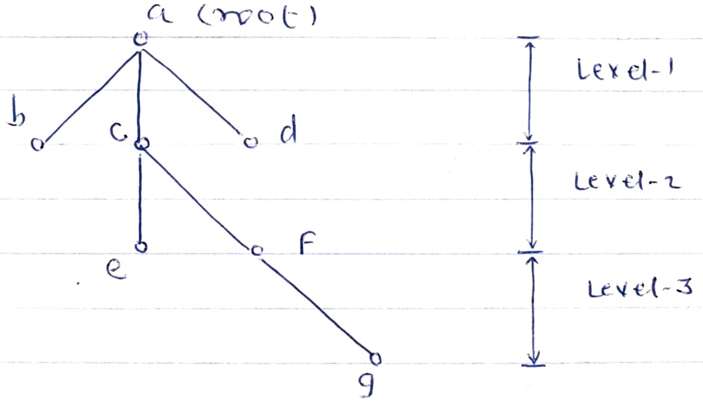
(2) The height of the tree is the maximum of the levels of its vertices.

(3) In rooted tree, level of vertex y is greater than x $\Rightarrow$ y is below x.

- if there is edge joing x to y then
- x is father of y &
- y is son of x
- Two vertices are called brothers if they are sons of same vertex.

- y is called descendent of x if there is path $P = (x\ v_1\ v_2\ v_3 \cdots v_{n-1}\ y)$ from x to y and x is called ancestor of y

(4) The degree of tree is the maximum degree of the nodes in tree.



G

degree of tree (G) = 3

a = root

(b, c, d) & (e, f) = brothers

a - father of (b, c, d)

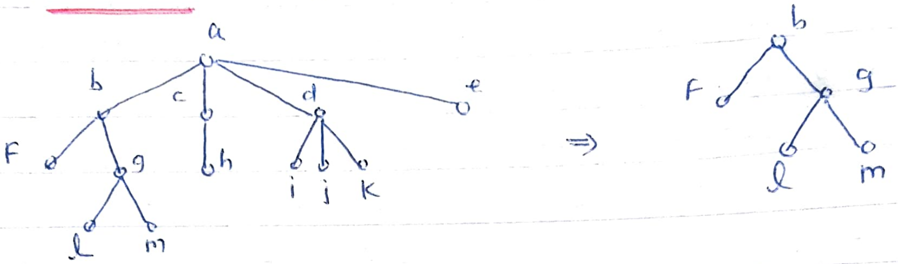
c - father of (e, f)

g - descendent of a

a - ancestor of g, e etc

(5) A forest is a set of disjoint trees. If the root is removed, then forest can be formed.

⑥ In a rooted tree T, a vertex-x, together with all its descendants, is called the subtree of T rooted at x

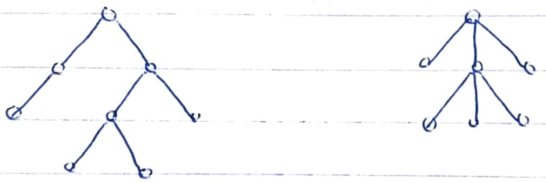


T

T' (subtree)

⑦ A rooted tree in which every interior node has atmost m-sons is called m-ary tree

- A m-ary tree is called regular tree or Full m-ary tree if every branch node has exactly m-sons.



G1 G2

G_1 = 2-ary tree but not regular

G_2 = 3-ary tree & also regular tree

Theorem

A regular m-ary tree with i-interior nodes has $(mi+1)$ nodes at all

i.e

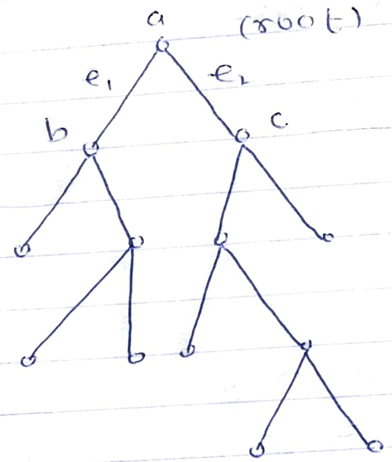
$$\boxed{n = (mi + 1)}$$

m - ary
i - intermediate nodes

- Binary Tree -

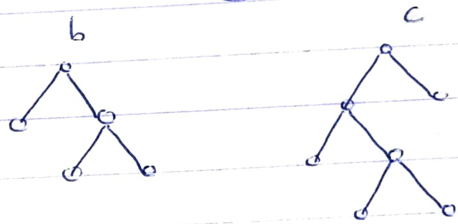
- In Binary Tree, every internal vertex has at most 2 sons.
- A binary tree is a Full or regular binary tree if each internal vertex has exactly 2 sons.

Let T be a full binary tree with height greater than zero & a as root.
- Deleting root a along with its edges produces two disjoint trees.
- These disjoint binary trees are called left-subtree & Right-subtree of the root 'a'



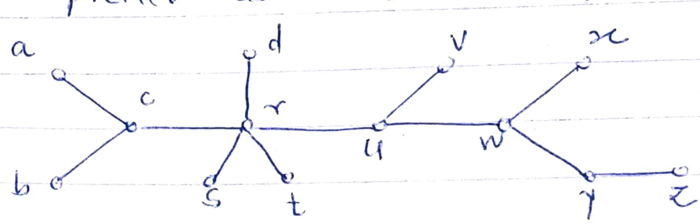
Full-binary tree contains $\left(\frac{n+1}{2}\right)$ number of leaves

(T)



left-subtree (T_1) Right subtree (T_2)

Ex(1) Consider the tree as shown in fig.
i> which of the vertices if any are cut-points?
ii> find all the vertices at level three, if the vertex picked as root is U & W.



-i> c, r, u, w, y — cut-points
ii> if U = root then a, b & z
if W = root then d, s, t & c

Ex (2) What is the total no. of nodes in full binary tree with 20 leaves?

$\Rightarrow$ Case-1

$n = mi + 1$

Let n = total nodes $\quad n = 20 + i \Rightarrow i = n - 20$

then no. of internal nodes $i = (n-20)$

In full binary tree $m=2$

$n = 2i + 1$

$n = 2(n-20) + 1$

$n = 2n - 40 + 1$

$n = 39$

$\therefore$ 39 nodes

Case-2

In full binary tree,

no. of leaves $= \left(\frac{n+1}{2}\right)$

$\therefore \quad 20 = \frac{n+1}{2}$

$40 = n+1$

$\therefore \quad n = 39$

$\therefore$ total 39 nodes.

Ex (3) Does there exist a ternary tree with exactly 21 nodes?

$\Rightarrow$

Here $m=3$ (ternary tree)

$n = 21$

Now $n = mi + 1$

$21 = 3i + 1$

$20 = 3i$

Solution to above eq^n is not integer

Thus, there is no such a tree exists

Ex (4) If there are 60 contestants in a single elimination tournament, how many matches are played?

⇒

- Single elimination tournament represented by full binary tree ($m=2$)

- According to the problem, there are 60 contenstants (leaves)
- Let, i be the no. of internal vertices.

∴ total vertices $n = 60 + i$

Now,

$$n = mi + 1$$

$$(60 + i) = 2i + 1$$

$$i = 59$$

∴ 59 mathches

Ex (5) 19 lamps are to be connected to a single electrical outlet, using extension chords, each of which has 4 outlets.

Find the no. of extension chords needed and draw the corresponding tree.

⇒

Tree with 4 outlets (i.e $m=4$)

19 lamps ∴ $n = 19 + i$

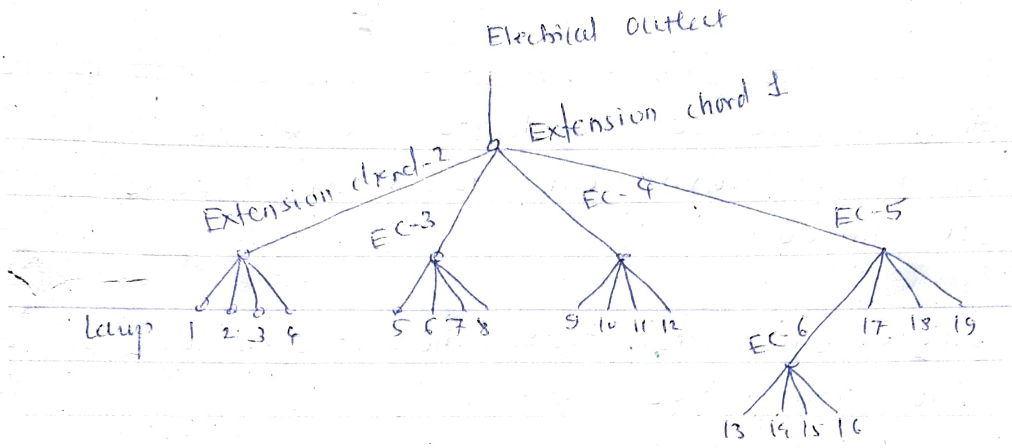
Now, by theorem

$$n = mi + 1$$

$$(19 + i) = 4i + 1$$

$$i = 6$$

∴ Hence, six extension chords are required to connect 19 lamps with a single outlet.

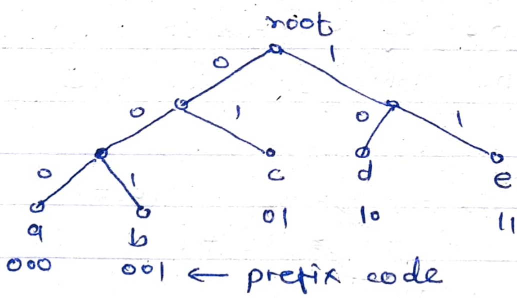


* **Prefix Codes & Binary Search Trees**

A set of sequence is said to be a prefix code if no sequence in the set is a prefix of another sequence in the set

Ex. {01, 10, 11, 000, 001} – Prefix code but
{1, 00, 01, 000, 0001} – Not prefix code
because the sequence 00 is a prefix of sequence 0001.

To obtain prefix code, full binary tree is used as shown in fig. where 0 is assigned to left branch & 1 is assigned to right branch.



Optimal Tree

Let T be any full binary tree and let $w_1, w_2, w_3 \dots w_t$ be the weights of the terminal vertices (leaves).

Then, weight (w) of the binary tree is

$$\boxed{W(T) = \sum_{i=1}^{t} w_i l_i}$$

$l_i \rightarrow$ length of path from root

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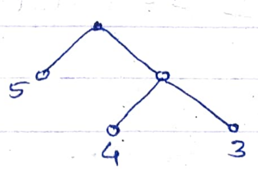
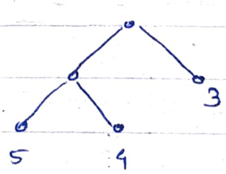
The full binary tree is called an optimal tree if its weight is minimum.



(T_1)



(T_2)

Weight of $T_1 = w(T_1) = (5\times1)+(4\times2)+(3\times2)$

$= 5+8+6$

$\therefore w(T_1) = 19$

Weight of $T_2 = w(T_2) = (5\times2)+(4\times2)+(3\times1)$

$= 10+8+3$

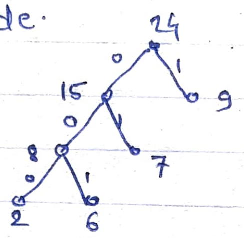
$w(T_2) = 21$

$\therefore w(T_1) \le w(T_2) \quad \therefore T_1$ is optimal tree

65

* **Optimal prefix codes**

A binary prefix code obtained from a optimal tree is called optimal prefix code.



optimal prefix codes {000, 001, 01, 1}

* **Huffman Algorithm to find optimal tree**

Let $w_1, w_2, \ldots w_t$ be the weights of leaves & it is required to construct optimal binary tree.

Algorithm

Step ① Arrange the weights in increasing order.

Step ② Select two leaves with minimum weight w_1 & w_2.

- Add new node i.e parent node with weight $(w_1 + w_2)$

Step ③ Repeate step no. ② untill no weight remains.

Step ④ Tree obtained in this way is optimal tree for given weights & stop.

* **Binary Search Tree**

Used to solve searching problems.

Let $K_1, K_2, K_3 \ldots K_n$ be the n-items in ordered list which are known as keys.

& $K_1 < K_2 < K_3 \ldots < K_n$

Given item 'x', the problem is to determine whether 'x' is equal to one of the keys or 'x' falls between ~~x~~ K_i & K_{i+1}

Ex. x is less than K_1, OR
x is equal to K_1, OR
x is greater than K_1 but less than K_2
and so on.

This problem can be solved by **Binary Search Tree**

n - branch nodes $(K_1, K_2, K_3 \cdots K_n)$
$(n+1)$ - leaf nodes $(K_0, K_1, \cdots K_n)$

It is convenient to take middle of key (K_i) from $K_1, K_2, \cdots K_n$ as root.

For branch node with label K_i,
- left sub-tree contains vertices with label $< K_i$ &
- right subtree contains label $\geq K_i$

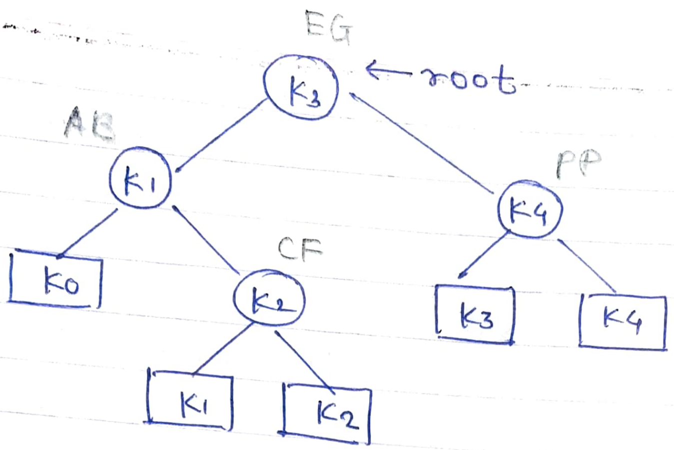
Circle - denotes branch nodes &
Square - denotes leaf nodes

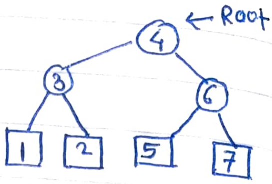
Search Procedure

- Compare 'x' with root K_i
  If 'x' is equal to K_i, stop
  If $x < K_i$, → compare 'x' with left son of K_i
  If $x > K_i$, → compare 'x' with right son of K_i

Such comparision continues for succesive branch nodes until either x mathces a key or leaf is reached.

Ex.





Let AB, CF, EG, PP be the keys k_1, k_2, k_3 & k_4
& item to be search is BB

① Compare BB with k_3 (i.e EG)

② Since BB < EG
Compare BB with k_1 (i.e AB)

③ Since BB > AB
Compare BB with k_2 (i.e CF)

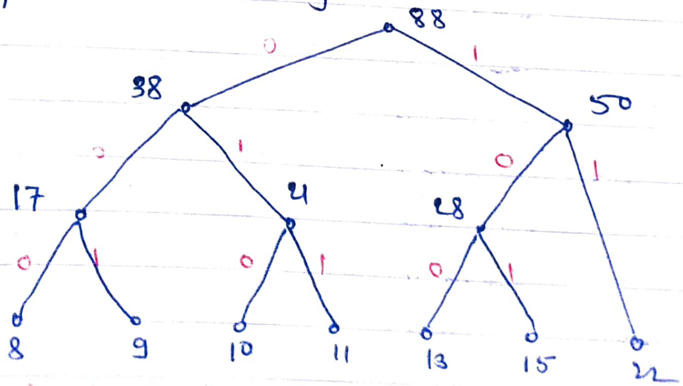
④ Since BB < CF
the leaf labeled k_1 is reached

It means item 'BB' is larger than AB (i.e k_1) but less than CF (i.e k_2)

Ex. ① Construct optimal tree for the weights 8, 9, 10, 11, 13, 15, 22.
Find the weight of the optimal tree.

⇒

By Huffman Algorithm.

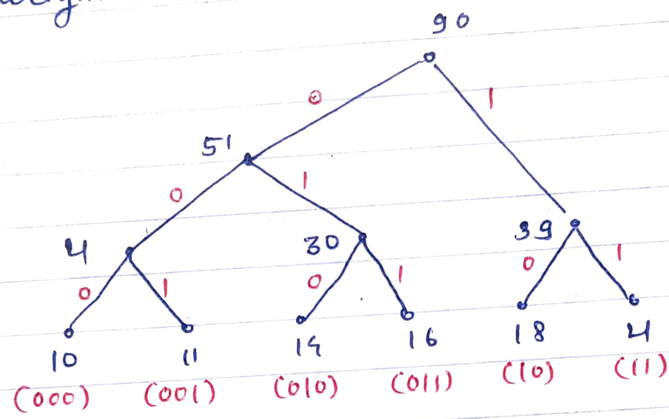


optimal code → (000) (001) (010) (011) (100) (101) (11)

$$\text{Weight of tree} = (8\times3)+(9\times3)+(10\times3)+(11\times3)+(13\times3)+(15\times3)+(22\times2)$$
$$= 242$$

Ex. ② Construct optimal binary prefix code for the weights 10, 11, 14, 16, 18, 21.

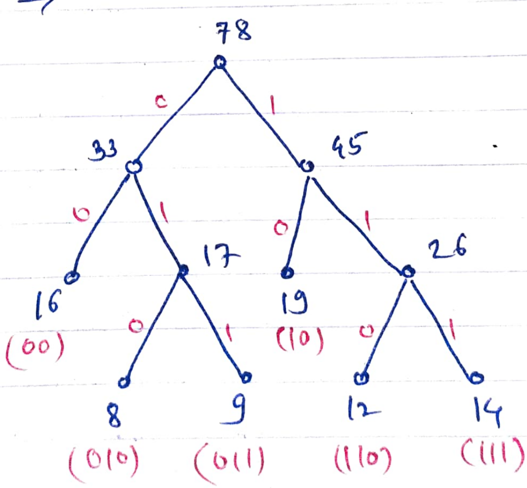
⇒



weight		optimal binary prefix code
10	→	000
11	→	001
14	→	010
16	→	011
18	→	10
21	→	11

Ex. ③ Construct optimal binary prefix code for the weights – 8, 9, 12, 14, 16, 19

⇒

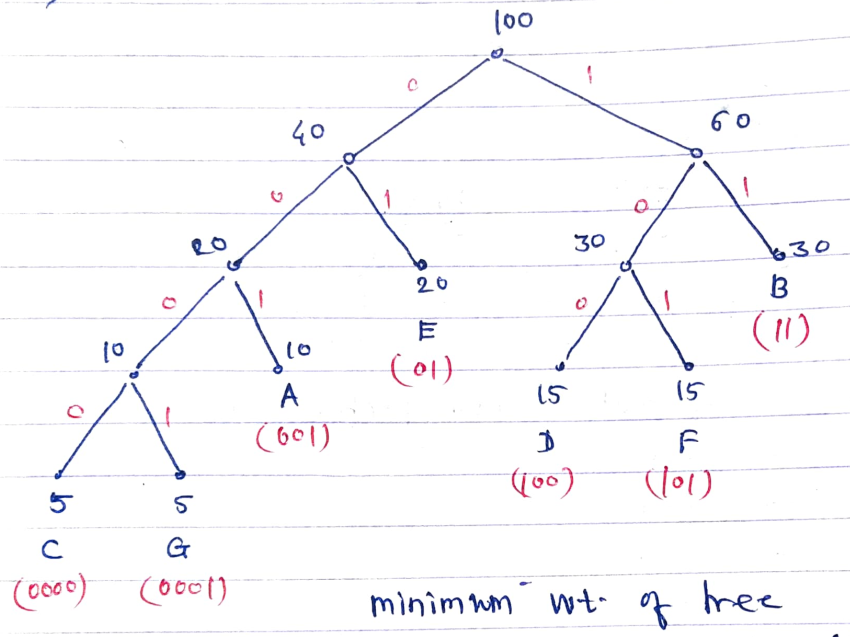


weight		prefix Code
8	→	010
9	→	011
12	→	110
14	→	111
16	→	00
19	→	10

Ex. ④ Suppose data items A, B, C, D, E, F & G occur with the following probability distribution:

Data item	A	B	C	D	E	F	G
Probability	10	30	5	15	20	15	5

⟹ Construct Huffman code. What is the min. wt. path length?



minimum wt. of tree

$= (5\times4) + (5\times4) + (10\times3) + (20\times2) + (15\times3) + (15\times3) + (30\times2)$

∴ minimum wt. = 260

The minimum weight path length for vertices

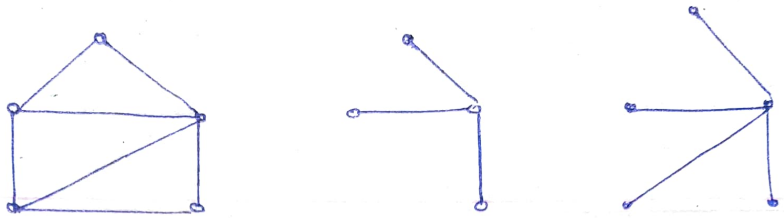
$A \to 3$	$D \to 3$	$G \to 4$
$B \to 2$	$E \to 2$	
$C \to 4$	$F \to 3$	

* Spanning Trees

A tree of a graph is a subgraph of the graph which is a tree.

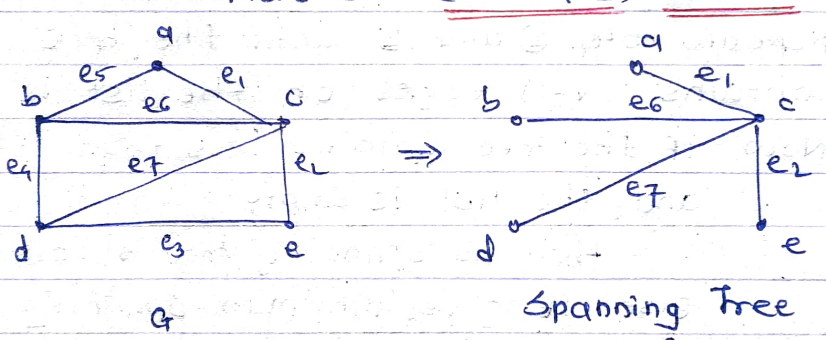
A spanning tree of a connected graph is a spanning subgraph of the graph which is a tree.

Ex. Fig. ⓑ & ⓒ Spanning tree of the graph in Fig. ⓐ. Fig. ⓒ shows only tree of graph in Fig ⓐ (



ⓐ ⓑ ⓒ Spanning Tree

- A branch of a tree is an edge of the graph that is in the tree.
- A chord or link of a tree is an edge of the graph that is not in a tree.
  ↳ The set of the chords of a tree is referred to as the complement of the tree.
- For a connected graph with 'e' edges and 'v' vertices, there are (v-1) branches in any spanning tree.
  ∴ There are (e-v+1) chords.



Spanning Tree
branches = $\{e_1, e_2, e_6, e_7\}$
chords = $\{e_3, e_4, e_5\}$

* Minimum Spanning Tree

For weighted graph, the weight of a spanning tree is defined to be the sum of the weights of the branches of the tree.

A minimum spanning tree is one with minimum weight.

Algorithms to find out minimum spanning tree

1. Kruskal's Algorithm
2. Prim's Algorithm

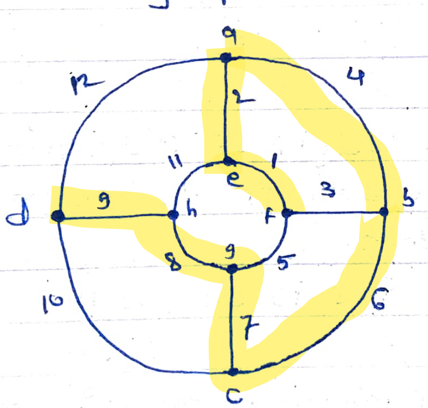
* Kruskal's Algorithm

This technique was proposed by Joseph Kruskal in 1956.

Algorithm.

1. List all the edges of the graph G in increasing order of weights.
2. Select edge with minimum weight from the list and add it to the spanning tree if the inclusion of the edge does not make a circuit.
   - If the selected edge makes the circuit, then remove it from the list.
3. Repeate step ① and ② until the tree contains (V-1) edges or the list is empty.
4. Now, if the tree contains less than (V-1) edges and the list is empty then no spanning tree is possible Else it gives minimum spanning tree.

Ex. ① Determine the minimum spanning tree for the graph G shown in Fig. using Kruskal's algorithm.



⇒

Edge	Weight	Selection of Edge
e-f	1	Yes
e-a	2	Yes
f-b	3	Yes
a-b	4	NO
f-g	5	Yes
b-c	6	Yes
g-c	7	NO
g-h	8	Yes
h-d	9	Yes
d-c	10	NO
e-h	11	NO
a-d	12	NO

DSGT-4 H.E-Semyavanshi (10)

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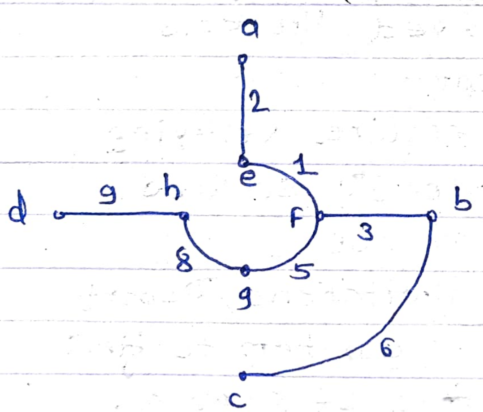
Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

(Start From here only)



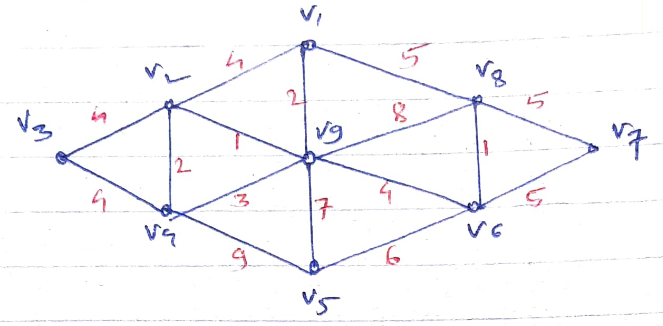
minimum spanning tree

weight of minimum spanning tree

$= 1+2+3+5+6+8+9$

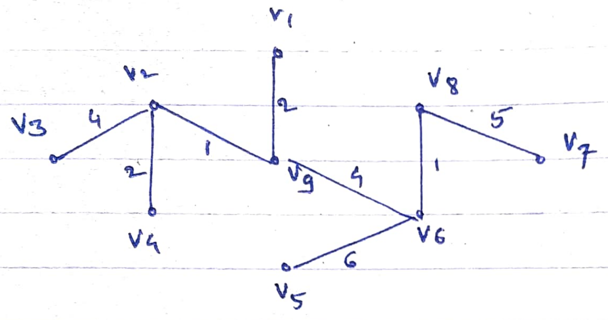
$= 34$

Ex. ② Determine the minimum spanning tree of graph G using kruskals algorithm.



⇒

minimum spanning tree



weight of tree = 25

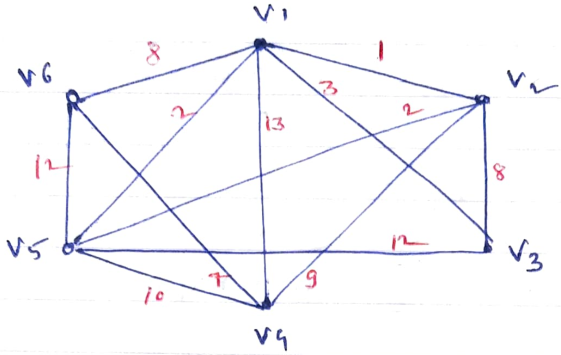
Kruskals algorithm requires sorting of all the edges in order of increasing weight. Because of sorting involved, Kruskals algorithm is not efficient.

Also, the algorithm requires verifying at each step whether a newly selected edge forms a circuit or not.

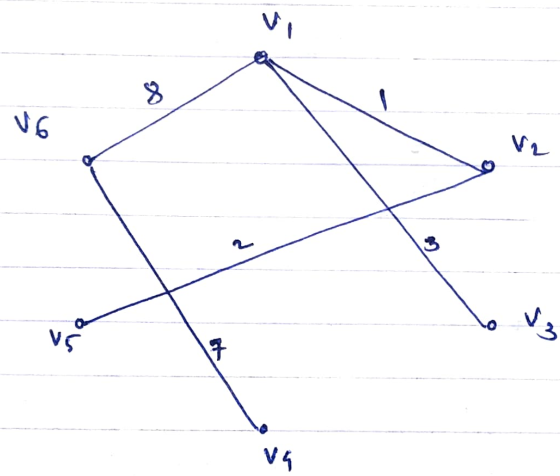
To overcome these limitations, Robert Prim has proposed an algorithm to find a minimum spanning tree of a graph.

Prims algorithm requires no sorting of edges but constructs a minimal spanning tree by successively connecting the partially formed tree to its nearest neighbor.

Ex. ② Use kruskal's Algo. to find minimum spanning tree for the graph shown in fig.



⇒



minimum spanning tree

weight of tree $= 1+2+3+7+8 = 21$

* **Prims Algorithm**

Let $G(V,E)$ be a connected weighted graph. construct the minimum spanning tree T as follows:

Step ① Take vertex V_0 in the graph G.

set $T = \{\{V_0\}, \phi\}$

Step ② Find edge $e_1 = (V_0, V_1)$ in E such that its one end vertex V_0 is in T and its weight is minimum.

Adjoin the vertex V_1 and edge e_1 to T

i.e $T = \{\{V_0, V_1\}, \{e_1\}\}$

Step ③ choose the next edge $e_i = (V_i, V_j)$ in such a way that its one end vertex V_i is in T &

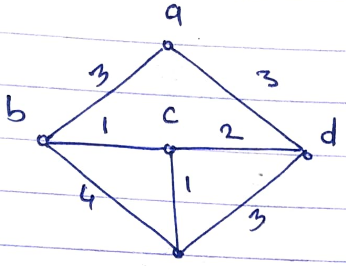
other end vertex V_j is not in T & weight of e_i is minimum & it should not form the circuit.

Adjoin the edge e_i and vertex V_j to T.

Step ④ Repeate step ③ until T contains all the vertices of G.

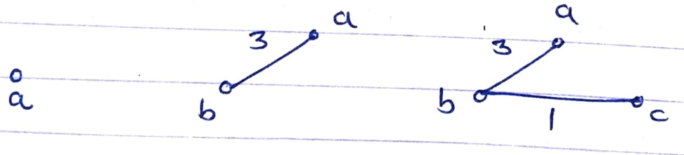
The set T will give minimum spanning tree of the graph G.

Ex. ① Find the minimum spanning tree for the following graph by using prims algorithm.

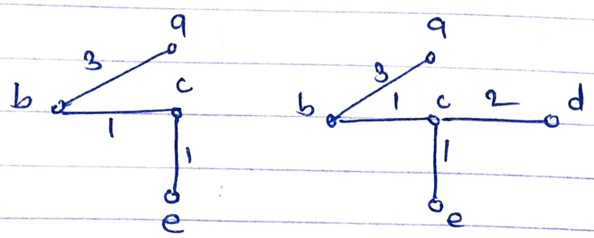


⇒

Starting from vertex 'a', the minimum spanning tree of the given graph can be obtained by using prims algo. as follows:



Step ① ② ③

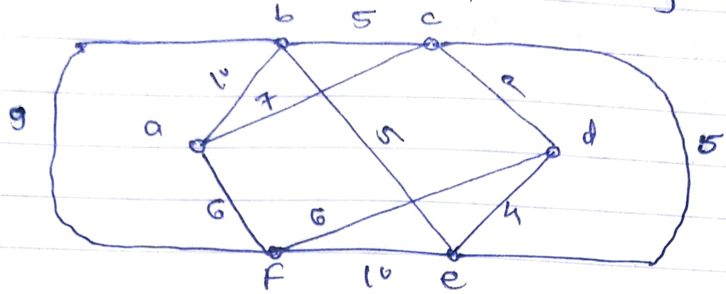


④ ⑤

minimum spanning tree

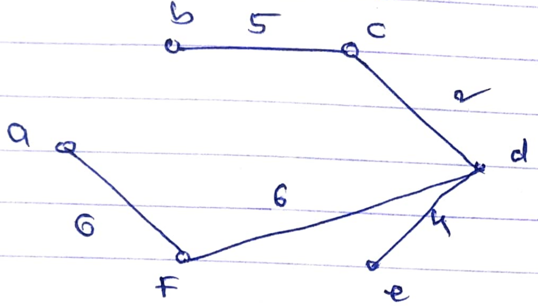
Total weight = 3+1+1+2 = 7

E② Determine the minimum spaning tree.



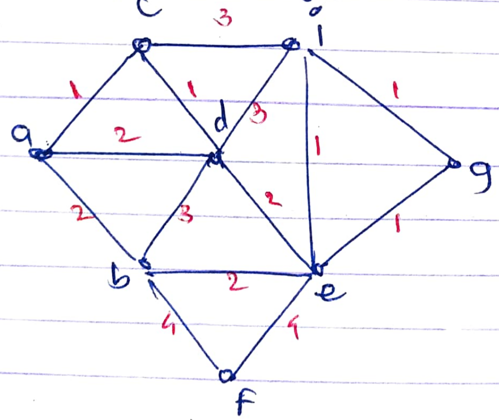
⇒

By using prims algorithm

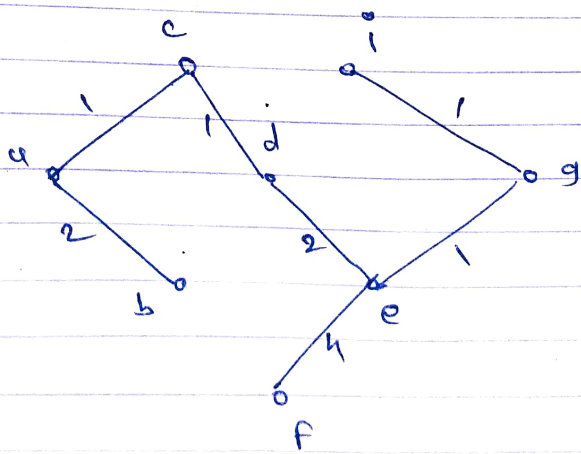


total cost = 5+2+4+6+6 = 23

Ex③ Use prims algo. to construct mini. spaning tree



⇒



total wt

= 2+1+1+2+4

+1+1

= 12