Unit 2

Relations and Function.

* A Common notion of a relation is a type of association ~~that~~ exists between ~~two~~ or more objects.

For ex - 1) x is a father of y

2) x was born in the city y in the year z.

* Product set or Cartesian product.

Let A & B be non empty sets we define the product. set or the Cartesian product $A \times B$ as.

$A = \{2, 3\}$

$B = \{a, b\}$

$A \times B = \{(a,b) \mid a \in A \ \& \ b \in B\}$

If $A = \phi$ or $B = \phi$ then $A \times B = \phi$

Q. $A = \{a, b\}$, $B = \{4, 5, 6\}$ Find $A \times B$, $B \times A$, $A \times A$, $B \times B$, $A \times A \times B$.

$\rightarrow$ $A \times B = \{(a,4)\ (a,5)\ (a,6)\ (b,4)\ (b,5)\ (b,6)\}$

$B \times A = \{(4,a)\ (5,a)\ (6,a)\ (4,b)\ (5,b)\ (6,b)\}$

$A \times A = \{(a,a)\ (a,b)\ (b,a)\ (b,b)\}$

$B \times B = \{(4,4)\ (4,5)\ (4,6)\ (5,4)\ (5,5)\ (5,6)$

$(6,4)\ (6,5), (6,6)\}$

$A = \{a, b\}$, $A = \{a, b\}$, $B = \{4, 5, 6\}$

$A \times A \times B = \{(a,a,4), (a,b,4)\ (a,a,5)\ (a,a,6)$

$(a,b,5)\ (a,b,6)\ (b,b,4)\ (b,b,5)$

$(b,b,6)\ (b,a,4)\ (b,a,5)\ (b,a,6)\}$

Q4.1) Let $A = \{1, 2\}$. $B = \{1, 2, 3\}$ Find

$(A \times B) \cap (B \times A)$ (- common pair)

$\rightarrow$ $(A \times B) = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\}$

$(B \times A) = \{(1,1)\ (1,2)\ (2,1)\ (2,2), (3,1)\ (3,2)\}$

$(A \times B) \cap (B \times A) = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\}$

* Basic Concepts of a relation ~~set~~. Let.

$\{A_1, A_2, \ldots, A_n\}$ be a finite Collection of a set. A subset R of $A_1 \times A_2 \ldots A_n$ is called an n-ary relation.

* Binary relation.

Let A & B be non empty sets then a binary relation R From A to B is subset of $A \times B$ is $(R \subseteq A \times B)$ the domain of R denoted by $D(R)$ is the set of elements in a that are related to sub elements in B i.e

$D(R) = \{a \in A \mid \text{For Some } b \in B\ (a,b) \in R\}$

The range of R denoted by $Rn(R)$ is a set of elem in B that are related to some elements in A.

$Rn(R) = \{b \in A \mid \text{for some } a \in A, (a,b) \in R\}$

Qu. Let $A = \{2, 3, 4, 5\}$ & Let R be the relation on A such that $a<b$ find $D(R)$ & $Rn(R)$

→ given $a<b$

$D(R)$ & $Rn(R) = ?$

$A = \{2, 3, 4, 5\}$ → so $a<b$.

(a, b)

$R = \{(2,3)\ (2,4)\ (2,5)\ (3,4)\ (3,5)\ (4,5)\}$

[Domain = a]

[range = b]

$D(R) = \{2, 3, 4\}$

$Rn(R) = \{3, 4, 5\}$

Complement of a Relation.

A relation as a set has its Complement which is define as.

$\overline{R} = \{(a,b) \mid (a,b) \notin R\}$ i.e. $a \not R b$

Qu. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$ ~~Let R~~

Let $R = \{(1,a)\ (1,b)\ (2,c)\ (3,a)\ (4,b)\}$ &

$S = \{(1,b)\ (1,c)\ (2,a)\ (3,b)\ (4,b)\}$

Find 1) $\overline{R}$ & $\overline{S}$ 2) Verify Demorgan's law.

→ $A \times B = \{(1,a)\ (1,b)\ (1,c)\ (2,a)\ (2,b)\ (2,c)$
$(3,a)\ (3,b)\ (3,c), (4,a)\ (4,b), (4,c)\}$

Demorgan's law

$\overline{A \cup B} = \overline{A} \cap \overline{B}$.

$\overline{R} = \{(1,c)\ (2,a)\ (2,b)\ (3,c)\ (4,a)\ (4,c)\}$

$\overline{S} = \{(1,a)\ (2,b)\ (3,c)\ (4,a)\ (4,c)\}$

$\overline{S} = \{(1,a)\ (2,b)\ (2,c)\ (3,a), (3,c)\ (4,a)\ (4,c)\}$

$\overline{R \cup S} = \overline{R} \cap \overline{S}$

$R \cup S = \{(1,a)\ (1,b)\ (1,c)\ (2,a)\ (2,c)\ (3,a)$
$(3,b)\ (4,b)\}$

$\overline{R \cup S} = \{(2,b)\ (3,c)\ (4,a)\ (4,c)\}$ — ①

$\overline{R} \cap \overline{S} = \{(2,b)\ (3,c)\ (4,a)\ (4,c)\}$

From ① & ②

$\therefore \overline{R \cup S} = \overline{R} \cap \overline{S}$.

Converse of a relation.

given a relation from A to B.

One may define a relation from B to A as. Follows.

Let R be a relation from A to B then the converse of R denoted by (R^c) is a relation from B to A define as $R^c = \{(b,a)\ (a,b) \in R\}$

$R^c \subseteq B \times A$.

$A = \{1, 2, 3, 4\}$ $B = \{a, b, c\}$

$R = \{(1,a)\ (3,a)\ (3,c)\}$

Find 1) R^c 2) $D(R^c)$ 3) $Rn(R^c)$.

→ $R^c = \{(a,1)\ (a,3)\ (c,3)\}$

$D(R^c) = \{a, c\}$

$Rn(R^c) = \{1, 3\}$

* Matrix representation of a relation:

Q1) $A=\{a,b,c,d\}$ & $B=\{1,2,3\}$
$R=\{(a,1)(a,2)(b,1)(c,2)(d,1)\}$ Find the relation Matrix.

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array} \quad 4\times 3$$

Q4 2) Let $A=\{1,2,3,4,8\}=B$, aRb iff $a+b\le 9$ Find the relation Matrix:
$R=\{(1,2)(1,3)(1,4)(1,8)(1,1)(2,1)(2,2)(2,3)(2,4)$
$(3,1)(3,2)(3,3)(3,4)(4,1)(4,2)(4,3)(4,4)$
$(8,1)\}$

$$MR = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 8 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Qu. Let $A=\{a,b,c,d\}=B$.

$$MR = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

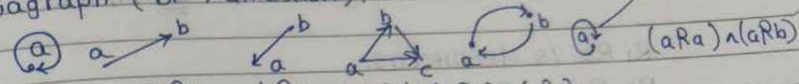
$R=\{(a,a)(a,b)(a,d)(b,b)(b,d)(c,a),(c,b)$
$(c,d)(d,b)(d,c)(d,d)\}$

* Graphical Representation of a Relation:

If A is a finite set & R is a relation on A it is a finite set & R is a relation on A it is possible to represent R pictorially by means of a graph.

The elements of A are represented by points or circles Called as nodes or vertices (edgesires & nodes).

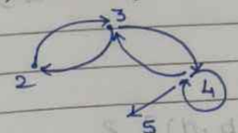
* Diagraph (Di → direction)



aRa aRb bRa (aRb)∧(bRc)∧(aRc)

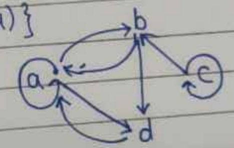
Qu Let $A=\{2,3,4,5\}$ & $R\{(2,3)(3,2)(3,4)(3,5)(4,3)(4,4)(4,5)\}$. Draw its digraph / Directed graph.

→



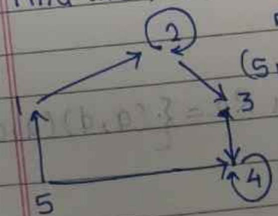
Qu Let $A=\{a,b,c,d\}$ & $MR=\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ draw.

→ $R=\{(a,a)(a,b)(a,d)(b,a)(b,d)(c,d)$
$(c,c)(d,a)\}$



Qu. Find the Relation determine by diagram & give its matrix

→

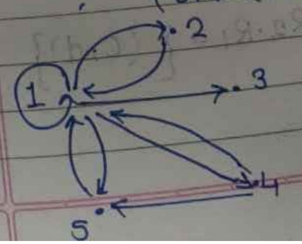


$R=\{(1,2)(2,2)(2,3)(3,4)(4,4)(5,1)$
$(5,4)\}$

$$MR = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Qu. Let $A=\{1,2,3,4,5\}$ & $R=\{(1,1)(1,2)(2,1)(1,3)(1,4)$
$(5,1)(1,5)(4,1)$ draw digraph of R.

→



$$MR = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

* Composition of Binary Relation :-
Let R_1 Be a relation From A ~~to~~ B & R_2 Be a relation From B to C. The Composite Relation From A to C is denoted by $R_1 \cdot R_2$ or $R_1 R_2$.

$R_1 R_2$ is defined as.

$R_1 . R_2 = \{ (a,c) \mid a \in A \wedge c \in C \wedge \exists b [b \in B \wedge (a,b) \in R_1 \wedge (b,c) \in R_2] \}$

Let $A = \{a, b, c, d\}$

$R_1 = \{ (a,a), (a,b) (b,d) \}$ &

$R_2 = \{ (a,d), (b,c), (b,d), (c,d) \}$.

Find 1) $R_1 R_2$.
2) $R_2 R_1$
3) R_1^2
4) R_2^2
5) R_2^3.

How to Find $R_1 R_2$.

R_1	R_2	$R_1 R_2$
a a	a d	(a,d)
a b	b c	(a,c)
a b	b d	(a,d)

1) $\therefore R_1 R_2 = \{ (a,d) (a,c) \}$.

2)

R_2	R_1	$R_2 . R_1$
(a,d)	–	=
(b,c)	–	=
(b,c)	–	=
(c,d)		$\boxed{(c,d)}$

2) $R_2 . R_1 = \{ (c,d) \}$

3) R_1^2

R_1	R_1	$R_1 . R_1$
(a,a)	(a,a)	(a,a)
(a,b)	(b,d)	(a,d)
(a,a)	(a,b)	(a,b).

• $R_1^2 = \{ (a,a), (a,d), (a,b) \}$.

R_2^2.

R_2	R_2	$R_2 R_2$
(a,d)	–	
(b,c)	(c,d)	(b,d)
(b,d)	–	
(c,d)	– b,c	(c,c)
(c,b)	(b,d)	(c,d).

• $R_2^2 = \{ (b,d), (c,c) (c,d) \}$

• R_2^3.

R_1R_2 R_1 $R_1R_2R_1$

Qu. Let $A = \{2,3,4,5,6\}$

Let R_1, R_2 Be relation on A Such that

$R_1 = \cancel{R}\{(a,b) \mid a-b=2\}$

$R_2 = \{(a,b) \mid a+1=b \text{ or } a=2b\}$

Find the Composite Relation,

R_1R_2, R_2R_1, $R_1R_2R_1$, R_1^2.

1) R_1R_2.

$a-b=2$

$4-2=2$

$5-3=2$

$6-4=2$

$\cancel{R_1}R_2 = \{(4,2)($

$a+1=b$	$a+1=b$	$4+1=b$	$5+1=b$
$2+1=b$	$3+1=b$	$5=b$	$6=b$
$3=b$	$4=b$	(a,b)	(a,b)
(a,b)	(a,b)	$(4,5)$	$(5,6)$
$(2,3)$	$(3,4)$		

$R_1 = \{(6,4)\ (4,2)\ (5,3)\}$

$R_2 = \{(2,3)\ (3,4)\ (4,5)\ (5,6)\ (4,2)\ (6,3)\}$

$R_1R_2 =$

R_1	R_2	R_1R_2.
$(6,4)$	$(4,5)$	$(6,5)$
$(\cancel{4,5}$		

$R_1R_2 = \{(4,3)\ (5,4)\ (6,5)\ (6,2)\}$

2) $R_2R_1 = \{(3,2)\ (4,3)\ (5,4)\}$.

3) $R_1R_2R_1 = \{(6,3)\ (5,2)\}$

4) $R_1^2 = \{(6,2)\}$.

5) ~~RT~~ (R converse) $R_1^C = \{(2,4)\ (3,5)\ (4,6)\}$

6) $R_2^C = \{(3,2)\ (4,3)\ (5,4)\ (6,5)\ (2,4)\ (3,6)\}$.

7) $(R_1R_2)^C = \{(3,4)\ (4,5)\ (5,6)\ (2,6)\}$

8) $(R_2R_1)^C = \{(2,3)\ (3,4)\ (4,5)\}$

9) $(R_1R_2R_1)^C = \{(3,6)\ (2,5)\}$

* Equivalence classes.

Let R be an equivalence relation on a set A. For every $a \in A$.
Let $[a]_R$ denote the set $\{x \in A \mid xRa\}$

then $[a]_R$ is called equivalence class of A with respect to R.
The rank of R is the number of distinct equivalence classes of R. If the number of classes are finite otherwise it is infinite.

Qu. Let $A = \{a, b, c\}$

$R = \{(a,a), (b,b) (c,c), (a,b) (b,a)\}$
where, R is clearly an equivalence Relation. Determine the equivalence classes and the Rank of R.

$[a] = \{a, b\}$ —— ①

$[b] = \{b, a\}$

$[c] = \{c\}$ —— ②

The rank of given relation R is 2.

Qu. Let $A = \{1, 2, 3, 4\}$

$R = \{(1,1) (1,2) (1,3) (2,1) (2,2) (3,1) (2,3) (3,2) (3,3) (4,4)\}$

Show that the given Relation is an equivalence Relation find its classes and Rank.

→ 1) Reflexive
$(1,1) \in R$, & $(2,2) \in R$, & $(3,3) \in R$
$(4,4) \in R$.
Therefore the relation is reflexive.

Symmetric.
$(1,2) \in R$ & $(2,1) \in R$.
$(3,1) \in R$ & $(1,3) \in R$.
$(2,3) \in R$ & $(3,2) \in R$.

$[1] = \cancel{\{2,1\}}\ \{1,2,3\}$ —— ①
$[2] = \cancel{\{1,3\}}\ \{1,2,3\}$
$[3] = \cancel{\{1,2\}}\ \{1,2,3\}$ —— ②
$[4] = \{4\}$

∴ The rank of R is is 2

* Partition-

A partition of a non-empty set A is a Collection of $\{A_1, A_2, \ldots A_n\}$ Such that. We denote a partition of A by symbol π

~~i) A =~~
i) $A = \bigcup_{i=1}^{n} A_i$

2) $A_i \cap A_j = \phi$

we denote a partition of A by symbol 'π'
an element of a partition is Called a Block of
The rank of π is the number of blocks of

→ $A = \{1, 2, 3\}$

$\pi_1 = \{\{1, 2\}, \{3\}\}$ Rank = 2

$\pi_2 = \{\{1, 3\}, \{2\}\}$ 2

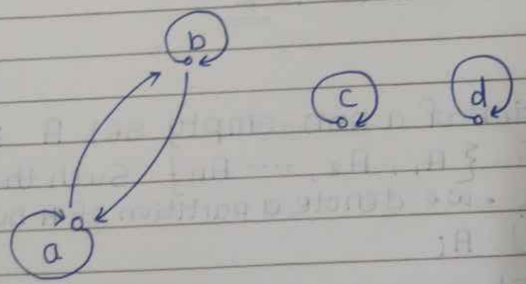
$\pi_3 = \{\{2, 3\}, \{1\}\}$ 2

$\pi_4 = \{\{1\}, \{2\}, \{3\}\}$ 3

Qu Let $A = \{a, b, c, d\}$ and $\pi = \{\{a, b\} \{c\} \{d\}\}$

Find the equivalance relation induced by π. and draw its diagraph.

→ $R = \{(a,a)\ (a,b)\ (b,a)\ (b,b)\ (c,c)\ (d,d)\}$

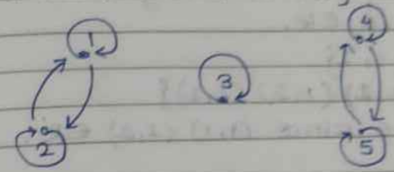


2) $A = \{1, 2, 3, 4, 5\}$

$\pi_{\cancel{R}} = \{\{1, 2\} \{3\} \{4, 5\}\}$

$R = \{$ ~~(1,1) (1,2) (1,3) (1,4) (1,5) (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (3,1) (3~~

$R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,3)\ (4,5)\ (4,4)\ (5,4)\ (5,5)\}$



* Transitive Clouser

Transitive clouser of a relation R is the Smallest transitive relation Containing R we denote transitive clouser of R by R^*

$R^* = R \cancel{\cup R} \cup R^2 \cup R^3 \ldots R^n$.

Back

Special properties of Binary Relation.

1) Reflexive relation:
R is reflexive if for every element $a \in A$, $(a,a) \in R$.
aRa

ex - $A = \{1, 2\}$
$R = \{(1,1) (1,2) (2,2)\}$
R is reflexive since $(1,1)$ $(2,2) \in R$.

2) Irreflexive Relation:
R is said to be irreflexive if for every element $a \in A$ $a \not R a$ i.e $(a,a) \notin R$.
ex - $A = \{a, b\}$ $R = \{(a,b) (b,a)\}$
R is Irreflexive since (a,a) $(b,b) \notin R$.

3) Symmetric Relation.
R is said to be symmetric if whenever aRb then bRa.
ex - $A = \{a, b\}$
$R = \{(a,b)\ (b,a)\ (a,a)\}$
R is symmetric since $(a,b) \in R$ & $(b,a) \in R$

4) Antisymmetric Relation.
R is said to be antisymmetric if whenever aRb then $b \not R a$.
ex -
$A = \{4, 3\}$
$R = \{(4,4)\ (4,3)\ (3,3)\}$
R is Antisymmetric since $(4,3) \in R$ & $(3,4) \notin R$

5) Transistive property.
R is said to be transistive if whenever aRb & bRc then aRc.
ex.
$A = \{3, 4, 5\}$
$R = \{(3,3) (3,4) (4,5) (3,5) (4,4)\}$
R is transistive since,
$(3,4) \in R$ & $(4,5) \in R$ then $(3,5) \in R$

★ Equivalence Relation:
A binary relation are on a set A is called equivalence relation if it is reflexive symmetric & transistive.

Qu. $A = \{a, b, c, d\}$
$R = \{(a,a)\ (a,b)\ (c,c)\ (d,d)\ (b,b)\ (b,d)\}$
determine weather the given weather given condition is an equivalence relation.

1) Reflexive : $\because (a,a)\ (b,b)\ (c,c)\ (d,d) \in R$.
$\therefore$ R is reflexive.

2) Symmetric : $\because (a,b)$ & $(b,d) \in R$ but (b,a) & $(d,b) \notin R$
$\therefore$ R is not symmetric

3) The given Relation R is not an equivalence Relat

Qu. let $A = \{p, q, r\}$ & $M_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Determine weather the given relation R is an equ relation.

$\rightarrow$ $R = \{(p,p)\ (q,q)\ (q,r)\ (r,q)\ (r,r)\}$

1) Reflexive: $\because (p,p)\ (q,q)\ (r,r) \in R$ the given relation is reflexive relation.

2) Symmetric : $\because (q,r) \in R$ & $(r,q) \in R$
Then the relation is symmetric relation.

3) Transitive : $(q,r) \in R$. $(r,q) \in R$ then $(q,q) \in R$

$\therefore$ The Given relation is equivalence relation

Qu. • Determine weather the given relation R is an equivalence relation.

44 46 64 55 54 45
56 65 66

$R = \{(4,4)\ (4,6)\ (5,4)\ (5,5)\ (6,4)\ (5,6)\ (4,5)$
$(6,5)\ (6,6)\}$

1) Reflexive : $(4,4)\ (5,5)\ (6,6) \in R$ given Relation is reflexive.

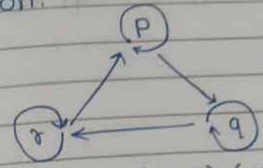
2) Symmetric.
$(4,6) \in R$ and $(6,4) \in R$
$(5,4) \in R$ and $(4,5) \in R$
$(5,6) \in R$ and $(6,5) \in R$.
$\therefore$ Then given relation is symmetric.

3) Transitive
$(4,6) \in R$ & $(6,4) \in R$ $(6,4) \in R$
$(5,4) \in R$ and $(4,5) \in R$.
$\ldots\ (6,5) \in R$

$\therefore$ Then given Relation is transitive.

$\therefore$ The given relation is equivalence relation.

Qu determine whether the given relation are in an equivalence relation.



$R = \{(p,p)\ (r,p)\ (p,q)\ (r,r)\ (q,q)\ (q,r)\}$

1) Reflexive - $(p,p)\ (q,q)\ (r,r) \in R$
$\therefore$ given relation is reflexive.

2) Symmetric:
$(r,p) \in R$ and but $(p,r) \notin R$
$\therefore$ It is not symmetric relation.
Therefore, it is not equivalence relation.

* Composite of Binary Relation.
Let R_1 be the relation from A to B & R_2 be a relation from B to C.
relation from A to C is denote
The Composite relation from A to C is define as
$R_1.R_2$ or R_1R_2 is

$R_1R_2 = \{(a,c) \mid a \in A \wedge c \in C \wedge \exists b [b \in B \wedge (a,b) \in R_1 \wedge (b,c) \in R_2]\}$

Qu. Let $A = \{a, b, c, d\}$

$R_1 = \{(a,a)\ (a,b)\ (b,d)\}$ and

$R_2 = \{(a,d)\ (b,c)\ (b,d)\ (c,b)\}$

Find 1) R_1R_2 2) R_1^2 3) R_2^2 4) R_2^3 5) R_2R_1

→ $\therefore R_1R_2 = \{(a,d)\ (a,c)\}$

R_1	R_2	R_1R_2
a a	a d	$\underline{\underline{ad}}$
a b	b c	$\underline{\underline{ac}}$
a b	b d	$\underline{\underline{ad}}$

$\therefore R_1^2 = R_1.R_2 = \{(a,a)\ (a,b)\ (a,d)\}$

$\therefore R_2^2 = R_2.R_2 = \{(b,b)\ (c,d)\ (c,c)\}$

$\therefore R_2R_1 = \{(c,d)\}$

$\therefore \underset{(R_2^2 . R_2)}{\underline{R_2 . R_2 . R_2}} = \{(b,c)\ (b,d)\ (c,d)\}$

Qu. Let $A = \{2, 3, 4, 5, 6\}$

Let R_1, R_2 be Relation on A such that.

$R_1 = \{(a,b) \mid (a-b) = 2\}$ &

$R_2 = \{(a,b) \mid (a+1) = b \text{ or } a = 2b\}$ find

the Composite relation R_1R_2, R_2R_1, $R_1R_2R_1$.

not necessary

→ $R = \{(2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)\ (3,2)\ (3,3)$
$(3,4)\ (3,5)\ (3,6)\ (4,2)\ (4,3)\}$

$R_1 = \{(4,2)\ (6,4)\ (5,3)\}$

$R_2 = \{(2,3)\ (3,4)\ (4,5)\ (5,6)\ (4,2)\ (6,3)\}$

1) $R_1 . R_2 = \{(4,3)\ (6,5)\ (6,2)\ (5,4)\}$

2) $R_2 . R_1 = \{(3,2)\ (4,3)\ (5,4)\}$

3) $R_1 . R_2 . R_1 = \{(6,3)\ (5,2)\}$

4) $R_1 . R_1 = \{(6,2)\}$

5) $R_1^c = \{(2,4)\ (4,6)\ (3,5)\}$

6) $R_2^2 = \{(3,2)\ (4,3)\ (5,4)\ (6,5)\ (2,4)\ (3,6)\}$

7) $(R_1R_2)^c = \{(3,4)\ (5,6)\ (2,6)\ (4,5)\}$

* Transistive Clouser:

→ Transistive clouser

$R^* = R \cup R^2 \cup R^3 \cup R^4 \dots \cup R^n$.

Que) Let $A = \{1,2,3,4\}$ and $R = \{(1,2)\ (2,3)\ (3,4)\}$ be a relation on A find R^* and draw its diagraph.

→ $R = \{(1,2)\ (2,3)\ (3,4)\}$

- $R^2 = R \circ R = \{(1,2)\ (2,3)\ (3,4)\}\ \{(1,2)\ (2,3)\ (3,4)\}$

  $R^2 = \{(1,3), (2,4)\}$

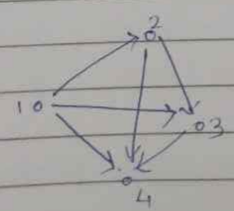
- $R^3 = R^2 \circ R = \{(1,3)\ (2,4)\}\ \{(1,2)\ (2,3)\ (3,4)\}$.

  $R^3 = \{(1,4)\}$.

- $R^* = R \cup R^2 \cup R^3$

  $R^* = \{(1,2)\ (2,3)\ (3,4)\ (1,3)\ (2,4)\ (1,4)\}$

Ou.



* Warshall's Algorithm:

$A = \{1,2,3,4\}$

$R = \{(1,2)\ (2,4)\ (1,3)\ (3,2)\}$.

Qu Find the transitive Clouser by warshall's Algorithm.

→
$$W_0 = M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$W_1 \Rightarrow a_k = 1$. $k = 1$.

where, 1 is not an interior vertex. for any path in R.

therefore.

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

W_2, $a_k = 2$, $k = 2$.

∵ 2 is an interior vertex For the path from

$(1,2)\ (2,4) = (1,4)$

$(3,2)\ (2,4) = (3,4)$.

So add 1 for the position (1,4) and (3

therefore,

$$W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

W_3, $a_k = 3$, $k = 3$.
where 3 is an interior vertex. (1,3) (3,2) = (1,2)

	1	2	3	4
$W_3 =$ 1	0	1	1	1
2	0	0	0	1
3	0	1	0	1
4	0	0	0	0

W_4, $a_k = 4$, $k = 4$. $W_4 = M_{R^*}$.

	1	2	3	4
$W_4 =$ 1	0	1	1	1
2	0	0	0	1
3	0	1	0	0
4	0	0	0	0

$R^* = \{(1,2)(1,3)(1,4)(2,4)(3,2)\}$

* Warshall's Algorithm:

2) Find the transitive clouser R by Warshall's Algo
where, $A = \{1, 2, 3, 4, 5, 6\}$
$R = \{(x, y) \mid |x - y| = 2\}$

$R = \{(1,3), (3,1) (2,4) (4,2) (3,5) (5,3) (4,6) (6,4)\}$

	1	2	3	4	5	6
1	0	0	1	0	0	0
$W_0 =$ 2	0	0	0	1	0	0
3	1	0	0	0	1	0
4	0	1	0	0	0	1
5	0	0	1	0	0	0
6	0	0	0	1	0	0

For, W_1, $a_k = 1$, $k = 1$.
1 is an interior vertex (3,1) (1,3) = (3,3). For the path. so add 1 For the position(3, 3.)

	1	2	3	4	5	6
1	0	0	1	0	0	0
$W_1 =$ 2	0	0	0	1	0	0
3	1	0	1	0	1	0
4	0	1	0	0	0	1
5	0	0	1	0	0	0
	0	0	0	1	0	0

For W_2, $a_k = 2$, $k = 2$.
2 is an interior vertex (4,2) (2,4) = (4,4).
So add 1 for the position (4, 4)

	1	2	3	4	5	6
$W_2 =$ 1	0	0	1	0	0	0
2	0	0	0	1	0	0
3	1	0	$\boxed{1}$	0	1	0
4	0	1	0	$\boxed{1}$	0	1
5	0	0	1	0	0	0
6	0	0	0	1	0	0

For W_3, $a_k = 3$, $k = 3$.
3 is an interior vertex For the path (1,3) (3,1) = (
So add 2 for pos^n (1,1) & (5,5) (5,3) (3,5) = (
(1,3) (3,5) =
(5,3) (3,1) =

4 P – (1,3) (3,1) (5,3) (

	1	2	3	4	5	6
$W_3 =$ 1	$\boxed{1}$	0	1	0	$\boxed{1}$	0
2	0	0	0	1	0	0
3	1	0	$\boxed{1}$	0	1	0
4	0	1	0	1	0	1
5	$\boxed{1}$	0	1	0	$\boxed{1}$	0
6	0	0	0	1	0	0

For W_4, $a_k = 4$, $k = 4$.
4 is an interior vertex for path
(2,4) (4,2) (6,4) (4,6).

(2,4) (4,2) = (2,2).
(2,4) (4,6) = (2,6).
(6,4) (4,2) = (6,2).
(6,4) (~~4,6~~ (4,2) = (6,2)

$$W_6 = W_5 = W_4 = \begin{array}{c|c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

For W_5 & W_6, $a_k = 5$, $k = 5$. , $a_k = 6$, $k = 6$.
5 is an interior vertex for path. & 6 is an interior vertex for path
(3,5) (5,3) = (3,3)
(4,6) (6,4) = (4,4)

$$MR^* = W_4 = W_5 = W_6 = \begin{array}{c|c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

For position 5 and 6 the entries are already present so need to W_5 and W_6 entries again.

$R^* = \{$(1,1) (~~1,1~~) (1,3) (1,5) (2,2) (2,4) (2,6) (3,1) (3,3) (3,5) (4,2) (4,4) (4,6) (5,1) (5,3) (5,5) (6,2) (6,4) (6,6)$\}$.

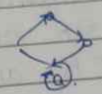
* **Partial Ordering Relations:**
An ordered relation is a transitive relation on a set by means of which we can compare elements of set.
Define - A binary relation R on a non empty set A is a partial order, if R is reflexive, antisymmetric and transitive.

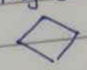
Ex - 1) The relation '≤' is a partial order on the set of real number.

2) The relation of "being a subset" is a partial order on any collection of subsets of a set i.e. the ordered pair $(P(A), \subseteq)$ is a Poset
power set of A, ∈ to subset, partially ordered set.

* **Hasse Diagram:-**

POSET Hasse diag.	equivalence rel.^n E.R. Diagram.
1) Reflexivity ×	
2) Direction ×	
3) looping omitted ×	





A Hasse diagram is a simpler version of a digraph encorporating the following Rules ~~all~~
1) All arrow heads that appear ~~on~~ on the edges are omited
2) Loops are ommited
3) Transitivity are omitted.

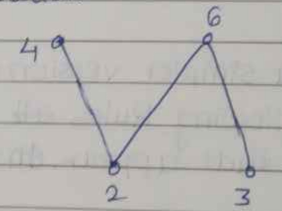
Qu1) Let $A = \{2, 3, 4, 6\}$ aRb if $\frac{a}{b}$ show that
show that R is partial order and draw its Hasse diagram.

→ $R = \{(2,4)\ (2,6)\ (3,6)\ (2,2)\ (4,4)\ (6,6)\ (3,3)\}$.

i) R is reflexive since.
$(2,2)\ (3,3)\ (4,4)\ (6,6) \in R$.

ii) Antisymmetric -
R is antisymmetric since.
$(2,4) \in R$ and $(4,2) \notin R$.
$(2,6) \in R$ and $(6,2) \notin R$.
$(3,6) \in R$ and $(6,3) \notin R$.

iii) Transitive -
R is transitive, since
$(2,4) \in R$ and $(4,4) \in R$ then $(2,4) \in R$.
$(3,3) \in R$ and $(3,6) \in R$ then $(3,6) \in R$.
∴ Hence the given Relation is partial order relation.

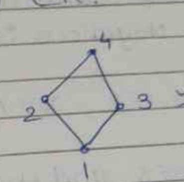


Qu2) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,1)\ (1,2)\ (2,2)\ (2,4),\ (1,3)\ (3,3),\ (3,4)\ (1,4)\ (4,4)\}$
then show that R is Partial order and draw its Hasse Diagram.

→ i) R is Reflexive since,
$(1,1)\ (2,2)\ (3,3)\ (4,4) \in R$.

ii) R is ~~reflexive~~ Antisymmetric since,
$(1,2) \in R$ $(2,1) \notin R$
$(2,4) \in R$ $(4,2) \notin R$
$(3,4) \in R$ $(4,3) \in R$.

iii) Transitive.
R is transitive since.
$(1,2) \in R$. and $(2,4)$ then $(2,4) \in R$.



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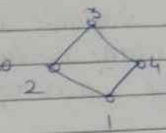

Unit 1 topic

* Normal forms :

* Properties -
1) logical Equivalence involving Conditional statement

1) $p \to q \equiv \sim p \vee q$
2) $p \to q \equiv \sim q \to \sim p$ } - Imp
3) $p \vee q = \sim p \to q$.
4) $p \wedge q \equiv \sim(p\, q \to \sim p)$
5) $\sim(p \to q) \equiv p \wedge \sim q$
6) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
7) $(p \to r) \wedge (q \to r) \equiv (p \wedge q) \to r$
8) $(p \to r) \vee (q \to r) \equiv (p \vee q) \to r$.