DSGT-2 H. E. Suryavanshi (2)

The Shirpur Educaton Society's

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High Caliber Technical Education in an Environment that Promotes Excellence

TEST (I / II) / PRELIMINARY EXAMINATION

Name of Candidate : ______________________

(IN BLOCK LETTERS) (Surname) (First Name) (Middle Name)

Year : FE / SE / TE / BE Branch : ________ Division : ________ Roll No. : ________

Semester : I / II Name of Subject : ______________________

Total Supplements : 1 + ______ = ______

Signature of Student Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

Signature of Examiner : Signature of Moderator :

(Start From here only)

* DSGT $\boxed{\text{Unit-2}}$

* **Basic concept of Relation** -

Let $\{A_1, A_2, \cdots A_n\}$ be a finite collection of sets. R is subset of $A_1 \times A_2 \times \cdots \times A_n$ is called n-ary relation on $\{A_1, A_2, \cdots A_n\}$

If $R = \phi$, R → void / empty relation

If $R = A_1 \times A_2 \times \cdots \times A_n$, R → universal relation

If $A_i = A$ for all i, R → n-ary relation

If $n = 1, 2, $ or 3 → Unary, Binary, Ternary rel^n

Ex. ① Let $A = \{1, 2, 3\}$ R = "x is less than y"

∴ $R = \{(1,2)\ (1,3)\ (2,3)\}$

Then R = Binary Relation

* Binary Relation

Let A & B non-empty sets. Then binary Relation R from A to B is subset of $A \times B$

i.e $R \subseteq A \times B$

Domain : $D(R)$

- It is a set of elements in A that are related to some elements in B i.e

$$D(R) = \{ a \in A \mid \text{for some } b \in B, (a,b) \in R \}$$

Range : $Rn(R)$

- It is the set of elements in B, that are related to some elements in A i.e

$$Rn(R) = \{ b \in B \mid \text{for some } a \in A, (a,b) \in R \}$$

Ex. ①: Let $A = \{2, 3, 4, 5\}$
let R be the relation on A (aRb iff $a<b$)
Find $D(R)$ & $Rn(R)$

$\Rightarrow$

$A = \{2, 3, 4, 5\}$

$R = \{(2,3)\ (2,4)\ (2,5)\ (3,4)\ (3,5)\ (4,5)\}$

$\therefore D(R) = \{2, 3, 4\}$

$\therefore Rn(R) = \{3, 4, 5\}$

* Complement of a Relation

The complement of relation R, denoted by $\bar{R}$ is as

$$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$$

i.e $a\bar{R}b$ iff $a \not R b$

Ex ① Let $A = \{1, 2, 3, 4\}$ & $B = \{a, b, c\}$

$R = \{(1, a)\ (1, b)\ (2, c)\ (3, a)\ (4, b)\}$

and

$S = \{(1, b)\ (1, c)\ (2, a)\ (3, b)\ (4, b)\}$

Find i> $\bar{R}$ & $\bar{S}$

ii> verify De Morgan's laws for R & S

$\Rightarrow$

i> $A \times B = \{(1, a)\ (1, b)\ (1, c)\ (2, a)\ (2, b)\ (2, c)\ (3, a)\ (3, b)\ (3, c)\ (4, a)\ (4, b)\ (4, c)\}$

$(A \times B) - R \rightarrow \therefore \bar{R} = \{(1, c)\ (2, a)\ (2, b)\ (3, b)\ (3, c)\ (4, a)\ (4, c)\}$

&

$(A \times B) - S \rightarrow \therefore \bar{S} = \{(1, a)\ (2, b)\ (2, c)\ (3, a)\ (3, c)\ (4, a)\ (4, c)\}$

ii> De Morgans Law

$$\overline{R \cup S} = \bar{R} \cap \bar{S} \quad \&$$
$$\overline{R \cap S} = \bar{R} \cup \bar{S}$$

$\therefore R \cup S = \{(1, a)\ (1, b)\ (1, c)\ (2, a)\ (2, c)\ (3, a)\ (3, b)\ (4, b)\}$

$(A \times B) - R \cup S \hookrightarrow \overline{R \cup S} = \{(2, b)\ (3, c)\ (4, a)\ (4, c)\}$

$\bar{R} \cap \bar{S} = \{(2, b)\ (3, c)\ (4, a)\ (4, c)\}$

Hence $\overline{R \cup S} = \bar{R} \cap \bar{S}$

Now $R \cap S = \{(1, b)\ (4, b)\}$

$\overline{R \cap S} = \{(1, a)\ (1, c)\ (2, a)\ (2, b)\ (2, c)\ (3, a)\ (3, b)\ (3, c)\ (4, a)\ (4, c)\}$

$\bar{R} \cup \bar{S} = \{(1, a)\ (1, c)\ (2, a)\ (2, b)\ (2, c)\ (3, a)\ (3, b)\ (3, c)\ (4, a)\ (4, c)\}$

Hence, $\overline{R \cap S} = \bar{R} \cup \bar{S}$

* Converse of a Relation

Given relation from A to B, is R

The converse relation of R, denoted by R^c is the relation from B to A, defined as

$$R^c = \{ (b,a) \mid (a,b) \in R \}$$

clearly

$$R^c \subseteq B \times A$$

Theorem

Let R, S be relations from A to B. Then

(i) $(R^c)^c = R$

(ii) $(R \cup S)^c = R^c \cup S^c$

(iii) $(R \cap S)^c = R^c \cap S^c$

Ex. (i)

Let $A = \{1, 2, 3, 4\}$ & $B = \{a, b, c\}$

Given $R = \{ (1,a) (3,a) (3,c) \}$

Find (i) R^c (ii) $D(R^c)$ (iii) $R_n(R^c)$

$\Rightarrow$ (i) $R = \{ (1,a) (3,a) (3,c) \}$

$R^c = \{ (a,1) (a,3) (c,3) \}$

(ii) $D(R^c) = \{a, c\} = R_n(R)$

(iii) $R_n(R^c) = \{1, 3\} = D(R)$

Ex (2)

Let $A = \{2, 3, 4, 6\}$. Let R & S be relations on A that

$R = \{ (a,b) \mid a = b+1 \text{ or } b = 2a \}$

and

$S = \{ (a,b) \mid a \text{ divides } b \}$

find $(R \cap S)^c$

$\Rightarrow \quad A = \{2, 3, 4, 6\}$

$$R = \{ (3,2)\ (4,3)\ (2,4)\ (3,6) \}$$

&

$$S = \{ (2,2)\ (2,4)\ (2,6)\ (3,3)\ (3,6)\ (4,4)\ (6,6) \}$$

$$\therefore R \cap S = \{ (2,4)\ (3,6) \}$$

Then

$$(R \cap S)^c = \{ (4,2)\ (6,3) \}$$

=x=

* Composition of Binary Relations

Composit Relation

— Relations that are formed from an existing sequence of relations.

Ex.

Relationship of grand father who is fathers / mothers father

Definition

Let R_1 be relation from A to B

R_2 be relation from B to C

Then,

composit relation from A to C, denoted by $R_1 \cdot R_2$ (or $R_1 R_2$)

Theorems

① R_1, R_2 & R_3 be relations from A to B, B to C and C to D.

Then $(R_1 \cdot R_2) \cdot R_3 = R_1 \cdot (R_2 \cdot R_3)$

② R_1, R_2 be relations from A to B & B to C

Then

$$(R_1 \cdot R_2)^c = R_2^c \cdot R_1^c$$

Ex. 1 $A = \{a, b, c, d\}$

$R_1 = \{(a,a)\ (a,b)\ (b,d)\}$ &

$R_2 = \{(a,d)\ (b,c)\ (b,d)\ (c,b)\}$

Find $R_1 \cdot R_2$, $R_2 \cdot R_1$, R_2^2 & R_2^3

$\Rightarrow$

$R_1 \cdot R_2 = \{(a,d)\ (a,c)\ (a,d)\}$

$R_2 \cdot R_1 = \{(c,d)\}$

$R_1^2 = \{(a,a)\ (a,b)\ (a,d)\}$

$R_2^2 = \{(b,b)\ (c,c)\ (c,d)\}$

$R_2^3 = \{(b,c)\ (b,d)\ (c,b)\}$

Ex. 2

$A = \{1, 2, 3, 4\}$ Let R_1 defined as

$R_1 = \{(x,y) \mid x+y=5\}$ let R_2 defined as

$R_2 = \{(x,y) \mid y-x=2\}$

Verify that $(R_1 \cdot R_2)^c = R_2^c \cdot R_1^c$

$\Rightarrow$

$A = \{1,2,3,4\}$

$R_1 = \{(1,4)\ (2,3)\ (3,2)\ (4,1)\}$

$R_2 = \{(1,3)\ (2,4)\}$

$R_1 \cdot R_2 = \{(3,4)\ (4,3)\}$

$(R_1 \cdot R_2)^c = \{(4,3)\ (3,4)\}$

$R_1^c = \{(4,1)\ (3,2)\ (2,3)\ (1,4)\}$

$R_2^c = \{(3,1)\ (4,2)\}$

$R_2^c \cdot R_1^c = \{(3,4)\ (4,3)\}$ Hence $(R_1 \cdot R_2)^c = R_2^c \cdot R_1^c$

Ex. ③ $A = \{2, 3, 4, 5, 6\}$

$R_1 = \{(a, b) \mid a - b = 2\}$

$R_2 = \{(a, b) \mid a + 1 = b \text{ or } a = 2b\}$

$\Rightarrow$ Find ① $R_1 \cdot R_2$ ② $R_2 \cdot R_1$ ③ $R_1 \cdot R_2 \cdot R_1$

④ R_1^2 ⑤ $R_1 \cdot R_2^2$

$\Rightarrow$

$A = \{2, 3, 4, 5, 6\}$

$R_1 = \{(4,2)\ (5,3)\ (6,4)\}$

$R_2 = \{(2,3)\ (3,4)\ (4,5)\ (5,6)\ (4,2)\ (6,3)\}$

① $R_1 \cdot R_2 = \{(4,3)\ (5,4)\ (6,2)\ (6,5)\}$

② $R_2 \cdot R_1 = \{(3,2)\ (5,4)\ (4,3)\}$

③ $R_1 \cdot R_2 \cdot R_1 = R_1 \cdot (R_2 \cdot R_1)$

$= \{(5,2)\ (6,3)\}$

=x=

* **Matrix Relation (M_R)** — matrix Representation of Relation

$R \subseteq A \times B$

$\therefore M_R$ (A – Rows, B – columns)

Ex. ① $A = \{a, b, c, d\}$

$B = \{1, 2, 3\}$

and

$R = \{(a,1)\ (a,2)\ (b,1)\ (c,2)\ (d,1)\}$

Find the relation matrix

$\Rightarrow$

M_R : (4 rows, 3 columns)

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Ex. 2 $A = \{1, 2, 3, 4, 8\}$

$B = \{1, 4, 6, 9\}$

Let aRb iff $a \mid b$ (a divides b)

Find the relation matrix

$\Rightarrow$

$R = \{(1,1)\ (1,4)\ (1,6)\ (1,9)\ (2,4)\ (2,6)$
$(3,6)\ (3,9)\ (4,4)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 6 & 9 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Ex. 3 Let $A = \{a, b, c, d\}$ and

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$ Find R

$\Rightarrow$

$R = \{(a,a)\ (a,b)\ (b,c)\ (b,d)\ (c,c)\ (c,d)$
$(d,a)\ (d,c)\}$

Ex. 4 $A = \{1, 2, 3, 4, 8\} = B$; aRb iff $a + b \le 9$

Find M_R

$\Rightarrow$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,8)\ (2,1)\ (2,2)\ (2,3)$
$(2,4)\ (3,1)\ (3,2)\ (3,3)\ (3,4)\ (4,1)\ (4,2)$
$(4,3)\ (4,4)\ (8,1)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 8 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

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* Relation Matrix Operation

Boolean matrix → having entry either 0 or 1.

$A = [a_{ij}]$

$B = [b_{ij}]$ be $m \times n$ boolean matrix

① $A + B = [c_{ij}]$ where,

$c_{ij} = 1$ if $a_{ij} = 1$ or $b_{ij} = 1$

$= 0$ if a_{ij} and b_{ij} — both zero

② $A \cdot B = [d_{ij}]$ where,

$d_{ij} = 1$ if $a_{ij} = b_{ij} = 1$

$= 0$ if $a_{ij} = 0$ or $b_{ij} = 0$

i.e (Rows x Columns)

Ex ① $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$A + B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ & $A \cdot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

* Properties of Relation of Matrix

Let R_1 : relation from A to B
R_2 : $B \to C$

1. $M_{R_1 \cdot R_2} = M_{R_1} \cdot M_{R_2}$

2. $M_{R^c} =$ transpose of M_R

3. $M_{(R_1 \cdot R_2)^c} = M_{R_2^c \cdot R_1^c} = M_{R_2^c} \cdot M_{R_1^c}$

Ex. ①

$A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1)\ (1,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\ (4,2)\}$

and

$R_2 = \{(3,1)\ (4,4)\ (2,3)\ (2,4)\ (1,1)\ (1,4)\}$

verify ① $M_{R_1 \cdot R_2} = M_{R_1} \cdot M_{R_2}$

② $M_{R_1^c} =$ transpose of M_{R_1}

③ $M_{(R_1 \cdot R_2)^c} = M_{R_2^c} \cdot M_{R_1^c}$

$\Rightarrow$

① $M_{R_1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ & $M_{R_2} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_1 = \{(1,1)\ (1,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\ (4,2)\}$

$R_2 = \{(1,1)\ (1,4)\ (2,3)\ (2,4)\ (3,1)\ (4,4)\}$

$R_1 \cdot R_2 = \{(1,1)\ (1,4)\ (1,3)\ (2,1)\ (2,4)\ (3,4)\ (4,4)\ (4,1)\ (4,3)\}$

$$M_{R_1 \cdot R_2} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad \text{--- (1)}$$

$$M_{R_1} \cdot M_{R_2} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad \text{--- (2)}$$

from (1) & (2)

$$M_{R_1 \cdot R_2} = M_{R_1} \cdot M_{R_2}$$

(2) $R_1^c = \{(1,1)\ (2,1)\ (3,2)\ (4,2)\ (4,3)\ (1,4)\ (2,4)\}$

$$M_{R_1^c} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \text{transpose of } M_{R_1}$$

(3) $(R_1 \cdot R_2)^c = \{(1,1)\ (4,1)\ (3,1)\ (1,2)\ (4,2)\ (4,3)\ (4,4)\ (1,4)\ (3,4)\}$

$$M_{(R_1 \cdot R_2)^c} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M_{R_2^c} \cdot M_{R_1^c} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= M_{R_2^c} \cdot M_{R_1^c}$$

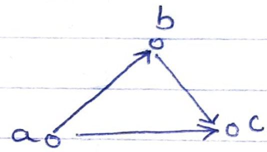
* Graphical Representation of a Relation -

If A is finite set & R is relation on A, then R is represented by graph using points / circles, called nodes or vertices

Ex. ① aRb  ② aRa 

③ $aRb \wedge bRa$ 

④ $aRb \wedge bRb$ 

⑤ $aRb \wedge bRc \wedge aRc$ 

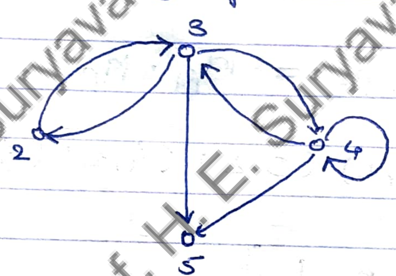
Examples

① $A = \{2, 3, 4, 5\}$

$R = \{(2,3)(3,2)(3,4)(3,5)(4,3)(4,4)(4,5)\}$

draw its digraph.

$\Rightarrow$

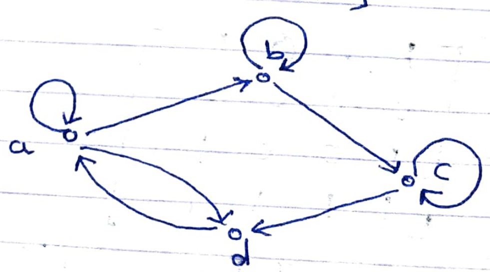


② $A = \{a, b, c, d\}$ and $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

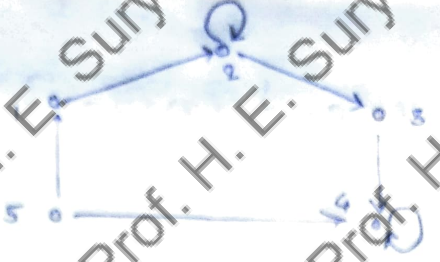
draw the digraph of R

$\Rightarrow$

$R = \{(a,a)(a,b)(a,d)(b,b)(b,c)(c,c)(c,d)(d,a)\}$



③ Find the relation determined by the digraph and give its matrix.



$\Rightarrow$

$A = \{1, 2, 3, 4, 5\}$

$R = \{(1,2)\ (2,2)\ (2,3)\ (3,4)\ (4,4)\ (5,4)\ (5,1)\}$

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

* Special Properties of Binary Relations -

Let, R be a Relation on a set A

① Reflexive Relation (Every element related to itself)

- R is reflexive if for every element $a \in A$, aRa i.e $(a,a) \in R$
- R is not reflexive if for some element $a \in A$, $a \not R a$ i.e $(a,a) \in R$

Ex ① $A = \{a, b\}$ & $R = \{(a,a)\ (a,b)\ (b,b)\}$

$\therefore$ R is reflexive

Ex ② $A = \{1, 2\}$ & $R = \{(1,1)\ (1,2)\}$

$\therefore$ R is not-reflexive since $(2,2) \notin R$

② Irreflexive Relation (Elements cannot related to itself)

- R is irreflexive if for every element $a \in A$, $a \not R a$ i.e $(a, a) \notin R$.

Ex ① $A = \{1, 2\}$ & $R = \{(1,2)\ (2,1)\}$

$\therefore$ R irreflexive since $(1,1)\ (2,2) \notin R$

Ex. ② $A = \{1, 2\}$ & $R = \{(1,2), (2,2)\}$

$\therefore$ R is not-irreflexive. Since $(2,2) \notin R$

also,

R is not-reflexive Since $(1,1) \notin R$

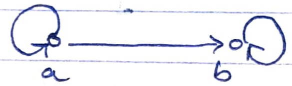
Note - If R - Reflexive

then M_R - have digonal entries

- If R - Irreflexive :

then M_R - digonal elements zero

Diagraph of Reflexive Relation



③ Symmetric Relation

- R is Symmetric if whenever aRb then bRa
- R is not-Symmetric if for some a & b ∈ A, aRb but $b \not R a$

Note

R - Symmetric then M_R is Symmetric matrix

i.e If $A_{ij} = 1$ then $A_{ji} = 1$

If $A_{ji} = 0$ then $A_{ij} = 0$ (For $i \neq j$)

Ex. ① A = Set of people.

If aRb - a is friend of b

then obviously b is related to a

$\therefore$ "friend" - Symmetric Relation

Ex. ② A = Set of people

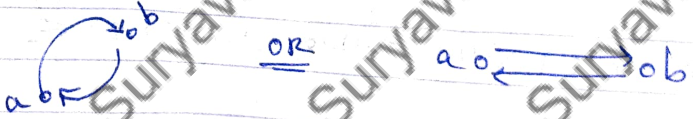
aRb if a is brother of b

$\therefore$ Not-Symmetric

Since, b can be Sister of a

It is Symmetric if A = Set of males

④ Diagraph of Symmetric Relation



$(a,b) \in R$ but $(b,a) \notin R$

④ Asymmetric Relation

{ R is asymmetric if it is both anti-symmetric & irreflexive }

- R is asymmetric if whenever aRb, then $b \not{R} a$
- R not-asymmetric if for some a and b, we have both aRb, & bRa

Ex. ① $A = \{2, 4, 5\}$ R – "is a divisor of"

Then

$R = \{ (2,2)\,(2,4)\,(4,4)\,(5,5) \}$

$\therefore$ R is not-asymmetric since $\{(2,2), (4,4), (5,5)\} \in R$

Ex. ② $A = \mathbb{R}$ Set of Real no.

Let R = Relation '$<$'

Then $a < b \rightarrow b \not< a$.

⑤ Anti-Symmetric Relation –

- R is Anti-Symmetric if whenever aRb & bRa then $a = b$
- R not Anti-Symmetric if we have elements $a, b \in A$ such that $a \neq b$ but both aRb and bRa

Ex. ① $A = \mathbb{R}$, let R be the Relation '$\leq$'.

Then $a \leq b$ and $b \leq a \rightarrow a = b$

Hence '$\leq$' is Anti-Symmetric Relation

Ex. ② $A = \{1, 2, 3\}$ and $R = \{ (1,2)\,(2,1)\,(2,3) \}$

R – Not-Anti-Symmetric since $(1,2)\,(2,1) \in R$

R – Not-Symmetric since $(2,3) \in R$ but $(3,2) \notin R$

R – Not-Asymmetric since $(1,2)$ & $(2,1) \in R$

⑥ Transitive Relation

R is transitive if aRb, bRc then aRc

R – Not-transitive if aRb, bRc but $a \not{R} c$

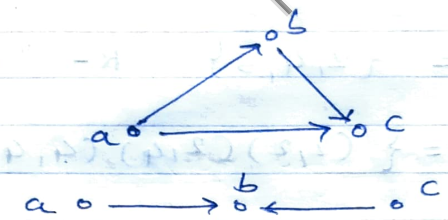
Ex. ① A → Set of people & R = "brother of"

Then 'a' → brother of b, b → brother of c

then a → brother of c

∴ transitive relation

Diagraph



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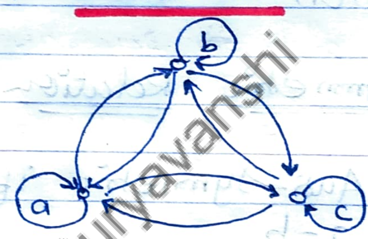
* Equivalence Relation

A binary relation R on set A is called equivalence relation if it is reflexive, symmetric and transitive.

diagraph



Ex. ①

$A = \{a, b, c, d\}$

$R = \{(a,a)\ (b,a)\ (b,b)\ (c,c)\ (d,d)\ (d,c)\}$

⇒ Determine whether R – Equivalence relⁿ.

R – reflexive since $(a,a)(b,b)(c,c)$ & (d,d)

R – Not-Symmetric since $(b,a) \in R$

but $(a,b) \notin R$

∴ R – Not-Equivalence relation ①

Ex. ②

Let $A = \{a, b, c\}$ and let $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ ②

Determine wheather R is an equivalence relation.

⇒

$R = \{(a,a)\ (b,b)\ (b,c)\ (c,b)\ (c,c)\}$

R : Reflexive since $(a,a)(b,b)(c,c) \in R$

R : Symmetric since $(b,c) \in R$ & $(c,b) \in R$

R : transitive since

(b,b) & $(b,c) \in R$ implies $(b,c) \in R$

(b,c) & $(c,b) \in R$ implies $(b,b) \in R$ &

DSGT-2   H. E. Suryavanshi   (1)

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Semester : I / II Name of Subject : ______________________________

Total Supplements : 1 + ______ = ______

Signature of Student   Signature of Supervisor

Question Number	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											
Marks out of											

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* Properties of Equivalence Relations

(1) If R_1 & R_2 are equivalence relations on set A then $R_1 \cap R_2$ is an equivalence relations

(2) If R_1 & R_2 are equivalence Relations, it is not necessary that $R_1 \cup R_2$ is also an equivalence relation.

=x=

* Equivalence Classes -

Let, R be an equivalence Relation on a set A. For every $a \in A$, let $[a]_R$ denote that the set $\{x \in A \mid x R a\}$.

Then, $[a]_R$ is called equivalence class of a with respect to R.

$= [a]_R \neq \phi$ since $a \in [a]_R$

Rank of R is the no. of distinct equivalence classes of R if the no. of classes is finite; otherwise the rank is said to be finite.

Theorems

Let R be an equivalence relⁿ. on set A. Then following hold:

① For all $a, b \in A$, either $[a] = [b]$ or $[a] \cap [b] = \phi$

② $A = \bigcup_{a \in A} [a]$

Ex. ①

Let $A = \{a, b, c\}$ & let $R = \{(a,a) (b,b) (c,c) (a,b) (b,a)\}$

Where R is equivalence relation

⟹

Equivalence classes of elements of A

$[a] = \{a, b\}$

$[b] = \{b, a\} = [a]$

$[c] = \{c\}$

∴ Then rank of R is $\underline{\underline{2}}$

Ex. ② $A = \{1, 2, 3, 4\}$ &

$R = \{(1,1) (1,2) (1,3) (2,1) (2,2) (3,1) (2,3) (3,2) (3,3) (4,4)\}$

- Show that R is an equivalence relation & determine the equivalence classes & hence find the rank of R

⟹

R: reflexive since $(1,1) (2,2) (3,3) (4,4) \in R$

R: Symmetric $(1,2), (2,1) \in R$

Similarly, $(2,3), (3,2), (1,3)(3,1) \in R$

R: transitive

Since $(1,2)$ & $(2,1) \in R$

implies $(1,1) \in R$

Similarly,

$(1,3)(3,1) \in R \rightarrow (1,1) \in R$
$(2,3)(3,2) \in R \rightarrow (2,2) \in R$
$(3,1)(1,3) \in R \rightarrow (3,3) \in R$
$(3,2)(2,1) \in R \rightarrow (3,1) \in R$

hence,

R is equivalence Relation.

$\therefore$ Equivalence classes of A are

$[1] = \{1,2,3\}$
$[2] = \{1,2,3\} = [1]$
$[3] = \{1,2,3\} = [1]$
$[4] = \{4\}$

$\therefore$ there are two distinct equivalence classes. Hence Rank of R is 2

* Partitions

- The concept of partition is closely Related to the equivalence Reln.

Defn:

- A partition of non-empty set A is collection of sets {A1, A2, ... An} such that

i> $A = \bigcup_{i=1}^{n} A_i$

ii> $A_i \cap A_j = \phi$, for $i \neq j$ (i.e sets Ai are mutually disjoint)

- The partition of A is denoted by π. (pi)
- An element of partition is called a block.
- The Rank of π is the no. of blocks of π.

Ex.① $A = \{1,2,3\}$

Then $\pi_1 = \{\{1,2\}, \{3\}\}$ partition of A

& $\pi_2 = \{\{1,3\}, \{2\}\}$

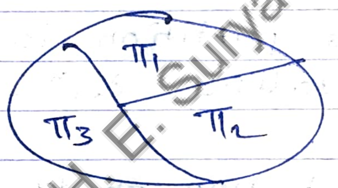
$\pi_3 = \{\{1\}, \{2\}, \{3\}\}$ & so on

Ex. ② Let Z = Set of all integers
E = Set of all even integers
O = Set of all odd integers
Then
$\{E, O\}$ is a partition of Z.

Ex. ③ The rooms (Flats) in a building block form a partition.

Ex. ④ The main m/m of multi-programmed computer System is partitioned & a seperate prog. is stored in each block of the partition.

Following diagram represents partition of se



Theorem

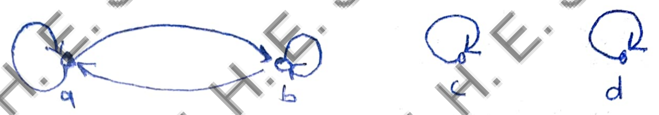
① Let, A be non-empty set & R an equivalence relations & ~~partitions~~ ~~one~~ on A., then the set of equivalence classes $\{[a]_R \mid a \in A\}$ constitutes a partition of A. i.e (A/R)

② Let, A be non-empty set, & let π be a partition of A. Then π includes an equivalence relation on A.

Ex. ① Let $A = \{a, b, c, d\}$
$\pi = \{\{a, b\}, \{c\}, \{d\}\}$

Find the equivalence relation induced by π & construct its graph

$\Rightarrow$ $R = \{(a,a)\ (b,b)\ (a,b)\ (b,a)\ (c,c)\ (d,d)\}$

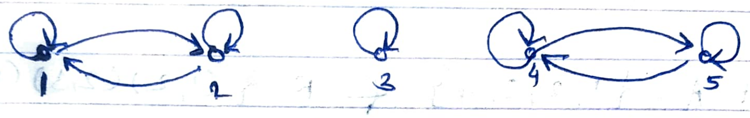


Ex. ② Let $A = \{1, 2, 3, 4, 5\}$

$\pi = \{ \{1,2\} \ \{3\} \ \{4,5\} \}$

Find the equivalence Relation determined by π & draw its diagraph.

$\Rightarrow$ $R = \{ (1,1) (2,2) (1,2) (2,1) (3,3) (4,4) (5,5) (4,5) (5,4) \}$



—x—

Let R be an equivalence Relation on A. We denote by A/R the partition induced by R. Hence a partition of A is called as a quotient set of A.

—x—

Ex. ① Let $A = \{1, 2, 3\}$ &

$R = \{ (1,1) (2,2) (1,3) (3,1) (3,3) \}$

Find A/R (partition).

$\Rightarrow$

$R = \{ (1,1) (2,2) (1,3) (3,1) (3,3) \}$

$[1] = \{1, 3\}$

$[2] = \{2\}$

$[3] = \{1, 3\} = [1]$

from Theorem ① i.e $\{ [a]_R \mid a \in A \}$

$A/R = \{ \{1,3\} \ \{2\} \}$

—x—

* Compatible Relation

- A Relation R on a set A is said to be compatible if it is reflexive & symmetric.

Ex. ① All equivalence Relations are compatible Relations.

Ex. ② The Relation of "being friend of" is a compatible Relation.

* Transitive Closure

- The transitive closure of a relⁿ R is the smallest transitive relation containing R, & denoted by R^*.

Method to find the transitive closure

- Let A be set $|A| = n$, & R be relation on A.

Then

$$R^* = R \cup R^2 \cup R^3 \cup \cdots \cup R^n$$

Ex. ① Let $A = \{1, 2, 3, 4\}$ & $R = \{(1,2)(2,3)(3,4)\}$ R be a relation on A.
Find R^* & draw its digraph.

$\Rightarrow$ $R = \{(1,2)(2,3)(3,4)\}$ | $|A| = 4$ ∴ upto R^4 only

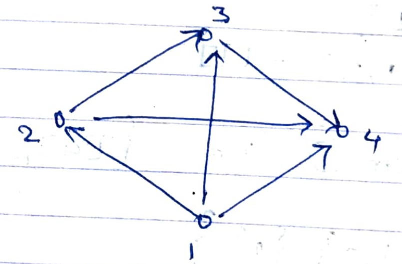
$R^2 = R \cdot R = \{(1,3)(2,4)\}$

$R^3 = R \cdot R^2 = \{(1,4)\}$

$R^4 = \phi$

Hence $R^* = R \cup R^2 \cup R^3$
$= \{(1,2)(2,3)(3,4)(1,3)(2,4)(1,4)\}$

Diagraph of R^*



Ex. ② Let $A = \{a, b, c, d\}$,
$R_1 = \{(a,a)(b,b)(a,c)(a,b)\}$ &
$R_2 = \{(a,a)(b,d)(d,c)\}$

Find $(R_1 \cup R_2)^*$ & draw its diagraph.

$\Rightarrow$ $R = R_1 \cup R_2 = \{(a,a)(b,b)(c,c)(a,b)(b,d)(d,c)\}$

Note $|A| = 4$ so go upto R^4

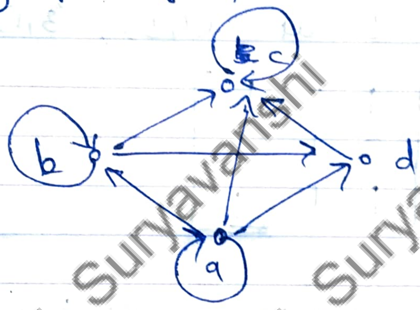
$R^2 = R \cdot R = \{(a,b)(b,d)(a,d)(b,c)(a,a)(b,b)(c,c)$
$(d,c)\}$

$R^3 = R \cdot R^2 = \{(a,a)(a,b)(a,c)(a,d)(b,b)(b,c)$
$(b,d)(d,c)(c,c)\}$

$R^4 = R \cdot R^3 = \{(a,a)(a,b)(a,c)(a,d)(b,b)(b,c)(b,d)$
$(c,c)(d,c)\}$

$R^* = R \cup R^2 \cup R^3$
$= \{(a,a)(b,b)(c,c)(a,b)(b,d)(d,c)$
$(a,d)(b,c)(a,c)\}$

Diagraph of R^*



* <u>Warshall Algorithm</u>

For large sets of relations, finding transitive closure of a relation, by computing various powers of R or products of the relation matrix MR, is quite impractical.

Ex.① Use warshall Algo. to find the transitive closure of R, where $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ & A=

$\Rightarrow$ From matrix M_R,

$R = \{ (1,1)(1,3)(2,2)(3,1)(3,2) \}$

Now, $W_0 = \cancel{M_R}\ M_R$

for W_1, 1 is interior-vertex for path from (1,1) to (1,3), (3,1) to (1,1) & (3,1) to (1,3)

So so set 1 at (1,3), (3,1) & (3,3)

$\therefore W_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & \underline{1} \end{bmatrix}$

for W_2, 2 is interior-vertex for path from (3,2) to (2,2) so set 1 at (3

which is already present $\therefore W_2 = W$

For W_3, 3 is interior-vertex for path From (1,3) to (3,1) & (1,3) to (3,2)

So set 1 at (1,1) & (1,2)

$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore$ ~~W_R~~ $M_{R^*} = W_3$

$R^* = \{ (1,1)\ (1,2)\ (1,3)\ (2,2)\ (3,1)\ (3,2)\ (3$

EDSGT-2 H.E. Suryawanshi (5)

TEST (I / II) / PRELIMINARY EXAMINATION

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(IN BLOCK LETTERS) (Surname) (First Name) (Middle Name)

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Warshall Algorithm

Ex① let $A = \{1,2,3,4\}$ & $R = \{(1,2)(2,4)(1,3)(3,2)\}$
find transitive closure of R (i.e R^*) using warshall algorithm.

$\Rightarrow$

$$W_0 = M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

for W_1 (i.e find in R, whether 1 is interior vertex or not)

$\therefore$ 1 in not interior vertex in R

$W_1 = W_0$

For W_2, 2 is interior-vertex for path from (1,2) (2,4) & (3,2) (2,4)
So we have 1 in position (1,4), (3,4)

$$\text{hence, } W_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For W_3, 3 is interior-vertex in path (1,3) (3,2). So set 1 at (1,2)
but, it already exist.
Hence $W_3 = W_2$

For W_4, 4 is not interior-vertex in R
hence, $W_4 = W_3$

$$\& \text{ So } M_R^* = W_4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ex. ② Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ & let R be a relation on A whose matrix is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

⇒

from M_R

$R = \{(a_1\ a_1)\ (a_1\ a_4)\ (a_2\ a_2)\ (a_3\ a_4)$
$(a_3\ a_5)\ (a_4\ a_1)\ (a_5\ a_2)\ (a_5\ a_5)\}$

Now

$W_0 = M_R$

For W_1, a_1 is interior-vertex
for path $(a_1\ a_1)$ to $(a_1\ a_4)$
& $(a_4\ a_1)$ to $(a_1\ a_4)$
So set 1 at $(a_1\ a_4)$ & $(a_4\ a_4)$
where 1 already exist at $(a_1\ a_4)$
So set 1 at only $(a_4\ a_4)$

$$\therefore W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

For W_2, a_2 is interior vertex (a_5, a_2) to $(a_2 a_2)$
Set 1 at (a_5, a_2) which is already exist
so $W_2 = W_1$

For W_3, a_3 is not-interior vertex
so $W_3 = W_2$

For W_4, a_4 is interior vertex for path $(a_1\ a_4)$ to $(a_4\ a_1)$ & $(a_3\ a_4)$ to $(a_4\ a_1)$
So set 1 at $(a_1 a_1)$ & $(a_3\ a_1)$

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

For W_5, a_5 interior vertex $(a_3 a_5)$ to $(a_5 a_2)$
so set 1 at $(a_3\ a_2)$

$$W_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Hence
$$M_R^* = W_5$$

Home work

Ex ① Find transitive closure of R by Warshall algo, when $A = \{1, 2, 3, 4, 5, 6\}$ & $R = \{(x, y) \mid |x - y| = 2\}$

Hint
$R = \{(1,3)\ (3,1)\ (2,4)\ (4,2)\ (3,5)\ (5,3)\ (4,6)\ (6,4)\}$

* Partial Ordering Relation's

Def^n. A binary relation R on set A is a partial order if R is
- reflexive,
- anti-symmetric &
- transitive

The ordered pair (A, R) is called partially-ordered set or poset.

Ex. ① Rel^n "<" is partial order on the set of real numbers.

* Hasse Diagrams

A poset can be depicted by Hasse Diagram which includes following rules
① All arrow heads on edges are omitted.
② Loops are omitted as reflexivity is implied by def^n. of partial order
③ Arc/edge not present for transitivity

——x——

From Dictionary
A digraph which represents finite poset, in which nodes are elements of the poset & arrows represents the order relation between elements called Hasse Dig.

——x——

Ex. ① Let $A = \{2, 3, 4, 6\}$ & $R = \{(a,b) \mid a \text{ divides } b\}$. Show that R is partial order & draw its Hasse diagram.

$\Rightarrow$

$$R = \{(2,2)(2,4)(2,6)(3,3)(3,6)(4,4)(6,6)\}$$

$\therefore$ R — Reflexive relation
Since $(2,2)(3,3)(4,4)(6,6) \in R$

R : anti-Symmetric

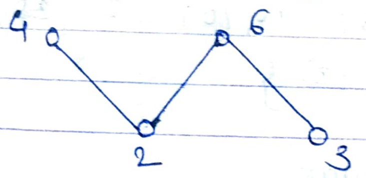
Since, if $a|b$, $b \nmid a$ unless $a=b$

R : transitive

Since, $a|b$ & $b|c$ implies $a|c$

Hence, R — partial order relation

∴ Hasse Diagram for R



Ex. ② $A = \{1,2,3,4\}$ and

$R = \{(1,1)(1,2)(2,2)(2,4)(1,3)(3,3)(3,4)(1,4)(4,4)\}$

Show that R — partial order relation & draw its Hasse diagram

⟹

R — Reflexive

Since $(1,1)(2,2)(3,3)(4,4) \in R$

R — Anti-Symmetric

Since, ~~(1,2) ∈ R but (2,1) ∉ R~~

$(1,1) \in R \to 1=1$ ~~(1,3) ∈ R but (3,1) ∉ R~~

$(2,2) \in R \to 2=2$ ~~(1,4) ∈ R but (4,1) ∉ R~~

$(3,3) \in R \to 3=3$ ~~(2,4) ∈ R but (4,2) ∉ R~~

$(4,4) \in R \to 4=4$ ~~(3,4) ∈ R but (4,3) ∉ R~~

R — transitive

Since, $(1,1)(1,2) \in R$ & $(1,2) \in R$

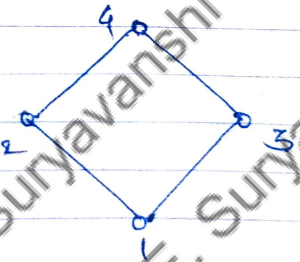
$(2,2)(2,4) \in R$ & $(2,4) \in R$

$(1,3)(3,3) \in R$ & $(1,3) \in R$

$(3,3)(3,4) \in R$ & $(3,4) \in R$

$(1,4)(4,4) \in R$ & $(1,4) \in R$

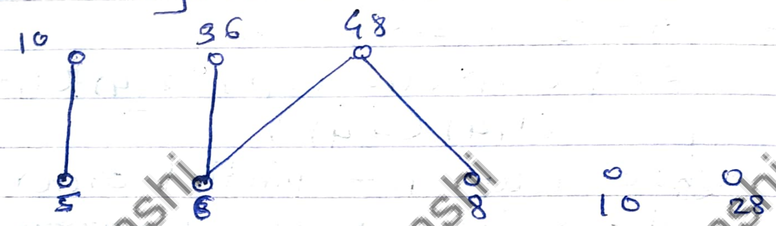
Now, Hasse diagram of R



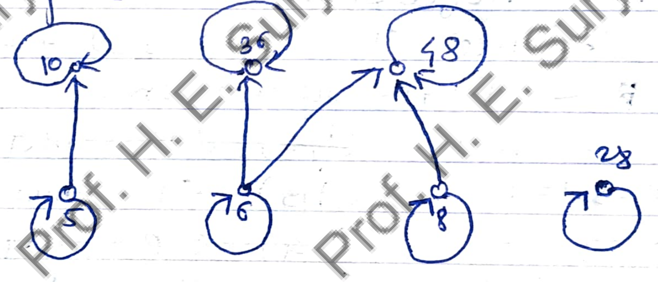
Ex. ③ Let R be relation on set $A = \{5, 6, 8, 10, 28, 36, 48\}$ & $R = \{(a,b) \mid a \text{ is divisor of } b\}$ Draw Hasse diagram & compare it with diagraph. Determine whether R is reflexive, transitive & symmetric

$\Rightarrow$ $R = \{(5,5)\ (5,10)\ (6,6)\ (6,36)\ (6,48)\ (8,8)\ (8,48)\ (10,10)\ (28,28)\ (36,36)\ (48,48)\}$

Hasse Diagram



Diagraph



R – Reflexive
– Since (5,5) (6,6) (8,8) (10,10) (28,28) $\in R$
– Similarly (36,36) (48,48) $\in R$

R – Not Symmetric
– Since (5,10) $\in R$ but (10,5) $\notin R$
(6,36) $\in R$ but (36,6) $\notin R$
& so on for (6,48) (8,48)

* Chains & Anti-chains

Let $(A, \leq)$ be a poset. A subset of A is called a **chain** if every pair of elements in the subset are related.

In any chain with finite no. of elements $\{a_1, a_2 \cdots a_k\}$, there is an element a_1 that is less than every element in the chain. & there is element a_2 which is less than every element except a_1

$\therefore$ So we have sequence $a_1 \leq a_2 \leq a_3 \leq \cdots \leq a_k$

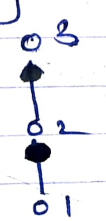
$\therefore$ The no. of elements in the chain is called **length of chain**.

If A itself is chain, then the poset $(A, \leq)$ called a totally ordered set or linearly ordered set.

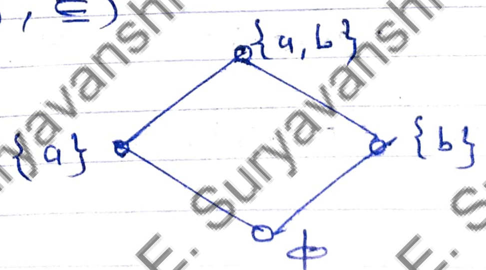
Anti-chain

A subset of A is called anti-chain if no two distinct elements in the subset are related.

Ex. ① Let $A = \{1, 2, 3\}$ & let the partial order $\leq$ mean "less than or equal to". Then $(A, \leq)$ is a chain & its Hasse diagram is



Ex. ② Let $A = \{a, b\}$ & consider its poset $(P(A), \subseteq)$



Chains

① $\{\phi, \{a\}\}$

② $\{\{a\}, \{a, b\}\}$

③ $\{\phi, \{a\}, \{a, b\}\}$

④ $\{\phi, \{b\}\}$

⑤ $\{\{b\} \{a, b\}\}$

⑥ $\{\phi, \{b\}, \{a, b\}\}$

Anti-chains

$\{\{a\}, \{b\}\}$

length of longest element is 3

* Lattice

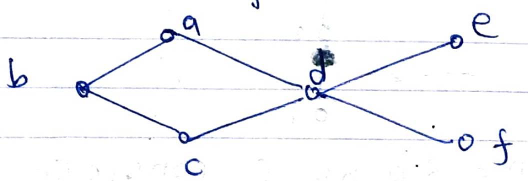
Let $(A, \leq)$ be a poset with partial order $\leq$.

① **Maximal & Minimum elements**

An element $a \in A$ is called maximal element if there is no element $b \in A$ such that $b \neq a$ & $a \leq b$.

An element $c \in A$ is called minimal element if there is no element $d \in A$ such that $d \neq c$ & $d \leq a$

Ex. ① consider the poset whose Hasse digram



Maximal elements = a, e

Minimal elements = c, f

DSGT - 2 H.E. Suryawanshi ⑥

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(Start From here only)

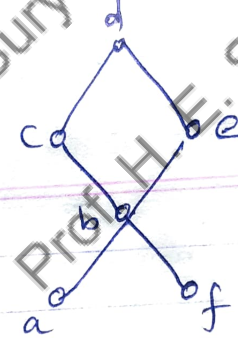
② Upper Bound & Lower Bound

Let a, b be elements in a poset $(A, \leq)$. An element c is said to be an upper bound of a & b if $a \leq c$ & $b \leq c$.

An element c is said to be least upper bound (lub) of a & b if c is upper bound of a & b and if there is no other upper bound 'd' of a & b such that $d \leq c$.

Similarly an element e is said to be a lower bound of a & b if $e \leq a$ & $e \leq b$; & e is called greatest lower bound (glb) of a & b if there is no other lower bound f of a, b such that $e \leq f$.

Ex.



① Upper bounds for $\{c,e\} \rightarrow d$

lub $\{c,e\} \rightarrow d$

lower bounds for $\{c,e\}$ are $\rightarrow b, a, \& f$

glb $\{c,e\} = b$

② Upper bounds for $\{a,f\} \rightarrow b, c, e, d$

lub $\{a,f\} = b$

lower bounds for (a,f) — doesn't exist.

③ Upper bounds for $\{b,d\} \rightarrow d$

lower bound for $\{b,d\} \rightarrow b, a \& f$

glb $\{b,d\} = b$

---

* Functions

let A & B be non-empty sets.
A function f from A to B, denoted by $f: A \rightarrow B$, is a relation from A to B such that for every $a \in A$, there exists a unique $b \in B$ such that $(a,b) \in f$

Normally, if $(a,b) \in f$, we write $f(a) = b$

If $f(a) = b$ &
$f(a) = c$ then $b = c$

In general, function is only many-to-one or one-to-one relation

A one-to-many relation is not function.

① **Domain** $D(f)$

Set A is called as domain of f.

② **Codomain**

Set B is called as co-domain of F

③ **Range** $R(f)$

Set $\{f(a) \mid a \in A\}$ which is subset of B called as range of f.

a – element 'a' is called argument of 'f'
f(a) – is called the value of function

Functions are also called as mappings or transformations, since they can be thought of as rules for assigning to each element $a \in A$, the unique element $f(a) \in B$.

$\therefore$ f(a) is ~~pr.~~ image of 'a' &
a is pre-image of ~~f(a)~~ b i.e. f(a)

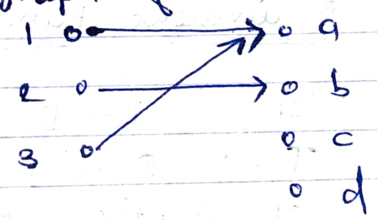
Ex.① $A = \{1, 2, 3\}$ & $B = \{a, b, c, d\}$

let $F : A \to B$ be defined as

$F(1) = a$
$F(2) = b$
$F(3) = a$

F is function
& $R(f) = \{a, b\}$

Graph of F

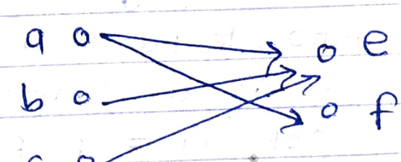


Ex ② Let

$A = \{a, b, c\}$ & $B = \{e, f\}$

Let $R = \{(a, e)\ (b, e)\ (a, f)\ (c, e)\}$

Graph of R



$\therefore$ R is not a function
since $f(a) = e$ & $f(a) = f$

Note = Function is one-to-one or
many-to-one

* partial functions

- consider domain of a function as subset of another set known as source & codomain as target.
  ∴ The function has set A as its domain but is not defined for some arguments.
- let, A & B be two sets. A partial function f with domain A & codomain B is any function from A' to B where $A' \subseteq A$.
  - for any element $x \in A - A'$, the value of $f(x)$ is undefined
- A function which is not partial is sometimes called as total function.

Ex. (1) $f : R \rightarrow R$ & $f(x) = 1/x$
It is partial function because it is undefined for $x = 0$ value

Ex. (2) $f : R \rightarrow R$ & $f(x) = \sqrt{x}$
It is partial function, as $\sqrt{x}$ is not defined for $x < 0$ in R

* Equivalent Functions (identical)

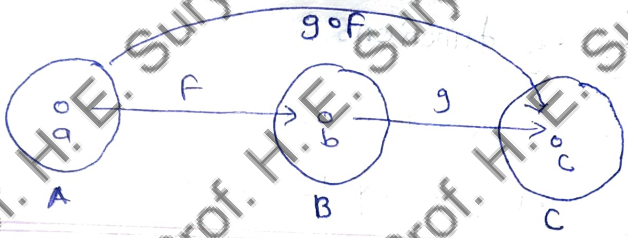
let, $f : A \rightarrow B$ & $g : C \rightarrow D$ be functions. Then f & g are said to be equivalent or identical only if $A = C$, $B = D$ &
$f(a) = g(a)$ for all $a \in A$

* Composite function

let, $f : A \rightarrow B$ & $g : B \rightarrow C$ be two functions. Then composite function of f & g denoted as gof is a relation from $A \rightarrow C$
where $gof(a) = g(f(a))$.
& $gof : A \rightarrow C$ is also a function.

Note -

gof is defined only when the range of f is a subset of the domain of g.



Ex. ① let $f: \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = x^2 + 2x + 2$ &

$g: \mathbb{Z} \to \mathbb{Z}$ defined as $g(x) = x - 1$

Then

$gof: \mathbb{Z} \to \mathbb{Z}$ defined as

$$gof(x) = g(f(x)) = (x^2+2x+2) - 1$$
$$= (x+1)^2$$

and $fog: \mathbb{Z} \to \mathbb{Z}$ defined as

$$fog(x) = f(g(x)) = (x-1)^2 + 2(x-1) + 2$$
$$= x^2 + 1$$

$fof: \mathbb{Z} \to \mathbb{Z}$ defined as

$$fof(x) = f(f(x)) = f(x^2+2x+2)$$
$$= (x^2+2x+2)^2 + 2(x^2+2x+2) + 2$$

$gog: \mathbb{Z} \to \mathbb{Z}$ defined as

$$gog(x) = g(g(x)) = (x-1) - 1 = x - 2$$

Ex. ② let $f: \mathbb{Z} \to \mathbb{R}$ defined as $f(x) = \frac{(x+1)}{2}$ &

$g: \mathbb{R} \to \mathbb{R}$ be defined as $g(x) = x^2$

Then

$gof: \mathbb{Z} \to \mathbb{R}$ defined as

$$gof(x) = g(f(x)) = g\left(\frac{x+1}{2}\right) = \left(\frac{x+1}{2}\right)^2$$

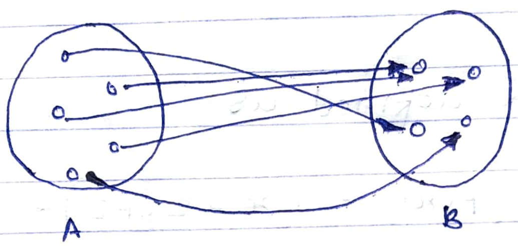
$$gof(x) = \frac{(x+1)^2}{4}$$

* Special types of functions

let $f: A \to B$ be function

① Surjective (onto / many-to-one)

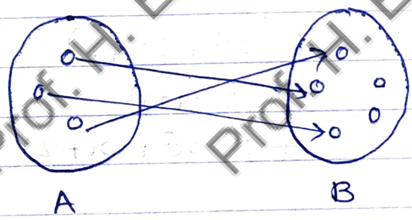
f is called surjective function
if $f(A) = B$
i.e range of f is equal to codomain of f



1. many-to-one
2. Range <= Codomain
3. Every "B" has one matching "A"
- There won't be a "B" left out

② Injective (one-to-one)

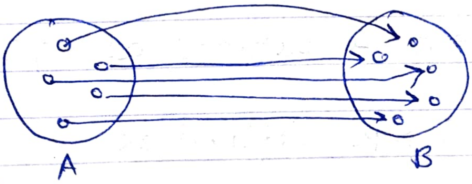
f is called injective function
if for elements $a, a' \in A$, $a \neq a'$ implies
$f(a) \neq f(a')$
or if $f(a) = f(a')$ then $a = a'$



1. one-to-one
2.

③ Bijection (one-to-one / many-to-one)

f is called bijection if f is both surjective & injective.



Theorem ①
let $f: A \to B$ & $g: B \to C$ be functions.
Then i> If f & g surjective, then $g \circ f \to$ surjective
ii> If f & g injective, then $g \circ f \to$ injective
iii> If f & g bijective, then $g \circ f \to$ bijective

Theorem ② Let $f : A \to B$ & $g : B \to C$ functions.

Then i> If $g \circ f$ surjective, then $g \to$ surjective

ii> If $g \circ f$ injective, then $f \to$ injective

iii> If $g \circ f$ bijective, then $g \to$ surjective &
$f \to$ injective

* **Inverse function (f^{-1})**

The concept of inverse function is analogous to that of the converse of relation

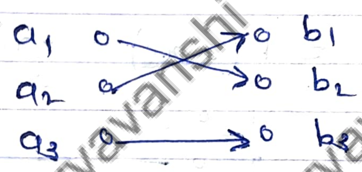
Let,

$f : A \to B$ be bijection from A to B.

$f^{-1} : B \to A$

Ex ① $f : \{a_1, a_2, a_3\} \to \{b_1, b_2, b_3\}$

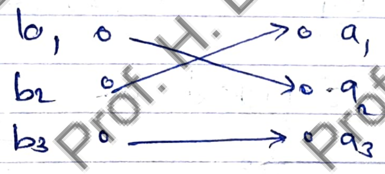
defined as



f

Then

f^{-1} is given by graph



properties of inverse function

① $(f^{-1})^{-1} = f$

② If f & g - bijective function $A \to B$, $B \to C$

then

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

Ex. ① functions f, g, h are defined on a set $X=\{1,2,3\}$ as $f=\{\cancel{(1,2)}\ (1,2)\ (2,3)\ (3,1)\}$

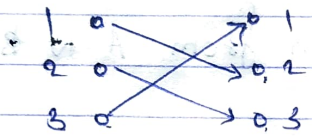
$g=\{(1,2)\ (2,1)\ (3,3)\}$

$h=\{(1,1)\ (2,2)\ (3,1)\}$

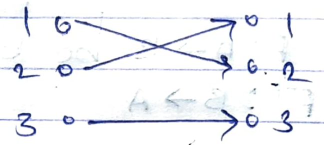
① Find fog, gof. Are they equal?

② Find fogoh and fohog.

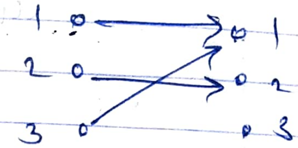
⇒ Graphically f, g & h depicted as follows



f



g

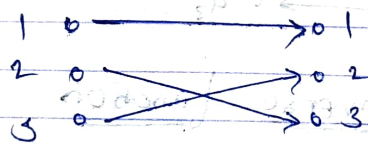


h

(i) $fog = f(g(x))$ graphically depicted as

$fog = \{(1,3)\ (2,2)\ (3,1)\}$

$gof = g(f(x))$ depicted as



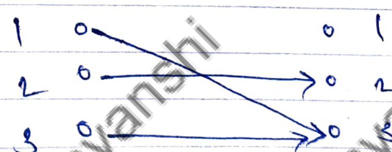
$gof = \{(1,1)\ (2,3)\ (3,2)\}$

$\therefore\ fog \neq gof$

(ii) $fogoh = (fog)oh$ depicted as

OR

$f(g(h(x))) = fogoh$



$fogoh = \{(1,3)\ (2,2)\ (3,3)\}$

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Signature of Student Signature of Supervisor

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Marks Obtained											
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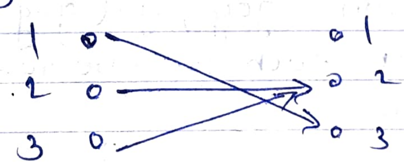
$F \circ h \circ g = F(h(g(x)))$ or $F \circ (h \circ g)$



$F \circ h \circ g = \{(1,3)(2,2)(3,2)\}$

Ex (2) $A = \{a, b, c, d\}$

$B = \{s, t, u\}$

$C = \{l, m, n\}$

obtain the composition of following functions

$F: A \to B$ & $g: B \to C$

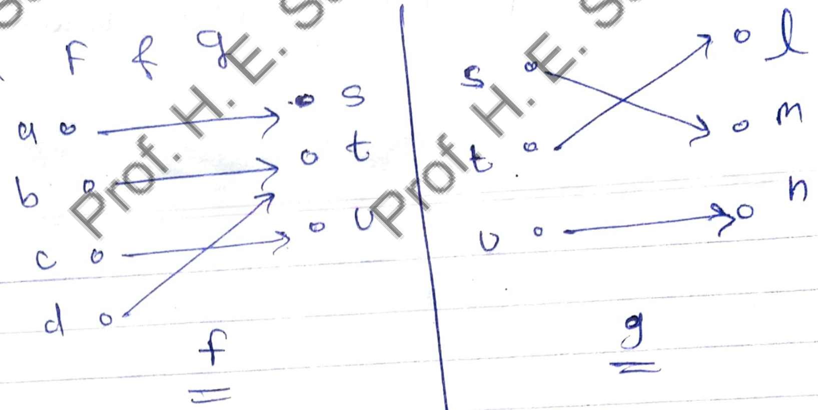
Where,

$F = \{(a,s)(b,t)(c,u)(d,t)\}$

$g = \{(s,m)(t,l)(u,n)\}$

Find $g \circ f$

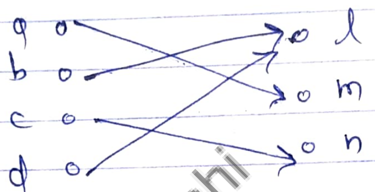
E ⇒ Given F & g



Now,

composition of F & g i.e gof

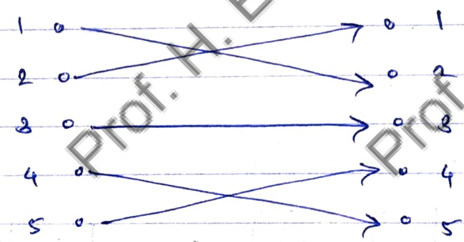
i.e $g(F(x))$



$gof = \{(a,m)(b,l)(c,n)(d,l)\}$

Ex (4) Let $A = \{1,2,3,4,5\}$

$g: A \to A$ is as shown in the fig.

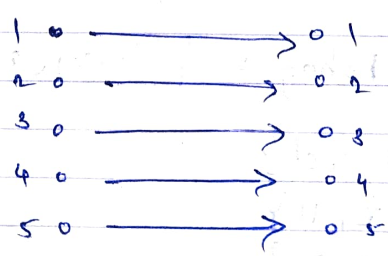


Find the composition gog, $go(gog)$.

Determine whether each is one-to-one or onto function

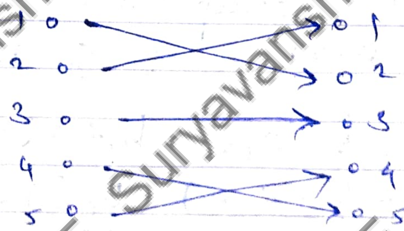
⇒

gog



gog is one-to-one function

$go(gog)$



one-one & onto function

Ex. 5 let $f(x) = x+2$

$g(x) = x-2$ &

$h(x) = 3x$ for $x \in R$

where R = Set of Real no.

Find gof, fog, fof, gog, foh, hog, hof, $fohog$.

$\Rightarrow$

$f(x) = x+2$

$g(x) = x-2$

$h(x) = 3x$

$gof = g(f(x)) = (x+2)-2 = x$

$fog = f(g(x)) = (x-2)+2 = x$

$fof = f(f(x)) = (x+2)+2 = x+4$

$gog = g(g(x)) = (x-2)-2 = x-4$

$foh = f(h(x)) = (3x)+2 = 3x+2$

$hog = h(g(x)) = 3(x-2) = 3x-6$

$hof = h(f(x)) = 3(x+2) = 3x+6$

$fohog = f(h(g(x)))$

$\therefore h(g(x)) = 3(x-2) = 3x-6$

$fohog = f(3x-6)$

$= (3x-6)+2$

$= 3x-4$

Ex. 6 If $f(x) = x^2+1$ and $g(x) = x+2$ are functions from R to R, where R is the set of real numbers. Find fog & gof.

$\Rightarrow$

$f(x) = x^2+1$

$g(x) = x+2$

Now, $fog = f(g(x)) = (x+2)^2+1 = x^2+4x+5$

& $gof = g(f(x)) = (x^2+1)+2 = x^2+3$

Ex. 7 Let $f(x) = 2x+3$

$g(x) = 3x+4$,

$h(x) = 4x$ for $x \in R$, where

R = set of real numbers.

Find $g \circ f$, $f \circ g$, $f \circ h$, $h \circ f$, $g \circ h$

$\Rightarrow$

$f(x) = 2x+3$

$g(x) = 3x+4$

$h(x) = 4x$

$g \circ f = g(f(x)) = 3 \cdot (2x+3) + 4$
$= 6x + 13$

$f \circ g = f(g(x)) = 2 \cdot (3x+4) + 3$
$= 6x + 11$

$f \circ h = f(h(x)) = 2(4x) + 3$
$= 8x + 3$

$h \circ f = h(f(x)) = 4(2x+3)$
$= 8x + 12$

$g \circ h = g(h(x)) = 3(4x) + 4$
$= 12x + 4$

* Pigeonhole Principle

If A & B are finite sets & bijection exists from A to B.

then their cardinalities are same.
Hence, if A & B are any two sets such that $|A| > |B|$,
then no bijection can exists from A to B.

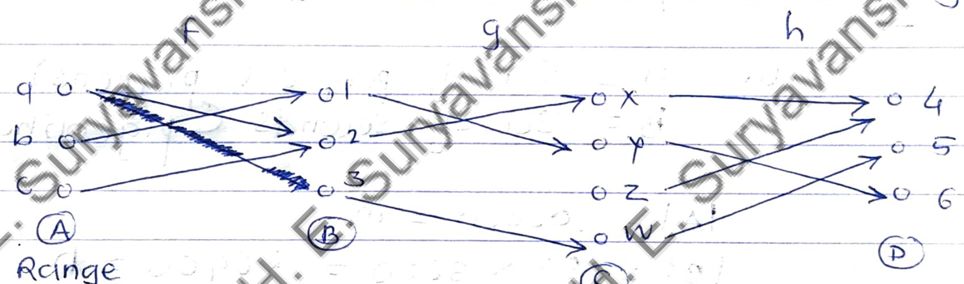
This fact is stated as '**Pigeon hole principle**'

The principle states that if there are $n+1$ pigeons & only n-pigeonholes then two pigeons will share the same hole.

Theorem: If $(n+1)$ objects are put into n boxes, then at least one box contains two or more objects.

Ex. Among 13 people there are two who have their birthdays in the same month.

Ex. ⑧ The functions $f: A \to B$; $g: B \to C$, $h: C \to D$ are defined in the following diagram. Determine the range of each function. state which functions are into and which are onto. Draw the diagram of composite function $(h \circ g \circ f)$



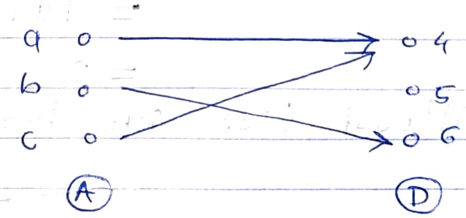
⇒ Range

$R(f) = \{1, 2\}$ f is into-function

$R(g) = \{x, y, w\}$ g is into-function

$R(h) = \{4, 5, 6\}$ h is onto-function

$(h \circ g \circ f)$ is



$h \circ g \circ f$

* Functions-

Binary relation $R: A \to B$, is said to be a function if for every element a in A, there is unique element b in B so that $(a, b) \in R$.

notation $R(a) = b$ | b is image of a

Set A is called domain &
B is called Rang.

The notion of function is a formalizati of the notion of associating or assigning an element in the range to each of the element in domain.

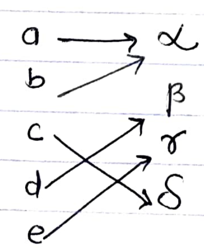
Function Types

① **Onto / Surjection-**
If every element of B is the image of one or more elements of A.

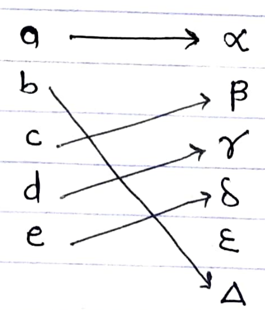
② **One-to-one / Injection-**
If no two elements of A having same image

③ **One-to-one onto / Bijection-**
If it is both an onto and one to one

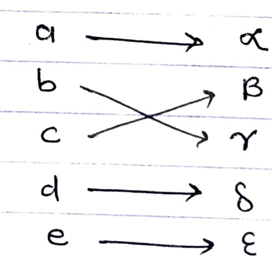
Example



① onto / Surjection



② one-to-one / Injection



③ one-to-one onto / Bijection