

## Unit 2: NFA with $\epsilon$ -transitions

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(null moves)  
It is an NFA including transitions on empty input i.e ' $\epsilon$ ' (epsilon).

- Formal Definition:

NFA with null moves is denoted by a five-tuple  $(Q, \Sigma, \delta, q_0, F)$

$Q$  : Finite set of states

$\Sigma$  = finite input alphabet

$q_0$  = Initial state contained in  $Q$

$F$  = Set of Final states

i.e  $F \subseteq Q$

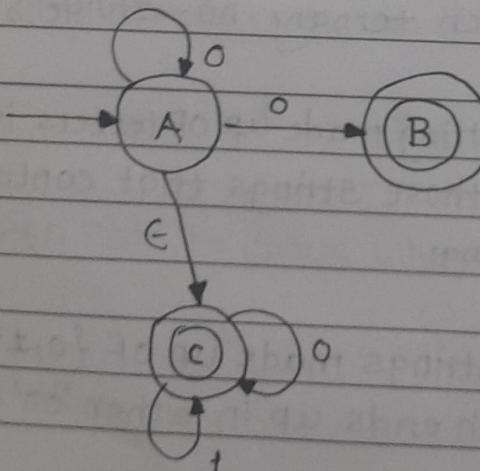
$\delta$  = state fun mapping from

$Q \times (\Sigma \cup \{\epsilon\})$  to  $2^Q$

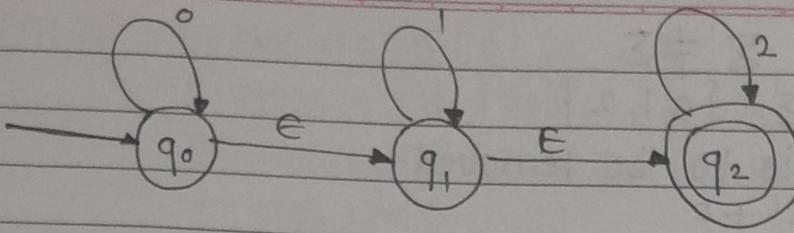
i.e  $Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$

Ex:-

The foll<sup>n</sup> fig. shows that there is one transition from state 'A' to state 'C'.

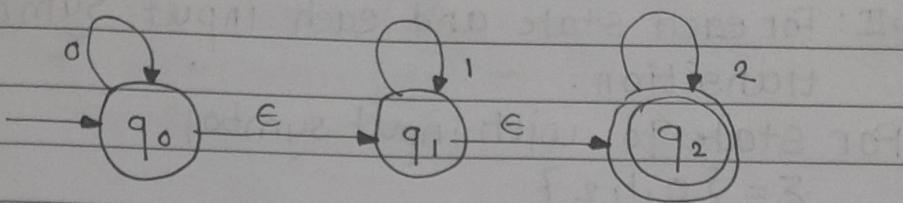


In the above <sup>below</sup> fig. there is Null-transition from state  $q_0 \rightarrow q_1$  and from state  $q_1 \rightarrow q_2$



The meaning of Null transition means transition of empty input of length '0' (zero).

Construction of NFA with  $\epsilon$ -moves to NFA without  $\epsilon$ -moves to equivalent NFA without Null-moves.



$\epsilon$ -closure ( $q_0$ )

$\epsilon$ -closure ( $q_1$ )

$\epsilon$ -closure ( $q_2$ )

$\epsilon$ -closure of state is define as, set of states 'X' such that, there is path from present state to 'X' having label ' $\epsilon$ '

$\epsilon$ -closure ( $q_0$ ) = all states with zero length from given state ' $q_0$ '.

$$= \{q_0, q_1, q_2\}$$

$\epsilon$ -closure ( $q_1$ ) =  $\{q_1, q_2\}$

$\epsilon$ -closure ( $q_2$ ) =  $\{q_2\}$ .

Ex. Given NFA with  $\epsilon$ -moves is  $M = (Q, \Sigma, q_0, f)$  where

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1, 2\}$$

$q_0$  = is initial state

$$f = \{q_2\}.$$

Step 1: Assume Equivalent NFA without  $\epsilon$ -moves

$$M' = (Q', \Sigma', q_0', f', d')$$

$$\text{Where } Q' = Q = \{q_0, q_1, q_2\}$$

Rule for finding state function  $\delta'$  of required NFA  
 is  $\delta'(q, a) = \epsilon\text{-closure}(\delta(\delta^T(q, \epsilon), a))$   
 $\delta'(q, \epsilon) = \epsilon\text{-closure}(q)$

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$$\Sigma' \neq \Sigma$$

i.e.  $\Sigma' = \{0, 1, 2\}$

' $\epsilon$ ' is to be removed.

$$F' \neq F$$

i.e. (calculated later)

$$Q = Q$$

$$\delta' = Q \times \Sigma' \rightarrow 2^Q$$

Step II: For each state and each input symbol find transition.

For state  $q_0$  with input symbol

$$\Sigma' = \{0, 1, 2\}$$

$$\delta'(q_0, 0) = \epsilon\text{-closure}(\delta(\delta^T(q_0, \epsilon), 0))$$

$$\begin{aligned} ① \quad \delta'(q_0, 0) &= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_0, \epsilon), 0)) \\ &= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 0)) \\ &= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)) \\ &= \epsilon\text{-closure}(q_0 \cup q_1 \cup q_2) \\ &\approx \epsilon\text{-closure}(q_0) \\ \delta'(q_0, 0) &= q_0, q_1, q_2 \end{aligned}$$

$$\begin{aligned} ② \quad \delta'(q_0, 1) &= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_0, \epsilon), 1)) \\ &= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 1)) \\ &= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)) \\ &= \epsilon\text{-closure}(q_0 \cup q_1 \cup q_2) \\ &\approx \epsilon\text{-closure}(q_1) \\ \delta'(q_0, 1) &= q_1, q_2 \end{aligned}$$

$$\begin{aligned} ③ \quad \delta'(q_0, 2) &= \epsilon\text{-closure}(\delta(\delta^{\wedge}(q_0, \epsilon), 2)) \\ &= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 2)) \\ &= \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2)) \\ &= \epsilon\text{-closure}(q_0 \cup q_1 \cup q_2) \\ &\approx \epsilon\text{-closure}(q_2) \\ \delta'(q_0, 2) &= q_2 \end{aligned}$$

$$\begin{aligned}
 ④ \quad \delta(q_1, 0) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_1, \epsilon), 0)) \\
 &= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 0)) \\
 &= \epsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_2, 0)) \\
 &= \epsilon\text{-closure}(\{\}) \\
 &= \emptyset
 \end{aligned}$$

$$\boxed{\delta(q_1, 0) = \emptyset}$$

$$\begin{aligned}
 ⑤ \quad \delta(q_1, 1) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_1, \epsilon), 1)) \\
 &= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 1)) \\
 &\rightarrow \epsilon\text{-closure}(q_1 \cup \{q_2\}) \\
 &= \epsilon\text{-closure}(q_1) \\
 &= q_1, q_2.
 \end{aligned}$$

$$\begin{aligned}
 ⑥ \quad \delta(q_2, 1) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_2, \epsilon), 1)) \\
 &= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 1)) \\
 &= \epsilon\text{-closure}(q_2) \\
 &= q_2.
 \end{aligned}$$

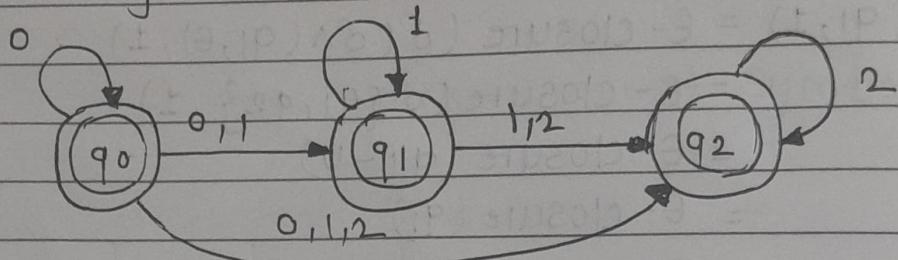
$$\begin{aligned}
 ⑦ \quad \delta(q_2, 0) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_2, \epsilon), 0)) \\
 &= \epsilon\text{-closure}(q_2, 0) \\
 &= q_2
 \end{aligned}$$

$$\begin{aligned}
 ⑧ \quad \delta(q_2, 1) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_2, \epsilon), 1)) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 ⑨ \quad \delta(q_2, 2) &= \epsilon\text{-closure}(\delta(\delta^\wedge(q_2, \epsilon), 2)) \\
 &= q_2
 \end{aligned}$$

$\Sigma$	0	1	2
Q	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$q_0$	$\emptyset$	$\{q_1, q_2\}$	$\{q_2\}$
$q_1$	$\emptyset$	$\{q_1, q_2\}$	$\{q_2\}$
$q_2$	$\{q_2\}$	$\emptyset$	$\{q_2\}$

Transition Diagram:



In above, transition Diag.  $q_0$  is final state because  $\epsilon$ -closure ( $q_0$ ) contains  $q_2$  which is the final state of given NFA with  $\epsilon$ -moves.

$q_1$  is also final state because  $\epsilon$ -closure ( $q_1$ ) contains  $q_2$  which is final state of given NFA with  $\epsilon$ -moves. Similarly  $q_2$  is also final state as per given NFA with  $\epsilon$ -moves.

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Conversion of NFA to DFA:

Q.1.

Convert NFA  $M = (\{q_0, q_1\}, \{0, 1\} \delta, \{q_0\}, \{q_1\})$  where  $\delta$  is shown in below table. Equivalent to DFA.

Q	$\Sigma$	0	1
$q_0$	$\{q_0, q_1\}$	$\{q_1\}$	
$q_1$	$\emptyset$	$\{q_0, q_1\}$	

From given

$$\textcircled{1} \quad A = q_0$$

$$\textcircled{2} \quad \delta(A, 0) = \delta(q_0, 0) \\ = q_0, q_1 - B$$

$$\textcircled{3} \quad \delta(A, 1) = \delta(q_0, 1) \\ = q_1 - C$$

$$\textcircled{4} \quad \delta(B, 0) = \delta(\{q_0, q_1\}, 0) \\ = \delta(q_0, 0) \cup \delta(q_1, 0) \\ = q_0, q_1 \cup \emptyset \\ = B.$$

$$\textcircled{5} \quad \delta(B, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) \\ = q_1 \cup \{q_0, q_1\} \\ = q_0 q_1.$$

$$\textcircled{6} \quad \delta(C, 0) = \delta(q_1, 0) \\ = \emptyset$$

$$\textcircled{7} \quad \delta(C, 1) = \delta(q_1, 1) \\ = q_0, q_1 - B.$$

	Z	0	1
Q			
A	B	C	
B	B	B	
C	$\emptyset$	B	

$\Sigma$	$\delta$	$\delta$
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_1\}$
$\{q_1\}$	$\emptyset$	$\{q_0, q_1\}$

Q.2 Convert NFA  $M = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$   
 Where  $\delta$  is shown in below table to an equivalent DFA.

$\Sigma$	a	b
$q_0$	$\{q_0, q_1\}$	$\{q_2\}$
$q_1$	$\{q_0\}$	$\{q_1\}$
$q_2$	-	$\{q_0, q_1\}$

Let DFA =  $(\{A, B, C, \dots, F\}, \{0, 1\}, \delta, A, \{H\})$

From given

$$\textcircled{1} \quad A = q_0.$$

$$\textcircled{2} \quad \delta(A, a) = \delta(q_0, a) \\ = \{q_0, q_1\} \rightarrow B$$

$$\textcircled{3} \quad \delta(A, b) = \delta(q_0, b) \\ = q_2 \rightarrow C$$

$$\textcircled{4} \quad \delta(B, a) = \delta(q_0, q_1), a \\ = \delta(q_0, a) \cup \delta(q_1, a) \\ = q_0, q_1 \cup q_0 \\ = q_0 q_1 \rightarrow B$$

$$\begin{aligned}
 ⑤ \quad d(B, b) &= d(q_0, q_1), b \\
 &= d(q_0, b) \cup d(q_1, b) \\
 &= q_2 \cup q_1 \\
 &= q_2 q_1 - D
 \end{aligned}$$

$$\begin{aligned}
 ⑥ \quad d(C, a) &= d(q_2, q) \\
 &= -
 \end{aligned}$$

$$\begin{aligned}
 ⑦ \quad d(C, b) &= d(q_2, b) \\
 &= q_0 q_1 - B
 \end{aligned}$$

$$\begin{aligned}
 ⑧ \quad d(D, a) &= d(q_2, q_1), a \\
 &= d(q_2, a) \cup d(q_1, a) \\
 &= - \cup q_0 \\
 &= q_0 \\
 &= A
 \end{aligned}$$

$$\begin{aligned}
 ⑨ \quad d(D, b) &= d(q_2, q_1), b \\
 &= q_0 q_1 \cup q_1 \\
 &= q_0 q_1 - B
 \end{aligned}$$

$\Sigma$	a	b	$\Sigma$	a	b
S			S		
A	B	CPU	{q0}	{q0q1}	{q2}
B	AB	D	{q0, q1}	{q0, q1}	{q2, q1}
C	-	B	{q2}	-	{q0q1}
D	<del>q0A</del>	B	{q2q1}	{q0}	{q0q1}

Q. Construct DFA equivalent to  $M = (\{q_0, q_1, q_2, q_3\}, \{q, b\}, \delta, q_0, \{q_3\})$   
 Where  $\delta$  is given below table.

$s \setminus \Sigma$	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$q_0$
$\{q_1\}$	$\{q_2\}$	$\{q_1\}$
$\{q_2\}$	$\{q_3\}$	$\{q_3\}$
$\{q_3\}$	-	$\{q_2\}$

Let DFA =  $(\{A, B, C, \dots, F\}, \{q, b\}, \delta, A, \{F\})$ .

Soln: from given

$$\textcircled{1} \quad A = q_0$$

$$\textcircled{2} \quad \delta(A, a) = \delta(q_0, a) \\ = q_0 q_1 - B$$

$$\textcircled{3} \quad \delta(A, b) = \delta(q_0, b) \\ = q_0 - A$$

$$\textcircled{4} \quad \delta(B, a) = \delta(q_0, q_1), a \\ = \delta(q_0, a) \cup \delta(q_1, a) \\ = q_0 q_1 \cup q_2 \\ = q_0 q_1 q_2 - C$$

$$\textcircled{5} \quad \delta(B, b) = \delta(q_0, q_1), b \\ = \delta(q_0, b) \cup \delta(q_1, b) \\ = q_0 \cup q_1 \\ = q_0 q_1 - B$$

$$\begin{aligned}
 ⑥ \quad d(c, q) &= d(q_0 q_1 q_2, q) \\
 &= d(q_0, q) \cup d(q_1, q) \cup d(q_2, q) \\
 &= q_0 \cup q_1 \cup q_2 \cup q_3 \\
 &= q_0, q_1, q_2, q_3 — D
 \end{aligned}$$

$$\begin{aligned}
 ⑦ \quad d(c, b) &= d(q_0 q_1 q_2, b) \\
 &= d(q_0, b) \cup d(q_1, b) \cup d(q_2, b) \\
 &= q_0 \cup q_1 \cup q_3 \\
 &= q_0 q_1 q_3 — E
 \end{aligned}$$

$$\begin{aligned}
 ⑧ \quad d(D, q) &= d(q_0, q_1, q_2, q_3, a) \\
 &= d(q_0, q) \cup d(q_1, q) \cup d(q_2, q) \cup d(q_3, q) \\
 &= q_0 \cup q_1 \cup q_2 \cup q_3 — \\
 &= D
 \end{aligned}$$

$$\begin{aligned}
 ⑨ \quad d(D, b) &= d(q_0, q_1, q_2, q_3, b) \\
 &= d(q_0, b) \cup d(q_1, b) \cup d(q_2, b) \cup d(q_3, b) \\
 &= q_0 \cup q_1 \cup q_3 \cup q_2 \\
 &= D
 \end{aligned}$$

$$\begin{aligned}
 ⑩ \quad d(E, q) &= d(q_0, q_1, q_3, q) \\
 &= d(q_0, q) \cup d(q_1, q) \cup d(q_3, q) \\
 &= q_0 \cup q_1 \cup q_2 \\
 &= q_0 q_1 q_2 — C
 \end{aligned}$$

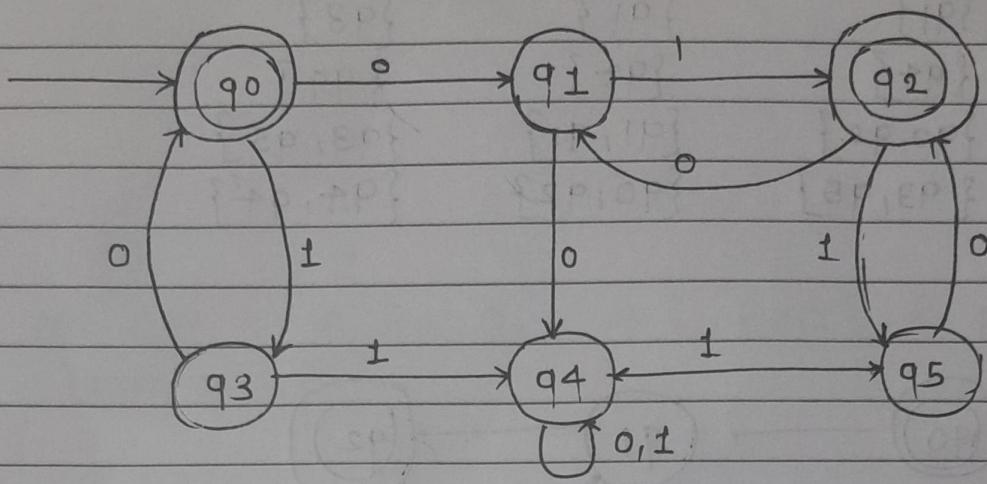
$$\begin{aligned}
 ⑪ \quad d(E, b) &= d(q_0, q_1, q_3, b) \\
 &= d(q_0, b) \cup d(q_1, b) \cup d(q_3, b) \\
 &= q_0 \cup q_1 \cup q_2 \\
 &= C.
 \end{aligned}$$

$\Sigma$	a	b
s		
A	B	A
B	C	B
C	D	E
D	D	D
E	C	C

$\Sigma$	a	b
s		
{90}	{9091}	{90}
{90, 91}	{909192}	{9091}
{9091, 92}	{90919293}	{909193}
{90, 91, 92, 93}	{90919293}	{90919293}
{90, 91, 93}	{909192}	{909192}

## Minimization of FSM:

Q. Construct minimum state automaton described by following Transition Diagram.



Soln:

$\Sigma$	0	1
q0	q1	q3
q1	q4	q2
q2	q1	q5
q3	q0	q4
q4	q4	q4
q5	q2	q4

$$\Pi_0 = \{ \{q_0, q_2\}, \{q_1, q_3, q_4, q_5\} \}$$

$$\Pi_1 = \{ \{q_0, q_2\}, \{q_1\}, \{q_3, q_5, q_4\} \}$$

$$\Pi_2 = \{ \{q_0, q_2\}, \{q_1\}, \{q_3, q_5\}, \{q_4\} \}$$

Q. Convert NFA ( $\{p, q, r, s\}$ ,  $\{0, 1\}$ ,  $\delta$ ,  $p$ ,  $\{s\}$ ) to its equivalent DFA where  $\delta$  is shown in below table.

$Q/E$	0	1
p	p, q	p
q	r	r
r	s	-
s	s	s

Let DFA =  $\{A, B, C, D, \dots, F\}$ ,  $\{0, 1\}$ ,  $\delta$ ,  $p$ ,  $\{s\}\}$ .

From given

$$\textcircled{1} A = \bigoplus p.$$

$$\textcircled{2} \delta(A, 0) = \delta(p, 0)$$

$$= p, q \rightarrow B$$

$$\textcircled{3} \delta(A, 1) = \delta(p, 1)$$

$$= p \rightarrow A$$

$$\textcircled{4} \delta(B, 0) = \delta((p, q), 0)$$

$$= \delta(p, 0) \cup \delta(q, 0)$$

$$= p, q \cup r$$

$$= p, q, r \rightarrow C$$

$$\textcircled{5} \delta(B, 1) = \delta((p, q), 1)$$

$$= \delta(p, 1) \cup \delta(q, 1)$$

$$= p \cup r$$

$$= p, r \rightarrow D$$

$$\textcircled{6} \delta(C, 0) = \delta((p, q, r), 0)$$

$$= \delta(p, 0) \cup \delta(q, 0) \cup \delta(r, 0)$$

$$= p, q \cup r \cup s$$

$$= p, q, r, s \rightarrow E$$

$$\begin{aligned}
 7) \quad \delta(C_{11}) &= \delta((p, q, r)_{11}) \\
 &= \delta(p, 1) \cup \delta(q, 1) \cup \delta(r, 1) \\
 &= p \cup r - \\
 &= p, r - D
 \end{aligned}$$

$$\begin{aligned}
 8) \quad \delta(D_0) &= \delta((p, r)_0) \\
 &= \delta(p, 0) \cup \delta(r, 0) \\
 &= p, q \cup S \\
 &= p, q, s - E
 \end{aligned}$$

$$\begin{aligned}
 9) \quad \delta(D_1) &= \delta((p, r)_{11}) \\
 &= \delta(p, 1) \cup \delta(r, 1) \\
 &= p \cup - \\
 &= p - A
 \end{aligned}$$

$$\begin{aligned}
 10) \quad \delta(E_0) &= \delta((p, q, r, s)_0) \\
 &= \delta(p, 0) \cup \delta(q, 0) \cup \delta(r, 0) \cup \delta(s, 0) \\
 &= p, q, r \cup S \cup S \\
 &= p, q, r, s - E
 \end{aligned}$$

$$\begin{aligned}
 11) \quad \delta(E_1) &= \delta((p, q, r, s), 1) \\
 &= \delta(p, 1) \cup \delta(q, 1) \cup \delta(r, 1) \cup \delta(s, 1) \\
 &= p, r, s - G
 \end{aligned}$$

$$12) \quad \delta(F_0) = \delta(S_0) = \phi$$

$\delta$	$Q/e$	$0$	$1$	$Q/e$	$0$	$1$
A	B	C		$\{p\}$	$\{q, r\}$	$\{q\}$
B	D	E		$\{q, r\}$	$\{r, s\}$	$\{r\}$
C	E	B		$\{q\}$	$\{r\}$	$\{q, r\}$
D	F	A		$\{r, s\}$	$\{s\}$	$\{p\}$
E	F	A		$\{r\}$	$\{s\}$	$\{p\}$
F	$\phi$	A		$\{s\}$	-	$\{p\}$

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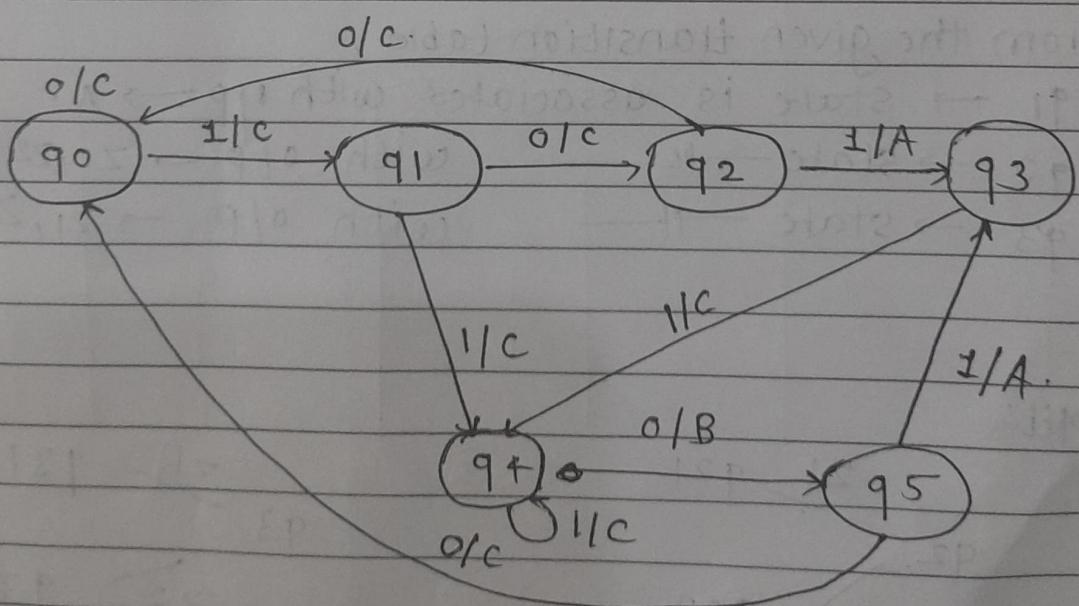
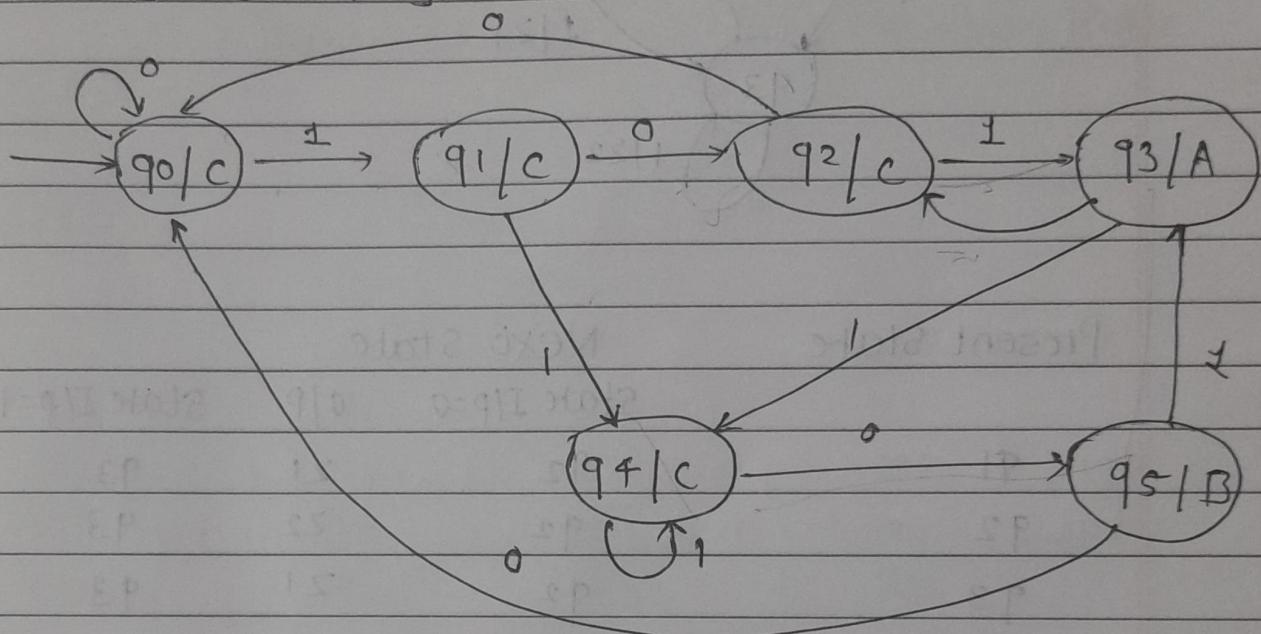
Design Moore and Mealy for binary input sequence if it end with 101 outputs A and if ends with 110 output B otherwise C.

Soln:

In above given problem

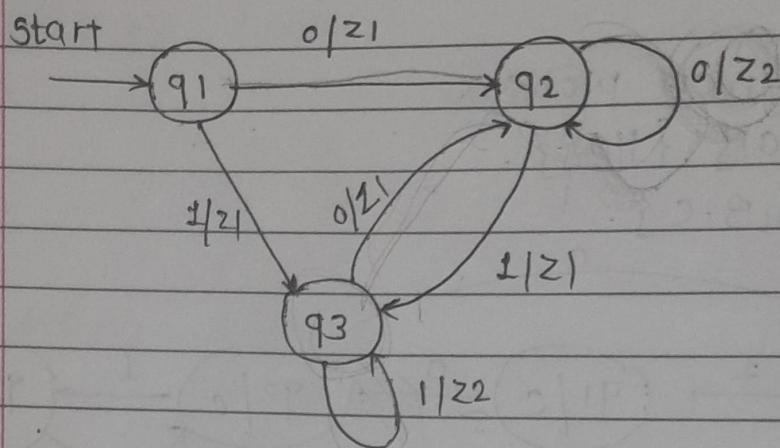
$$\Sigma = \{ \{101\}, \{110\} \}$$

$$\Delta = \{A, B, C\}$$



Mealy Diagram.

Q. Construct moore machine equivalent to Mealy machine given below.



Present State

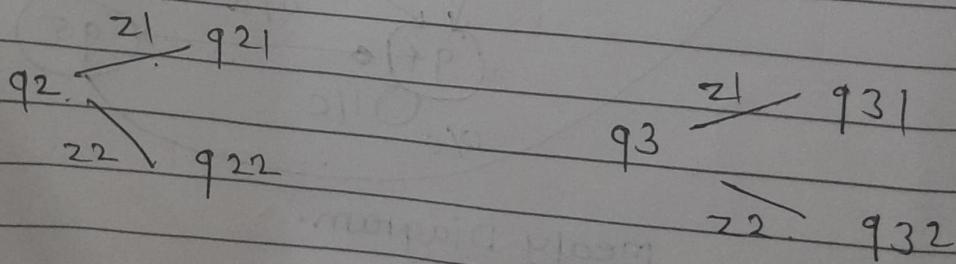
Next State

	State I/p=0	O/P	State I/p=1	O/P
q1	q2	z1	q3	z1
q2	q2	z2	q3	z1
q3	q2	z1	q3	z2

From the given transition table,

 $q_1 \rightarrow$  state is associated with  $0/p \rightarrow \lambda$ . $q_2 \rightarrow$  state  $\text{--} p \text{--}$ with  $0/p \rightarrow z_1, z_2$  $q_3 \rightarrow$  state  $\text{--} p \text{--}$ with  $0/p \rightarrow z_1, z_2$ 

Split:



Present State

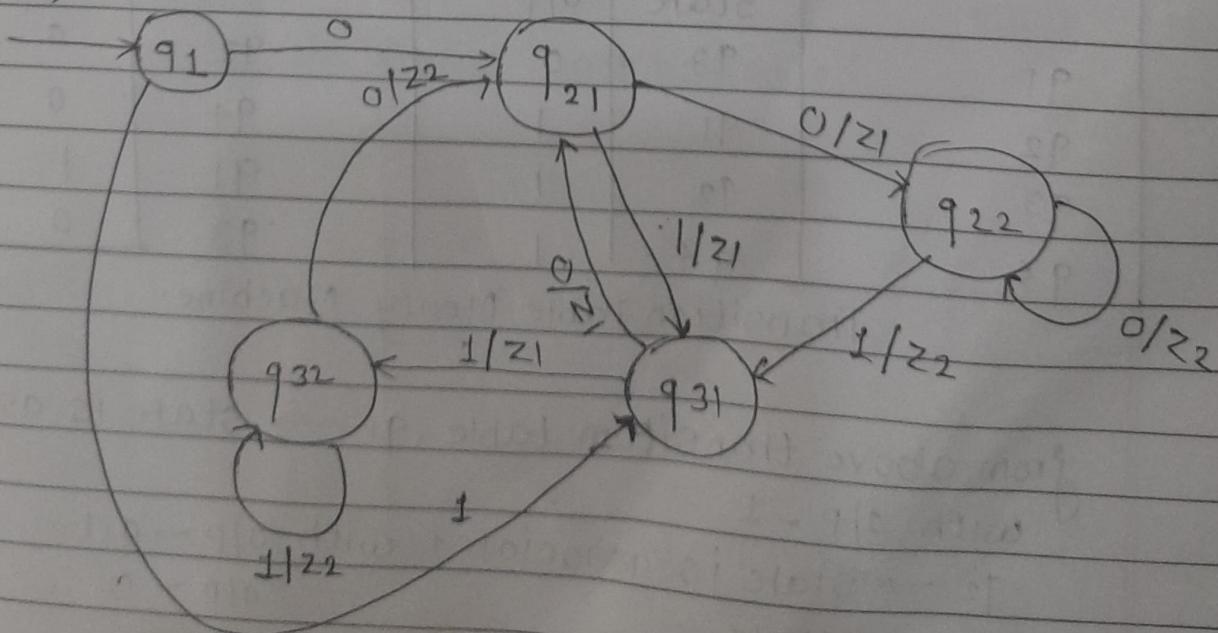
	$v/p = 0$	$o/p$	state $i/p = 1$	$o/p$
q1	q21	z1	q31	z1
q21	q22	z2	q31	z1
q22	q22	z2	q31	z1
q31	q21	z1	q32	z2
q32	q21	z1	q32	z2

Table: Revised Mealy M/C Transition Table

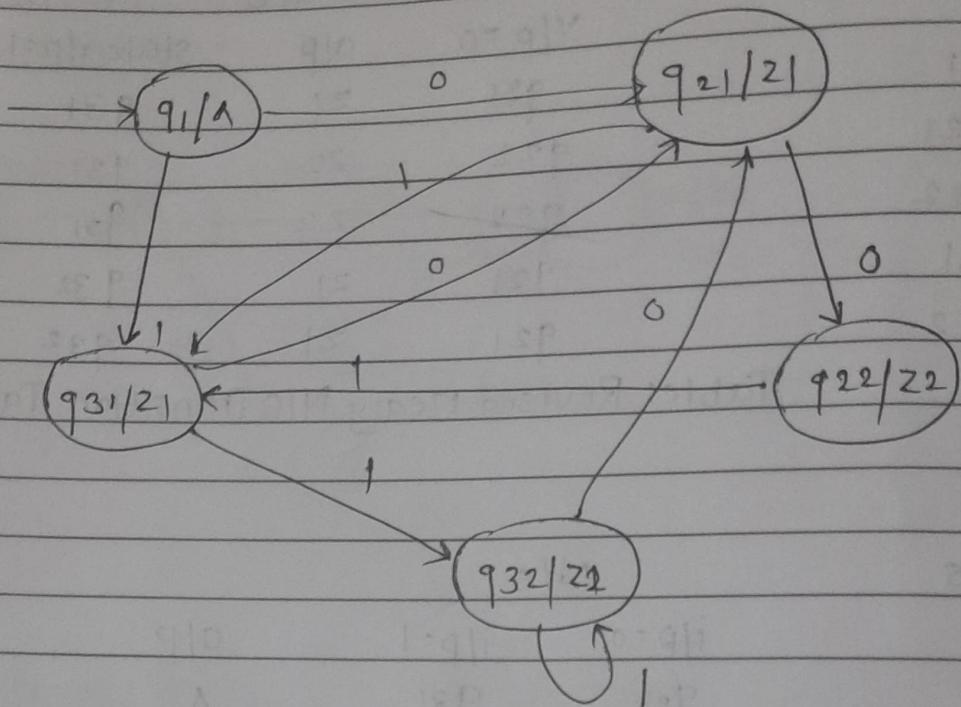
P/S

N/S

	$i/p = 0$	$i/p = 1$	$o/p$
q1	q21	q31	1
q21	q22	q31	z1
q22	q22	q31	z2
q31	q21	q32	z1
q32	q21	q32	z2



Moore Diagram.



Q.

Consider Mealy machine describe by given transition table. consider moore machine which eq. to mealy machine

Present state	Next state			
	ip = 0		ip = 1	
	state	o/p	state	o/p
q1	q3	0	q2	0
q2	q1	1	q4	0
q3	q2	1	q1	1
q4	q4	1	q3	0

Transition Table Mealy Machine.

from above transition table,  $q_1 \rightarrow$  state is associated with o/p - 1

$q_2 \rightarrow$  state is associated with o/p - 0, 1

$q_3 \rightarrow$  state

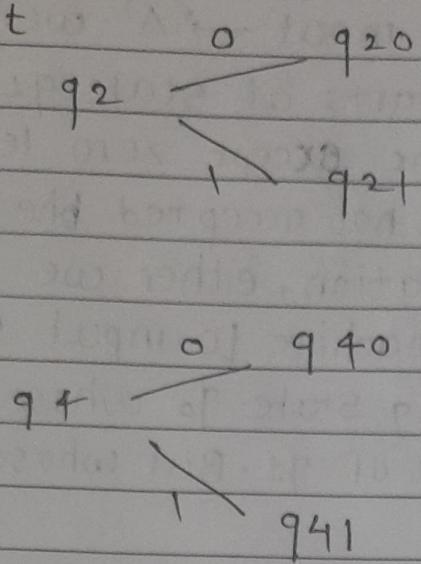
$q_4$

o/p - 0

o/p - 0, 1

As  $q_2$  &  $q_4$  state having two outputs 0 & 1.

∴ Split



Present state

Next state

	I/P = 0	I/P = 1	O/P	I/P = 0	I/P = 1	O/P
$q_1$	$q_3$	$q_1$	0	$q_{20}$	$q_{40}$	0
$q_{20}$	$q_1$	$q_1$	1	$q_{40}$	$q_1$	0
$\cancel{q_{20}}$	$q_{21}$	$q_1$	1	$q_1$	$q_1$	1
$q_{21}$	$q_1$	$q_1$	1	$q_{40}$	$q_3$	0
$q_{40}$	$q_{41}$	$q_1$	1	$q_3$	$q_3$	0
$q_{41}$	$q_{41}$	$q_1$	1	$q_3$	$q_3$	0

Revised Mealy Machine Table:

P/S

N/S

	I/P = 0	I/P = 1	O/P
$q_1$	$q_3$	$q_{20}$	1
$q_{20}$	$q_1$	$q_{40}$	0
$q_{21}$	$q_1$	$q_{40}$	1
$q_3$	$q_{21}$	$q_1$	0
$q_{40}$	$q_{41}$	$q_3$	0
$q_{41}$	$q_{41}$	$q_3$	1

Trans. Table Moore M/C.

In above table of Moore machine, the initial state  $q_1$  is associated with output  $\rightarrow 1$ . this means that with input  $\rightarrow '1'$  we get an output of 1. if the machine starts at state  $q_1$ .

Thus, this moore machine except zero length sequence (Null sequence) which is not accepted by the mealy machine. To overcome this situation, either we must neglect the response of moore machine to input ' $1$ '. or we must add new starting state  $q_0$  whose state transition are identical with those of  $q_1$ . But whose output is 0.

Ps

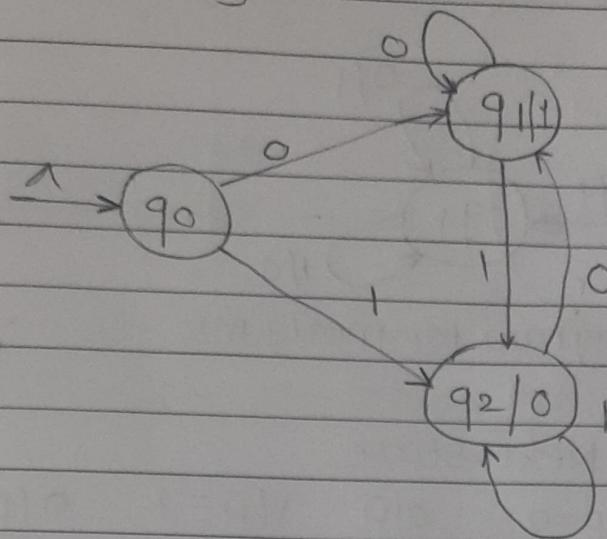
Ns

	$i/p=0$	$i/p=1$	$o/p$
$q_0$	$q_3$	$q_{20}$	0
$q_1$	$q_3$	$q_{20}$	1
$q_{20}$	$q_1$	$q_{40}$	0
$q_{21}$	$q_1$	$q_{40}$	1
$q_3$	$q_{21}$	$q_1$	0
$q_{40}$	$q_{41}$	$q_3$	0
$q_{41}$	$q_{41}$	$q_3$	1

Revised Moore Machine Diagram.

Q.

Design moore machine generate one's complement of given binary number.



Transition Diagram for moore m/c.

NS

PS	i/p=0	i/p=1	o/p
q0	q1	q2	0
q1	q1	q2	1
q2	q1	q2	0

Transition for moore m/c.

For ex. take binary i/p as 101

state      i/p                  Next state            o/p  
 $q_0 \rightarrow 1 \rightarrow q_2 \rightarrow 0$

$q_2 \xleftarrow{1} \xrightarrow{0} q_1 \xrightarrow{1} 1$

$q_1 \xleftarrow{0} \xrightarrow{1} q_2 \xrightarrow{0}$

$q_0 \rightarrow 1 \rightarrow q_2 \rightarrow 0$        $q_1 \rightarrow 1 \rightarrow q_2 \rightarrow 0$

$q_2 \rightarrow 0 \rightarrow q_1 \rightarrow 1$

Q.

Design mealy machine 2's complement of given binary number.

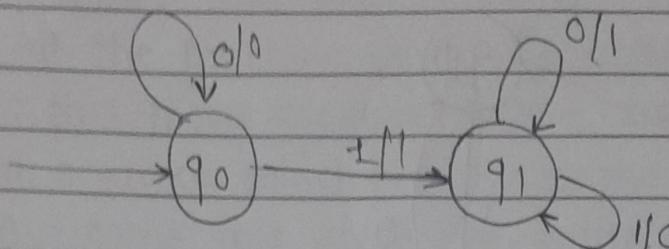


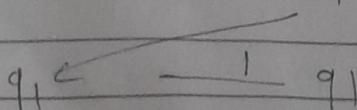
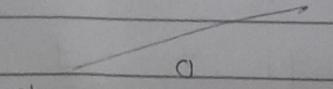
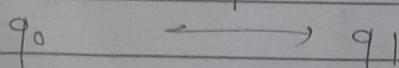
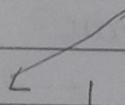
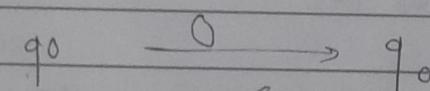
Fig. Tran. diagram for mealy m/c

Present state	Next state			
	$i/p=0$	$i/p$	$i/p=1$	$i/p$
$q_0$	$q_0$	$0$	$q_1$	$1$
$q_1$	$q_1$	$1$	$q_0$	$0$

Fig. Transition Table for mealy m/c

For ex. take binary  $i/p$  as 1010

PS



Read it from  
bottom to top.

⇒ 0110.

Q. Design Mealy machine one's complement of given binary no.

Soln:

Q. Design Moore machine to determine the residue (Remainder) mod 3 for binary number.

Q. Consider moore machine equivalent mealy machine define by the below table.

PS	NS	$a=0$		$a=1$	
		state	o/p	state	o/p
→ 91	91	91	1	92	0
92	94	94	1	94	1
93	92	92	1	93	1
94	93	93	0	91	1

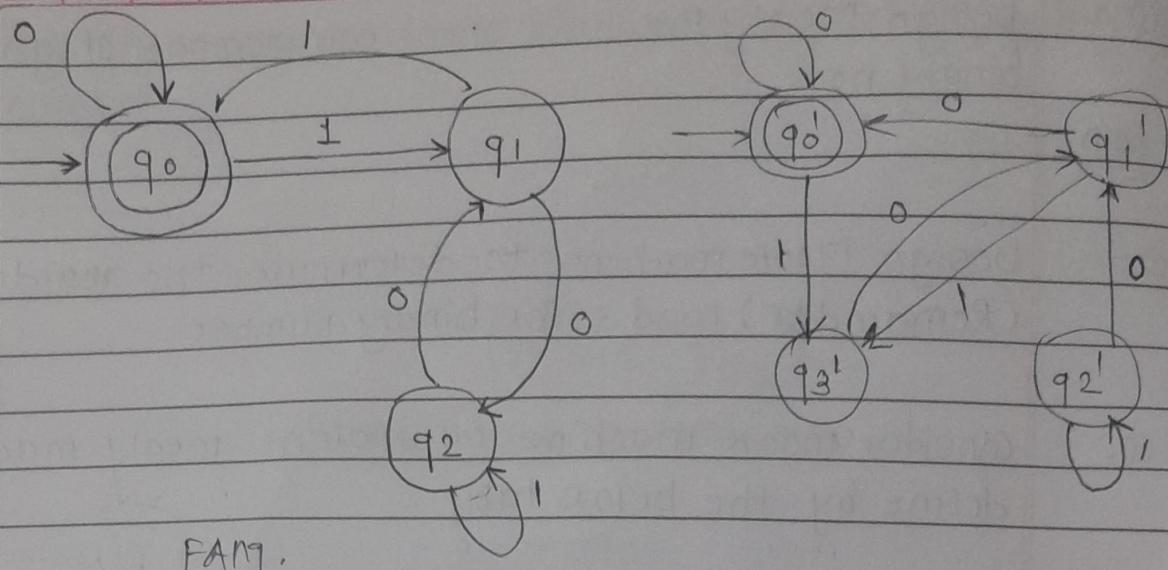
Q. Explain equivalence of two FSM.

OR.

~~Page no. 158~~ Explain Moore's Algorithm for FSM equivalence.

5.13 & 5.16.

Q. Consider the following two DFA  $M$  and  $M'$  over  $\{0,1\}$ . Given in below fig. Determine whether  $M$  and  $M'$  are equivalent.



For given FA's  $\Sigma = \{0, 1\}$  i.e  $|\Sigma| = 2 \neq n$ .

$\therefore$  It is necessary to create  $n+1$  columns labelled so  $n+1 = 2+1 = 3$ .

$\therefore$  We have to create three columns labelled  $(V, V')$ ,  $(V_0, V_0')$  &  $(V_1, V_1')$ .

Below table shows comparisons betn M and M'

	0	1
$(V, V')$	$(V_0, V_0')$	$(V_1, V_1')$
$(q_0, q_0')$	$(q_0, q_0')$	$(q_1, q_3')$
$(q_1, q_3')$	$(q_2, q_1')$	$(q_0, q_2')$

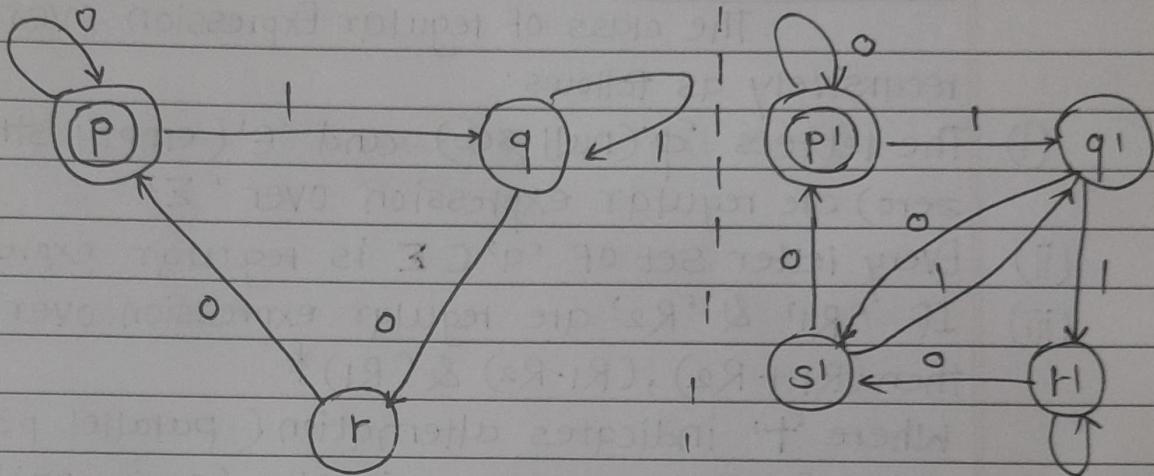
As we start with  $(q_0, q_0')$  now on 0 input symbol it goes to  $(q_0, q_0')$  where these are final states and on 1 input symbol it goes  $(q_1, q_3')$  both are non-final state.

$\therefore$  We can proceed for new column which is not there i.e  $(q_1, q_3')$  place it in the column 1, repeat the procedure for state  $(q_1, q_3')$  For input symbol '0' it goes to  $(q_2, q_1')$  both are non-final state.

But for input symbol '1'  $(q_1, q_3')$  it goes to  $(q_0, q_2')$  here  $q_0$  = final state of M &  $q_2'$  = Non-final state of M'.

$\therefore$  We have to stop & declare that the given two FSM's  $M$  and  $M'$  are not equivalent.

- Q. Consider the following two DFA's  $M$  and  $M'$  over  $\{0,1\}$  given in below Fig. Determine whether  $M$  and  $M'$  are equivalent



$$\Sigma = \{0, 1\}$$

$$\text{i.e } |\Sigma| = 2 = n$$

$\therefore$  We have to create three columns labelled.

$$(V, V') , (V_0, V_{0'}) \text{ & } (V_1, V_{1'})$$

Below table shows comparison b/w  $M$  and  $M'$

$(V, V')$	$< (V_0, V_{0'})$	$(V_1, V_{1'})$
$(P, p')$	$(P, p')$	$(q, q')$
$(q, q')$	$(r, s')$	$(q, r')$
$(q, r')$	$(l, s')$	$(q, r')$

As we start with  $(P, p')$  now '0' input symbol it goes to  $(P, p')$  where these are final states. and on '1' input symbol it goes  $(q, q')$  both are non-final states.

$\therefore$  We