Unit-4 : Grammers

Definition: Grammer can be defined as set of formal rules for generating synthetically correct sentence from a particular language for which it is written.

Formal Def^n: A phrase structure grammer ~~it~~ is defined by a quadraple $G = \{V, T, P, S\}$ where,

V: Finite set of non-terminals (variables)
T: Finite set of terminals
P: Finite set of Production of the form $\alpha \rightarrow \beta$
S: Non-terminal called as starting symbol.

where, α :, β : consists of any no. of terminals as well as non-terminals & are usually called as (termed as) sentencial form.

* Derivations:

There are two types of derivations:

① Left most derivation (LMD)
② Right most derivation (RMD)

① Left Most derivation

If at each step in a derivation, a production is applied to the left most variable (non-termal) then the derivation is called as left-most derivation.

For eg. Grammer G may consist of:

$$(\{E\}, \{+, *, id\}, \{P\}, \{E\})$$

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where P is

(1) $E \rightarrow E+E$
(2) $E \rightarrow E*E$
(3) $E \rightarrow id$

Suppose, we want to generate 'id+id'

$E \rightarrow E+E$ (from Production Rule 1)
$E \rightarrow id+E$ (from Production Rule 3)
$\boxed{E \rightarrow id+id}$ (from Production Rule 3)

(2) Right Most derivation

If at each step in a derivation, a production is applied to the right most variable (non-terminal) then the derivation is called as Right-most derivation.

For eg. Let the Grammer G be consist of ({E}, {+, *, id}, P, E)

where P is

(1) $E \rightarrow E+E$
(2) $E \rightarrow E*E$
(3) $E \rightarrow id$

Suppose, we want to generate string 'id + id * id':

$E \rightarrow E+E$ ... by Production Rule 1
$E \rightarrow E+E*\underline{E}$ ... by Production Rule 2
$E \rightarrow E+\underline{E}*\underline{id}$ ... by —"— 3
$E \rightarrow \underline{E}+\underline{id}*id$ .. —"—
$E \rightarrow \underline{id}+id*id$ ... —"—

OR

$E \rightarrow E * E$ ... By P.R. 2

$E \rightarrow E * id$ ... By P.R. 3

$E \rightarrow E + E * id$ ... By P.R. 1

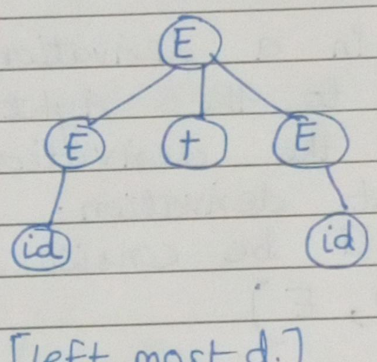
$E \rightarrow E + id * id$ ... By P.R. 3

$E \rightarrow id + id * id$ ... By P.R. 3

* Derivation Tree [Rule / Pass Tree]

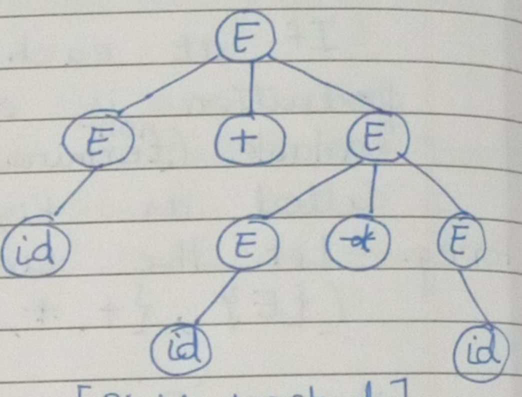
It is a graphical representation or description of how the sentence has been derived or generated using the grammer G

(i)



[Left most d.]

(ii)



[Right most d.]

For eg → 1) Write the left most & Right most derivation for string aaabbabbba using the following grammer.

$S \rightarrow aB \mid bA$ —— ①

$A \rightarrow a \mid aS \mid bAA$ —— ②

$B = b \mid bS \mid aBB$ —— ③

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From given

Q2, ~~Left most derivation~~.

(1) $S \rightarrow aB$

(2) $S \rightarrow bA$

(3) $A \rightarrow a$

(4) $A \rightarrow aS$

(5) $A \rightarrow bAA$

(6) $B \rightarrow b$

(7) $B \rightarrow bS$

(8) $B \rightarrow aBB$

~~aaabbabb~~ba

Here, V are ⇒ S, B, A.
T are ⇒ a, b.
There are 8 production rules

Left most derivation,

$S \rightarrow aB$ By P.R. (1)

$S \rightarrow aaBB$ By P.R. (8)

$S \rightarrow aaaBB$ By P.R. (8)

$S \rightarrow aaabB$ By P.R. (6)

$S \rightarrow aaabbS$ By P.R. ~~(4)~~ (7)

$S \rightarrow aaabbaB$ By P.R. (1)

$S \rightarrow aaabbabS$ By P.R. (7)

$S \rightarrow aaabbabbbA$ By P.R. (2)

$S \rightarrow aaabbabbba$ By P.R. (3)

(8) (8) 7 2 3 6 7 2 3

2] Right most derivation.

$S \rightarrow a\underline{B}$ by P.R. (1)
$S \rightarrow aa\underline{B}B$ by P.R. (8)
$S \rightarrow aaBb\underline{S}$ by P.R. (7)
$S \rightarrow aaBbbA$ by P.R. (2)
$S \rightarrow aaBbba$ by P.R. (3)
$S \rightarrow aaaBBbba$ by P.R. (8)
$S \rightarrow aaaB\cancel{b}bSbba$ by P.R. (7)
$S \rightarrow aaaBbaBbba$ by P.R. (1)
$S \rightarrow aaaBbabbba$ By P.R. (6)
$S \rightarrow aaabbabbba$ By P.R. (6)

OR

$S \rightarrow aB$ — 8,7,2,3,8,6,7,2,3
$S \rightarrow a$

Q.2] Write the left most & right most derivation for 00110101 using following grammer.

$S \rightarrow 0B \mid 1A$
$A \rightarrow 0 \mid 0S \mid 1AA$
$B \rightarrow 1 \mid 1S \mid 0BB$

Also draw derivation tree.

→ From above,

(1) $S \rightarrow 0B$
(2) $S \rightarrow 1A$
(3) $A \rightarrow 0$
(4) $A \rightarrow 0S$
(5) $A \rightarrow 1AA$
(6) $B \rightarrow 1$
(7) $B \rightarrow 1S$
(8) $B \rightarrow 0BB$

Left most derivation.

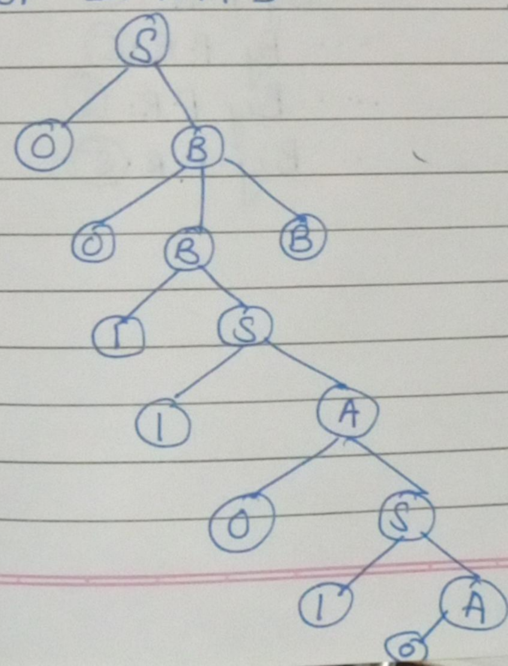
S → 0B — by P.R. 1
S → 00BB — by P.R. 8
S → 001B — by P.R. 6
S → 0011S — by P.R. 7
S → 00110B — by P.R. 1
S → 001101S — by P.R. 7
S → 0011010B — by P.R. 1
S → 00110101 — by P.R 6

Right most derivation

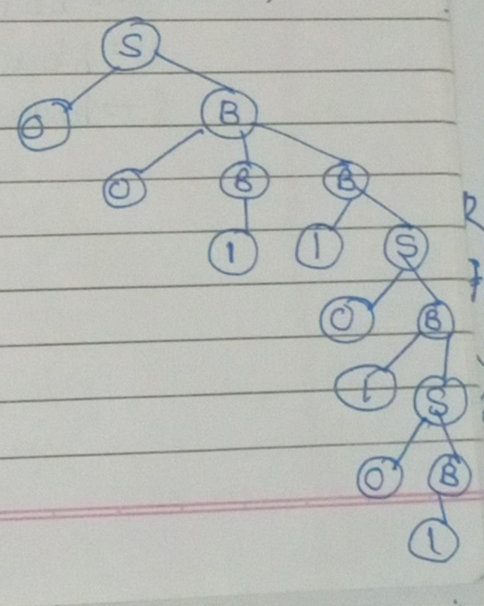
øoxxø1ox

S → 0B — by P.R. 1
S → 00BB — by P.R. 8
S → 00B1 — by P.R 6
S → 001S1 — by P.R. 7
S → 0011A1 — by P.R. 2
S → 00110S1 — by P.R: 4
S → 001101A1 — by P.R. 2
S → 00100101 — by P.R. 3

D.T for ~~LA~~ R.M.D



D.T. from L.M.D.



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Q.3] Consider the following CFG :

$S \rightarrow XX$

$X \rightarrow XXX \mid bX \mid Xb \mid a$

~~Point~~ Find LMD & RMD with derivation tree for the string bbaaaab.

→ Given → From above ;

① $S \rightarrow XX$

② $X \rightarrow XXX$

③ $X \rightarrow bX$

④ $X \rightarrow Xb$

⑤ $X \rightarrow a$

Left most derivative :

$S \rightarrow XX$ ... by P.R. ①

$S \rightarrow bXX$ ... by P.R. ③

$S \rightarrow bbXX$ ... by P.R. ③

$S \rightarrow bbaX$ ... by P.R. ⑤

$S \rightarrow bbaXb$ ... by P.R. ④

$S \rightarrow bbaXXXb$ ... by P.R. ②

$S \rightarrow bbaaXXb$ ... By P.R. ⑤

$S \rightarrow bbaaaXb$ ... By P.R. ⑤

$S \rightarrow bbaaaab$ ... By P.R. ⑤

Right most derivative :

S → XX ... By P.R. 1

S → XXb ... By P.R. 4

S → XXXXb ... By P.R. 2

S → XXXab ... By P.R. 5

S → XXaab ... By P.R. 5

S → Xaaab — by P.R. 5

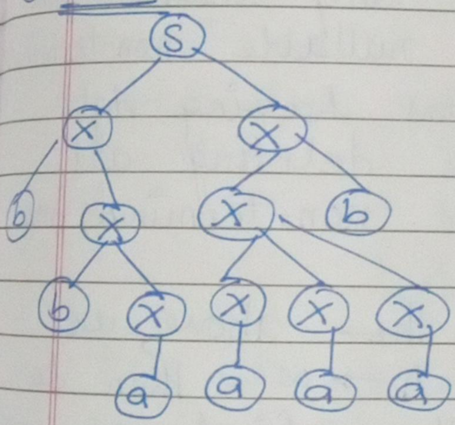
S → ~~b~~X~~X~~aaab ... by P.R. 3

S → bbXaaab ... by P.R. 3

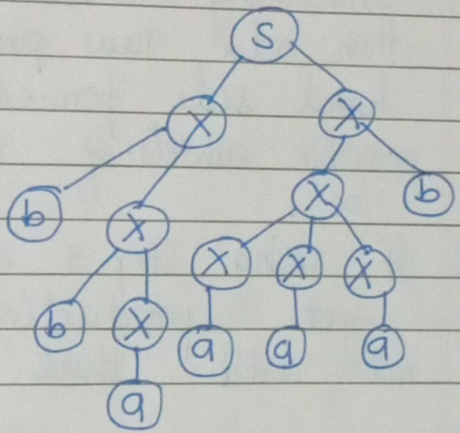
S → bbaaaab ... by P.R. 5

Tree

1 L.M.D.



2 R.M.D.



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* Minimization of Context Free Grammer (CFG) [Type-2]

[1] ELIMINATION OF 'ε'- PRODUCTION

A production of the form $A \rightarrow \epsilon$ where 'A' is any non-terminal (variable) is called as null-production [ε-production].

ELIMINATION PROCEDURE :

(1) Delete all ε-productions from the grammer.
(2) Identify nullable non-terminals.
(3) If there is a production of the form $A \rightarrow \alpha$, where α is any sentential form containing atleast one nullable non-terminal, then add new productions having right hand side formed by deleting all possible subsets of nullable non-terminals from 'α'.

(4) If using Step 3 ~~above~~ ~~the~~ above, we get production of $A \rightarrow \alpha$ then do not add that to the final grammer.

Eg. → 1) Eliminate ε-productions from grammer G where G consisting of following productions:

$S \rightarrow Xa$
$X \rightarrow aX \mid bX \mid \epsilon$

Sol^n → From given,

(1) $S \rightarrow Xa$
(2) $X \rightarrow aX$
(3) $X \rightarrow bX$
(4) $X \rightarrow \epsilon$

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(1) Delete all ϵ-productions from the above grammer. After ~~grammer~~ deleting ϵ-production we get,

$$\boxed{\begin{array}{l} S \rightarrow Xa \\ X \rightarrow aX \\ X \rightarrow bX \end{array}}$$

(2) Identify nullable non-terminals in the given grammer G, we found the production as

$$\boxed{X \rightarrow \epsilon}$$

$\therefore$ X is a nullable non-terminal.

(3) Identify nullable non-terminal in step 1 which should occur on R.H.S. of the production rule.

$$\boxed{\begin{array}{l} S \rightarrow \underline{X}a \\ X \rightarrow a\underline{X} \\ X \rightarrow b\underline{X} \end{array}}$$

Put the value of nullable non-terminal X on R.H.S. as null. We get new production rule as:

$$\boxed{\begin{array}{l} S \rightarrow a \\ X \rightarrow a \\ X \rightarrow b \end{array}}$$

(4) So the final grammer without ϵ-production is as follows:

$$\boxed{\begin{array}{l} S \rightarrow Xa \mid a \\ X \rightarrow aX \mid bX \mid a \mid b \end{array}}$$

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Q2] Eliminate ϵ- productions from the grammer G consisting :-

$$S \longrightarrow ABA$$
$$A \longrightarrow aA \mid \epsilon$$
$$B \longrightarrow bB \mid \epsilon$$

→ (1) From given,
first delete all ϵ-productions from the above grammer G.

$$\boxed{\begin{array}{l} S \longrightarrow ABA \\ A \longrightarrow aA \\ B \longrightarrow bB \end{array}}$$

(2) Identify nullable non-terminal from the given grammer.
As

$$A \longrightarrow \epsilon$$
$$\& \quad B \longrightarrow \epsilon$$

∴ A & B both are nullable non-terminal.

(3) Identify nullable non-terminals in Step (1) i.e. A & B : which should occur on R.H.S. of the production rule.

$$\boxed{\begin{array}{l} S \longrightarrow ABA \\ A \longrightarrow aA \\ B \longrightarrow bB \end{array}}$$

Put the value of nullable non-terminal A & B as null on R.H.S. of the production rule.

$S \rightarrow AB\cancel{A} \mid \cancel{A}BA \mid A\cancel{B}A \mid A\cancel{B}\cancel{A} \mid \cancel{A}B\cancel{A} \mid \cancel{A}\cancel{B}\cancel{A}$

$$\boxed{\begin{array}{l} S \rightarrow BA \mid AB \mid B \mid AA \mid A \mid \epsilon \\ A \rightarrow a \\ B \rightarrow b \end{array}}$$

(4) So, the final grammer without ϵ-productⁿ is as follows:

$$\boxed{\begin{array}{l} S \rightarrow BA \mid AB \mid B \mid AA \mid A \mid ABA \\ A \rightarrow a \mid aA \\ B \rightarrow b \mid bA \end{array}}$$

∵ As in the above Step (3) we got 1 production rule as $S \rightarrow \epsilon$. As per the elimination rule, we should drop it in the final step.

Q.3] Eliminate ϵ-production from the given grammer G.

$$\begin{array}{l} S \rightarrow XY \\ X \rightarrow Zb \\ Y \rightarrow bW \\ Z \rightarrow AB \\ W \rightarrow Z \\ A \rightarrow aA \mid bA \mid \epsilon \\ B \rightarrow Ba \mid Bb \mid \epsilon \end{array}$$

→ (1) From given,
first delete all the ϵ-productions from above grammer G,

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$$\boxed{\begin{array}{l} S \rightarrow XY \\ X \rightarrow Zb \\ Y \rightarrow bW \\ Z \rightarrow AB \\ W \rightarrow Z \\ A \rightarrow aA \mid bA \\ B \rightarrow Ba \mid Bb \end{array}}$$

(2) Identify nullable non-terminals from the given grammer:
As,

$$A \rightarrow \epsilon$$
&
$$B \rightarrow \epsilon$$

$\therefore$ A & B both are nullable non-terminals.

(3) Identify nullable non-terminals from Step (1) i.e. A & B which should occur on R.H.S. of the production rule.

$$\boxed{\begin{array}{l} Z \rightarrow AB \\ A \rightarrow aA \\ A \rightarrow bA \\ B \rightarrow Ba \\ B \rightarrow Bb \end{array}}$$

Put the values of ~~no~~ nullable non-terminals A & B as null on R.H.S. of the production rule.

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$$\boxed{\begin{array}{l} S \rightarrow XY \\ X \rightarrow Zb \\ Y \rightarrow bW \\ Z \rightarrow A \mid B \\ W \rightarrow z \\ A \rightarrow a \mid b \\ B \rightarrow a \mid b \end{array}}$$

4) So, the final grammer without ϵ-production is as follows:

$$\boxed{\begin{array}{l} S \rightarrow XY \\ X \rightarrow Zb \\ Y \rightarrow bW \\ Z \rightarrow AB \mid A \mid B \\ W \rightarrow z \\ A \rightarrow aA \mid a \mid bA \mid b \\ B \rightarrow Ba \mid a \mid Bb \mid b \end{array}}$$

[2]

* <u>Removal of Useless Symbol / Reduced Grammer</u>

1.1] Remove the useless symbol from the given context free grammer (CFG) is 'G' given as:

$$\begin{array}{l} S \rightarrow AB \mid CA \\ B \rightarrow BC \mid AB \\ A \rightarrow a \\ C \rightarrow aB \mid b \end{array}$$

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Sol^n → ① Assume, starting symbol in the given grammer as 'S'.

② In the above grammer, non-terminal 'A' & 'C' generates a string of terminals such that

$A \rightarrow a$
& $C \rightarrow b$

③ But, in the above grammer, non-terminal 'B' which cannot generate any string of terminals. So, it is an useless symbol. So, we should drop it (elimate) all the productions rules where non-terminal B occurs.

Following production-rules are dropped from the above grammer:

$S \rightarrow AB$
$B \rightarrow BC \mid AB$
$C \rightarrow aB$

④ So, the final grammer without useless symbol is as follows:

$S \rightarrow CA$
$A \rightarrow a$
$C \rightarrow b$

Q.2] Remove the useless symbol from the given context-free grammer G :

$S \rightarrow aB \mid bX$

$A \rightarrow BAd \mid bSX \mid a$

$B \rightarrow aSB \mid bBX$

$X \rightarrow SBD \mid aBx \mid ad$

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→ (1) Assume string symbol in the given grammer as S.
In the above grammer, non-terminals 'A' & 'X' generates a string of terminals such that

$$\boxed{\begin{array}{l} A \rightarrow a \\ X \rightarrow ad \end{array}}$$

~~But in the above grammer, non-terminal 'X' which cannot~~

(2)

(1) $S \rightarrow aB$
(2) $S \rightarrow bX$
(3) $A \rightarrow BAd$
(4) $A \rightarrow bSX$
(5) $A \rightarrow a$
(6) $B \rightarrow aSB$
(7) $B \rightarrow bBA$
(8) $X \rightarrow SBD$
(9) $X \rightarrow aBx$
(10) $X \rightarrow ad$

But in the above grammer, non-terminal D, 'B' which cannot generate any string of terminals. So, It is an useless symbol. So, we should drop all production-rules where non-terminals 'B', 'D' occurs.
Following production rule are dropped from the above grammer:

~~$S \rightarrow bX$~~
~~$A \rightarrow bSX$~~

$$\boxed{\begin{array}{l} S \rightarrow aB \\ A \rightarrow BAd \\ B \rightarrow aSB \\ B \rightarrow bBA \\ X \rightarrow SBD \\ X \rightarrow aBx \end{array}}$$

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(4) So the ~~final~~ remaining useful strings are :

$$\boxed{\begin{array}{l} S \rightarrow bX \\ A \rightarrow bSX \mid a \\ X \rightarrow ab \end{array}} \quad \textcircled{*}$$

But 'A' is not reachable from the starting symbol 'S' so, drop the non-terminal 'A' too. i.e. all prod-rules of non-terminal 'A'. as (*)

(5) Therefore, the final grammer without useless symbols is :

$$\boxed{\begin{array}{l} S \rightarrow bX \\ X \rightarrow ab \end{array}}$$

Q.3] Remove the useless symbol from given CFG 'G' :

$S \rightarrow A11B \mid 11A$
$S \rightarrow B \mid 11\}$'
$A \rightarrow 0$
$B \rightarrow BB$.

→ (1) Assume, starting symbol is the given grammer as 'S'.

(2) In the given grammer i.e.

(1) $S \rightarrow A11B$
(2) $S \rightarrow 11A$
(3) $S \rightarrow B$
(4) $S \rightarrow 11$
(5) $A \rightarrow 0$
(6) $B \rightarrow BB$

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the non-terminals 'A' & 'S' generate a string of terminals such that

$$\boxed{\begin{array}{l} A \to 0 \\ S \to 11 \end{array}}$$

(3) But, in the above grammer, non-terminal 'B' which cannot generate any string of terminals. So, it is an useless symbol. So, we should drop (eliminate) all the productions rules where non-terminal B occurs. Following prod. rules are dropped:

$$\boxed{\begin{array}{l} S \to A11B \\ S \to B \\ B \to BB \end{array}}$$

(4) So, after dropping these prod. rules we get,

$$\boxed{\begin{array}{l} S \to 11A \\ S \to 11 \\ A \to 0 \end{array}}$$

So the final grammer is as follows

$$\boxed{\begin{array}{l} S \to 11A \mid 11 \\ A \to 0 \end{array}}$$

[3]

* Removal of unit production

A production of the form $A \rightarrow B$ where A & B both are non-terminals are called as unit productions.
All the other productions (including ϵ-productions) are non-unit productions.

Example :

Q.1] Remove unit production from the given grammer 'G' :

$$S \rightarrow aX \mid Yb$$
$$X \rightarrow S$$
$$Y \rightarrow bY \mid b$$

Sol^n → Given:
(1) $S \rightarrow aX$
(2) $S \rightarrow Yb$
(3) $X \rightarrow S$
(4) $Y \rightarrow bY$
(5) $Y \rightarrow b$

(1) In the above grammer ~~their~~ there is one unit production rule $\boxed{X \rightarrow S}$

(2) We should put the value of non-terminal X as S, so the production rule becomes:

$$\boxed{S \rightarrow aS}$$

(3) So, the final grammer without unit production is as :

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$$\boxed{\begin{array}{l} S \rightarrow aS \mid yb \\ y \rightarrow by \mid b \end{array}}$$

Q.2] ~~Remove~~ Eliminate unit production from given grammer G:

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow C \mid b$
$C \rightarrow D$
$D \rightarrow E$
$E \rightarrow a$

There is a chain of above unit production
$B \rightarrow C \rightarrow D \rightarrow E$

→ Given

1. $S \rightarrow AB$
2. $A \rightarrow a$
3. $B \rightarrow C$
4. $B \rightarrow b$
5. $C \rightarrow D$
6. $D \rightarrow E$
7. $E \rightarrow a$

1) In the above grammer there ~~is~~ are unit productions rule :

$$\boxed{\begin{array}{l} D \rightarrow E \\ C \rightarrow D \\ B \rightarrow C \end{array}}$$

i.e.

$$\left.\begin{array}{l} B \rightarrow C \\ C \rightarrow D \\ D \rightarrow E \\ B \rightarrow E \\ \boxed{B \rightarrow a} \end{array}\right\}$$

2) We should put the value of non-terminal B as a

$$\boxed{S \xrightarrow{D} Aa}$$

3) So, the final grammer without unit production is :

$$\boxed{\begin{array}{l} S \rightarrow Aa \\ A \rightarrow a \\ B \rightarrow C \mid b \\ C \rightarrow D \\ D \rightarrow E \\ E \rightarrow a \end{array}}$$

There is a chain of above

As $E \rightarrow a$, so we should add new production rule as $B \rightarrow C \rightarrow D \rightarrow E$

$\therefore$ $\boxed{B \rightarrow a}$

(4) So the final grammer without unit production is as follows.

$$\boxed{\begin{array}{l} S \rightarrow Aa \\ A \rightarrow a \\ B \rightarrow a \mid b \end{array}}$$

Q.2] Remove the unit production from the following CFG:

$S \rightarrow AA$

$A \rightarrow B \mid BB$

$B \rightarrow abB \mid b \mid bb$

Solⁿ→ Given:

(1) $S \rightarrow AA$ (4) $B \rightarrow abB$

(2) $A \rightarrow B$ (5) $B \rightarrow b$

(3) $A \rightarrow BB$ (6) $B \rightarrow bb$

(1) In the above grammer there is an unit production $\boxed{A \rightarrow B}$

(2) We should put the value of non-terminal A as B so the production rule becomes,

$A \rightarrow abB \mid b \mid bb \mid \underline{BB}$

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$BB \Rightarrow abBb \mid babB \mid abBbb \mid bbabB \mid bbb$

$\therefore A \rightarrow abB \mid b \mid bb \mid abBb \mid babB \mid abBbb \mid$
$bbabB \mid bbb$

(3) So the final grammer without unit production is,

$$\boxed{\begin{array}{l} S \rightarrow AA \\ A \rightarrow abB \mid b \mid bb \mid abBb \mid babB \mid abBbb \\ A \rightarrow bbabB \mid bbb \end{array}}$$

Chomsky Normal Form (CNF)

Any context free language without 'ϵ' is generated by a grammer in which all the productions are of the form $A \rightarrow BC$ or $A \rightarrow a$ where, A, B, C are variables (non-terminal) & 'a' is a terminal.
This type of grammer is said to be in CNF.

Example:

Q.1] Convert the following CFG to CNF:
$S \rightarrow aSa \mid bSb \mid a \mid b \mid aa \mid bb$

Sol^n → Given →

(1) $S \rightarrow aSa$ (4) $S \rightarrow b$
(2) $S \rightarrow bSb$ (5) $S \rightarrow aa$
(3) $S \rightarrow a$ (6) $S \rightarrow bb$

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Take 1st production Rule,

$S \to aSa$

Assume, $\boxed{R_1 \to a}$ — (i)

$S \to R_1 S R_1$

right side ko put kiya

Assume, $\boxed{R_2 \to S R_1}$ — (ii)

$\therefore \boxed{S \to R_1 R_2}$ — (iii)

Take Production Rule - 2,

$S \to bSb$

Assume, $\boxed{R_3 \to b}$ — (iv)

$S \to R_3 S R_3$ left side ko put kiya

Assume, $\boxed{R_4 \to R_3 S}$ — (v)

$\therefore \boxed{S \to R_4 R_3}$ — (vi)

Take production Rule - 3

$\boxed{S \to a}$ — (vii)

It is already in CNF

Take production Rule - 4

$\boxed{S \to b}$ — (viii)

It is already in CNF

Take production Rule - 5

$S \to aa$

As $R_1 \to a$

$\therefore \boxed{S \to R_1 R_1}$ — (ix)

Take production Rule - 6

$S \rightarrow bb$

As $R_3 \rightarrow b$

$\therefore$ $\boxed{S \rightarrow R_3 R_3}$ — (x)

So the final grammer in CNF is as follows -

$$\boxed{\begin{array}{l} S \rightarrow R_1R_2 \mid R_4R_3 \mid R_1R_1 \mid R_3R_3 \mid a \mid b \\ R_1 \rightarrow a \\ R_3 \rightarrow b \\ R_4 \rightarrow R_3 S \end{array}}$$

Q.2] Convert the following grammar to CNF :

$S \rightarrow bA \mid aB$

$A \rightarrow bAA \mid aS \mid a$

$B \rightarrow aBB \mid bS \mid a$

→ Given :

(1) $S \rightarrow bA$	(5) $A \rightarrow a$
(2) $S \rightarrow aB$	(6) $B \rightarrow aBB$
(3) $A \rightarrow bAA$	(7) $B \rightarrow bS$
(4) $A \rightarrow aS$	(8) $B \rightarrow a$

Take production Rule - 1

$S \rightarrow bA$

Assume, $\boxed{R_1 \rightarrow b}$ — (i)

$\therefore$ $\boxed{S \rightarrow R_1 A}$ — (ii)

Take production rule - 2

$S \rightarrow aB$

Assume, $\boxed{R_2 \rightarrow a}$ — (iii) $\therefore$ $\boxed{S \rightarrow R_2 B}$ — (iv)

Take production Rule 3

$A \to bAA$

$\because R_1 \to b$

$\therefore A \to R_1 A A$

Assume, $\boxed{R_3 \to R_1 A}$ — (v)

$\therefore \boxed{A \to R_3 A}$ — (vi)

Take production - Rule - 4

$A \to aS$

$R_2 \to a$

$\therefore \boxed{A \to R_2 S}$ — (vii)

Take production Rule - 5

$\boxed{A \to a}$ — (viii)

It is already in CNF

Take production Rule - 6

$B \to aBB$

$\because R_2 \to \cancel{b}\ a$

$B \to R_2 B B$

Assume, $\boxed{R_4 \to R_2 B}$ — (ix)

$\therefore \boxed{B \to R_4 B}$ — (x)

Take production Rule - 7

$B \to bS$

As $R_1 \to b$

$\therefore \boxed{B \to R_1 S}$ — (xi)

Take Production Rule - 8

$\boxed{B \rightarrow a}$ — (xii)

It is already in CNF.

So, the final grammer in CNF is as follows -

$$\boxed{\begin{array}{l} S \rightarrow R_1A \mid R_2B \\ A \rightarrow R_3A \mid R_2S \mid a \\ B \rightarrow R_4B \mid R_1S \mid a \\ R_1 \rightarrow b \\ R_2 \rightarrow a \\ R_3 \rightarrow R_1A \\ R_4 \rightarrow R_2B \end{array}}$$

Q3] Convert the following grammer into CNF -

$S \rightarrow ABA$
$A \rightarrow aA \mid \epsilon$
$B \rightarrow bB \mid \epsilon$

Sol^n → Given - ① $S \rightarrow ABA$
② $A \rightarrow aA \mid \epsilon$
③ $B \rightarrow bB \mid \epsilon$

(1) from the given grammer, first delete all the ϵ - productions

∴ ① $S \rightarrow ABA$
② $A \rightarrow aA$
③ $B \rightarrow bB$

(2) Identify the nullable non-terminals.

As $A \rightarrow \epsilon$ and $B \rightarrow \epsilon$ ∴ A & B are nullable non-terminals.

(3) Identify nullable non-terminals from Step (1) i.e. A & B which should occur on R.H.S of the production Rule:

$$\boxed{\begin{array}{l} S \rightarrow ABA \\ A \rightarrow aA \\ B \rightarrow bB \end{array}}$$

Put values of non-terminals A & B as null on RHS of the production Rule,

$$\boxed{\begin{array}{l} S \rightarrow BA \mid AB \mid AA \\ A \rightarrow a \\ B \rightarrow b \end{array}}$$

(4) So, the final grammer without ϵ production is:

$$\boxed{\begin{array}{l} S \rightarrow ABA \mid BA \mid AB \mid AA \mid \cancel{AB} \mid A \mid B \\ A \rightarrow aA \mid a \\ B \rightarrow bB \mid b \end{array}}$$

Now Production Rules :-

(1) $S \rightarrow ABA$
(2) $S \rightarrow BA$
(3) $S \rightarrow AB$
(4) $S \rightarrow AA$
(5) $A \rightarrow aA$
(6) $A \rightarrow a$
(7) $B \rightarrow bB$
(8) $B \rightarrow b$
(9) $S \rightarrow A$
(10) $S \rightarrow B$

Take production rule (1)

$S \rightarrow ABA$

Assume, $\boxed{R_1 = AB}$ — (i)

$\boxed{S \rightarrow R_1 A}$ — (ii)

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Take production Rule (2)

$\boxed{S \to BA}$ — (iii)

It is already in CNF.

Take production Rule (3)

$\boxed{S \to AB}$ — (iv)

It is already in CNF

Take production Rule (4)

$\boxed{S \to AA}$ — (v)

It is already in CNF

Take production Rule (5)

$A \to aA$

Assume $\boxed{R_2 \to a}$ — (vi)

$\therefore$ $\boxed{A \to R_2 A}$ — (vii)

Take production Rule (6)

$\boxed{A \to a}$ — (viii)

It is already in CNF

Take production Rule (7)

$B \to bB$

~~Assume~~ $\boxed{R_3 \to b}$ — (ix)

$\therefore$ $\boxed{B \to R_3 B}$ — (x)

Take production Rule (8)

$\boxed{B \to b}$ — (xi)

It is already in CNF.

to be continued →

Rule for CNF:
1. NT $\rightarrow$ NT. NT
2. NT $\rightarrow$ T

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So, the final grammer in CNF is as follows:

$S \rightarrow R_1A \mid BA \mid AB \mid AA \mid R_2A \mid R_3B \mid a \mid b$
$A \rightarrow R_2A \mid a$
$B \rightarrow R_3B \mid b$
$R_1 \rightarrow AB$
$R_2 \rightarrow a$
$R_3 \rightarrow b$

continue.

Take Production-Rule (9)

$S \rightarrow A$

As it is an unit production Rule we have to eliminate it,

$\therefore S \rightarrow aA \mid a$

~~Assume~~, $R_2 \rightarrow a$

$\therefore \boxed{S \rightarrow R_2A}$ — (xii)

Take Production Rule (10).

$S \rightarrow B$

As it is an unit production Rule, we have to eliminate it,

$S \rightarrow bB \mid b$

As, $R_3 \rightarrow b$

$\therefore \boxed{S \rightarrow R_3B}$ — (xiii)

Also,

$\boxed{S \rightarrow a}$ — (xiv)
$\boxed{S \rightarrow b}$ — (xv)

Q] Convert the following grammer to GNF.

$S \to AB$

$A \to BS \mid b$

$B \to SA \mid a$

$\to$ Given (1) $S \to AB$ (4) $B \to SA$

(2) $A \to BS$ (5) $B \to a$

(3) $A \to b$

Replacing S by A_1, A by A_2 & B by A_3,

$\therefore$ (1) $A_1 \to A_2 A_3$

~~$A_3 \to A_1 A_2$~~ (2) ~~$A_2 \to A_3 A_1 \mid b$~~ (3)

(4) $A_3 \to A_1 A_2 \mid a$ (5)

(1) $A_1 \to A_2 A_3$

(2) $A_2 \to A_3 A_1$

(3) $A_2 \to b$

(4) $A_3 \to A_1 A_2$

(5) $A_3 \to a$

~~$A_j \to A_j X$~~

$A_i \to A_j X$

$(i < j)$

then don't take it.

Take 4th ~~P.R.~~ & 5th P.R.

$A_3 \to A_1 A_2 \mid a$

By 1st P.R.

$A_3 \to A_2 A_3 A_2 \mid a$

By 3rd & 2nd P.R.

$A_3 \to A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a$

As $A_3 \to A_3 A_1 A_3 A_2$

by $A_i \to A_j X$

$i = 3$ & $j = 3$

$\therefore i = j$ not true

∴ Apply Left Recursion

As $A_3 \rightarrow bA_3A_2 \mid a$

$A_3 \rightarrow bA_3A_2Z \mid aZ \mid bA_3A_2 \mid a$

$Z \rightarrow A_1A_3A_2Z \mid A_1A_3A_2$

Take 2nd P.R.

$A_2 \rightarrow A_3A_1 \mid$

$A_2 \rightarrow bA_3A_2ZA_1 \mid aZA_1 \mid bA_3A_2A_1 \mid aA_1$

Take 1st P.R.

$A_1 \rightarrow \underline{A_2}\,\underline{A_3}$

$A_1 \rightarrow bA_3A_2ZA_1A_3 \mid aZA_1A_3 \mid bA_3A_2A_1A_3 \mid aA_1A_3$

~~$A_1 \rightarrow$~~

As $Z \rightarrow A_1A_3A_2Z \mid A_1A_3A_2$

$Z \rightarrow A_1A_3A_2Z$

$Z \rightarrow \underline{bA_3A_2ZA_1A_3}A_3A_2Z \mid aZA_1A_3A_3A_2Z \mid bA_3A_2A_1A_3A_3A_2Z \mid aA_1A_3A_3A_2Z \mid$

$bA_3A_2ZA_1A_3A_3A_2 \mid bA_3A_2A_1A_3A_3A_2 \mid$

$aZA_1A_3A_3A_2 \mid aA_1A_3A_3A_2$

Imp

Q] Write short note on Chomsky Hierarchy
or Describe diff types of grammer with eg

Chomsky Hierarchy

[1] TYPE 0 [Unrestricted Grammer]
[2] TYPE 1 [Context sensitive Grammer]
[3] TYPE 2 [Context free Grammer]
[4] TYPE 3 [Regular Grammer]
- [i] Left Linear Grammer $A \to Bw$, $A \to \epsilon$ or $A \to w$
- [ii] Right Linear Grammer $A \to wB$, $A \to \epsilon$ or $A \to w$

* EQUIVALENCE OF RIGHT LINEAR GRAMMER & LEFT LINEAR GRAMMER.

Q] Convert the foll. RLG to equivalent LLG.

$S \to bB$
$B \to bC$
$B \to aB$
$C \to a$
$B \to b$

Solⁿ → **Step 1:** Convert the above RLG to transition diagram (transition graph) in which initial state will be labelled by start symbol 'S' & final state will be labelled by 'ε.' This transition diagram is as follows :

$\therefore \to S \to bB$
$B \to bC$
$B \to aB$
$C \to a\epsilon$
$B \to b\epsilon$

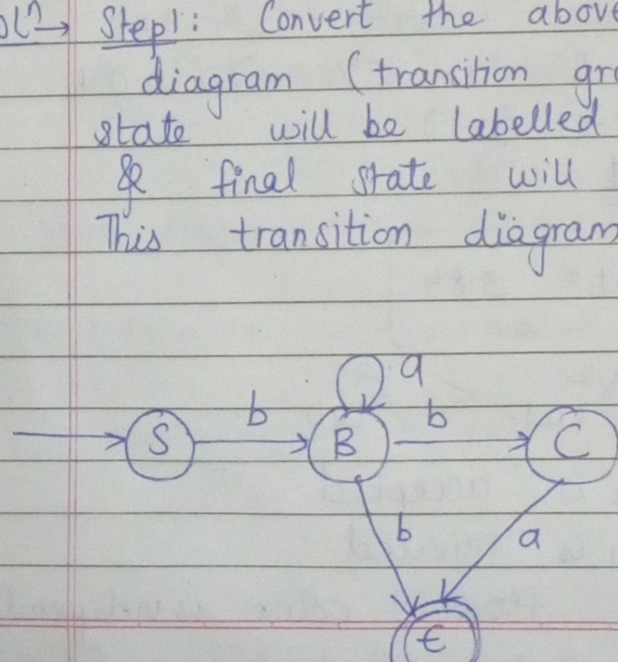


Fig. Representatⁿ of TG for RLG

Step2 → After interchanging posⁿ of initial & final state & reversing the directions of all transitions of Transition graph above we get foll. T.G. as :

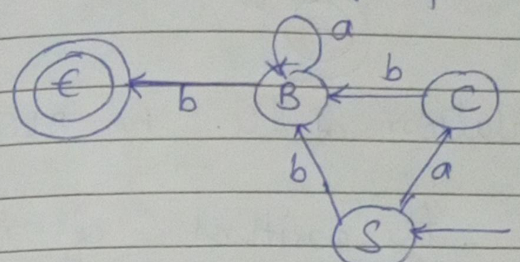


fig. T.G. after interchanging posⁿ.

Step3 → From this above TG we can write LLG as follows:

$S \rightarrow Ca$

$S \rightarrow Bb$

$C \rightarrow Bb$

$B \rightarrow Ba$

$B \rightarrow \epsilon b$ or $B \rightarrow b$

Q.2] Convert the foll. RLG to its equivalent LLG.

$S \rightarrow 0A \mid 1B$

$A \rightarrow 0C \mid 1A \mid 0$

$B \rightarrow 1B \mid 1A \mid 1$

$C \rightarrow 0 \mid 0A$.

→ Step 1 : Convert the above RLG into transition diagram (transition graph) in which initial state will be labelled by start symbol 'S' & final state is labelled by 'ε'

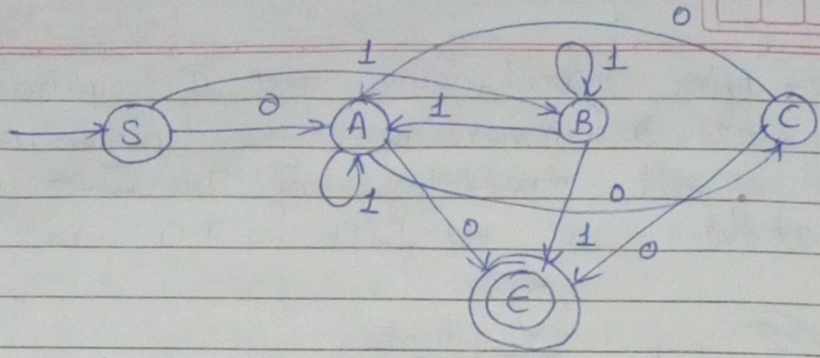


Fig. Representation of T.G. for RLG.

<u>Step 2</u>: Interchanging the positions of initial & final states & reversing the dirⁿ of all transitions.

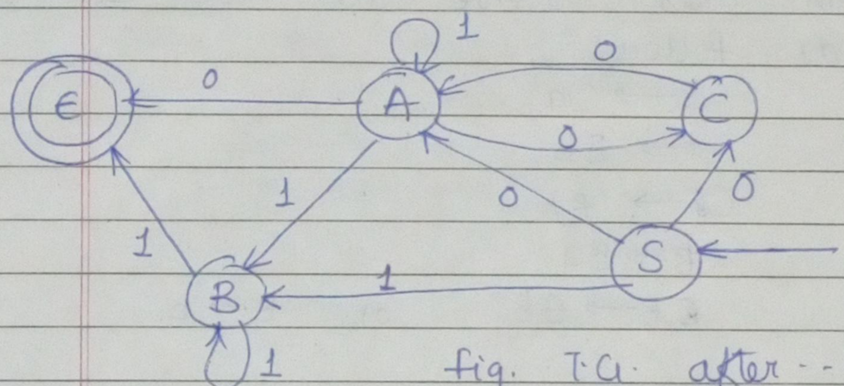


fig. T.G. after ...

<u>Step 3</u>: from the above tr. diag, we can write LLG as:

$S \rightarrow C0 \mid A0 \mid B1$

$A \rightarrow A1 \mid C0 \mid B1 \mid \epsilon 0$

$B \rightarrow \epsilon 1 \mid B1$

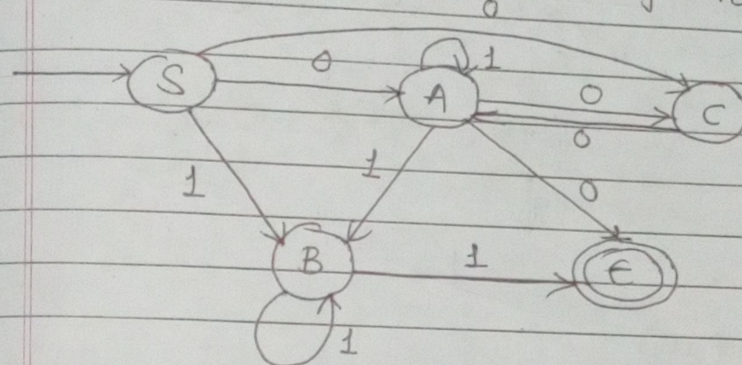
$C \rightarrow A0$

Q] Convert following L.L.G to equival. R.L.G.

$S \rightarrow B1 \mid A0 \mid C0$
$A \rightarrow C0 \mid A1 \mid B1 \mid 0$
$B \rightarrow B1 \mid 1$
$C \rightarrow A0$

→ ① Convert the given LLG to transition diag. in which transtition state will be labelled by start symbol 'S' & final state with 'ϵ'

Fig. Transition diag. for LLG



② After interchanging posⁿ of initial & final state reversing the dirⁿ of all transitions we get,

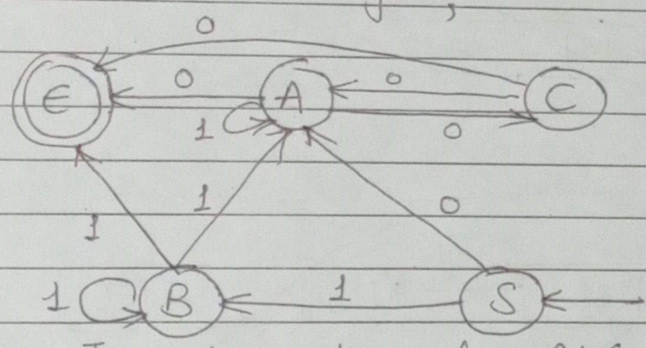


fig. Transition diag. for RLG

③ From the above T.G. we can write RLG as follows :-

$S \rightarrow 1B \mid 0A$
$B \rightarrow 1 \mid 1A \mid 1B$
$A \rightarrow 0C \mid 0 \mid 1A$
$C \rightarrow 0 \mid 0A$

* Equivalence of RG & FA.

Eg. 1] ~~Point~~ Find the equivalent DFA accepting the regular language defined by the RLG as given below:

$$S \rightarrow 0A \mid 1B$$
$$A \rightarrow 0C \mid 1A \mid 0$$
$$B \rightarrow 1B \mid 1A \mid 1$$
$$C \rightarrow 0 \mid 0A$$

Sol^n →

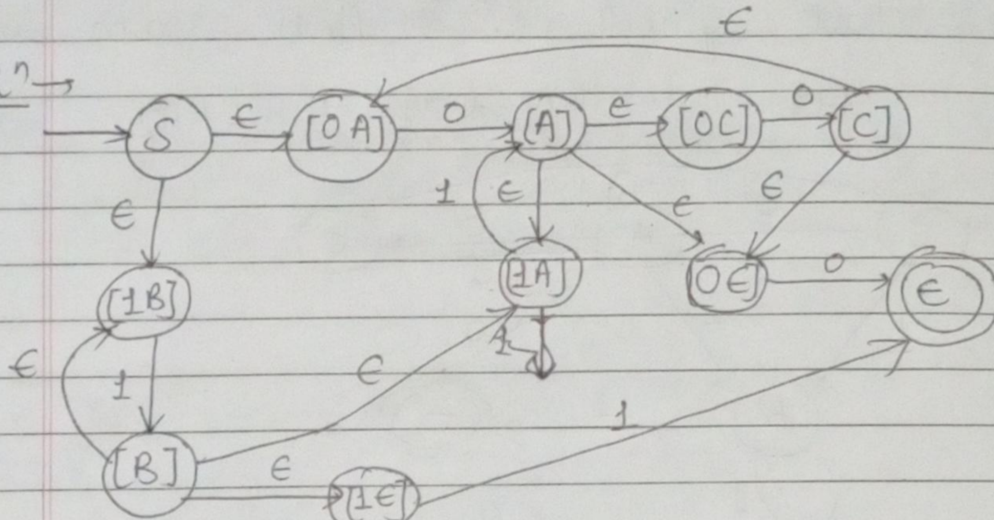
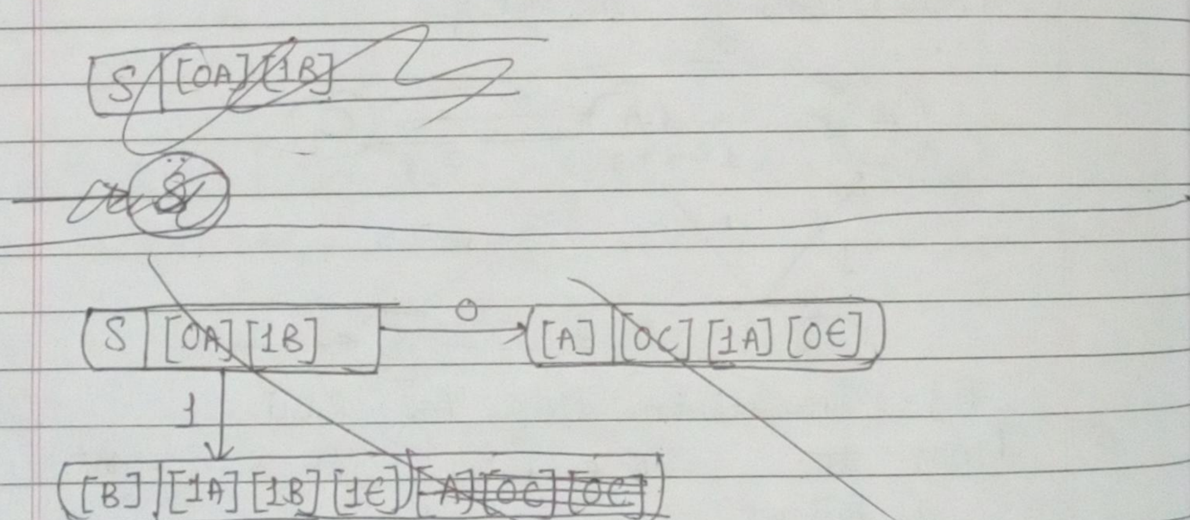
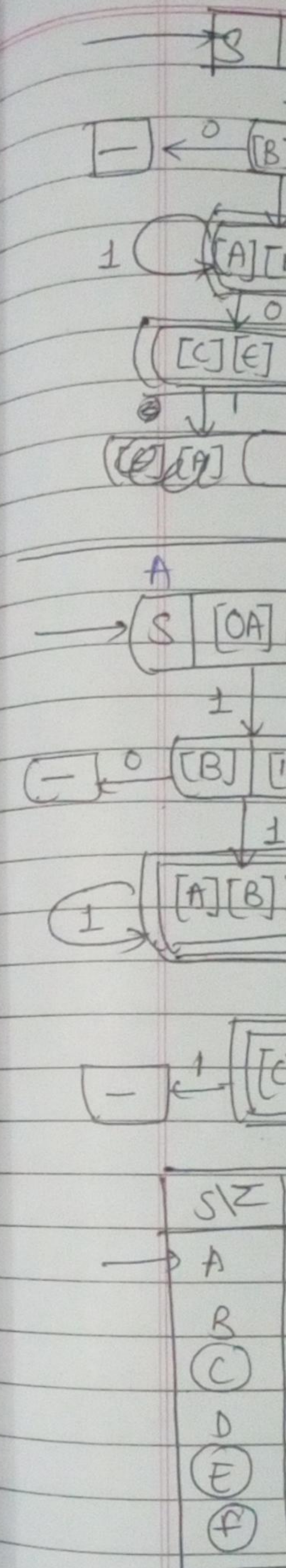
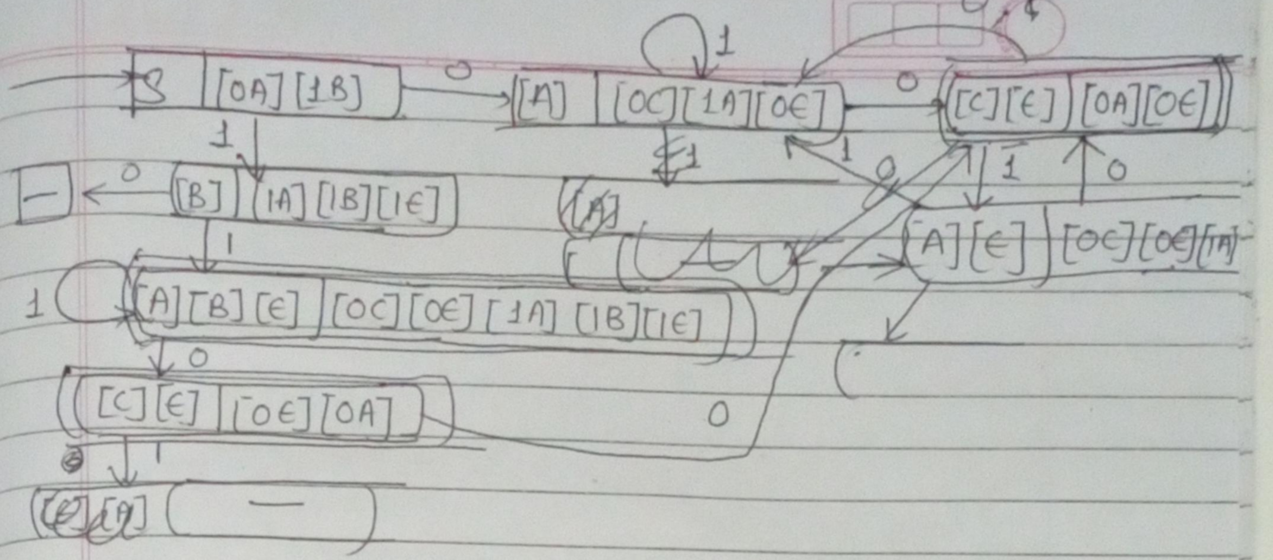


Fig. NFA with ε-moves for given GR. [RLG]







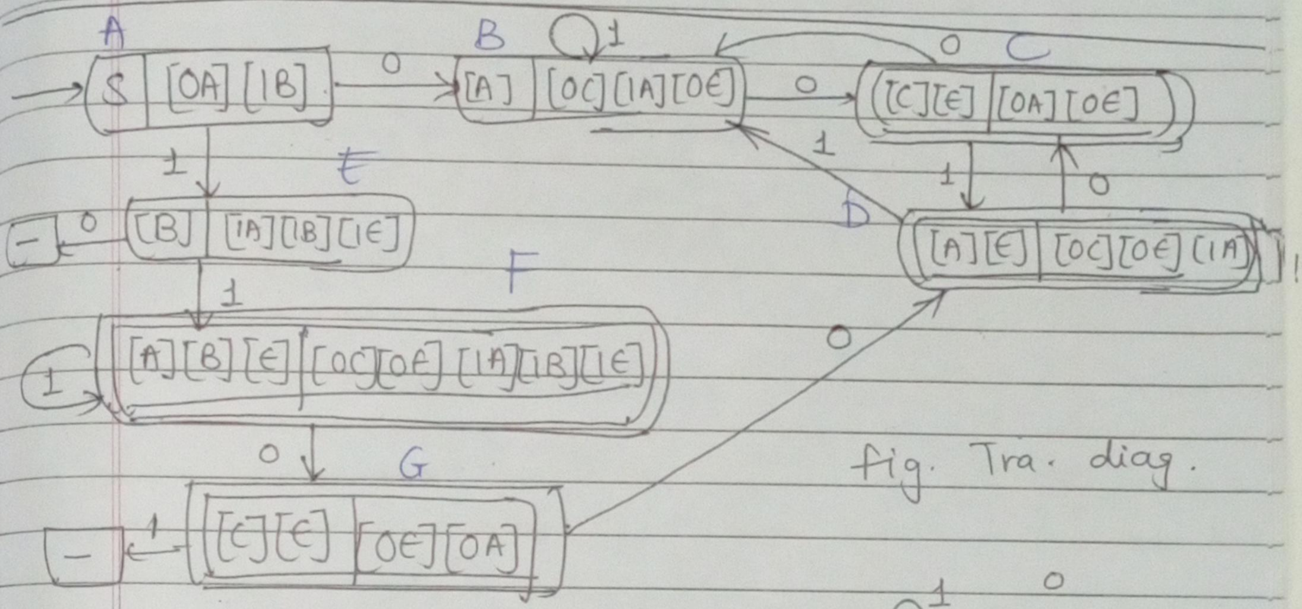
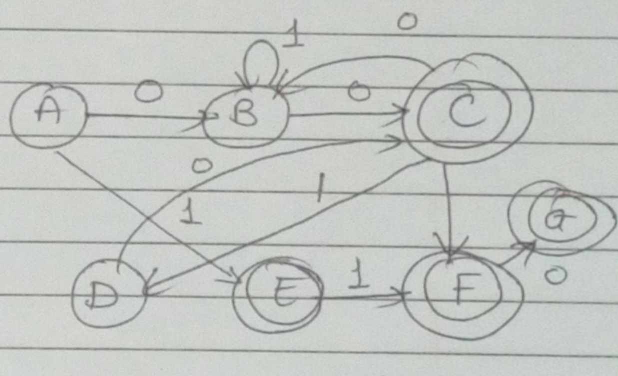


fig. Tra. diag.

S\Σ	0	1
→ A	B	D
B	C	B
C	E	F—
D	—	F
E	C	B
F	C	F



~~Transition diag.~~

Q1] Construct DFA accepting a regular language generated by LLG :-

$S \rightarrow Ca \mid Bb$

$C \rightarrow Bb$

$B \rightarrow Ba \mid b$

Sol^n →

Step 1 → We should reverse the R.H.S of the productions in above G_L to get some Right linear grammer.

After reversing the production :

$S \rightarrow aC \mid bB$

$C \rightarrow bB$

$B \rightarrow aB \mid b\epsilon$

Step 2 → From this above grammer construct NFA with ϵ moves as below :

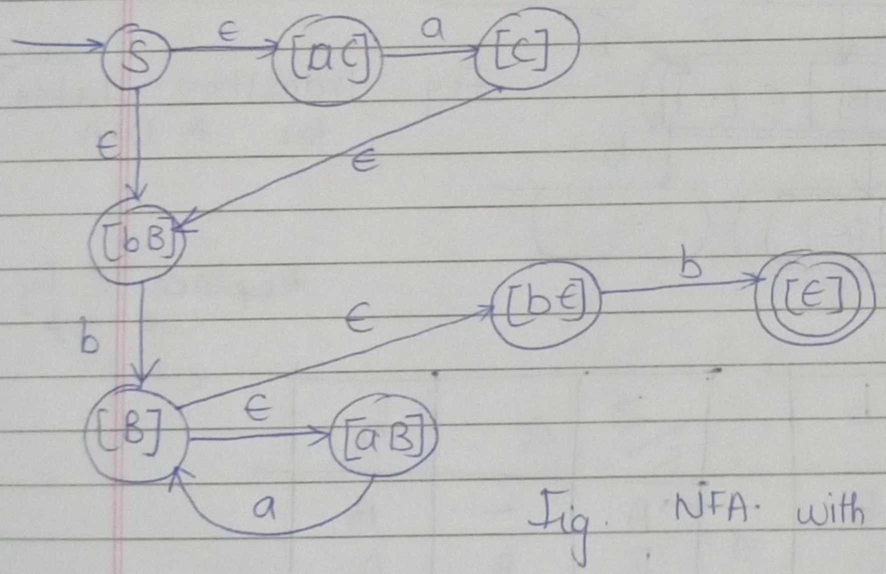


Fig. NFA. with ϵ - moves

Step3 → Interchange the final state with starting state & reverse the directions.

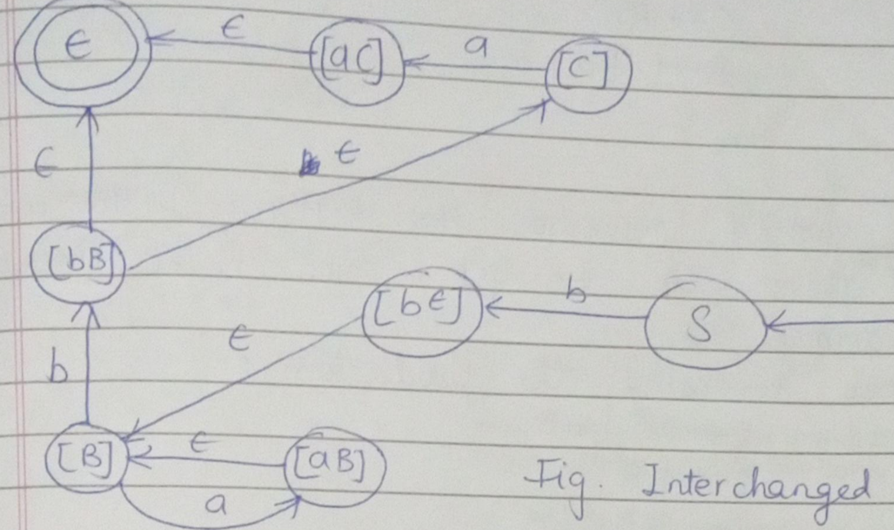


Fig. Interchanged state diag.

Step4 →

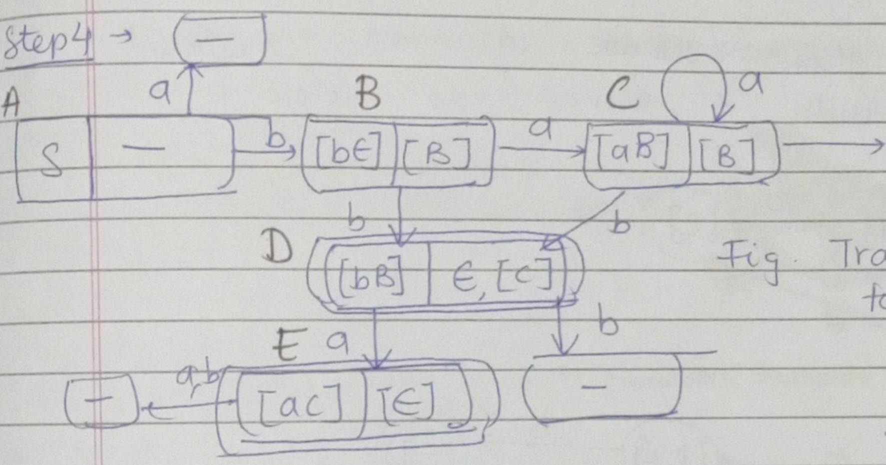


Fig. Transition ~~Table~~ diag. for DFA.

Replace C by B we get

Step5.

S \ Σ	a	b
→ A	—	B
B	C	D
C	C	D
D	E	—
E	—	—

Fig. Transition Table for DFA.

⇒

S \ Σ	a	b
→ A	—	B
B	B	D
D	E	—
E	—	—

Fig. R. TT for DFA.

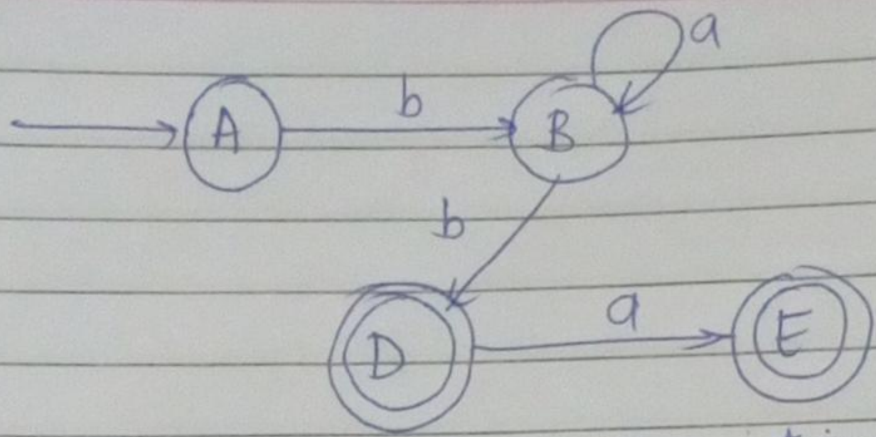


Fig. Transition Diagram for DFA.

Q2] Construct DFA accepting the same RL defined by the RLG:

$S \rightarrow bB$

$B \rightarrow bC \mid aB \mid b$

$C \rightarrow a$