Shri Vile Parle Kelavani Mandal's

DWARKADAS J. SANGHVI COLLEGE OF ENGINEERING

(Autonomous College Affiliated to the University of Mumbai)

NAAC Accredited with "A" Grade (CGPA : 3.18)

Academic Year (2021-22)

Year: 2 Semester: IV

Program: B. Tech. (Computer Engg.) **Max. Marks: 75**

Subject: Engineering Mathematics-IV **Time: 10: 30 am to 1:30 pm**

Date: **Duration: 3 Hours**

REGULAR EXAMINATION

ANSWER KEY

Question No.		Max. Marks

Q1 (a) **[7]**

We have $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

For 3x3 matrix, we consider

$\therefore \quad \phi(A) = \alpha_2 A^2 + \alpha_1 A + \alpha_0 I$

$\therefore \quad A^{75} = \alpha_2 A^2 + \alpha_1 A + \alpha_0 I \quad \cdots (1)$

Now find characteristic roots of A

$\therefore \quad |A - \lambda I| = 0$

$\Rightarrow \quad \lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0 \quad \cdots (2)$

where $S_1 = 1, \quad S_2 = -1, \quad S_3 = |A| = -1$

$\therefore \quad \lambda^3 - \lambda^2 - \lambda + 1 = 0$

$\Rightarrow \quad \lambda_1 = -1, \quad \lambda_2 = 1 = \lambda_3$

Now replace A by λ in eq^n (1)

$\Rightarrow \quad \lambda^{75} = \alpha_2 \lambda^2 + \alpha_1 \lambda + \alpha_0 \quad \cdots (3)$

Put $\lambda_1 = -1$ in eq^n (3)

$\Rightarrow \quad -1 = \alpha_2 - \alpha_1 + \alpha_0 \quad \cdots (4)$

Put $\lambda_2 = 1$ in eq^n (3)

$\Rightarrow \quad 1 = \alpha_2 + \alpha_1 + \alpha_0 \quad \cdots (5)$

Now since we get repeated root, so taking derivative of eq^n (3) w.r.t 'λ' & put $\lambda = 1$

$\Rightarrow \quad 75\lambda^{74} = 2\alpha_2\lambda + \alpha_1$

$\Rightarrow \quad 75 = 2\alpha_2 + \alpha_1 \quad \cdots (6)$

by solving (4), (5) & (6), we get

$\alpha_0 = -37, \quad \alpha_1 = 1, \quad \alpha_2 = 37$

Put it in eq^n (1)

$A^{75} = \alpha_2 A^2 + \alpha_1 A + \alpha_0 I = 37\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 37\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad A^{75} = \begin{bmatrix} 1 & 0 & 0 \\ 38 & 0 & 1 \\ 37 & 1 & 0 \end{bmatrix}$

	OR	
Q1 (a)	We have $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ $\therefore \lvert A-\lambda I \rvert = 0$ $\Rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - \lvert A \rvert = 0$ where $S_1 = 6$, $S_2 = 11$, $\lvert A \rvert = 6$ $\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$ $\therefore X_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $X_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$, $X_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ $\therefore P = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$ $P^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix}$ $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	[7]

Q1 (b) **[8]**

We have

$$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$\therefore \; |A - \lambda I| = 0 \;\Rightarrow\; \lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

where $S_1 = 4$, $S_2 = -20$ & $|A| = 35$

$\therefore \; \lambda^3 - 4\lambda^2 - 20\lambda - 35 = 0$ &

by using Cayley-Hamilton thm

$A^3 - 4A^2 - 20A - 35I = 0$ ---- (1)

multiply by A^{-1}, we get

$A^2 - 4A - 20I - 35A^{-1} = 0$

$A^{-1} = \frac{1}{35}\{A^2 - 4A -$ [illegible] $\}$

$$A^{-1} = \frac{1}{35}\left\{\begin{bmatrix}20 & 23 & 23\\15 & 22 & 37\\10 & 9 & 14\end{bmatrix} - 4\begin{bmatrix}1 & 3 & 7\\4 & 2 & 3\\1 & 2 & 1\end{bmatrix} - \begin{bmatrix}20 & 0 & 0\\0 & 20 & 0\\0 & 0 & 20\end{bmatrix}\right\}$$

$$\therefore A^{-1} = \frac{1}{35}\begin{bmatrix}-4 & 11 & -5\\-1 & -6 & 25\\6 & 1 & -10\end{bmatrix}$$

Now we have $B = A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I$

divide B by characteristic eq^n

$$\therefore B = (A^3 - 4A^2 - 20A - 35I)(A^4 + A) + (2A + I)$$

using Cayley-Hamilton thm

$$\Rightarrow B = 2A + I$$

$$B = 2\begin{bmatrix}1 & 3 & 7\\4 & 2 & 3\\1 & 2 & 1\end{bmatrix} + \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\therefore B = \begin{bmatrix}3 & 6 & 14\\8 & 5 & 6\\2 & 4 & 3\end{bmatrix}$$

Q2 (a) [7]

We have Pdf $f(x) = 3x^2,\ 0 \le x \le 1$

Since $P(X \le a) = P(X > a)$

$$\Rightarrow 2P(X \le a) = 1$$

$$\therefore P(X \le a) = 1/2$$

$$\therefore \int_0^a 3x^2\,dx = 1/2$$

$$\therefore 3\left[\frac{x^3}{3}\right]_0^a = 1/2$$

$$[a^3 - 0] = 1/2$$

$$a = 0.7937$$

Also

$$P(X > b) = 0.05$$

$$\Rightarrow \int_b^1 3x^2\,dx = 0.05$$

$$\therefore 3\left[\frac{x^3}{3}\right]_b^1 = 0.05$$

$$\therefore 1 - b^3 = 0.05$$

$$\therefore b^3 = 0.95$$

$$\therefore b = 0.9830$$

	Now $E(x) = \int_0^1 x(3x^2)\,dx$ $= \int_0^1 3x^3\,dx$ $= 3\left[\frac{x^4}{4}\right]_0^1$ $= \frac{3}{4}[1-0] = 3/4$ and $Var(X) = \int_0^1 3x^4\,dx - \{E(x)\}^2$ $= 3\left[\frac{x^5}{5}\right]_0^1 - (3/4)^2$ $= \frac{3}{5}[1-0] - \frac{9}{16}$ $\therefore Var(x) = 0.0375$	
	OR	

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Q2 (a)	We have x : -2 -1 0 1 2 5 $P(X=x)$: 0.1 k 0.2 $2k$ 0.3 $3k$ $\therefore \sum x P(x) = 1$ $\Rightarrow -0.2 - k + 2k + 0.6 + 9k = 1$ $\Rightarrow 10k + 0.4 = 1$ $\therefore k = 0.06$ Now $P(X \geq 2) = P(X=2) + P(X=3)$ $= 0.3 + 3(0.06) = 0.48$ & $P(-2<X<2) = P(X=-1) + P(X=0) + \cdots + P(X=1)$ $(0.06) + (0.2) + (0.12)$ $= 0.2$[illegible]	[7]

Q2 (b) [8]

Let Probability that the businessman goes to hotel x is $P(x) = 0.2$

Probability that the businessman goes to hotel y is $P(y) = 0.5$

Probability that the businessman goes to hotel z is $P(z) = 0.3$

Assume that A is the event of all the faulty plumbings then

$P(A/x) = 0.05$, $P(A/y) = 0.04$ & $P(A/z) = 0.08$

Probability that faulty plumbing is assigned to hotel z is

$$P(z/A) = \frac{P(z).P(A/z)}{P(x)\,P(A/x) + P(y)\,P(A/y) + P(z).P(A/z)}$$

$$= \frac{(0.3)(0.08)}{(0.2)(0.05) + (0.5)(0.04) + (0.3)(0.08)}$$

$$= \frac{0.024}{0.054}$$

$\therefore P(z/A) = 0.444$

Q3 (a) [7]

Given $n = 5$, $p = 0.5$, $q = 0.5$, $N = 800$

(i) $P(x=3) = {}^5C_3 (0.5)^3 (0.5)^2 = 0.3125$

$\therefore$ Total No. of families would expect 3 boys $= 0.3125 \times 800$
$= 250$

(ii) $P(x=5) = {}^5C_5 (0.5)^5 (0.5)^0 = 0.03125$

Total No of families would expect 5 girls $= 0.03125 \times 800$
$= 25$

(iii) $P(x=2 \text{ or } x=3) = P(x=2) + P(x=3)$

$= {}^5C_2 (0.5)^2 (0.5)^3 + {}^5C_3 (0.5)^3 (0.5)^2$

$= 0.3125 + 0.3125 = 0.625$

Total [illegible] families would [illegible] = 0.625 [illegible]

(ii) $P(X \geq 1) = P(X=1) + P(X=2) + \cdots + P(X=5)$

$= 1 - P(X=0)$

$= 1 - {}^5C_0 (0.5)^0 (0.5)^5$

$= 1 - 0.03125 = 0.96875$

$\therefore$ Total No of families would expect at least one boy

$= 0.96875 \times 800 = 775$

OR

Q3 (a) [7]

The mean no. of mistakes

$$= \left\{ \frac{90 + 84 + 36 + 36 + 15 + 6}{300} \right\}$$

$$= \frac{267}{300} = 0.89$$

No. of Mistakes	Probability	Theoretical Freq^n.
0	$\frac{e^{-0.89}(0.89)^0}{0!} = 0.411$	$0.411 \times 300 \approx 123$
1	$\frac{e^{-0.89}(0.89)^1}{1!} = 0.365$	$0.365 \times 300 \approx 110$
2	$\frac{e^{-0.89}(0.89)^2}{2!} = 0.163$	$0.163 \times 300 \approx 49$
3	$\frac{e^{-0.89}(0.89)^3}{3!} = 0.048$	$0.048 \times 300 \approx 14$
4	$\frac{e^{-0.89}(0.89)^4}{4!} = 0.011$	$0.011 \times 300 \approx 03$
5	$\frac{e^{-0.89}(0.89)^5}{5!} = 0.002$	$0.002 \times 300 \approx 01$
6	$\frac{e^{-0.89}(0.89)^6}{6!} = 0.0003$	$0.0003 \times 300 \approx 00$

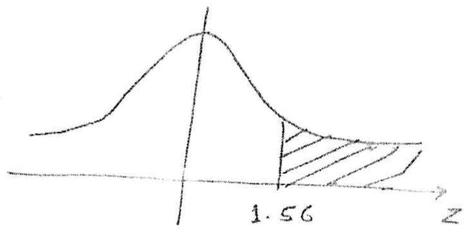
Q3 (b) [8]

Let X be the random variable which denotes the lifetime of a certain kind of batteries

given : $\mu = 400$, $\sigma = 45$

(i) Let $x_1 = 470$

$$\therefore \quad z_1 = \frac{470 - 400}{45} = 1.56$$



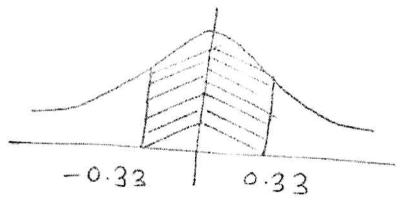
$$P(X \geq 470) = 0.5 - P(0 < z < 1.56)$$
$$= 0.5 - 0.4406$$
$$= 0.0594$$

$\therefore$ % of batteries with lifetime of at least 470 hours $= 5.94\%$

(ii). Let $x_2 = 385$, $x_3 = 415$

$$z_2 = \frac{385 - 400}{45} = -0.33$$

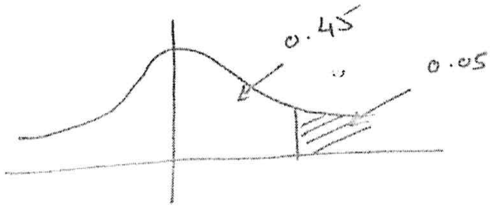
$$z_3 = \frac{415 - 400}{45} = 0.33$$



$$\therefore \quad P(385 < X < 415) = P(-0.33 < z < 0) + P(0 < X < 0.33)$$
$$= 0.1293 + 0.1293$$
$$= 0.2586$$

$\therefore$ Proportion of batteries with lifetime betn 385 & 415 hours $= 25.86\%$

iii) $P(x > x_4) = 0.05$



$0.05 = 0.5 - P(0 \leq z \leq z_4)$

$\Rightarrow P(0 \leq z \leq z_4) = 0.45$

$\Rightarrow z_4 = \dfrac{x_4 - 400}{45} \quad \Rightarrow 1.65 = \dfrac{x_1 - 400}{45}$

$\therefore x_4 = 474.25 \simeq 474$ hours

Q4 (a)

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A < \mu_B$

given that $\bar{x}_1 = 170$; $\bar{x}_2 = 178$

$n_1 = 400$; $n_2 = 800$

$\sigma_1 = 6$; $\sigma_2 = 8$

$$\therefore \quad Z = \cancel{170}\ \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\Rightarrow \quad Z = \frac{170 - 178}{\sqrt{\frac{6^2}{900} + \frac{8^2}{800}}}$$

$$\therefore \quad Z = -19.4029$$

$$\therefore \quad |Z|_{cal} = 19.4029$$

Here problem belongs to one tailed & Consider L.O.S as 5%.

$$\therefore \quad |Z|_{tab} = 1.64$$

$$\Rightarrow \quad |Z|_{cal} > |Z|_{tab}$$

$\therefore$ H_0 is rejected

[7]

$\therefore$ We can say that persons in country B are taller than those of country A.

OR

Q4 (a) [7]

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
9	10	-2.75	-0.43	7.5625	0.1849
11	12	-0.75	1.57	0.5625	2.4649
13	10	1.25	-0.43	1.5625	0.1849
11	14	-0.75	3.57	0.5625	12.7449
15	09	3.25	-1.43	10.5625	2.0449
09	08	-2.75	-2.43	7.5625	5.9049
12	10	0.25	-0.43	0.0625	0.1849
14	—	2.25	—	5.0625	—
94	73			33.5	23.5449

$\bar{x} = 11.75$ & $\bar{y} = 10.43$

$$\therefore \quad \sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1+n_2-2}}$$

$$\sigma = 2.0948$$

$H_0 : \bar{x} = \bar{y}$

$H_1 : \bar{x} \neq \bar{y}$

$$t = \frac{\bar{x}-\bar{y}}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$\therefore t = \frac{11.75 - 10.43}{2.0948\sqrt{\frac{1}{8}+\frac{1}{7}}}$$

$$\therefore t = \frac{1.32}{1.0842}$$

$\therefore t = 1.2174 \quad \therefore |t|_{cal} = 1.2174$

Consider problem belongs ~~too~~ to two tailed & L.O.S. as 5% $\therefore |t|_{tab} = 2.16$

with D.f. = 13

$\therefore |t|_{cal} < |t|_{tab}$

$\therefore H_0$ is accepted, we can say that difference betn mean is not significant

Q4 (b) **[8]**

H_0: Intelligence is associated with clothing

H_1: Intelligence is not associated with clothing

Intelligence	Clothing: Poorly clad	Well clad	Very well clad
Dull	$\frac{400 \times 240}{1200} = 80$	$\frac{500 \times 240}{1200} = 100$	$\frac{300 \times 240}{1200} = 60$
Intelligent	$\frac{400 \times 600}{1200} = 200$	$\frac{500 \times 600}{1200} = 250$	$\frac{300 \times 600}{1200} = 150$
Very Intelligent	$\frac{400 \times 360}{1200} = 120$	$\frac{500 \times 360}{1200} = 150$	$\frac{300 \times 360}{1200} = 90$

$$\therefore \chi^2_{cal} = \frac{(72-80)^2}{80} + \frac{(90-100)^2}{100} + \frac{(78-60)^2}{60} + \frac{(184-200)^2}{200} + \frac{(305-250)^2}{250} + \frac{(111-150)^2}{150} + \frac{(144-120)^2}{120} + \frac{(105-150)^2}{150} + \frac{(111-90)^2}{90}$$

$$\therefore \chi^2_{cal} = 0.8 + 1 + 5.4 + 1.28 + 12.1 + 10.14 + 4.8 + 13.5 + 4.9$$

$$\therefore \chi^2_{cal} = 53.92$$

$$D.F. = (r-1)(c-1) = (3-1)(3-1) = 4$$

Consider L.O.S as ~~5~~ 1%

$\therefore \; \chi^2_{tab} = 13.277$

$\Rightarrow \; \chi^2_{cal} > \chi^2_{tab}$

$\Rightarrow$ H_0 is rejected

$\therefore$ Intelligence is not associated with clothing.

Q5 (a) [7]

We have Max $z = 5x_1 + 3x_2$

subject to $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$, $x_1, x_2 \geq 0$

Consider slack variable s_1, s_2

Max $z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

Subject to $3x_1 + 5x_2 + s_1 + 0s_2 = 15$

$5x_1 + 2x_2 + 0s_1 + s_2 = 10$

$x_1, x_2, s_1, s_2 \geq 0$

		c_j	5	3	0	0	
c_B	x_B	b	x_1	x_2	s_1	s_2	Min Ratio
0	s_1	15	3	5	1	0	$15/3 = 5$
0	s_2	10	$\boxed{5}$	2	0	1	$10/5 = 2$ ←
	$z_j - c_j$		-5 ↑	-3	0	0	

		c_j	5	3	0	0	
c_B	x_B	b	x_1	x_2	s_1	s_2	Min Ratio
0	s_1	9	0	$\boxed{19/5}$	1	$-3/5$	$45/19$ ←
5	x_1	2	1	$2/5$	0	$1/5$	$10/2$
	$z_j - c_j$		0	-1 ↑	0	1	

C_B	x_B	b_j	x_1	x_2	s_1	s_2
		c_j				
3	x_2	45/19	0	1	5/19	-3
5	x_1	20/19	1	0	-2/19	5/19
	$z_j - c_j$		0	0	5/19	16/19

$\therefore$ Here all $z_j - c_j \geq 0$ $\therefore$ Sol^n is

$x_1 = 20/19, \quad x_2 = \frac{45}{19}$

Max $z = 235/19$

OR

Q5 (a) [7]

$\text{Max } z' = -12x_1 - 20x_2$

Subject to $6x_1 + 8x_2 - s_1 + A_1 = 100$

$7x_1 + 12x_2 - s_2 + A_2 = 120$

$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$

		C_j	-12	-20	0	0	$-M$	$-M$	
C_B	B	X_B	x_1	x_2	s_1	s_2	A_1	A_2	Min Ratio
$-M$	A_1	100	6	8	-1	0	1	0	$100/8 = 12.5$
$-M$	A_2	120	7	$\boxed{12}$	0	-1	0	1	$120/12 = 10$ ←
		$z_j - c_j$	$-13M+12$	$-20M+20$ ↑	M	M	0	0	

		C_j	-12	-20	0	0	$-M$	
C_B	B	X_B	x_1	x_2	s_1	s_2	A_1	Min Ratio
$-M$	A_1	20	$\boxed{4/3}$	0	-1	$2/3$	1	15 ←
-20	x_2	10	$7/12$	1	0	$-1/12$	0	17.1428
		$z_j - c_j$	$-4M+1/3$ ↑	0	M	$-2M+5/3$	0	

		C_j	-12	-20	0	0
C_B	B	X_B	x_1	x_2	s_1	s_2
-12	x_1	15	1	0	$-3/4$	$1/2$
-20	x_2	$5/4$	0	1	$7/16$	$-3/8$
		$z_j - c_j$	0	0	$1/4$	$3/2$

Here all $z_j - c_j \geq 0$

∴ optimum Sol^n is at $x_1 = 15$, $x_2 = 5/4$

∴ $\text{Max } z' = -205$

$\text{Min } z = 205$

Q5 (b) [8]

x	y	x^2	y^2	xy
78	84	6084	7056	6552
36	51	1296	2601	1836
98	91	9604	8281	8918
25	60	625	3600	1500
75	68	5625	4624	5100
82	62	6724	3844	5084
90	86	8100	7396	7740
62	58	3844	3364	3596
65	53	4225	2809	3445
39	47	1521	2209	1833
650	660	47648	45784	45604

$$\bar{x} = \frac{650}{10} = 65, \quad \bar{y} = \frac{660}{10} = 66$$

$$\therefore r(x,y) = \frac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\sqrt{\frac{1}{n}\Sigma x^2 - \bar{x}^2} \cdot \sqrt{\frac{1}{n}\Sigma y^2 - \bar{y}^2}}$$

$$= \frac{\frac{1}{10}(45604) - (65)(66)}{\sqrt{\frac{47648}{10} - 65^2} \cdot \sqrt{\frac{45784}{10} - 66^2}}$$

$$= \frac{270.4}{(23.2336)(14.9131)}$$

$$\therefore r(x,y) = 0.7804$$