Theory of SET

* SETS

- A set is collection of objects
- An object is called an element or member of the set.
- The term class is also used to denote a set.
- A set may contain finite or infinite no. of elements
- A set is called empty or null set if it contains no element.
  - An empty set is denoted by ϕ.

* Notations

- A set is denoted by capital letters eg. A, B, C ... Z
- Elements of the set are denoted by small letters eg a, b, c

  If x is element of set A

  $\therefore$ " $x \in A$ " // $\in$: belongs to

  If x is not element of set A

  $\therefore$ " $x \notin A$ " //

Various ways of describing a set

① **Listing Method**

eg. $A = \{ \text{pencil, byte, } 5 \}$

$B = \{ 2, 4, 6, 8 \cdots \}$

② **Statement Form**

eg. A = The set of all equilateral triangles

B = The set of all prime ministers of India

③ **Set builder notation**

$A = \{ x \mid P(x) \}$

eg. $A = \{ x \mid x > 10 \}$

$B = \{ x \mid x \text{ is real and } x^8 - 5x^4 + 4 = 0 \}$

* **Some Special Sets** (Numbers sets)

N - Set of all natural no. $\{1, 2, 3 \cdots\}$

Z → Set of all integers $\{\cdots -2, -1, 0, 1, 2 \cdots\}$

Z^+ - Set of all positive integers $\{0, 1, 2, \cdots\}$

Q - Set of Rational no.

Q^+ - Set of non-negative Rational no.

IR - Set of Real no.

IR^+ - Set of complex no.

* **Subsets**

- If every element of a set A is also an element of set B, then A is subset of B. or A is contained in B. $(A \subseteq B)$
  - If A is not subset of B, then $(A \nsubseteq B)$

Ex. (1)

$A = \{1, 3, 6\}$ and $B = \{-1, 1, 2, 3, 4, 6\}$

$C = \{1, 2, 3\}$

Then $A \subseteq B$

$A \nsubseteq C$

Note: i> Every set is a subset of itself.

ii> Empty set is a subset of any set.

* **Universal Set**

- If all sets, considered during a specific discussion are subsets of a given set, then this set is called as the Universal Set & denoted by U.

* **Equality of Sets**

- Two sets A & B are equal if $A \subseteq B$ & $B \subseteq A$ implies $A = B$

Ex. A = { BASIC, COBOL, FORTRAN }

B = { FORTRAN, COBOL, BASIC }

then $A = B$

Ex. ① Let $A = \{a, b, \{a,b\}, \{\{a,b\}\}\}$

Identify each of the following statements as true or false.

i> $a \in A$ → TRUE — a is element of A

ii> $\{a\} \in A$ → FALSE — $\{a\}$ is not element but subset of A

iii> $\{a,b\} \in A$ → TRUE — $\{a,b\}$ is element of A & listed third in the set

iv> $\{\{a,b\}\} \subseteq A$ → TRUE

v> $\{a,b\} \subseteq A$ → TRUE

* SET OPERATIONS

① Complement of a Set

Let, A be the set.

Complement of A, denoted by $\bar{A}$ is defined as

$$\bar{A} = \{x \mid x \notin A\}$$

Ex.

i> If $A = \{x \mid x$ is a real no. and $x \leq 7\}$,

then $\bar{A} = \{x \mid x$ is real no. and $x > 7\}$

② Union of two sets

The union of two sets A and B is the set consisting of all elements which are in A, or in B or in both sets A and B. It is denoted by $A \cup B$.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Ex. i> $A = \{2, 4, 6, 8, 10\}$

$B = \{1, 2, 6, 8, 12, 15\}$

$\therefore A \cup B = \{1, 2, 4, 6, 8, 10, 12, 15\}$

Note:

$A \cup \phi = A$

$A \cup U = U$ (U : universal set)

$A \cup \bar{A} = U$

③ **Intersection of sets.**

The intersection of two sets A and B, denoted by $A \cap B$ is the set consisting of elements which are in A as well as in B.

$\therefore \quad A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

if $A \cap B = \phi$, **disjoint set.**

Ex. i> If $A = \{a, b, c, g\}$ & $B = \{d, e, f, g\}$

then $A \cap B = \{g\}$

Ex. ii> If $A = \{n \mid n \in N,\ 4 < n < 12\}$ &

$B = \{n \mid n \in N,\ 5 < n < 10\}$

then

$A \cap B = \{6, 7, 8, 9\} = B$

Note :

$A \cap \phi = \phi$

$A \cap \bar{A} = \phi$

$A \cap U = A$

④ **Difference of Sets** (Relative Complement)

Let, A & B be any two sets

The difference $A - B$ is the set defined as

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

is the (relative) complement of B in A

Similarly,

$B - A = \{x \mid x \in B \text{ and } x \notin A\}$

is the complement of A in B.

Ex. i> If $A = \{1, 2, 3, \ldots, 10\}$

$B = \{1, 3, 5, \ldots, 9\}$ i.e odd no.

then

$A - B = \{2, 4, 6, 8, 10\}$

$B - A = \{\phi\}$

* **properties of difference**

let A and B be any two sets, then

i> $\bar{A} = U - A$

ii> $A - A = \phi$

iii> $A - \bar{A} = A$,

$\bar{A} - A = \bar{A}$

iv> $A - \phi = A$

v> $A - B = A \cap \bar{B}$

vi> $A - B = B - A$ iff $A = B$

vii> $A - B = A$ iff $A \cap B = \phi$

viii> $A - B = \phi$ iff $A \subseteq B$

⑤ **Symmetric Differences**

Symmetric difference of two sets A & B, denoted by $A \oplus B$, is defined as

$$A \oplus B = \{ x / x \in A - B \text{ or } x \in B - A \}$$

In other words

$$A \oplus B = (A - B) \cup (B - A) \quad // \text{ OR } (A \cup B) - (A \cap B)$$

Ex. i> If $A = \{a, b, e, g\}$

$B = \{d, e, f, g\}$

then $A \oplus B = \{a, b, d, f\}$

ii> If $A = \{2, 4, 5, 9\}$

$B = \{x \in Z^+ \mid x^2 \leq 16\}$

then

$A \oplus B = \{0, 1, 3, 5, 9\}$

* **properties of Symmetric Difference**

i> $A \oplus A = \phi$

ii> $A \oplus \phi = A$

iii> $A \oplus U = \bar{A}$

iv> $A \oplus \bar{A} = U$

v> $A \oplus B = (A \cup B) - (A \cap B)$

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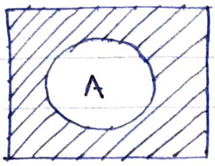
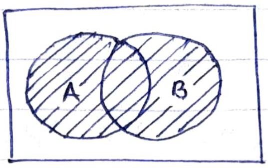
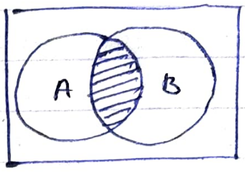
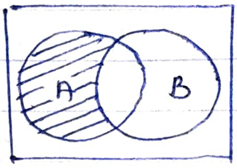
* <u>Representation of Set operations on Venn Diagrams</u>

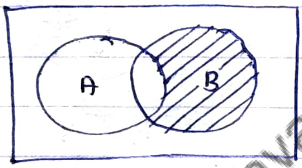


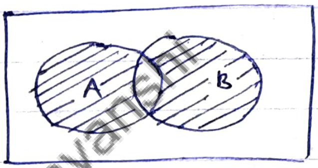






① $\bar{A}$ = ② $A \cup B$ = ③ $A \cap B$ = ④ $A - B$ =





⑤ $B - A$ = ⑥ $A \oplus B$ =

* <u>Algebra of Set Operations</u>

① <u>Commutativity</u> i> $A \cup B = B \cup A$
ii> $A \cap B = B \cap A$

② <u>Associativity</u>
i> $A \cup (B \cup C) = (A \cup B) \cup C \Rightarrow A \cup B \cup C$
ii> $A \cap (B \cap C) = (A \cap B) \cap C \Rightarrow A \cap B \cap C$

③ <u>Distributivity</u>
i> $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
ii> $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

④ <u>Idempotent Laws</u> i> $A \cup A = A$
ii> $A \cap A = A$

⑤ <u>Absorptions Laws</u> i> $A \cup (A \cap B) = A$
ii> $A \cap (A \cup B) = A$

⑥ De Morgans Laws

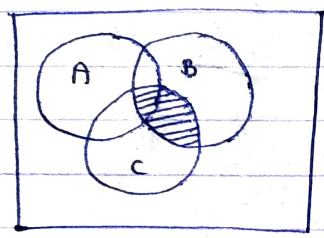
i> $\overline{A \cup B} = \bar{A} \cap \bar{B}$

ii> $\overline{A \cap B} = \bar{A} \cup \bar{B}$

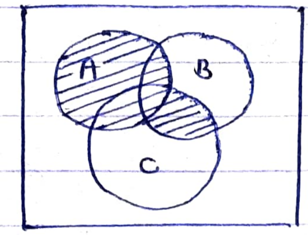
⑦ Double Complement $\bar{\bar{A}} = A$

For distributive Laws

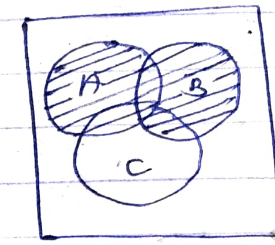
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$



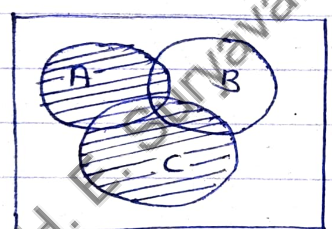
$B \cap C$



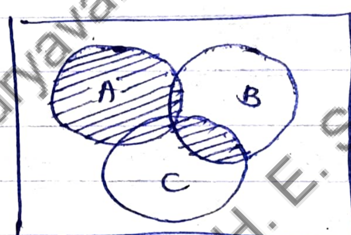
$A \cup (B \cap C)$



$A \cup B$



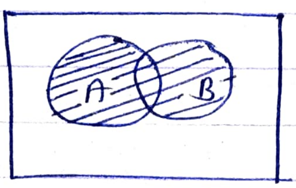
$A \cup C$



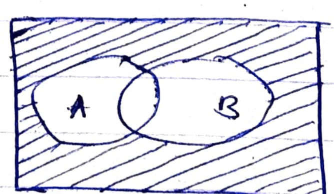
$(A \cup B) \cap (A \cup C)$

De Morgans Laws

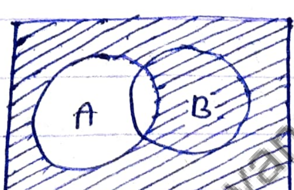
$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$



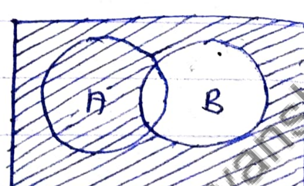
$A \cup B$



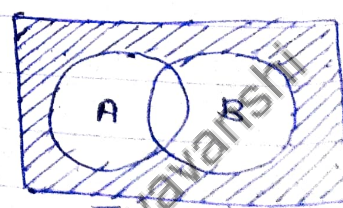
$\overline{A \cup B}$



$\bar{A}$



$\bar{B}$



$\bar{A} \cap \bar{B}$

Ex. ① let $U = \{ n \mid n \in N,\ n \le 15 \}$

$A = \{ n \mid n \in N,\ 4 < n < 12 \}$

$B = \{ n \mid n \in N,\ 8 < n < 15 \}$

$C = \{ n \mid n \in N,\ 5 < n < 10 \}$

find

$\bar{A} - \bar{B}$ and $\bar{C} - \bar{A}$

$\Rightarrow$

$U = \{ 1, 2, 3, 4, 5 \cdots\cdots 14, 15 \}$

$A = \{ 5, 6, 7, 8, 9, 10, 11 \}$

$B = \{ 9, 10, 11, 12, 13, 14 \}$

$C = \{ 6, 7, 8, 9 \}$

$\therefore\ \bar{A} = \{ 1, 2, 3, 4, 12, 13, 14, 15 \}$

$\bar{B} = \{ 1, 2, 3, 4, 5, 6, 7, 8, 15 \}$

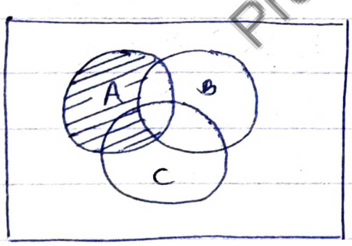
& $\bar{C} = \{ 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15 \}$

$\therefore\ \bar{A} - \bar{B} = \{ 12, 13, 14 \}$

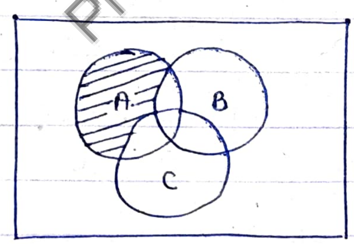
& $\bar{C} - \bar{A} = \{ 5, 10, 11 \}$

Ex. ② show that $(A-B)-C = A-(B\cup C)$ using Venn diagram.

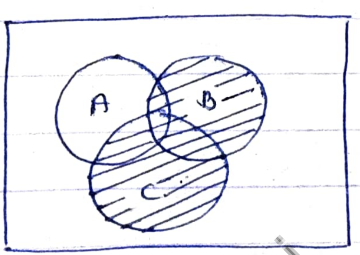
$\Rightarrow$



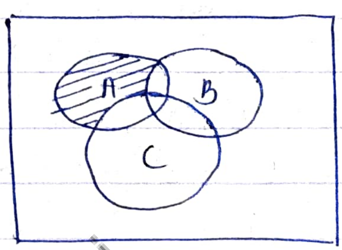
$A - B$



$(A-B)-C$



$B \cup C$

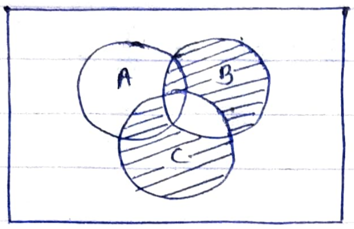


$A-(B\cup C)$

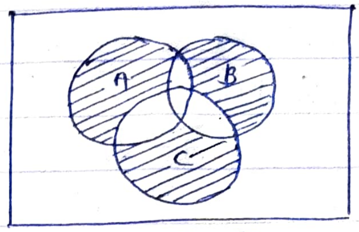
Ex. ③ using Venn diagram, prove or disprove

$$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$

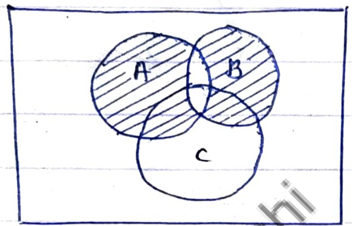
$\Rightarrow$



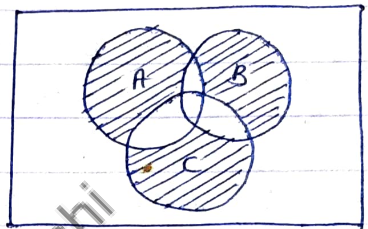
$B \oplus C$



$A \oplus (B \oplus C)$



$A \oplus B$



$(A \oplus B) \oplus C$

Ex. ④ i> Given that $A \cup B = A \cup C$, is it necessary that $B = C$?

ii> Given that $A \cap B = A \cap C$, is it necessary that $B = C$?

$\Rightarrow$

i> No,

let $A = \{1, 2, 3\}$ and $B = \{1\}$, $C = \{3\}$

$\therefore A \cup B = \{1, 2, 3\} = A \cup C$.

but

$B \neq C$

ii> No,

let $A = \{1, 2\}$, $B = \{2, 3, 4, 5\}$ and

$C = \{2, 6, 7\}$

then

$A \cap B = \{2\} = A \cap C$.

but

$B \neq C$.

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* Cardinality of Finite Set

Definition:

let A be a finite set.
The cardinality of A, denoted by $|A|$ is the no. of elements in set.

- If $A = \phi$, then $|A| = 0$
- If $A \subseteq B$, where B is finite set, then $|A| \leq |B|$

Theorem

① $|A \cup B| = |A| + |B|$ where A & B be finite sets

② $|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|$
where $A_1, A_2, \ldots A_n$ be a finite collection of mutually disjoint finite set.

③ let A & B two set
where A is finite set & B be any set (not necessarily finite set)
Then
$$|A - B| = |A| - |A \cap B|$$

④ Principle of Inclusion-Exclusion
let A & B be finite sets
Then,
$$|A \cup B| = |A| + |B| - |A \cap B|$$

⑤ Mutual Inclusion-Exclusion Principle for Three Sets
let A, B & C be finite sets
Then,
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Ex. ① In a survey, ~~of~~ 2000 people were asked whether they read India Today or Business Times. It was found that 1200 read India Today, 900 read Business Times and 400 read both.

Find how many read atleast one magazine and how many read neither. $A \cup B$ & $\overline{A \cup B}$

$\Rightarrow$

Let, A = Set of people who read India Today.

B = Set of people who read Business Times

Now,

$|U| = 2000$

$|A| = 1200$

$|B| = 900$

$|A \cap B| = 400$

we have to find out

$|U - (A \cup B)| = |U| - |A \cup B|$

$\therefore$ By the mutual inclusion - exclusion principle

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 1200 + 900 - 400$

$|A \cup B| = 1700$

$\therefore |U - (A \cup B)| = |U| - |A \cup B|$

$= 2000 - 1700$

$= 300$

Hence, 1700 read atleast one magazine and 300 read neither.

Ex. ② Among the integers 1 to 300, find how many are not divisible by 3, nor by 5.

Find also, how many are divisible by 5 but not by 7.

$\Rightarrow$ let A = set of integers divisible by 3
B = set of integers divisible by 5
C = set of integers divisible by 7

we have to find

$$|\bar{A} \cap \bar{B}| \text{ and } |A - C|$$

By De Morgans Law $|\bar{A} \cap \bar{B}| = \overline{A \cup B}$

$$\therefore |\overline{A \cup B}| = |U| - |A \cup B|$$

Given

$$|U| = 300$$

$$|A| = \left[\frac{300}{3}\right] = 100, \quad |B| = \left[\frac{300}{5}\right] = 60 \text{ ~~and~~}$$

$$|C| = \left[\frac{300}{7}\right] =$$

Now,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= 100 + 60 - \left[\frac{300}{3 \times 5}\right]$$

$$= 100 + 60 - \left[\frac{300}{15}\right]$$

$$|A \cup B| = 140$$

$$\therefore |\bar{A} \cap \bar{B}| = |\overline{A \cup B}| = |U| - |A \cup B|$$

$$= 300 - 140$$

$$= 160$$

Hence, 160 integers are not divisible by 3 nor by 5

Now

$$|A - C| = |A| - |A \cap C|$$

$$= 100 - \left[\frac{300}{3 \times 7}\right]$$

$$= 100 - \left[\frac{300}{21}\right] = 100 - 14 = 86$$

Hence,

86 integers are divisible by 3 but not by 7

Ex. (3) In a computer laboratory out of 6 computers

i) 2 have floating point arithmetic unit (FPA)

ii) 5 have magnetic disk memory

iii) 3 have graphic displays

iv) 2 have both floating point arithmetic unit and magnetic disk memory

v) 3 have both magnetic disk memory and graphics display

vi) 1 has both floating point arithmetic unit & graphic display

vii) 1 has FPA, magnetic disk m/m & graphic display

– How many have atleast one specification?

⟹

Let, A = set of computers having FPA unit

B = set of computers having magnetic disk m/m

C = –||– having graphics display

Then,

$|A| = 2$

$|B| = 5$

$|C| = 3$

$|A \cap B| = 2, \quad |B \cap C| = 3, \quad |A \cap C| = 1$

and $|A \cap B \cap C| = 1$

Now

$|A \cup B \cup C| = ?$

$$\therefore |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$= 2 + 5 + 3 - 2 - 3 - 1 + 1$$

$$\therefore |A \cup B \cup C| = 5$$

Hence, 5 computers out of 6, have atleast one specification.

Ex. (4) How many integers between 1 - 1000 are divisible by 2, 3, 5 or 7 ?

$\Rightarrow$

let A, B, C & D denote respectively set of integers from 1 to 1000 divisible by 2, 3, 5 or 7

$\therefore \quad |A| = [1000/2] = 500$

$\therefore \quad |B| = [1000/3] = 333$

$\therefore \quad |C| = [1000/5] = 200$

$\therefore \quad |D| = [1000/7] = 142$

$|A \cap B| = \left[\frac{1000}{6}\right] = 166$

$|A \cap C| = \left[\frac{1000}{10}\right] = 100$

$|A \cap D| = \left[\frac{1000}{14}\right] = 71$

$|B \cap C| = [1000/15] = 66$

$|B \cap D| = [1000/21] = 47$

$|C \cap D| = [1000/35] = 28$

$|A \cap B \cap C| = [1000/30] = 33$

$|B \cap C \cap D| = [1000/105] = 9$

$|A \cap C \cap D| = \lfloor 1000/70 \rfloor = 14$

$|A \cap B \cap D| = \lfloor 1000/42 \rfloor = 23$

$|A \cap B \cap C \cap D| = \lfloor 1000/210 \rfloor = 4$

Now

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| + |A \cap B \cap C \cap D|$$

$$= 500 + 333 + 200 + 142 - 166 - 100 - 71 - 66 - 47 - 28 + 33 + 23 + 14 + 9 + 4$$

$$= 780$$

Ex. 5 Among the integers 1 to 1000:

i> How many of them are not divisible by 3, nor by 5, nor by 7?

ii> How many are not divisible by 5 and 7 but divisible by 3?

⇒

i> Let, A, B, C denotes set of integers 1 to 1000 divisible by 3, 5 or 7 respectively.

Now,

we have to find $\bar{A} \cap \bar{B} \cap \bar{C}$

i.e integers not divisible by 3, 5 or 7

By Demorgans Law

$\bar{A} \cap \bar{B} \cap \bar{C} = \overline{(A \cup B \cup C)}$

Hence,

$|\bar{A} \cap \bar{B} \cap \bar{C}| = 1000 - |A \cup B \cup C| \longrightarrow (1)$

$|A| = [1000/3] = 333$

$|B| = [1000/5] = 200$

$|C| = [1000/7] = 142$

$|A \cap B| = [1000/15] = 66$

$|A \cap C| = [1000/21] = 47$

$|B \cap C| = [1000/35] = 28$

$|A \cap B \cap C| = [1000/105] = 9$

Hence

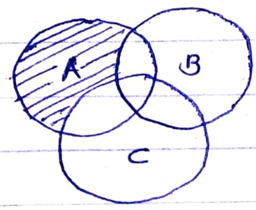
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$
$$= 333 + 200 + 142 - 66 - 28 - 47 + 9$$
$$|A \cup B \cup C| = 543$$

From (1)

$$|\bar{A} \cap \bar{B} \cap \bar{C}| = 1000 - |A \cup B \cup C|$$
$$= 1000 - 543$$
$$= 457$$

Hence, 457 integers are not divisible by 3, 5 or 7

ii> Consider Venn Diagram



Set of integers not divisible by 5 and 7 but by 3 i.e. $A \cap \bar{B} \cap \bar{C}$

$\therefore A \cap \bar{B} \cap \bar{C} = A \cap (\overline{B \cup C}) = A - (B \cup C)$

from venn diagram

OR $|A \cap \bar{B} \cap \bar{C}| = |A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$

$= 333 - 66 - 47 + 9$

$|A \cap \bar{B} \cap \bar{C}| = 229$

$\therefore |A - (B \cup C)| = |A| - |(A \cap B) \cup (A \cap C)|$

Now

$|(A \cap B) \cup (A \cap C)| = |A \cap B| + |A \cap C| - |A \cap B \cap C|$

$= 66 + 47 - 9$

$= 104$

$\therefore |A - (B \cup C)| = |A| - |(A \cap B) \cup (A \cap C)|$

$= 333 - 104$

$= 229$

Hence, 229 integers are not divisible by 5 & 7 but divisible by 3.

Ex. 6: An investigator interviewed 100 students to determine their preferences for the three drinks - Milk (M), Coffee (C) and Tea (T). He reported following:

10 students had all the three drinks,

20 had 'M' and 'C',

30 had 'C' and 'T'

25 had 'M' and 'T'

12 had 'M' only

5 had 'C' only and 8 had 'T' only.

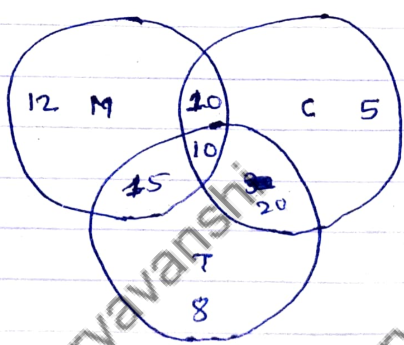
i) How many did not take any of three drinks

ii) How many take milk but not coffee?

iii) How many take tea and coffee but not milk?

$\Rightarrow$

Consider venn diagram



i) $|\bar{M} \cap \bar{C} \cap \bar{T}| = 100 - |M \cup C \cup T|$

$= \cancel{100 - [|M| + |C| + |T| - |M \cap C| - |M \cap T| - |C \cap T|}$
$\quad \cancel{+ |M \cap C \cap T|]}$

$= \cancel{100 - [12 + 5 + 8 - 20 - 25 - 30 + 10]}$

$= \cancel{100 - 40}$

$= 100 - [12 + 10 + 10 + 15 + 20 + 8 + 5]$

$= 100 - 80$

$= 20$

ii) Set of students taking milk but not coffee $(M - C)$

$\cancel{|M - C| = |M| + |C| - |M \cap C|}$

$\cancel{= 12 + 5 - 10}$

$\cancel{= 7}$

OR

$\cancel{|M - C| = 12 - 5 = 7}$

$|M - C| = 12 + 15 = 27$

From venn diagram

iii) Set of students taking tea & coffee, but not milk is $(T \cap C) - M$

$|(T \cap C) - M| = |T \cap C| - |T \cap C \cap M|$

$= 30 - 10$

$= 20$

Ex. (7) It was found that in first year of computer science of 80 students : 50 knows 'COBOL', 55 knows 'C', 46 knows 'PASCAL'.

- It was also known that 37 know 'C' & 'COBOL'
- 28 knows 'C' & 'PASCAL'
- 25 knows 'PASCAL' & 'COBOL'.
- 7 students know none of the language.

Find ① How many know all the three languages?
② How many know exactly two languages?
③ How many know exactly one language?

$\Rightarrow$

Let, B = set of students who know 'COBOL'
C = —"— know 'C'
P = —"— know 'PASCAL'

Now,

7 Students know none of the language

i.e $|\overline{B \cup C \cup P}| = 7$

$$\therefore |B \cup C \cup P| = |U| - |\overline{B \cup C \cup P}|$$
$$= 80 - 7$$
$$= 73$$

$\therefore$ 73 Students know at least one language

① How many know all three language i.e $|B \cap C \cap P| = ?$

Now

$$|B \cup C \cup P| = |B| + |C| + |P| - |B \cap C| - |B \cap P| - |C \cap P| + |B \cap C \cap P|$$

$$\therefore |B \cap C \cap P| = |B \cup C \cup P| - |B| - |C| - |P| + |B \cap C| + |B \cap P| + |C \cap P|$$

$$= 73 - 50 - 55 - 46 + 37 + 25 + 28$$
$$= 12$$

$\therefore$ 12 Students know all three languages

② How many know exactly two languages?

– Find out : Students who know 'COBOL' & 'C' but not 'PASCAL'

$$|B \cap C \cap \bar{P}| = |B \cap C| - |B \cap C \cap P|$$
$$= 37 - 12 = 25$$

– Find out : Students who know 'COBOL' & 'PASCAL' but not 'C'

$$|B \cap P \cap \bar{C}| = |B \cap P| - |B \cap P \cap C|$$
$$= 25 - 12 = 13$$

– Find out : Students who know 'PASCAL' & 'C' but not 'COBOL'

$$|\bar{B} \cap P \cap C| = |P \cap C| - |B \cap P \cap C|$$
$$= 28 - 12 = 16$$

∴ Hence, the no. of students who know exactly two languages

$= 25 + 13 + 16$

$= 54$ Students

③ How many knows exactly one language

∴ Find out : Students who know only COBOL & not 'PASCAL' & 'C'

$$\therefore |B \cap \bar{P} \cap \bar{C}| = |B| - |B \cap P| - |B \cap C| + |B \cap P \cap C|$$
$$= 50 - 25 - 37 + 12$$
$$= 0$$

Similarly students who knows only 'C'

$= 55 - 37 - 28 + 12 = 2$

And also, students who know Pascal

$= 46 - 28 - 25 + 12 = 5$

Hence,

No. of students who know exactly one language

$= 0 + 2 + 5 = 7$

Ex. (8) A college record gives following information-
119 Students enrolled in Introductory Computer Science;
of these 96 took data structures, 53 took
Foundations, 39 took Assembly language, 31 took
both foundations and Assembly language,
32 took both data structures & Assembly language,
38 took data structures & Foundations and
22 took all the three courses.

Is the information correct? Why?

$\Rightarrow$ let,

D, F & A denote the set of students who took Data Structure, Foundation & Assembly language.

Given $|D| = 96$, $|F| = 53$, $|A| = 39$
$|F \cap A| = 31$, $|D \cap A| = 32$, $|D \cap F| = 38$
$|F \cap A \cap D| = 22$

$\therefore |F \cup A \cup D| = |D| + |F| + |A| - |F \cap A| - |D \cap A| - |D \cap F| + |F \cap A \cap D|$

$= 96 + 53 + 39 - 31 - 32 - 38 + 22$

$= 109$ which is less than 119

Since, 119 students enrolled for the course, assuming that all these students had taken at least one course, the given info. is NOT correct

* Power Set

Let A be any set. The power set of A, denoted by P(A) is the set of all subsets of A.

Ex. (1) $A = \{a\}$ $\therefore P(A) = \{\phi, A\}$ i.e $\{\phi, \{a\}\}$

(2) $A = \{a, b\}$ $\therefore P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

Cardinality of Power Set $|P(A)|$

Let A be a finite set containing n-elements. Then the powerset of A has exactly 2^n elements

H.E. Suryavanshi

DSGT-1

* **Propositions (statements)**
  - A declarative sentence which is either true or false, but not both.

Ex. ① There are 7 days in week. → TRUE
② $2+2=5$. → FALSE
③ The earth is flat. → FALSE
④ It will rain tomorrow. → ?

∴ sentences which are not propositions -

Ex. ① $x+3=5$
② Bring that book!
③ When is your interview?

* **Notations**
  - Primary, primitive or atomic statements
    - sentences which cannot be further split or broken down into simpler sentence
    - primary statements denoted by lower-case letters ($p, q \ldots$)

* **Logical Connectives**
  - compound statements
    - Complicated statements can be obtained from the primary statements by using certain connecting words
    - Ex. Not, And, Or, but, while etc.

① **Negation**
- The negation of a statement is formed by either introducing the word "not" at proper place or by prefixing the statement with the phrase "It is not the case that".
- If "p" denotes a statement, the negation of "p" is denoted by "$\sim p$" or ($\bar{p}$).
- i.e. if p is true then $\sim p$ is false & vice-versa.

Ex. ① $p \rightarrow$ "I am going for walk"
$\sim p \rightarrow$ "I am not going for walk"
OR "It is not the case that I am going for a walk".

Ex. ② $q \rightarrow$ "3 is not a prime number"
$\sim q \rightarrow$ "3 is a prime number".

② Conjunction ("And")

If p and q are ~~compound~~ statements, then "$p \wedge q$" or "p and q" is called as conjunction of p & q.

Ex. ① p: The sun is shining.
q: The birds are singing.
Then $p \wedge q \rightarrow$ "The sun is shining and the birds are singing".

Ex. ② p: 2 is prime number
q: Ram is an intelligent boy.
Then $p \wedge q \rightarrow$ "2 is prime number and Ram is an intelligent boy".

- "but" & "while" are treated as equivalent to "and"

Translate the statement into symbolic form

Ex. ① Amar is poor but happy.
→ p: Amar is poor
q: Amar is happy.
∴ $p \wedge q$

Ex. ② We watch television while we have dinner
→ p: we watch television
q: we have dinner
∴ $p \wedge q$

③ **Disjunction ("or")**

If p & q are statements, then the compound statement "p or q" is called as the disjunction of p & q and is denoted by "$p \vee q$".

Ex. ① There is an error in the program or the data is wrong.

→ p : There is an error in the program

q : The data is wrong.

$\therefore$ $p \vee q$.

Ex. ② Either I will read a book or goto sleep.

→ p : I will read a book

q : I will goto sleep

$\therefore$ $p \vee q$

In Ex. ① "or" is inclusive – i.e at least one possibility exists or even both

In Ex. ② "or" is exclusive – i.e only one possibility can exists but not the both.

Notations = $\vee$: inclusive or

$\oplus$ $\underline{\vee}$: exclusive or

④ **Conditional ("If ... then")**

If p & q are statements, the compound statement "If p then q", denoted by $p \to q$ is called as conditional statement or implication.

$p \to q$

- where p is antecedent & q is consequent
- converse of "$p \to q$" is the conditional "$q \to p$"
- contrapositive of "$p \to q$" is the conditional "$\sim q \to \sim p$"
- inverse of "$p \to q$" is $\sim p \to \sim q$

p – Hypothesis

q - Conclusion

Ex. ① p : Hari works hard

q : Hari will pass the exam.

$\therefore$ $p \rightarrow q$: If Hari works hard, then he will pass the exam.

Ex. ② Give the converse & contrapositive of the conditional statement

"If it rains, then I carry an umbrella"

$\rightarrow$ let p : It rains

q : I carry an umbrella

Converse of $p \rightarrow q$ is

$q \rightarrow p$: If I carry an umbrella, then it rains

Contrapositive of $p \rightarrow q$ is

$\sim q \rightarrow \sim p$: If I do not carry an umbrella, then it does not rains.

Ex. ③ Write in symbolic form

"farmers will face hardship if the dry spell continues"

$\rightarrow$ let p : Farmers will face hardship

q : The dry spell continues

$\therefore$ $q \rightarrow p$

⑤ Biconditional ("If and only if") "iff"

If p & q are statements, the compound statement "p if and only if q", denoted by $p \leftrightarrow q$, is a biconditional statement

"If p then q, and conversely"

Ex. ① An integer is even if and only if it is divisible by 2.

② Two lines are parallel if and only if they have the same slope.

* Propositional or Statement Form

Ex. ① $\sim(p \vee q) \rightarrow p$

② $(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$ etc.

- A statement form has no fixed truth value
- It value depends on value assigned to its variables

Notations : Truth → "T" (1)

False → "F" (0)

—x—x—x—

Ex. ① using the following statements

p: Mohan is rich

q: Mohan is happy.

write the following statements in symbolic form

i> Mohan is rich but unhappy $(p \wedge \sim q)$

ii> Mohan is poor but happy $(\sim p \wedge q)$

iii> Mohan is neither rich nor happy $\sim(p \vee q)$ or $\sim p \wedge \sim q$

iv> Mohan is poor or he is both rich and unhappy

$\sim p \vee (p \wedge \sim q)$

Ex. ② p : Rajani is tall

q : Rajanis beautiful

write the following statements in symbolic form

i> Rajani is tall and beautiful $(p \wedge q)$

ii> Rajani is tall but not beautiful $(p \wedge \sim q)$

iii> It is false that Rajani is short or beautiful

$\sim(\sim p \vee q)$

iv> Rajani is tall or Rajani is short and beautiful

$p \vee (\sim p \wedge q)$

Ex. ③ p: I will study discrete mathematics
q: I will goto movie
r: I am in a good mood

Write the following statements in symbolic form

i> If I am not in a good mood, then I will go to a movie ($\sim r \rightarrow q$)

ii> I will not goto a movie & I will study discrete mathematics ($\sim q \wedge p$)

iii> I will goto a movie only if I will not study discrete mathematics ($q \rightarrow \sim p$)

iv> If I will not study discrete mathematics, then I am not in a good mood.
($\sim p \rightarrow \sim r$)

Ex. ④ Put the following statements into symbolic form

i> Whenever weather is nice, then only we will have a picnic

→

p : weather is nice
q : we will have a picnic

The statement is equivalent to " we will have a picnic only if the weather is nice"

$\therefore \quad q \rightarrow p$

ii> Program is readable only if it is well structured

→

p: program is readable
q: Program is well structured

$\therefore \quad p \rightarrow q$

iii> Unless he studies, he will fail in the exam.

→ p: He studies & q: He will fail in exam

" If he does not study, then he will fail in exam"

$\therefore \sim p \rightarrow q$

Ex. ⑤ p: I am bored

q: I am waiting for one hour

r: There is no bus

Translate the following into English

i> $(q \vee r) \rightarrow p$

"If I am waiting for one hour or there is no bus, then I get bored".

ii> $\sim q \rightarrow \sim p$

" If I am not waiting for one hour, then I am not bored".

iii> $(q \rightarrow p) \vee (r \rightarrow p)$

" If I am waiting for one hour then I am bored, or if there is no bus, then I am bored".

Ex. ⑥ Write the following statements in symbolic form

i> Gopal is intelligent and rich.

ii> Gopal is intelligent but not rich.

iii> Gopal is either intelligent or rich

→ let, p: Gopal is intelligent

q: Gopal is rich

i> $p \wedge q$

ii> $p \wedge \sim q$

iii> $p \vee q$

Write logical negations of above statements

i> $\sim (p \wedge q)$ OR $\sim p \vee \sim q$

ii> $\sim (p \wedge \sim q)$ OR $\sim p \vee q$

iii> $\sim (p \vee q)$ OR $\sim p \wedge \sim q$

* **Truth Tables**

A table giving all possible truth values of a statement form, corresponding to the truth values assigned to its variables, is called truth table.

- If a statement form consists of n-distinct variables, then the table will contains 2^n values
- Ex. $2^2 = 4$, $2^3 = 8$

p	~p
T	F
F	T

① Negation

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

② And

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

③ OR

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

④ conditional

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

⑤ Biconditional

p	q	p $\overline{\vee}$ q
T	T	F
T	F	T
F	T	T
F	F	F

⑥ Exclusive-OR

p	q	p × q
T	T	T
T	F	F
F	T	F
F	F	T

priority of operators

operator	Precedence
~	1
∧	2
∨	3
→	4
↔	5

Ex. 1 Construct the truth tables for the following statement form

i) $(\sim p \vee q) \rightarrow q$

ii) $\sim(p \wedge q) \vee (p \overset{\leftrightarrow}{\times} q)$

iii) $(\sim p \rightarrow r) \wedge (p \overset{\leftrightarrow}{\times} q)$

→ i)

p	q	$\sim p$	$\sim p \vee q$	$(\sim p \vee q) \rightarrow q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	F

ii)

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \times q$	$\sim(p \wedge q) \vee (p \times q)$
T	T	T	F	T	T
T	F	F	T	F	T
F	T	F	T	F	T
F	F	F	T	T	T

iii)

p	q	r	$\sim p$	$\sim p \rightarrow r$	$p \times q$	$(\sim p \rightarrow r) \wedge (p \times q)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

Ex. (2) If $p \to q$ is False, determine the truth value of $(\sim(p \wedge q)) \to q$

→

$p \to q$ is false when $p = T$ & $q = F$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$(\sim(p \wedge q)) \to q$
T	F	F	T	F

Ex. (3) If p and q are False propositions, find the truth value of $(p \vee q) \wedge (\sim p \vee \sim q)$

→

p	q	$p \vee q$	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$(p \vee q) \wedge (\sim p \vee \sim q)$
F	F	F	T	T	T	F

Ex. (4) If $p \to q$ is true, can we determine the truth value of $\sim p \vee (p \to q)$? Explain your answer.

→

p	q	$\sim p$	$p \to q$	$\sim p \vee (p \to q)$
T	T	F	T	T
F	T	T	T	T
F	F	T	T	T

Yes, it is possible to determine the truth value & its is "T".

Ex. (5) Let, p and q = T & r and s = F, find the truth values of following

i) $p \vee (q \wedge r)$ ii) $p \to (r \wedge s)$

→

i)

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	F	F	T

→

ii)

p	r	s	$r \wedge s$	$p \to (r \wedge s)$
T	F	F	F	F

Ex. ⑥ $p \to q$ is false, determine the truth value of:
$(\sim p \vee \sim q) \to q$

→

p	q	$\sim p$	$\sim q$	$(\sim p \vee \sim q)$	$(\sim p \vee \sim q) \to q$
T	F	F	T	T	F

Ex. ⑦ $p \to q$ is true, can you determine the value of
$\sim p \vee (p \leftrightarrow q)$

→

p	q	$\sim p$	$p \leftrightarrow q$	$\sim p \vee (p \leftrightarrow q)$
T	T	F	T	T
F	T	T	F	T
F	F	T	T	T

* Tautology

- A statement form is called Tautology if it always assums the truth value 'T' irrespective of truth values assigned to its variables.

Contradiction

- A statement form is called a contradiction if it always assums the truth value 'F' irrespective of the truth values assigned to its variables.

Contingency

- A statement form which is neither a tautology nor a contradiction is called a contingency.

Ex. ① $p \vee \sim p$ is tautology & $p \wedge \sim p$ is contradiction

→

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

Ex. ② $p \to p$ is tautology

p	p	$p \to p$
T	T	T
F	F	T

Ex. ③ construct truth tables to determine whether each of the following is a tautology, contradiction or contingency.

i> $p \to (q \to p)$

ii> $(p \land q) \to p$

iii> $(p \land q) \land \sim(p \lor q)$

iv> $(p \to q) \leftrightarrow (q \lor \sim p)$

⇒ i>

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Hence $p \to (q \to p)$ is tautology

ii>

p	q	$p \land q$	$(p \land q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Hence, $(p \land q) \to p$ is tautology

iii>

p	q	$p \land q$	$p \lor q$	$\sim(p \lor q)$	$(p \land q) \land \sim(p \lor q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Hence, $(p \land q) \land \sim(p \lor q)$ is contradiction

Ex. Show that $(p \wedge (p \to q)) \to q$ is a tautology, without using truth table.

→ we have to only show that,
$p \wedge (p \to q)$ is true, q is true since in the other cases $(p \wedge (p \to q)) \to q$ is anyway true.
Now, $p \wedge (p \to q)$ is T implies p is T and $p \to q$ is T.
These together means that q is T.
Hence, the required form is a tautology.

* **Equivalence of statement forms**

Two statement forms are logically equivalent if both have the same truth values, whatever may be the truth values assigned to the statements variable, occuring in both forms.

① **Idempotence**

p and $p \wedge p$ are logically equivalent.

p	$p \wedge p$
T	T
F	F

p and $p \to p$ are logically equivalent

② i> $p \wedge q$ and $q \wedge p$ are logically equivalent
ii> $p \vee q$ and $q \vee p$ are logically equivalent
iii> p and $\sim(\sim p)$ are logically equivalent

③ **Contrapositive** : $p \to q$ and $\sim q \to \sim p$ are equivalent

p	q	$p \to q$	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

④ Distributivity

$p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ equivalent

⑤ $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ equivalent

⑥ De Morgan's Law : $\sim(p \wedge q)$ and $\sim p \vee \sim q$

⑦ $\sim(p \vee q)$ and $\sim p \wedge \sim q$ equivalent

⑧ Elimination of conditional : $p \to q$ and $\sim p \vee q$

⑨ $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ which is logically equivalent to $(\sim p \vee q) \wedge (\sim q \vee p)$

* Logical Identities

1. $p \equiv p \vee p$
2. $p \equiv p \wedge p$
3. $p \vee q \equiv q \vee p$ } Commutative
4. $p \wedge q \equiv q \wedge p$
5. $p \vee (q \vee r) \equiv (p \vee q) \vee r$ } Associative
6. $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
7. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ } Distributivity
8. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
9. $p \equiv \sim(\sim p)$
10. $\sim(p \vee q) \equiv \sim p \wedge \sim q$ } De Morgans Law
11. $\sim(p \wedge q) \equiv \sim p \vee \sim q$
12. $p \vee \sim p \equiv$ Tautology
13. $p \wedge \sim p \equiv$ Contradiction
14. $p \vee (p \wedge q) \equiv p$
15. $p \wedge (p \vee q) \equiv p$

* Predicates -

① " x is tall and handsome".
② " $x + 3 = 5$ "
③ " $x + y \geq 10$ "

These sentences are not propositions, since they do not have truth value. However, if values are assigned to the variables, each of them becomes a proposition, which is either true or false.

- An assertion (sentence) that contains one or more variables is called a predicate; its truth value is predicted after assigning truth values to its variables.

Notations - $P(x_1, x_2, \dots x_n)$
where P is predicate containing n-variables $x_1, x_2 \dots x_n$ is called an n-place predicate.

Ex. ① " x is a city in India " $\longrightarrow$ $P(x)$
② " x is the father of y " $\longrightarrow$ $P(x, y)$
③ " $x + y \geq z$ " $\longrightarrow$ $P(x, y, z)$

- Here x, y, and z are variables or arguments.
- The value which the variables may assume constitute a collection / set called universe of disclosure.
- Value is then ~~bind~~ bound to the variable appearing in a predicate.
- A predicate becomes a proposition only when all its variables are bound.

Quantification

- method of binding individual variables in a predicate.
- The most common forms of quantification are universal and existential.

① Universal Quantifier

If $P(x)$ is a predicate with the individual variable x as argument, the the assertion "For all x, $P(x)$", which is interpreted as "for all values of x, the assertion $P(x)$ is true" is a statement in which the varsable x is said to be universally quantified.

Notation - $\forall$ "For all"/ "For every"/ "For each"

If $P(x)$ is true for every possible value of x, then $\forall x\, P(x)$ is true, otherwise false.

② Existential Quantifier

Suppose for the predicate $P(x)$, $\forall x\, P(x)$ is false, but there exists atleast one value of x for which $P(x)$ is true, then in this proposition, x is bound by existential quantification.

Notation - $\exists x\, P(x)$ means "there exist a value of x for which $P(x)$ is true".

Ex. ① Let $P(x)$: "$x+3=5$" predicate.

The proposition

$\exists x\, P(x)$ is true (by setting $x=2$)

but $\forall x\, P(x)$ is false.

Let, P(x, y) be two-place predicate

i> $\exists x \forall y\ P(x,y)$ – "There exists a value of x such that for all values of y, P(x,y) is true"

ii> $\forall y \exists x\ P(x,y)$ – "For each value of y, there exist an x, such that P(x,y) is true"

iii> $\exists x \exists y\ P(x,y)$ – "There exist a value of x & value of y such that P(x,y) is true"

iv> $\forall x \forall y\ P(x,y)$ – "For all values of x & y, P(x) is true"

* Negation of Quantified Statements

Ex. ① "All invited guests were present for dinner"

Negation – "All invited guests were not present for the dinner"

Let, x : x is guest

P(x) : x was present for dinner

$\therefore\ \forall x\ P(x)$ & negation is $\exists x\ (\sim P(x))$

Statement	Negation
$\forall x\ (P(x))$	$\exists x\ (\sim P(x))$
$\exists x\ (\sim P(x))$	$\forall x\ (P(x))$
$\forall x\ (\sim P(x))$	$\exists x\ (P(x))$
$\exists x\ P(x)$	$\forall x\ (\sim P(x))$

Ex. ① Negate the following in such a way that the symbol $\sim$ does not appear outside the square brackets

i> $\forall x\ [x^2 \geq 0]$

ii> $\exists x\ [x \cdot 2 = 1]$

iii> $\forall x \exists y\ [x + y = 1]$

Ans: i> $\exists x\ [x^2 < 0]$

ii> $\forall x\ [x \cdot 2 \neq 1]$

iii> $\exists x \forall y\ [x + y \neq 1]$

Ex. ② Transcribe the following into logical notation.
Let the universe of disclosure be the real no.

i> for any value of x, x^2 is non-negative.

ii> for every value of x, there is some value of y such that $x \cdot y = 1$

iii> For every values of x, there is some value of y such that $x - y = 1$.

iv> There are positive values of x & y, such that $x \cdot y > 0$

v> There is value of x such that if y is positive, then $x+y$ is negative

⇒

i> $\forall x \, [x^2 \geq 0]$

ii> $\forall x \, \exists y \, [x \cdot y = 1]$

iii> $\forall x \, \exists y \, [x - y = 1]$

iv> $\exists x \, \exists y \, [(x > 0) \wedge (y > 0) \wedge (x \cdot y > 0)]$

v> $\exists x \, \forall y \, [(y > 0) \rightarrow (x + y < 0)]$

Ex ③ For the universe of all integers, let $P(x)$, $Q(x)$, $R(x)$, $S(x)$ and $T(x)$ be the following statements:

$P(x)$: $x \geq 0$

$Q(x)$: x is even

$R(x)$: x is a perfect square

$S(x)$: x is divisible by 4

$T(x)$: x is divisible by 5

Write the following statements in symbolic forms:

- i> Atleast one integer is even
- ii> There exists a positive integer that is even
- iii> If x is even, then x is not divisible by 5.
- iv> No even integer is divisible by 5
- v> There exists an even integer divisible by 5
- vi> If x is even & x is a perfect square, then x is divisible by 4

$\Rightarrow$ i> $\exists x \; Q(x)$

ii> $\exists x \; [P(x) \wedge Q(x)]$

iii> $\forall x \; [Q(x) \rightarrow \sim T(x)]$

iv> $\forall x \; [Q(x) \rightarrow \sim T(x)]$

v> $\exists x \; [Q(x) \wedge T(x)]$

vi> $\forall x \; [Q(x) \wedge R(x) \rightarrow S(x)]$

Ex. 4 Write the following statements in symbolic form, using quantifiers

i> All students have taken a course in communication skills.

ii> There is a girl student in the class, who is also a sport person.

iii> Some students are intelligent, but not hardworking

$\Rightarrow$

Soln. i> let, $P(x)$: student x has taken a course in communication skills

$\therefore \; \forall x \; P(x)$

ii> let, $P(x)$: x is a student

$Q(x)$: x is a girl

$R(x)$: x is sport person

$\therefore \; \exists x \; [P(x) \wedge Q(x) \wedge R(x)]$

iii> Let, $P(x)$: x is intelligent

$Q(x)$: x is hardworking

$\therefore \; \exists x \; [P(x) \wedge \sim Q(x)]$

Ex. 5 Negate each of the following

i> $\forall x, \; |x| = x$

ii> $\exists x, \; x^2 = x$

$\Rightarrow$

i> $\exists x, \; |x| \neq x$

ii> $\forall x, \; x^2 \neq x$