



Academic Year (2021-22)
Year: 2 Semester: IV

Program: B. Tech. (Computer Engg.)

Max. Marks: 75

Subject: Formal Language And Automata Theory

Time: 10: 30 am to 1:30 pm

Date: 1st July 2022

Duration: 3 Hours

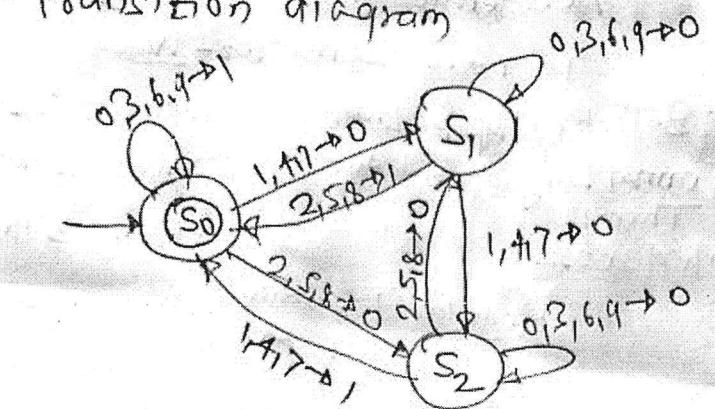
REGULAR EXAMINATION

ANSWER KEY

Question No.		Max. Marks																																
Q1 (a)	<p>-ASSUME -</p> <p>1. $I = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $I = \{(0, 3, 6, 9), (1, 4, 7), (2, 5, 8)\}$ $S = \{S_0, S_1, S_2\}$</p> <p>Where S_0 = for remainder 0 S_1 = for remainder 1 S_2 = for remainder 2</p> <p>2. State function (STF) $STF: I \times S \rightarrow S$</p> <table border="1"> <tr> <td>$S \setminus I$</td> <td>(0,3,6,9)</td> <td>(1,4,7)</td> <td>(2,5,8)</td> </tr> <tr> <td>S_0</td> <td>S_0</td> <td>S_1</td> <td>S_2</td> </tr> <tr> <td>S_1</td> <td>S_1</td> <td>S_2</td> <td>S_0</td> </tr> <tr> <td>S_2</td> <td>S_2</td> <td>S_0</td> <td>S_1</td> </tr> </table> <p>3. machine function (MAF) $MAF: I \times S \rightarrow \{0, 1\}$</p> <table border="1"> <tr> <td>$S \setminus I$</td> <td>(0,3,6,9)</td> <td>(1,4,7)</td> <td>(2,5,8)</td> </tr> <tr> <td>S_0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>S_1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>S_2</td> <td>0</td> <td>1</td> <td>0</td> </tr> </table>	$S \setminus I$	(0,3,6,9)	(1,4,7)	(2,5,8)	S_0	S_0	S_1	S_2	S_1	S_1	S_2	S_0	S_2	S_2	S_0	S_1	$S \setminus I$	(0,3,6,9)	(1,4,7)	(2,5,8)	S_0	1	0	0	S_1	0	0	1	S_2	0	1	0	[5]
$S \setminus I$	(0,3,6,9)	(1,4,7)	(2,5,8)																															
S_0	S_0	S_1	S_2																															
S_1	S_1	S_2	S_0																															
S_2	S_2	S_0	S_1																															
$S \setminus I$	(0,3,6,9)	(1,4,7)	(2,5,8)																															
S_0	1	0	0																															
S_1	0	0	1																															
S_2	0	1	0																															



Transition diagram



Q1 (b)

[10]

Present state	Next state			
	01p → 0	01p → 1	01p → 2	01p → 3
q1	q2	z1	q3	z1
q2	q2	z2	q3	z1
q3	q2	z1	q3	z2

Transition table for mealy machine
from above transition table

q_2 state is associated with $01p \rightarrow z_1 \neq z_2$

q_3 state is associated with $01p \rightarrow z_1 \neq z_2$

q_1 state is associated with $01p \rightarrow 1$

Assume

Split state $q_2 \xrightarrow{01p} q_{21} \rightarrow$ for $01p z_1$
 $\xrightarrow{01p} q_{22} \rightarrow$ for $01p z_2$

Split state $q_3 \xrightarrow{01p} q_{31} \rightarrow$ for $01p z_1$
 $\xrightarrow{01p} q_{32} \rightarrow$ for $01p z_2$



PS	NS		0/p	State 1/p=1	0/p
	State 1/p=0	0/p			
q_1	q_{21}	z_1	q_{31}	z_1	
q_{21}	q_{22}	z_2	q_{21}	z_1	
q_{22}	q_{22}	z_2	q_{31}	z_1	
q_{31}	q_{21}	z_1	q_{32}	z_2	
q_{32}	q_{21}	z_1	q_{32}	z_2	

Revised transition table for mealy M/c

PS	NS		0/P
	1/p=0	1/p=1	
q_1	q_{21}	q_{31}	λ
q_{21}	q_{22}	q_{31}	z_1
q_{22}	q_{22}	q_{31}	z_2
q_{31}	q_{21}	q_{32}	z_1
q_{32}	q_{21}	q_{32}	z_2

Transition table for moore M/c

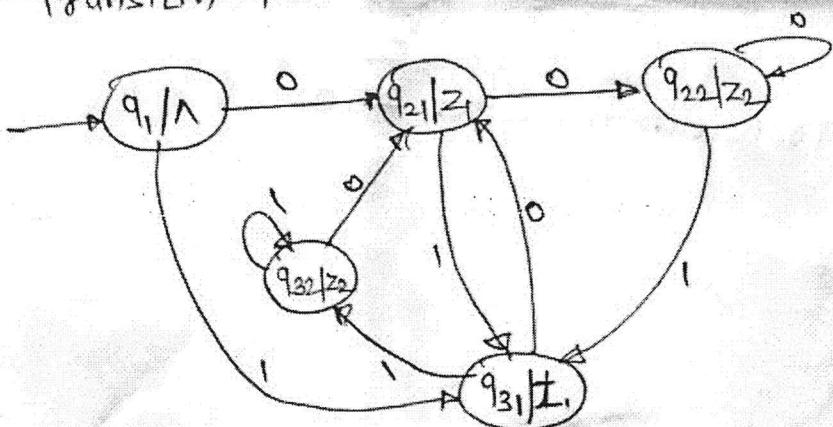


fig:- Transition diagram for moore M/c



Q1 (b)

OR

[10]

$$\Pi_0 = \left\{ \{q_3, q_4\}, \{q_0, q_1, q_2, q_5, q_6, q_7\} \right\}$$

$$\Pi_1 = \left\{ \{q_3, q_4\}, \{q_0, q_6\}, \{q_1, q_2, q_5, q_7\} \right\}$$

$$\Pi_2 = \left\{ \{q_3, q_4\}, \{q_0, q_6\}, \{q_1, q_2\}, \{q_5, q_7\} \right\}$$

$$\Pi_3 = \left\{ \{q_3, q_4\}, \{q_0\}, \{q_6\}, \{q_1, q_2\}, \{q_5, q_7\} \right\}$$

$$\Pi_4 = \left\{ \{q_0\}, \{q_1, q_2\}, \{q_3, q_4\}, \{q_5, q_7\}, \{q_6\} \right\}$$

Σ	a	b
$\{q_0\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_3, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_5, q_7\}$	$\{q_6\}$
$\{q_5, q_7\}$	$\{q_3, q_4\}$	$\{q_6\}$
$\{q_6\}$	$\{q_6\}$	$\{q_6\}$

Transition table for DFA



Q2 (a)

[8]

(i) Take 1st production rule

$$S \rightarrow ABC$$

Assume

$$[R_1 \rightarrow BC] \rightarrow ①$$

$$\therefore [S \rightarrow AR_1] \rightarrow ②$$

(ii) Take 2nd production rule

$$[A \rightarrow a] \rightarrow ③$$

It is already in CNF

(iii) Take 3rd production rule

$$[A \rightarrow b] \rightarrow ④$$

It is already in CNF

(iv) Take 4th production rule

$$B \rightarrow Bb$$

Assume

$$[R_2 \rightarrow b] \rightarrow ⑤$$

$$\therefore [B \rightarrow BR_2] \rightarrow ⑥$$

(v) Take 5th production rule

$$B \rightarrow aa$$

Assume

$$[R_3 \rightarrow a] \rightarrow ⑦$$

$$\therefore [B \rightarrow R_3 R_3] \rightarrow ⑧$$

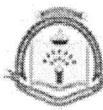
(vi) Take 6th production rule

$$C \rightarrow aC$$

As

$$R_3 \rightarrow a$$

$$\therefore [C \rightarrow R_2 C] \rightarrow ⑨$$



vii) Take 7th production rule

$$C \rightarrow CC$$

Assume

$$\boxed{R_4 \rightarrow C} \quad \text{--- (10)}$$

$$\therefore \boxed{C \rightarrow R_4 C} \quad \text{--- (11)}$$

viii) Take 8th production rule

$$C \rightarrow ba$$

As $R_2 \rightarrow b$

&

$$R_3 \rightarrow a$$

$$\therefore \boxed{C \rightarrow R_2 R_3} \quad \text{--- (12)}$$

\therefore The final grammar in CNF is
as follows

- | | | |
|----------------------------|--------------------------------|-------------------------------|
| (i) $R_1 \rightarrow BC$ | (v) $R_2 \rightarrow b$ | (ix) $C \rightarrow R_3 C$ |
| (ii) $S \rightarrow A R_1$ | (vi) $B \rightarrow BR_2$ | (x) $R_4 \rightarrow c$ |
| (iii) $A \rightarrow a$ | (vii) $R_3 \rightarrow a$ | (xi) $C \rightarrow R_4 C$ |
| (iv) $A \rightarrow b$ | (viii) $B \rightarrow R_3 R_3$ | (xii) $C \rightarrow R_2 R_3$ |



Q2 (b)

[7]

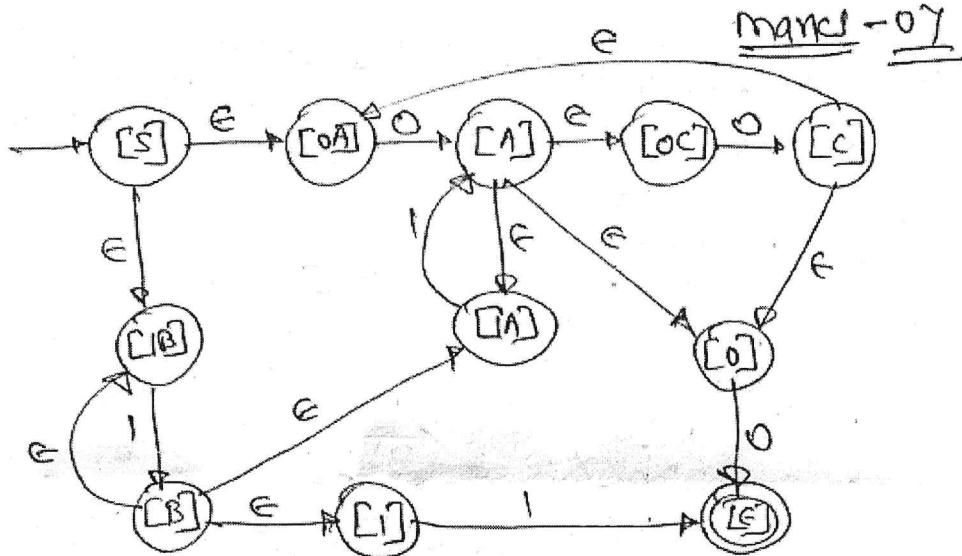


fig)- NFA with 'f' moves drawn for GR

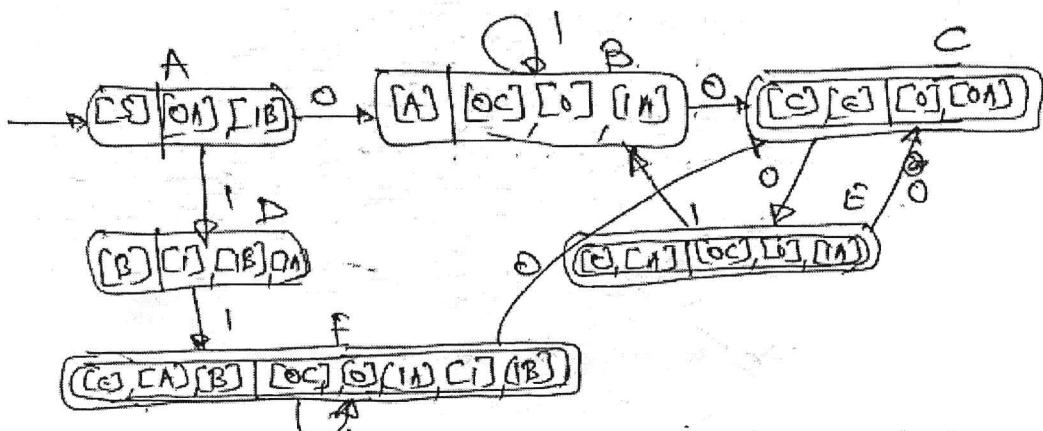
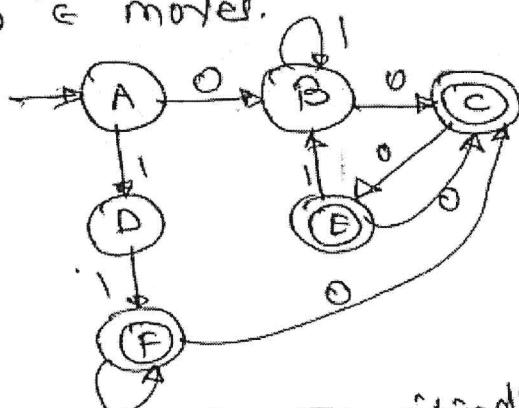


Fig:- DFA obtained by direct method from above NFA with ϵ moves.

	A	B	C	D
1	B	C	D	B
	C	E	-	-
	D	-	F	F
	E	C	B	
	F	F		



Transition Table for DFA

Fig:- Transition diagram for DFA



OR

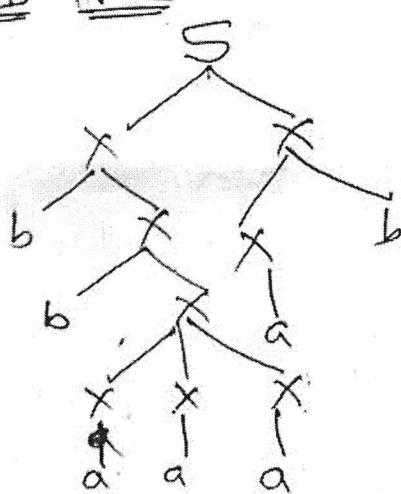
[7]

- 1) $S \rightarrow XX$
- 2) $X \rightarrow XXX$
- 3) $X \rightarrow bX$
- 4) $X \rightarrow Xb$
- 5) $X \rightarrow a$

LMD

$S \rightarrow XX \quad \overline{1}$
 $\rightarrow bXX \quad \overline{2}$
 $\rightarrow bbXX \quad \overline{3}$
 $\rightarrow bbXXX \quad \overline{2}$
 $\rightarrow bbaxxx \quad \overline{5}$
 $\rightarrow bbqaxx \quad \overline{5}$
 $\rightarrow bbaaxx \quad \overline{5}$
 $\rightarrow bbaaqx \quad \overline{4}$
 $\rightarrow bbaaqab \quad \overline{5}$

LMD Tree

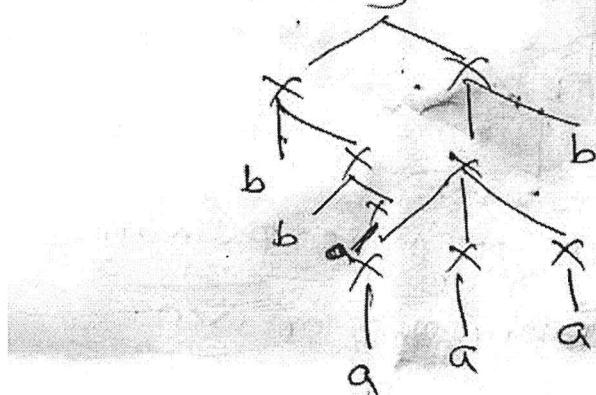




RMD

$S \rightarrow XX \quad \text{--- 1}$
 $\rightarrow X \underline{X} b \quad \text{--- 4}$
 $\rightarrow \underline{XXX} b \quad \text{--- 2}$
 $\rightarrow \underline{XXXab} \quad \text{--- 5}$
 $\rightarrow \underline{XXaab} \quad \text{--- 5}$
 $\rightarrow \underline{Xaab} \quad \text{--- 5}$
 $\rightarrow \underline{bXaab} \quad \text{--- 5}$
 $\rightarrow \underline{bbXaab} \quad \text{--- 3}$
 $\rightarrow \underline{bbbaaab} \quad \text{--- 5}$

RMD Tree



Q3 (a)

pictorial representation with simulation of
strong — many — ~~many~~ ⁰⁷
The language can be listed
as follows.

[7]

$$L = \{ \epsilon, aa, bb, aaa, abba, baab, bbbb, \\ aaaaa, bbbb bbbb \dots \}$$

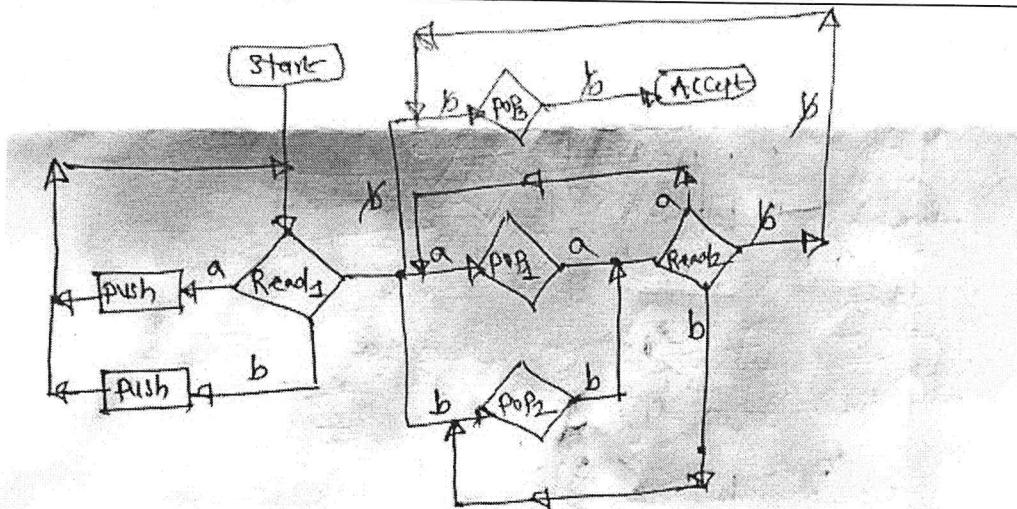


Fig:- PDA accepting even palindrome

Simulation of any string of even
palindrome of a's & b's

example			palindrome of a = b	
Current State	Stack of Content		Tape & Head	
Start	b		abba b b	
↓	b		↑	
Read ₁	b		a b b a b b	
↓			↑	
push a	a		abba b b	
↓			↑	
Read ₁	a		abb a b b	
↓			↑	
push b	ba		abba b b	
↓			↑	
pop ₂	a		abba b b	
↓			↑	
Read ₁	bab		abba b b	
↓			↑	
pop ₂	b		abba b b	
↓			↑	
Read ₂	a		abba b b	
↓			↑	
pop ₃			-	
		Accept		



Q3 (a)

OR

[7]

pictorial representation of simulation marks of

$$\begin{aligned} S &\rightarrow S \cdot S \\ &\rightarrow 4 + S \\ &\rightarrow 4 + 4 \cdot S \\ &\rightarrow 4 + 4 * 4 \end{aligned}$$

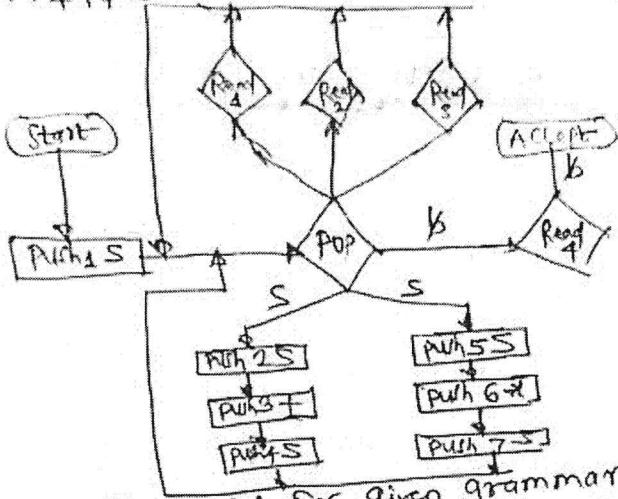


fig:- PDA for given grammar
simulation for the string $4+4*4$ which
is given grammar

Q3 (b)

[8]

Σ	$\{0, 3, 6\}$	$\{1, 4\}$	$\{2, 5, 8\}$	B
q_0	$q_0 R X$	$q_1 R Y$	$q_2 R Z$	$q_3 R B$
q_1	$q_1 R X$	$q_2 R Y$	$q_0 R Z$	$q_4 R B$
q_2	$q_2 R X$	$q_0 R Y$	$q_1 R Z$	$q_5 R B$
q_3	—	—	—	$q_6 N_0$
q_4	—	—	—	$q_6 N_1$
q_5	—	—	—	$q_6 N_2$
q_6	Accept	Reject	Reject	—

Assume: $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 q_0 — for remainder 0 , X — for $\{0, 3, 6\}$
 q_1 — for remainder 1 , Y — for $\{1, 4\}$
 q_2 — for remainder 2 , Z — for $\{2, 5, 8\}$



B₁B₂B₃B₁B₂B₃
B₂B₁B₃B₂B₁B₃
B₁B₂B₃B₁B₂B₃
B₂B₁B₃B₂B₁B₃
B₁B₂B₃B₁B₂B₃

B₁B₂B₃B₁B₂B₃
B₂B₁B₃B₂B₁B₃
B₁B₂B₃B₁B₂B₃
B₂B₁B₃B₂B₁B₃
B₁B₂B₃B₁B₂B₃

Accent.

Q4 (a)

[10]

- Q4 Given: $n > 0$
- (1) for $n=1$ length is = 5 many → ∞
- | | |
|-------|-----------------------------|
| $n=2$ | $\frac{11}{\text{---}} = 5$ |
| $n=3$ | $\frac{11}{\text{---}} = 7$ |
- from above each string generates the odd length of string sequence
- (2) Let the given ~~grammar~~ language is $L = \{a^n b^{n+1} \mid n > 0\}$ is regular
- (3) Let λ be a pumping lemma constant
- (4) Let w be as $a^{\lambda} b^{\lambda}$
- so $|w| = \lambda + \lambda$
 $|w| = 2\lambda + 1$
- (5) By pumping lemma $w = uvw$ for $1 \leq |v| \leq \lambda$ for $i \geq 0$ then $uv^i w$ is in L
- (6) Let $i=2$
 $1 \leq |v| \leq \lambda$
 $2\lambda + 1 + i \leq |uv^2w| \leq \lambda + 2\lambda + 1$
 $2\lambda + 2 \leq |uv^2w| \leq 3\lambda + 1$
- (7) $2\lambda + 1 \leq |uv^2w| \leq 3\lambda + 2$
- Let $\lambda = 1$
 $2 \times 1 + 1 \leq |uv^2w| \leq 3 \times 1 + 2$
 $3 \leq |uv^2w| \leq 5$
- We got value as $\frac{4}{4}$
This value is even



Let $\ell = 2$
 $2 \times 2 + 1 < |uv^2w| < 3 \times 2 + 2$

$$5 < |uv^2w| < 8$$

we got value as 6 & 7
 it is not odd

Then length of uv^2w is not always odd, that means uv^2w is not in 'L'.
 so, that the given language
 $\{a^n b^{n+1} | n > 0\}$ is not regular.

OR

[10]

Q4(a)

We get the following equations

$$q_0 = \Lambda \quad \text{--- (1)}$$

$$q_1 = q_0 1 + q_1 1 + q_3 1 \quad \text{--- (2)}$$

$$q_2 = q_1 0 + q_2 0 + q_3 0 \quad \text{--- (3)}$$

$$q_3 = q_2 1 \quad \text{--- (4)}$$

$$q_2 = q_1 0 + q_2 0 + q_2 1 0$$

$$= q_1 0 + q_2 (0 + 10)$$

By Arden's Th.

$$q_2 = q_1 0 (0 + 10)^*$$

$$\text{Now } q_1 = 1 + q_1 + q_2 1 1$$

$$= 1 + q_1 + q_1 0 (0 + 10)^* 1 1$$

$$q_1 = 1 + q_1 (1 + 0 (0 + 10)^* 1 1)$$

By Arden's Th.

$$q_1 = 1 (1 + 0 (0 + 10)^* 1 1)^*$$



$$q_3 = q_2$$

$$q_3 = 1(1+0(0+10)^*)^* 0(0+10)^*$$

$$\therefore q_3 = 1(1+0(0+10)^*)^* 0(0+10)^*$$

Q4 (b)

[5]

$$\begin{aligned}
&= \lambda + 1^*(011)^* (1^*(011)^*)^* \\
&= \lambda + P_1 P_1^* \quad (\text{where } P_1 = 1^*(011)^*) \\
&= P_1^* \quad \text{Using I}_g \\
&= (1^*(011)^*)^* \\
&= (P_2 + P_3)^* \quad \text{let } P_2 = 1, P_3 = 011 \\
&= (P_2 + P_3)^* \quad \text{Using I}_{II} \\
&= (1 + 011)^*
\end{aligned}$$

Q5 (a)

[5]

(i) Q.5
①;

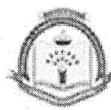
DPDA

NPDA

- | | |
|--|--|
| 1) It is less powerful than NPDA. | 1) It is more powerful than DPDA |
| 2) It is possible to convert every DPDA to a corresponding NPDA. | 2) It is not possible to convert every NPDA to a corresponding DPDA. |
| 3) The language accepted by DPDA is a subset of the language accepted by NPDA. | 3) The language accepted by NPDA is not a subset of the language accepted by DPDA. |
| 4) The language accepted by DPDA is DCFL. | 4) The language accepted by NPDA is called NCFL. |



Q5 (a) (ii)	<p>ii) Explanation with example</p>	[5]
Q5 (a) (iii)	<p><u>Q.5</u></p> <p>(a) There are two types of <u>Turing machine</u></p> <p>(i) multitape Turing machine</p> <p>(ii) Nondeterministic Turing machine</p> <p>i) <u>multitape TM</u></p> <p>The diagram illustrates a multitape Turing machine with multiple tapes. It shows Tape 1, Tape 2, and Tape n. Each tape is represented as a horizontal sequence of cells. A head is shown above Tape 1, labeled 'Heads'. Arrows point from the heads to the tapes. Below the tapes is a box labeled 'finite Control'.</p> <p>Fig:- multitape machine</p> <p>2) <u>Nondeterministic TM</u></p> <p>A nondeterministic TM M is seven tuple given below</p> $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ <p>where</p> <ul style="list-style-type: none"> Q: finite set of states of TM Γ: finite set of symbols called tape symbols. Σ: nonempty subset of Γ called i/p symbols. q_0: initial state B: Blank symbol $F \subseteq Q$ called as set of final states 	[5]



f - defined a mapping from $\mathcal{Q} \times \Gamma$ to $\mathcal{Q} \times \Gamma \times \{L, R, N\}$

Q5 (b)

- ① Delete all ' ϵ ' productions from given grammar G. After deleting ' ϵ ' productions we get

$$\begin{array}{l} S \rightarrow Xa \\ X \rightarrow \alpha X \\ X \rightarrow bX \end{array}$$

→ ①

- ② Identify nullable non-terminal in the given grammar

$X \rightarrow \epsilon$ so ' X ' is a nullable non-terminal

- ③ Identify nullable non-terminal which should occur on right hand side of the production rule.

$$\begin{array}{l} S \rightarrow Xa \\ X \rightarrow \alpha X \\ X \rightarrow bX \end{array}$$

After putting value of ' X ' as 'null' we get new production rule as

$$\begin{array}{l} S \rightarrow a \\ X \rightarrow a \\ X \rightarrow b \end{array}$$

→ ②

- ④ So the final grammar G as

$$\begin{array}{l} S \rightarrow Xa | a \\ X \rightarrow \alpha X | a \\ X \rightarrow bX | b \end{array}$$