String Matching

String Matching Algorithm is also called "String Searching Algorithm." This is a vital class of string algorithm is declared as "this is the method to find a place where one is several strings are found within the larger string."

We formalize the string-matching problem as follows. We assume that the text is an array $T[1..n]$ of length n and that the pattern is an array $P[1..m]$ of length $m \leq n$. We further assume that the elements of P and T are characters drawn from a finite alphabet Σ. For example, we may have $\Sigma = \{0, 1\}$ or $\Sigma = \{a, b, \ldots, z\}$. The character arrays P and T are often called ***strings*** of characters.

Referring to Figure 32.1, we say that pattern P ***occurs with shift*** s in text T (or, equivalently, that pattern P ***occurs beginning at position*** $s + 1$ in text T) if $0 \leq s \leq n - m$ and $T[s + 1..s + m] = P[1..m]$ (that is, if $T[s + j] = P[j]$, for $1 \leq j \leq m$). If P occurs with shift s in T, then we call s a ***valid shift***; otherwise, we call s an ***invalid shift***. The ***string-matching problem*** is the problem of finding all valid shifts with which a given pattern P occurs in a given text T.

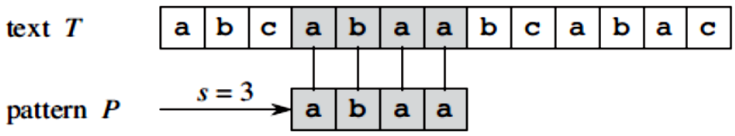


Figure 32.1 An example of the string-matching problem, where we want to find all occurrences of the pattern $P = \texttt{abaa}$ in the text $T = \texttt{abcabaabcabac}$. The pattern occurs only once in the text, at shift $s = 3$, which we call a valid shift. A vertical line connects each character of the pattern to its matching character in the text, and all matched characters are shaded.

Algorithm	Preprocessing time	Matching time
Naive	0	$O((n - m + 1)m)$
Rabin-Karp	$\Theta(m)$	$O((n - m + 1)m)$
Finite automaton	$O(m \lvert\Sigma\rvert)$	$\Theta(n)$
Knuth-Morris-Pratt	$\Theta(m)$	$\Theta(n)$

Figure 32.2 The string-matching algorithms in this chapter and their preprocessing and matching times.

Except for the naive brute-force algorithm, each string-matching algorithm performs some preprocessing based on the pattern and then finds all valid shifts;

we call this latter phase "matching." Figure shows the preprocessing and matching times for each of the algorithms. The total running time of each algorithm is the sum of the preprocessing and matching times. Rabin and Karp algorithm is good Although Θ ((n-m+1)m) worst-case running time of this algorithm is no better than that of the naive method, it works much better on average and in practice. It also generalizes nicely to other pattern matching problems. a string-matching with finite automata algorithm that begins by constructing a finite automaton specifically designed to search for occurrences of the given pattern P in a text. This algorithm takes O(m) preprocessing time, but only Θ (n) matching time. Another same, but much cleverer, Knuth-Morris-Pratt (or KMP) algorithm; it has the same Θ (n) matching time, and it reduces the preprocessing time to only Θ (m).

The naive string-matching algorithm

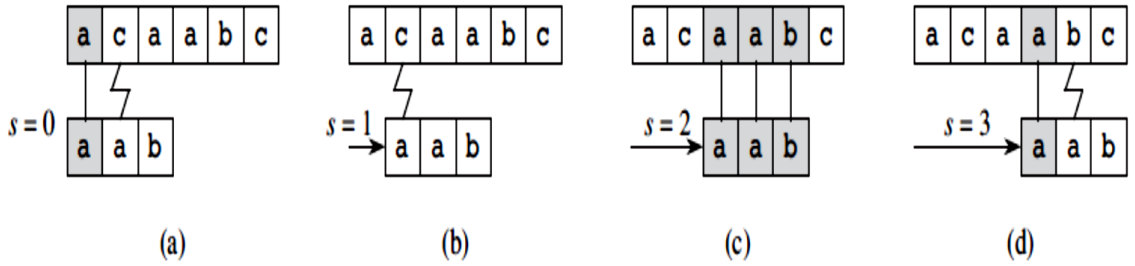
The naive algorithm finds all valid shifts using a loop that checks the condition $P[1..m] = T[s+1..s+m]$ for each of the $n-m+1$ possible values of s.

```
NAIVE-STRING-MATCHER(T, P)
1  n = T.length
2  m = P.length
3  for s = 0 to n − m
4      if P[1..m] == T[s + 1..s + m]
5          print "Pattern occurs with shift" s
```

Procedure NAIVE-STRING-MATCHER takes time $O((n-m+1)m)$, and this bound is tight in the worst case. For example, consider the text string $\mathtt{a}^n$ (a string of n a's) and the pattern $\mathtt{a}^m$. For each of the $n-m+1$ possible values of the shift s, the implicit loop on line 4 to compare corresponding characters must execute m times to validate the shift. The worst-case running time is thus $\Theta((n-m+1)m)$, which is $\Theta(n^2)$ if $m = \lfloor n/2 \rfloor$. Because it requires no preprocessing, NAIVE-STRING-MATCHER's running time equals its matching time.

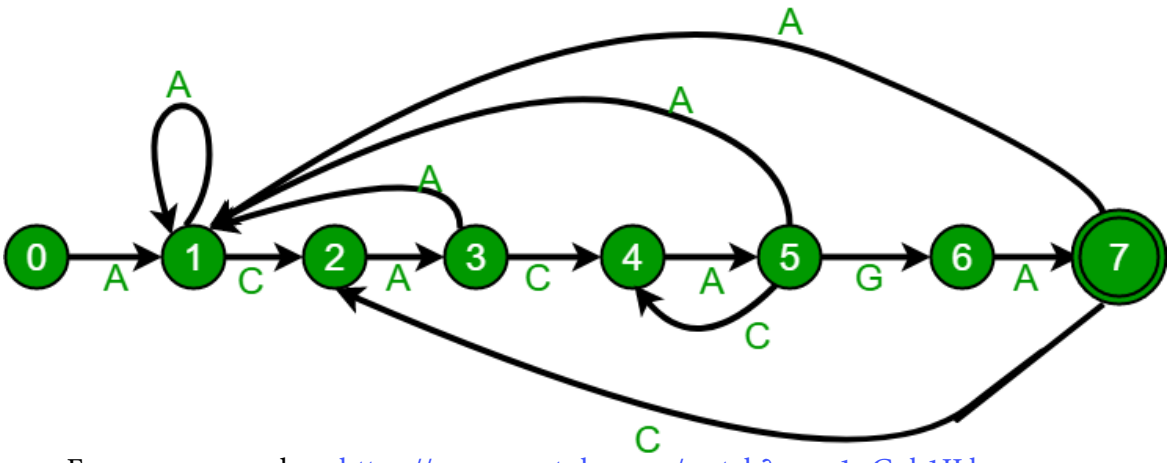
Example –

The operation of the naive string matcher for the pattern P = aab and the text T = acaabc. We can imagine the pattern P as a template that we slide next to the text. (a)–(d) The four successive alignments tried by the naive string matcher. In each part, vertical lines connect corresponding regions found to match (shown shaded), and a jagged line connects the first mismatched character found, if any. The algorithm finds one occurrence of the pattern, at shift s D 2, shown in part (c).



String matching with finite automata –

In FA based algorithm, we preprocess the pattern and build a 2D array that represents a Finite Automata. Construction of the FA is the main tricky part of this algorithm. Once the FA is built, the searching is simple. In search, we simply need to start from the first state of the automata and the first character of the text. At every step, we consider next character of text, look for the next state in the built FA and move to a new state. If we reach the final state, then the pattern is found in the text. The time complexity of the search process is O(n). Before we discuss FA construction, let us take a look at the following FA for pattern ACACAGA.



For more examples - https://www.youtube.com/watch?v=px1nG_b1JLk
https://www.youtube.com/watch?v=szHX26wC-AY

Number of states in FA will be M+1 where M is length of the pattern. The main thing to construct FA is to get the next state from the current state for every possible character. Given a character x and a state k, we can get the next state by considering the string "pat[0..k-1]x" which is basically concatenation of pattern characters pat[0], pat[1] ... pat[k-1] and the character x. The idea is to get length of the longest prefix of the given pattern such that the prefix is also suffix of "pat[0..k-1]x". The value of length gives us the next state. For example, let us see how to get the next state from current state 5 and character 'C' in the above

diagram. We need to consider the string, "pat[0..4]C" which is "ACACAC". The length of the longest prefix of the pattern such that the prefix is suffix of "ACACAC"is 4 ("ACAC"). So the next state (from state 5) is 4 for character 'C'.

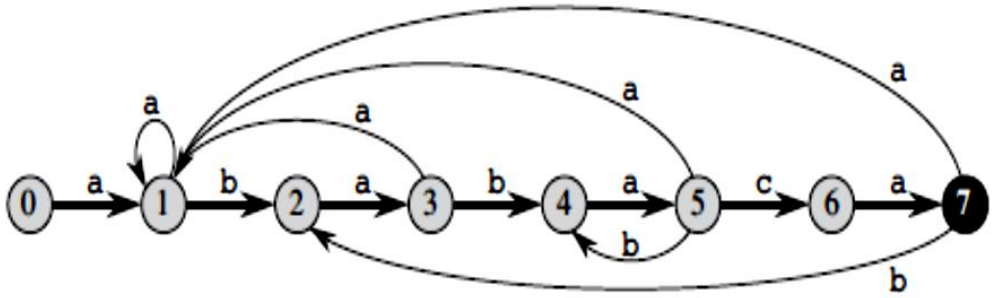
	character			
state	A	C	G	T
0	1	0	0	0
1	1	2	0	0
2	3	0	0	0
3	1	4	0	0
4	5	0	0	0
5	1	4	6	0
6	7	0	0	0
7	1	2	0	0

FINITE-AUTOMATON-MATCHER (T, δ, m)

```
1  n = T.length
2  q = 0
3  for i = 1 to n
4      q = δ(q, T[i])
5      if q == m
6          print "Pattern occurs with shift" i − m
```

From the simple loop structure of FINITE-AUTOMATON-MATCHER, we can easily see that its matching time on a text string of length n is $\Theta(n)$. This matching time, however, does not include the preprocessing time required to compute the transition function δ. We address this problem later, after first proving that the procedure FINITE-AUTOMATON-MATCHER operates correctly.

Consider how the automaton operates on an input text $T[1..n]$. We shall prove that the automaton is in state $\sigma(T_i)$ after scanning character $T[i]$. Since $\sigma(T_i) = m$ if and only if $P \sqsupset T_i$, the machine is in the accepting state m if and only if it has just scanned the pattern P. To prove this result, we make use of the following two lemmas about the suffix function σ.



(a)

state	input a	b	c	P
0	1	0	0	a
1	1	2	0	b
2	3	0	0	a
3	1	4	0	b
4	5	0	0	a
5	1	4	6	c
6	7	0	0	a
7	1	2	0	

(b)

i	–	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	–	a	b	a	b	a	b	a	c	a	b	a
state $\phi(T_i)$	0	1	2	3	4	5	4	5	6	7	2	3

(c)

Figure 32.7 (a) A state-transition diagram for the string-matching automaton that accepts all strings ending in the string ababaca. State 0 is the start state, and state 7 (shown blackened) is the only accepting state. A directed edge from state i to state j labeled a represents $\delta(i, a) = j$. The right-going edges forming the "spine" of the automaton, shown heavy in the figure, correspond to successful matches between pattern and input characters. The left-going edges correspond to failing matches. Some edges corresponding to failing matches are omitted; by convention, if a state i has no outgoing edge labeled a for some $a \in \Sigma$, then $\delta(i, a) = 0$. (b) The corresponding transition function δ, and the pattern string $P =$ ababaca. The entries corresponding to successful matches between pattern and input characters are shown shaded. (c) The operation of the automaton on the text $T =$ abababacaba. Under each text character $T[i]$ appears the state $\phi(T_i)$ that the automaton is in after processing the prefix T_i. The automaton finds one occurrence of the pattern, ending in position 9.

Computing the transition function

The following procedure computes the transition function δ from a given pattern $P[1..m]$.

```
COMPUTE-TRANSITION-FUNCTION(P, Σ)
1  m = P.length
2  for q = 0 to m
3      for each character a ∈ Σ
4          k = min(m + 1, q + 2)
5          repeat
6              k = k − 1
7          until P_k ⊐ P_q a
8          δ(q, a) = k
9  return δ
```

This procedure computes $\delta(q, a)$ in a straightforward manner according to its definition in equation (32.4). The nested loops beginning on lines 2 and 3 consider all states q and all characters a, and lines 4–8 set $\delta(q, a)$ to be the largest k such that $P_k \sqsupset P_q a$. The code starts with the largest conceivable value of k, which is $\min(m, q + 1)$. It then decreases k until $P_k \sqsupset P_q a$, which must eventually occur, since $P_0 = \varepsilon$ is a suffix of every string.

The running time of COMPUTE-TRANSITION-FUNCTION is $O(m^3 |\Sigma|)$, because the outer loops contribute a factor of $m |\Sigma|$, the inner **repeat** loop can run at most $m + 1$ times, and the test $P_k \sqsupset P_q a$ on line 7 can require comparing up

Note that the gap character may occur an arbitrary number of times in the pattern but not at all in the text. Give a polynomial-time algorithm to determine whether such a pattern P occurs in a given text T, and analyze the running time of your algorithm.

32.2 The Rabin-Karp algorithm

For more Examples - https://www.youtube.com/watch?v=qQ8vS2btsxI

Rabin and Karp proposed a string-matching algorithm that performs well in practice and that also generalizes to other algorithms for related problems, such as two-dimensional pattern matching. The Rabin-Karp algorithm uses $\Theta(m)$ preprocessing time, and its worst-case running time is $\Theta((n-m+1)m)$. Based on certain assumptions, however, its average-case running time is better.

This algorithm makes use of elementary number-theoretic notions such as the equivalence of two numbers modulo a third number. You might want to refer to Section 31.1 for the relevant definitions.

For expository purposes, let us assume that $\Sigma = \{0, 1, 2, \ldots, 9\}$, so that each character is a decimal digit. (In the general case, we can assume that each character is a digit in radix-d notation, where $d = |\Sigma|$.) We can then view a string of k consecutive characters as representing a length-k decimal number. The character string `31415` thus corresponds to the decimal number 31,415. Because we interpret the input characters as both graphical symbols and digits, we find it convenient in this section to denote them as we would digits, in our standard text font.

Given a pattern $P[1..m]$, let p denote its corresponding decimal value. In a similar manner, given a text $T[1..n]$, let t_s denote the decimal value of the length-m substring $T[s+1..s+m]$, for $s = 0, 1, \ldots, n-m$. Certainly, $t_s = p$ if and only if $T[s+1..s+m] = P[1..m]$; thus, s is a valid shift if and only if $t_s = p$. If we could compute p in time $\Theta(m)$ and all the t_s values in a total of $\Theta(n-m+1)$ time,[1] then we could determine all valid shifts s in time $\Theta(m) + \Theta(n-m+1) = \Theta(n)$ by comparing p with each of the t_s values. (For the moment, let's not worry about the possibility that p and the t_s values might be very large numbers.)

We can compute p in time $\Theta(m)$ using Horner's rule (see Section 30.1):

$$p = P[m] + 10\,(P[m-1] + 10(P[m-2] + \cdots + 10(P[2] + 10P[1])\cdots))\,.$$

Similarly, we can compute t_0 from $T[1..m]$ in time $\Theta(m)$.

[1] We write $\Theta(n-m+1)$ instead of $\Theta(n-m)$ because s takes on $n-m+1$ different values. The "+1" is significant in an asymptotic sense because when $m = n$, computing the lone t_s value takes $\Theta(1)$ time, not $\Theta(0)$ time.

To compute the remaining values $t_1, t_2, \ldots, t_{n-m}$ in time $\Theta(n-m)$, we observe that we can compute t_{s+1} from t_s in constant time, since

$$t_{s+1} = 10(t_s - 10^{m-1}T[s+1]) + T[s+m+1] \,. \tag{32.1}$$

Subtracting $10^{m-1}T[s+1]$ removes the high-order digit from t_s, multiplying the result by 10 shifts the number left by one digit position, and adding $T[s+m+1]$ brings in the appropriate low-order digit. For example, if $m = 5$ and $t_s = 31415$, then we wish to remove the high-order digit $T[s+1] = 3$ and bring in the new low-order digit (suppose it is $T[s+5+1] = 2$) to obtain

$$\begin{aligned} t_{s+1} &= 10(31415 - 10000 \cdot 3) + 2 \\ &= 14152 \,. \end{aligned}$$

If we precompute the constant 10^{m-1} (which we can do in time $O(\lg m)$ using the techniques of Section 31.6, although for this application a straightforward $O(m)$-time method suffices), then each execution of equation (32.1) takes a constant number of arithmetic operations. Thus, we can compute p in time $\Theta(m)$, and we can compute all of $t_0, t_1, \ldots, t_{n-m}$ in time $\Theta(n-m+1)$. Therefore, we can find all occurrences of the pattern $P[1\,..\,m]$ in the text $T[1\,..\,n]$ with $\Theta(m)$ preprocessing time and $\Theta(n-m+1)$ matching time.

Until now, we have intentionally overlooked one problem: p and t_s may be too large to work with conveniently. If P contains m characters, then we cannot reasonably assume that each arithmetic operation on p (which is m digits long) takes "constant time." Fortunately, we can solve this problem easily, as Figure 32.5 shows: compute p and the t_s values modulo a suitable modulus q. We can compute p modulo q in $\Theta(m)$ time and all the t_s values modulo q in $\Theta(n-m+1)$ time. If we choose the modulus q as a prime such that $10q$ just fits within one computer word, then we can perform all the necessary computations with single-precision arithmetic. In general, with a d-ary alphabet $\{0, 1, \ldots, d-1\}$, we choose q so that dq fits within a computer word and adjust the recurrence equation (32.1) to work modulo q, so that it becomes

$$t_{s+1} = (d(t_s - T[s+1]h) + T[s+m+1]) \bmod q \,, \tag{32.2}$$

where $h \equiv d^{m-1} \pmod q$ is the value of the digit "1" in the high-order position of an m-digit text window.

The solution of working modulo q is not perfect, however: $t_s \equiv p \pmod q$ does not imply that $t_s = p$. On the other hand, if $t_s \not\equiv p \pmod q$, then we definitely have that $t_s \neq p$, so that shift s is invalid. We can thus use the test $t_s \equiv p \pmod q$ as a fast heuristic test to rule out invalid shifts s. Any shift s for which $t_s \equiv p \pmod q$ must be tested further to see whether s is really valid or we just have a ***spurious hit***. This additional test explicitly checks the condition

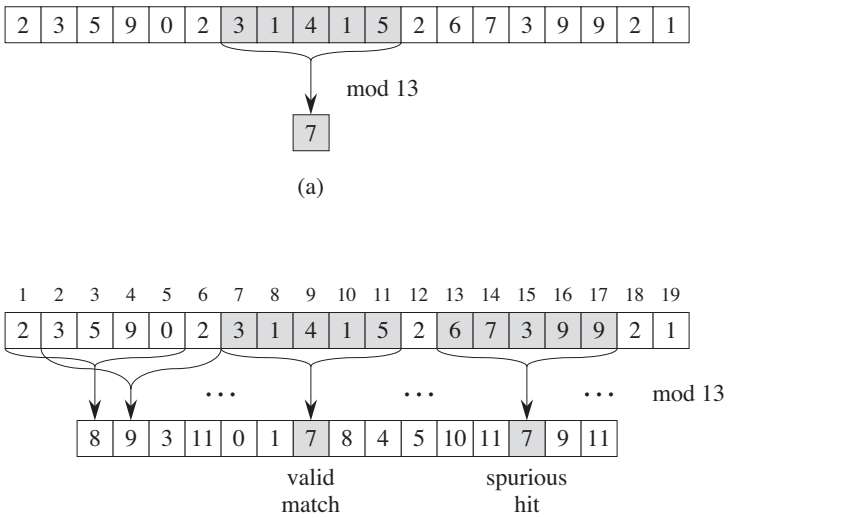


Figure 32.5 The Rabin-Karp algorithm. Each character is a decimal digit, and we compute values modulo 13. **(a)** A text string. A window of length 5 is shown shaded. The numerical value of the shaded number, computed modulo 13, yields the value 7. **(b)** The same text string with values computed modulo 13 for each possible position of a length-5 window. Assuming the pattern $P = 31415$, we look for windows whose value modulo 13 is 7, since $31415 \equiv 7 \pmod{13}$. The algorithm finds two such windows, shown shaded in the figure. The first, beginning at text position 7, is indeed an occurrence of the pattern, while the second, beginning at text position 13, is a spurious hit. **(c)** How to compute the value for a window in constant time, given the value for the previous window. The first window has value 31415. Dropping the high-order digit 3, shifting left (multiplying by 10), and then adding in the low-order digit 2 gives us the new value 14152. Because all computations are performed modulo 13, the value for the first window is 7, and the value for the new window is 8.

$P[1..m] = T[s+1..s+m]$. If q is large enough, then we hope that spurious hits occur infrequently enough that the cost of the extra checking is low.

The following procedure makes these ideas precise. The inputs to the procedure are the text T, the pattern P, the radix d to use (which is typically taken to be $|\Sigma|$), and the prime q to use.

```
RABIN-KARP-MATCHER(T, P, d, q)
 1  n = T.length
 2  m = P.length
 3  h = d^(m-1) mod q
 4  p = 0
 5  t_0 = 0
 6  for i = 1 to m                    // preprocessing
 7      p = (dp + P[i]) mod q
 8      t_0 = (dt_0 + T[i]) mod q
 9  for s = 0 to n - m                // matching
10      if p == t_s
11          if P[1..m] == T[s+1..s+m]
12              print "Pattern occurs with shift" s
13      if s < n - m
14          t_{s+1} = (d(t_s - T[s+1]h) + T[s+m+1]) mod q
```

The procedure RABIN-KARP-MATCHER works as follows. All characters are interpreted as radix-d digits. The subscripts on t are provided only for clarity; the program works correctly if all the subscripts are dropped. Line 3 initializes h to the value of the high-order digit position of an m-digit window. Lines 4–8 compute p as the value of $P[1..m] \bmod q$ and t_0 as the value of $T[1..m] \bmod q$. The **for** loop of lines 9–14 iterates through all possible shifts s, maintaining the following invariant:

Whenever line 10 is executed, $t_s = T[s+1..s+m] \bmod q$.

If $p = t_s$ in line 10 (a "hit"), then line 11 checks to see whether $P[1..m] = T[s+1..s+m]$ in order to rule out the possibility of a spurious hit. Line 12 prints out any valid shifts that are found. If $s < n - m$ (checked in line 13), then the **for** loop will execute at least one more time, and so line 14 first executes to ensure that the loop invariant holds when we get back to line 10. Line 14 computes the value of $t_{s+1} \bmod q$ from the value of $t_s \bmod q$ in constant time using equation (32.2) directly.

RABIN-KARP-MATCHER takes $\Theta(m)$ preprocessing time, and its matching time is $\Theta((n-m+1)m)$ in the worst case, since (like the naive string-matching algorithm) the Rabin-Karp algorithm explicitly verifies every valid shift. If $P = \mathtt{a}^m$

and $T = \mathtt{a}^n$, then verifying takes time $\Theta((n-m+1)m)$, since each of the $n-m+1$ possible shifts is valid.

In many applications, we expect few valid shifts—perhaps some constant c of them. In such applications, the expected matching time of the algorithm is only $O((n-m+1)+cm) = O(n+m)$, plus the time required to process spurious hits. We can base a heuristic analysis on the assumption that reducing values modulo q acts like a random mapping from Σ^* to $\mathbb{Z}_q$. (See the discussion on the use of division for hashing in Section 11.3.1. It is difficult to formalize and prove such an assumption, although one viable approach is to assume that q is chosen randomly from integers of the appropriate size. We shall not pursue this formalization here.) We can then expect that the number of spurious hits is $O(n/q)$, since we can estimate the chance that an arbitrary t_s will be equivalent to p, modulo q, as $1/q$. Since there are $O(n)$ positions at which the test of line 10 fails and we spend $O(m)$ time for each hit, the expected matching time taken by the Rabin-Karp algorithm is

$$O(n) + O(m(v + n/q)) ,$$

where v is the number of valid shifts. This running time is $O(n)$ if $v = O(1)$ and we choose $q \geq m$. That is, if the expected number of valid shifts is small ($O(1)$) and we choose the prime q to be larger than the length of the pattern, then we can expect the Rabin-Karp procedure to use only $O(n+m)$ matching time. Since $m \leq n$, this expected matching time is $O(n)$.

Exercises

32.2-1

Working modulo $q = 11$, how many spurious hits does the Rabin-Karp matcher encounter in the text $T = 3141592653589793$ when looking for the pattern $P = 26$?

32.2-2

How would you extend the Rabin-Karp method to the problem of searching a text string for an occurrence of any one of a given set of k patterns? Start by assuming that all k patterns have the same length. Then generalize your solution to allow the patterns to have different lengths.

32.2-3

Show how to extend the Rabin-Karp method to handle the problem of looking for a given $m \times m$ pattern in an $n \times n$ array of characters. (The pattern may be shifted vertically and horizontally, but it may not be rotated.)

to m characters. Much faster procedures exist; by utilizing some cleverly computed information about the pattern P (see Exercise 32.4-8), we can improve the time required to compute δ from P to $O(m\,|\Sigma|)$. With this improved procedure for computing δ, we can find all occurrences of a length-m pattern in a length-n text over an alphabet Σ with $O(m\,|\Sigma|)$ preprocessing time and $\Theta(n)$ matching time.

Exercises

32.3-1

Construct the string-matching automaton for the pattern $P = \texttt{aabab}$ and illustrate its operation on the text string $T = \texttt{aaababaabaababaab}$.

32.3-2

Draw a state-transition diagram for a string-matching automaton for the pattern $\texttt{ababbabbababbababbabb}$ over the alphabet $\Sigma = \{\texttt{a}, \texttt{b}\}$.

32.3-3

We call a pattern P ***nonoverlappable*** if $P_k \sqsupset P_q$ implies $k = 0$ or $k = q$. Describe the state-transition diagram of the string-matching automaton for a nonoverlappable pattern.

32.3-4 ⋆

Given two patterns P and P', describe how to construct a finite automaton that determines all occurrences of *either* pattern. Try to minimize the number of states in your automaton.

32.3-5

Given a pattern P containing gap characters (see Exercise 32.1-4), show how to build a finite automaton that can find an occurrence of P in a text T in $O(n)$ matching time, where $n = |T|$.

⋆ 32.4 The Knuth-Morris-Pratt algorithm

For more examples - https://www.youtube.com/watch?v=V5-7GzOfADQ

We now present a linear-time string-matching algorithm due to Knuth, Morris, and Pratt. This algorithm avoids computing the transition function δ altogether, and its matching time is $\Theta(n)$ using just an auxiliary function π, which we precompute from the pattern in time $\Theta(m)$ and store in an array $\pi[1\,..\,m]$. The array π allows us to compute the transition function δ efficiently (in an amortized sense) "on the fly" as needed. Loosely speaking, for any state $q = 0, 1, \ldots, m$ and any character

$a \in \Sigma$, the value $\pi[q]$ contains the information we need to compute $\delta(q, a)$ but that does not depend on a. Since the array π has only m entries, whereas δ has $\Theta(m|\Sigma|)$ entries, we save a factor of $|\Sigma|$ in the preprocessing time by computing π rather than δ.

The prefix function for a pattern

The prefix function π for a pattern encapsulates knowledge about how the pattern matches against shifts of itself. We can take advantage of this information to avoid testing useless shifts in the naive pattern-matching algorithm and to avoid precomputing the full transition function δ for a string-matching automaton.

Consider the operation of the naive string matcher. Figure 32.10(a) shows a particular shift s of a template containing the pattern $P = \texttt{ababaca}$ against a text T. For this example, $q = 5$ of the characters have matched successfully, but the 6th pattern character fails to match the corresponding text character. The information that q characters have matched successfully determines the corresponding text characters. Knowing these q text characters allows us to determine immediately that certain shifts are invalid. In the example of the figure, the shift $s + 1$ is necessarily invalid, since the first pattern character (`a`) would be aligned with a text character that we know does not match the first pattern character, but does match the second pattern character (`b`). The shift $s' = s + 2$ shown in part (b) of the figure, however, aligns the first three pattern characters with three text characters that must necessarily match. In general, it is useful to know the answer to the following question:

> Given that pattern characters $P[1..q]$ match text characters $T[s+1..s+q]$, what is the least shift $s' > s$ such that for some $k < q$,
>
> $$P[1..k] = T[s' + 1..s' + k]\,, \tag{32.6}$$
>
> where $s' + k = s + q$?

In other words, knowing that $P_q \sqsupset T_{s+q}$, we want the longest proper prefix P_k of P_q that is also a suffix of T_{s+q}. (Since $s' + k = s + q$, if we are given s and q, then finding the smallest shift s' is tantamount to finding the longest prefix length k.) We add the difference $q - k$ in the lengths of these prefixes of P to the shift s to arrive at our new shift s', so that $s' = s + (q - k)$. In the best case, $k = 0$, so that $s' = s + q$, and we immediately rule out shifts $s + 1, s + 2, \ldots, s + q - 1$. In any case, at the new shift s' we don't need to compare the first k characters of P with the corresponding characters of T, since equation (32.6) guarantees that they match.

We can precompute the necessary information by comparing the pattern against itself, as Figure 32.10(c) demonstrates. Since $T[s' + 1..s' + k]$ is part of the

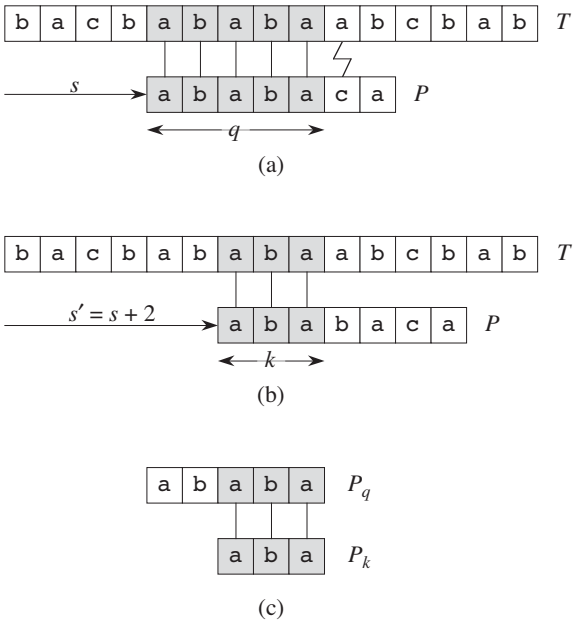


Figure 32.10 The prefix function π. **(a)** The pattern $P = \texttt{ababaca}$ aligns with a text T so that the first $q = 5$ characters match. Matching characters, shown shaded, are connected by vertical lines. **(b)** Using only our knowledge of the 5 matched characters, we can deduce that a shift of $s + 1$ is invalid, but that a shift of $s' = s+2$ is consistent with everything we know about the text and therefore is potentially valid. **(c)** We can precompute useful information for such deductions by comparing the pattern with itself. Here, we see that the longest prefix of P that is also a proper suffix of P_5 is P_3. We represent this precomputed information in the array π, so that $\pi[5] = 3$. Given that q characters have matched successfully at shift s, the next potentially valid shift is at $s' = s + (q - \pi[q])$ as shown in part (b).

known portion of the text, it is a suffix of the string P_q. Therefore, we can interpret equation (32.6) as asking for the greatest $k < q$ such that $P_k \sqsupset P_q$. Then, the new shift $s' = s + (q - k)$ is the next potentially valid shift. We will find it convenient to store, for each value of q, the number k of matching characters at the new shift s', rather than storing, say, $s' - s$.

We formalize the information that we precompute as follows. Given a pattern $P[1\,..\,m]$, the ***prefix function*** for the pattern P is the function $\pi : \{1, 2, \ldots, m\} \to \{0, 1, \ldots, m-1\}$ such that

$$\pi[q] = \max\{k : k < q \text{ and } P_k \sqsupset P_q\} \ .$$

That is, $\pi[q]$ is the length of the longest prefix of P that is a proper suffix of P_q. Figure 32.11(a) gives the complete prefix function π for the pattern `ababaca`.

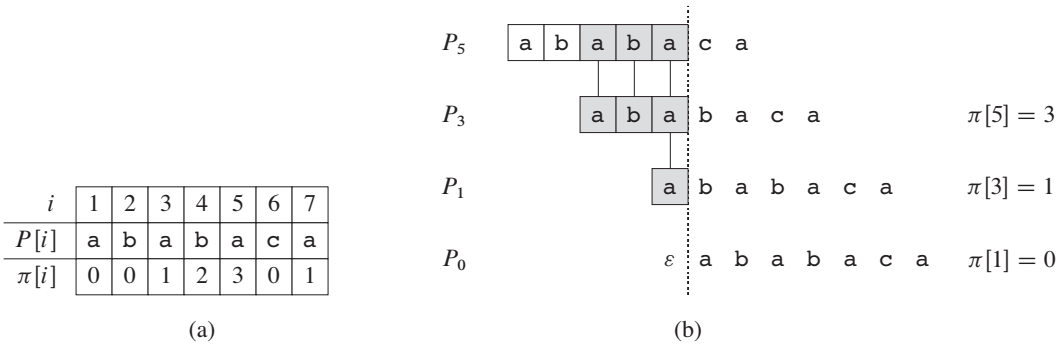


Figure 32.11 An illustration of Lemma 32.5 for the pattern $P = \texttt{ababaca}$ and $q = 5$. **(a)** The π function for the given pattern. Since $\pi[5] = 3$, $\pi[3] = 1$, and $\pi[1] = 0$, by iterating π we obtain $\pi^*[5] = \{3, 1, 0\}$. **(b)** We slide the template containing the pattern P to the right and note when some prefix P_k of P matches up with some proper suffix of P_5; we get matches when $k = 3$, 1, and 0. In the figure, the first row gives P, and the dotted vertical line is drawn just after P_5. Successive rows show all the shifts of P that cause some prefix P_k of P to match some suffix of P_5. Successfully matched characters are shown shaded. Vertical lines connect aligned matching characters. Thus, $\{k : k < 5 \text{ and } P_k \sqsupset P_5\} = \{3, 1, 0\}$. Lemma 32.5 claims that $\pi^*[q] = \{k : k < q \text{ and } P_k \sqsupset P_q\}$ for all q.

The pseudocode below gives the Knuth-Morris-Pratt matching algorithm as the procedure KMP-MATCHER. For the most part, the procedure follows from FINITE-AUTOMATON-MATCHER, as we shall see. KMP-MATCHER calls the auxiliary procedure COMPUTE-PREFIX-FUNCTION to compute π.

```
KMP-MATCHER(T, P)
 1  n = T.length
 2  m = P.length
 3  π = COMPUTE-PREFIX-FUNCTION(P)
 4  q = 0                                  // number of characters matched
 5  for i = 1 to n                         // scan the text from left to right
 6      while q > 0 and P[q + 1] ≠ T[i]
 7          q = π[q]                       // next character does not match
 8      if P[q + 1] == T[i]
 9          q = q + 1                      // next character matches
10      if q == m                          // is all of P matched?
11          print "Pattern occurs with shift" i − m
12          q = π[q]                       // look for the next match
```