Final Project Report

ORIE/STSCI4630/5630
(Operations Research Tools for Financial Engineering)

Adarsh Pandey (AP2459)
Atharwa Pandey (AP2467)
Rethyam Gupta (RG795)
Riley Gerald Burke (RGB234)

1 Stock Selection for the Technology Sector

To represent the technology sector for the years 2017–2022, we selected four key stocks: **Amazon (AMZN)**, **Microsoft (MSFT)**, **Apple (AAPL)**, and **Google (GOOGL, now Alphabet Inc.)**. These companies were chosen due to their significant market capitalization, representation of distinct aspects of the technology industry, and their stock price movements, which often mirrored broader trends within the sector.

1.1 Justification for Stock Selection

The selected companies were among the most influential in the technology sector, both in terms of innovation and financial performance. The reasons for their selection include:

- **Market Capitalization**: These companies were among the largest by market capitalization during 2017–2022. For example, Apple consistently ranked as the most valuable company globally, while Amazon and Microsoft were leaders in their respective domains, often shifting positions among the top three globally by market cap.

- **Broader Industry Movements**: The stock price movements of these companies closely tracked overall trends in the technology sector, making them strong representatives for sector analysis.

- **Growth and Innovation**: These companies exhibited exceptional growth during this period, driven by innovation in fields such as cloud computing, consumer electronics, and artificial intelligence. Their ability to consistently launch new products and services contributed to their prominence.

- **Diverse Business Models**: Each company represents a unique aspect of the technology industry, ranging from cloud computing and software to hardware and digital advertising, making the analysis comprehensive.

1.2 Overview of Selected Stocks

- **Amazon (AMZN)**: Amazon emerged as a leader in the technology sector, with its cloud computing arm, Amazon Web Services (AWS), becoming a significant driver of growth and profitability. AWS dominated the cloud market, contributing more than 10% of Amazon's revenue during this period. Additionally, Amazon's e-commerce dominance provided stability and diversification, making it a pivotal company for the sector.

- **Microsoft (MSFT)**: Microsoft solidified its position as a technology giant through its cloud computing platform Azure, which experienced rapid growth and rivaled AWS. The company's enterprise software solutions, such as Office 365, and its investments in gaming (Xbox) further expanded its business reach. Microsoft consistently achieved a high market capitalization, reflecting investor confidence in its growth strategies.

- **Apple (AAPL)**: Apple represented the pinnacle of consumer technology innovation. The iPhone remained its flagship product, driving a significant portion of its revenue. Apple's ecosystem, including wearables like the Apple Watch and services such as the App Store and Apple Music, demonstrated strong growth. Apple frequently reached or exceeded a $1 trillion market capitalization during this timeframe, highlighting its global impact.

- **Google (GOOGL)**: Google, under its parent company Alphabet Inc., dominated digital advertising, with its search engine and YouTube being key contributors to revenue. Google also expanded into cloud computing and artificial intelligence, diversifying its offerings. Its consistent revenue growth and influence on internet-related industries made it a key representative of the tech sector.

1.3 Conclusion

The selected companies—Amazon, Microsoft, Apple, and Google—represent the broader technology sector exceptionally well due to their size, influence, and alignment with overall industry movements during 2017–2022. Their diversity in business models and consistent growth make them ideal candidates for analyzing the cumulative performance of the technology industry.

1.4   Representing of the Technology Sector with NDXT

To represent the technology sector as a whole for the years 2017–2022, we utilized the **NASDAQ-100 Technology Sector Index (NDXT)**. This index was selected for its comprehensive coverage of the technology industry and its ability to reflect the performance of leading technology companies. Below, we detail the reasons for this choice and its relevance to the analysis.

1.5   Reasons for Selecting NDXT

The NASDAQ-100 Technology Sector Index is one of the most widely recognized benchmarks for the technology sector. It captures the broader dynamics of the industry, making it an ideal choice for this analysis. The key reasons for selecting NDXT include:

- **Comprehensive Coverage**: The NDXT includes the technology-focused companies within the NASDAQ-100, one of the most prominent indices globally. It covers a diverse range of technology sub-industries, such as software, hardware, semiconductors, cloud computing, and artificial intelligence.

- **Equal Weighted Index**: The NASDAQ-100 Technology Sector Index is an equal-weighted index based on the securities of the Nasdaq-100 Index that are classified as Technology according to the Industry Classification Benchmark (ICB) classification system. On February 22, 2006, the Nasdaq-100 Technology Sector Index began with a base value of 1000.00.

- **Sector Representation**: By focusing specifically on technology companies, the NDXT eliminates noise from non-tech industries and provides a cleaner representation of the sector's performance.

- **Historical Relevance**: During the 2017–2019 period, the technology sector experienced significant growth, driven by innovation in areas such as cloud computing, artificial intelligence, and mobile technology. The NDXT reflects these growth trends, capturing both the achievements of established companies and the potential of emerging players.

- **Correlation with Broader Markets**: The NDXT's performance closely tracks the movements of broader technology-focused indices, such as the NASDAQ Composite, but offers a more refined focus on the pure technology segment.

1.6 Daily Returns of Sector and Assets (2017–2019)

Table 1: First 9 Daily Returns of Sector and Assets (2017–2019)

Sector	AAPL	MSFT	GOOGL	AMZN
0.003	-0.001	-0.004	-0.000	0.004
0.002	0.005	0.000	0.006	0.030
0.008	0.011	0.009	0.015	0.020
0.006	0.009	-0.003	0.002	0.001
0.004	0.001	-0.000	-0.001	-0.001
0.005	0.005	0.009	0.005	0.004
-0.004	-0.004	-0.009	-0.000	0.018
0.006	0.002	0.001	0.002	0.004
-0.007	0.008	-0.003	-0.004	-0.009

Table 2: Last 10 Daily Returns of Sector and Assets (2017–2019)

Sector	AAPL	MSFT	GOOGL	AMZN
0.011	0.017	0.006	0.010	0.005
0.001	0.002	-0.005	-0.004	0.012
0.002	-0.002	-0.002	-0.002	-0.004
0.008	0.001	0.009	0.003	0.005
0.007	-0.002	0.011	-0.004	-0.003
0.001	0.016	0.000	-0.000	0.004
0.001	0.001	-0.000	-0.005	-0.002
0.004	0.002	0.008	0.013	0.044
-0.002	-0.000	-0.006	-0.006	-0.006
-0.007	0.006	-0.009	-0.011	-0.012

1.7   Cumulative Returns of Stocks and Sector

Cumulative Performance
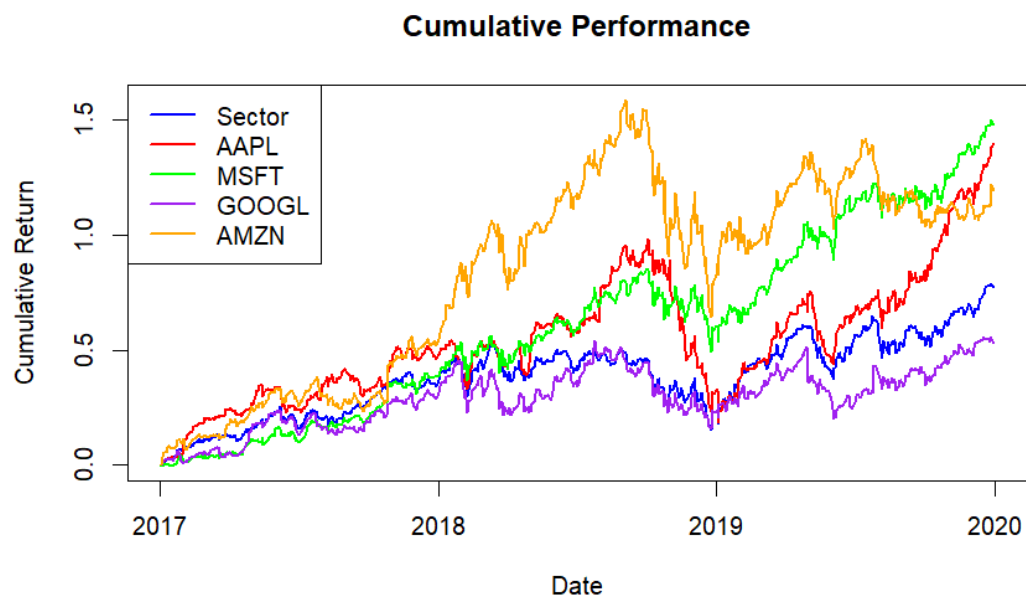
Figure 1: Cumulative Returns of the Stocks and the Sector (2017 - 2019).
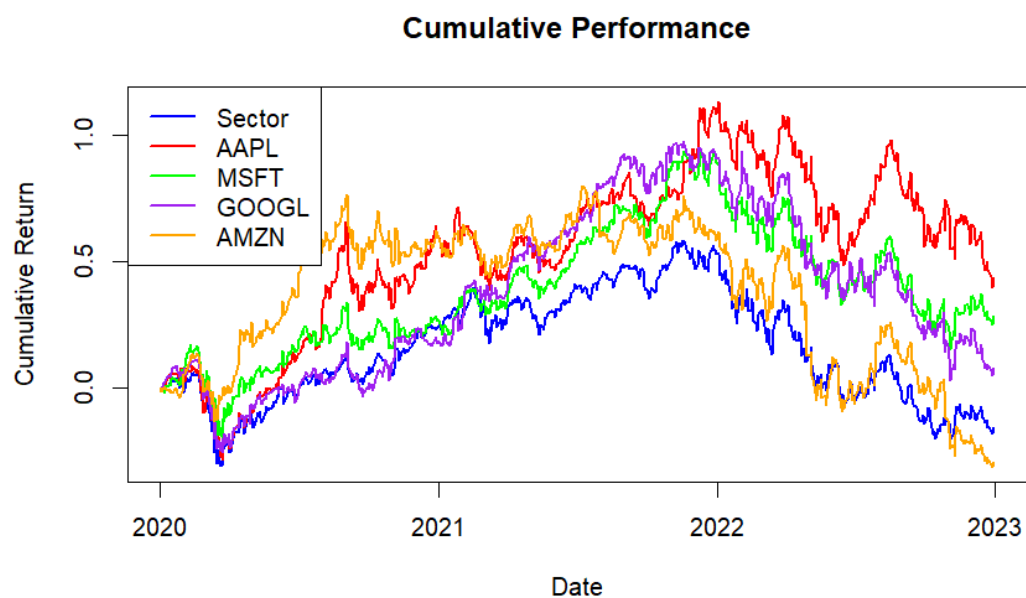
Cumulative Performance

Figure 2: Cumulative Returns of the Stocks and the Sector (2020 - 2022).

2 Mean and Std. Deviation of Daily Returns (2017-2019)

Table 3: Mean and Standard Deviation of Daily Returns (2017)

Mean Returns (2017)

Ticker	Mean_2017
NDXT	0.001 (0.121%)
AAPL	0.002 (0.156%)
MSFT	0.001 (0.133%)
GOOGL	0.001 (0.106%)
AMZN	0.002 (0.175%)

Standard Deviation (2017)

Ticker	SD_2017
NDXT	0.008
AAPL	0.011
MSFT	0.009
GOOGL	0.010
AMZN	0.013

Table 4: Mean and Standard Deviation of Daily Returns (2018)

Mean Returns (2018)

Ticker	Mean_2018
NDXT	-0.000 (-0.022%)
AAPL	-0.000 (-0.022%)
MSFT	0.001 (0.075%)
GOOGL	-0.000 (-0.003%)
AMZN	0.001 (0.100%)

Standard Deviation (2018)

Ticker	SD_2018
NDXT	0.015
AAPL	0.018
MSFT	0.018
GOOGL	0.018
AMZN	0.023

Table 5: Mean and Standard Deviation of Daily Returns (2019)

Mean Returns (2019)

Ticker	Mean_2019
NDXT	0.002 (0.153%)
AAPL	0.003 (0.251%)
MSFT	0.002 (0.181%)
GOOGL	0.001 (0.099%)
AMZN	0.001 (0.082%)

Standard Deviation (2019)

Ticker	SD_2019
NDXT	0.013
AAPL	0.017
MSFT	0.013
GOOGL	0.015
AMZN	0.014

2.1 Mean and Std. Deviation of Daily Returns (2020-2022)

Table 6: Mean and Standard Deviation of Daily Returns (2020)

Mean Returns (2020)

Ticker	Mean_2020
NDXT	0.001 (0.119%)
AAPL	0.002 (0.228%)
MSFT	0.001 (0.133%)
GOOGL	0.001 (0.098%)
AMZN	0.002 (0.213%)

Standard Deviation (2020)

Ticker	SD_2020
NDXT	0.025
AAPL	0.029
MSFT	0.028
GOOGL	0.024
AMZN	0.024

Table 7: Mean and Standard Deviation of Daily Returns (2021)

Mean Returns (2021)

Ticker	Mean_2021
NDXT	0.001 (0.095%)
AAPL	0.001 (0.118%)
MSFT	0.002 (0.167%)
GOOGL	0.002 (0.199%)
AMZN	0.000 (0.009%)

Standard Deviation (2021)

Ticker	SD_2021
NDXT	0.016
AAPL	0.016
MSFT	0.013
GOOGL	0.015
AMZN	0.015

Table 8: Mean and Standard Deviation of Daily Returns (2022)

Mean Returns (2022)

Ticker	Mean_2022
NDXT	-0.002 (-0.203%)
AAPL	-0.001 (-0.122%)
MSFT	-0.001 (-0.131%)
GOOGL	-0.002 (-0.198%)
AMZN	-0.003 (-0.273%)

Standard Deviation (2022)

Ticker	SD_2022
NDXT	0.026
AAPL	0.022
MSFT	0.022
GOOGL	0.024
AMZN	0.032

3 Portfolio Optimization

3.1 Mean-Variance Optimization

Markowitz Portfolio Optimization is a fundamental technique in Modern Portfolio Theory, aiming to construct an optimal portfolio by balancing risk and return. The process involves:

1. **Estimating Parameters:** Calculate expected returns and the covariance matrix of the assets.

2. **Optimization Problem:**

$$\min_{\mathbf{w}} \mathbf{w}^\top \Sigma \mathbf{w}, \quad \text{s.t.} \quad \mathbf{w}^\top \mathbf{1} = 1,$$

   where $\mathbf{w}$ are portfolio weights, Σ is the covariance matrix, μ is expected returns, and μ_p is the desired portfolio return.

The result is the **efficient frontier**, a curve representing the best risk-return combinations.

3.2 Tangency Portfolio

The Tangency Portfolio is the portfolio on the efficient frontier that maximizes the Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{E[R_p] - R_f}{\sigma_p},$$

where $E[R_p]$ is the portfolio's expected return, R_f is the risk-free rate, and σ_p is its standard deviation. Steps to calculate:

1. Compute excess returns: $\mu - R_f \mathbf{1}$.

2. Solve:

$$\max_{\mathbf{w}} \frac{\mathbf{w}^\top (\mu - R_f \mathbf{1})}{\sqrt{\mathbf{w}^\top \Sigma \mathbf{w}}}, \quad \text{s.t.} \quad \mathbf{w}^\top \mathbf{1} = 1.$$

The Tangency Portfolio combines risky assets with the risk-free asset, forming the Capital Market Line (CML), which provides superior risk-return trade-offs compared to the efficient frontier alone.

3.3 Application to Selected Stocks

For our analysis of **Apple (AAPL)**, **Microsoft (MSFT)**, **Amazon (AMZN)**, and **Google (GOOGL)**:

- We will calculate the Mean-Variance Optimized Portfolio based on historical returns and risks.

- We will construct the Tangency Portfolio using a risk-free rate of 2%.

This approach will demonstrate the risk-return trade-offs and optimal portfolio allocations.

3.4 Mean-Variance Optimization Analysis

The optimal portfolio weights for the Minimum Variance Portfolio (MVP), calculated using the historical returns and covariances of the four selected stocks, are as follows:

- **AAPL (Apple):** 26.0%

- **MSFT (Microsoft):** 50.2%

- **GOOGL (Google):** 30.8%

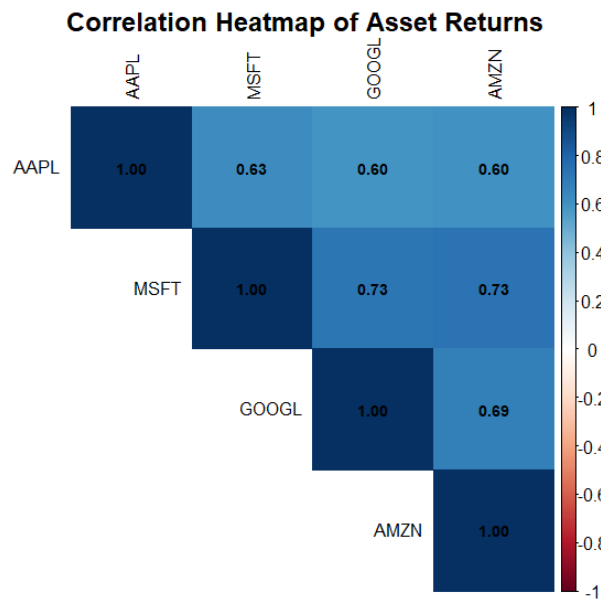- **AMZN (Amazon):** -7.1% (indicating a small short position)

Figure 3: Correlation Heatmap of Selected Stocks

3.5 Individual Asset Summaries

The performance metrics of the individual stocks are summarized in the table below:

Table 9: Performance Summary of Individual Assets

Asset	Daily Mean Return %	Daily Std. Deviation	Sharpe
AAPL	0.128	0.01558	1.227
MSFT	0.130	0.01364	1.419
GOOGL	0.067	0.01450	0.648
AMZN	0.119	0.01725	1.022

3.6 Portfolio and Sector Return Summary

The portfolio and sector performance are summarized in the table below:

Table 10: Portfolio and Sector Performance Summary

Metric	Portfolio	Sector
Daily Mean Return (%)	0.111	0.084
Daily Standard Deviation	0.01266	0.01262
Annualized Sharpe Ratio (2% Risk-Free Rate)	1.29	0.956

3.7 Discussion

- The Minimum Variance Portfolio (MVP) favors **MSFT** (50.2%) and **GOOGL** (30.8%) while maintaining a moderate allocation to **AAPL** (26.0%) and a small short position in **AMZN** (-7.1%).

- The portfolio's Sharpe Ratio (1.29) is significantly higher than that of the sector (0.956), indicating a superior risk-adjusted return.

- Among individual assets, **MSFT** has the highest Sharpe Ratio (1.419), reflecting the best risk-return trade-off. In contrast, **GOOGL** has the lowest Sharpe Ratio (0.648), suggesting relatively lower performance on a risk-adjusted basis.

3.8 Tangency Portfolio Results and Analysis

The Tangency Portfolio is constructed by maximizing the Sharpe Ratio, combining risky assets with the risk-free asset. Below are the results for the Tangency Portfolio:

- **Weights:**
    - AAPL: 49.93%
    - MSFT: 119.06%
    - GOOGL: -73.78% (short position)
    - AMZN: 4.79%

- **Portfolio Statistics:**
    - Daily Mean Return: 0.175%
    - Daily Standard Deviation: 1.612%
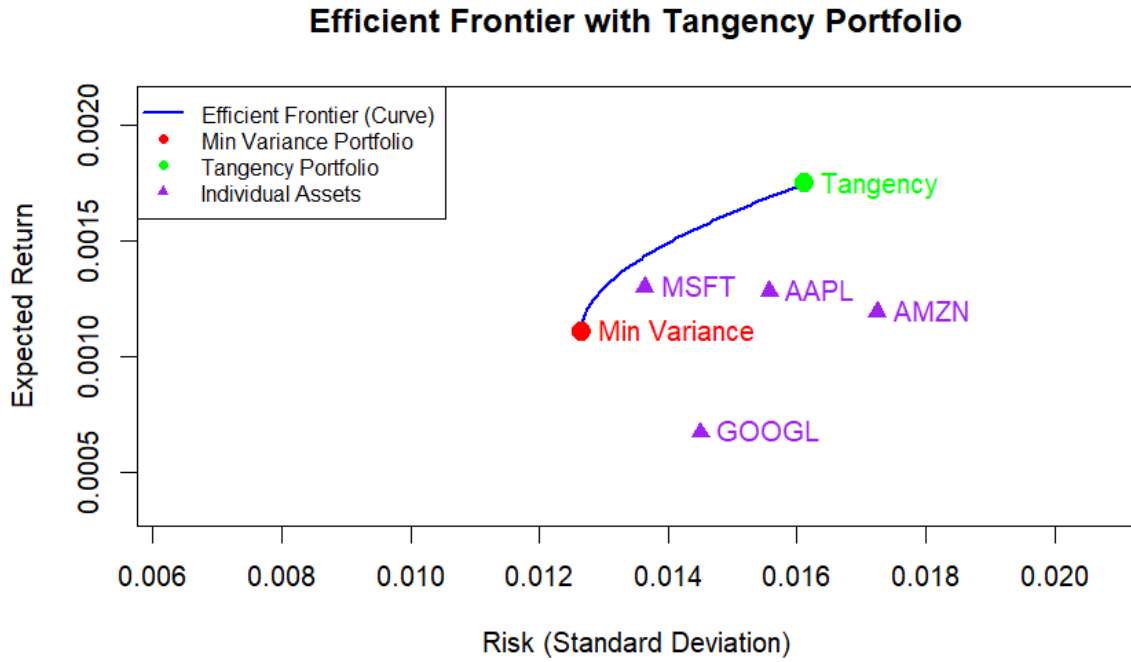   - Annualized Sharpe Ratio (2% risk-free rate): 1.643

Figure 4: Tangency Portfolio with Frontier and Individual Assets.

3.8.1 Discussion on Results

- **Portfolio Weights:** The Tangency Portfolio heavily favors **MSFT** with a weight of 119.06% and **AAPL** with 49.93%, while holding a short position in **GOOGL** (-73.78%). This allocation reflects the high Sharpe Ratio of MSFT and the relatively lower risk-return profile of GOOGL. A small positive weight is allocated to **AMZN**.

- **Efficiency of the Tangency Portfolio:** The Tangency Portfolio achieves an annualized Sharpe Ratio of 1.643, which is significantly higher than both the Sector (0.956) and the Minimum Variance Portfolio (1.29). This demonstrates its superior risk-adjusted performance. By leveraging the risk-free rate, the Tangency Portfolio provides an optimal balance between risk and return.

- **Risk-Free Rate Impact:** With a risk-free rate of 2%, the Sharpe Ratio reflects the portfolio's ability to generate excess returns per unit of risk. The high Sharpe Ratio indicates that the Tangency Portfolio effectively utilizes the risk-free asset to enhance overall returns.

4 Tangency Portfolio Vs. Sector Performance

4.1 Performance During Sample Period (2017–2019)

- **Outperformance:** The Tangency Portfolio exhibited significant outperformance over the sector (NDXT) during the sample period, demonstrating its efficiency in capturing higher returns while maintaining a relatively low level of risk.

- **Risk-Adjusted Returns:** The annualized Sharpe Ratio of the Tangency Portfolio (1.644) was markedly higher than the sector's (0.957), indicating superior risk-adjusted performance.

- **Volatility:** While the Tangency Portfolio had slightly higher daily volatility compared to the sector, the optimized allocation ensured this did not compromise its risk-return trade-off.

- **Portfolio Efficiency:** The Tangency Portfolio's allocation to high-performing assets, such as Microsoft (MSFT) and Apple (AAPL), contributed to its enhanced performance, while the short position in Google (GOOGL) improved diversification and risk management.
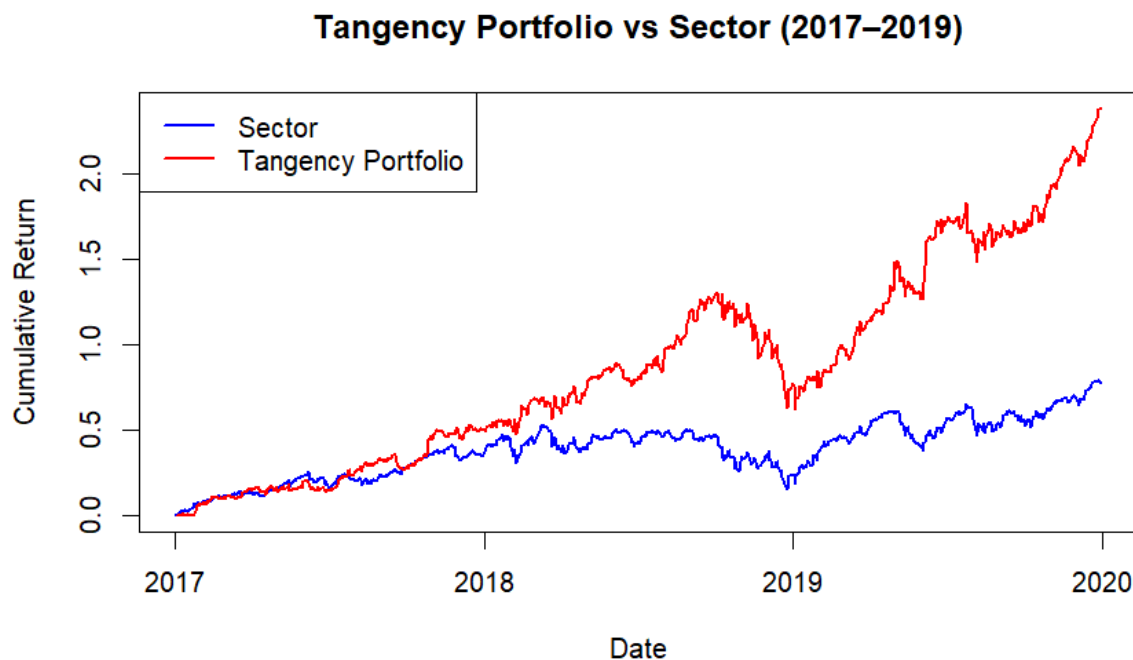
Figure 5: Tangency Portfolio vs Sector (2017–2019)

4.2 Performance in Out-of-Sample Period (2020–2022)

- **Risk-Adjusted Returns:** Despite significant market volatility caused by the COVID-19 pandemic, the Tangency Portfolio continued to outperform the sector in terms of cumulative returns while closely tracking its overall movement. The Sharpe Ratio of the Tangency Portfolio dropped to 0.453 during this period, reflecting the heightened volatility and lower overall returns compared to the previous period. However, it still significantly outperformed the sector, which had a negative Sharpe Ratio of -0.026, indicating underperformance on a risk-adjusted basis.

- **Market Rally and Corrections:** During the market rally of 2020 and 2021, the Tangency Portfolio offered higher returns than the sector. It also showed more resilience during the corrections of 2022, maintaining its advantage in cumulative returns.

- **Cumulative Return Advantage:** By the end of the period, the Tangency Portfolio maintained a cumulative return advantage of approximately 10–15%, highlighting its ability to outperform the sector index while reflecting its general performance trends.
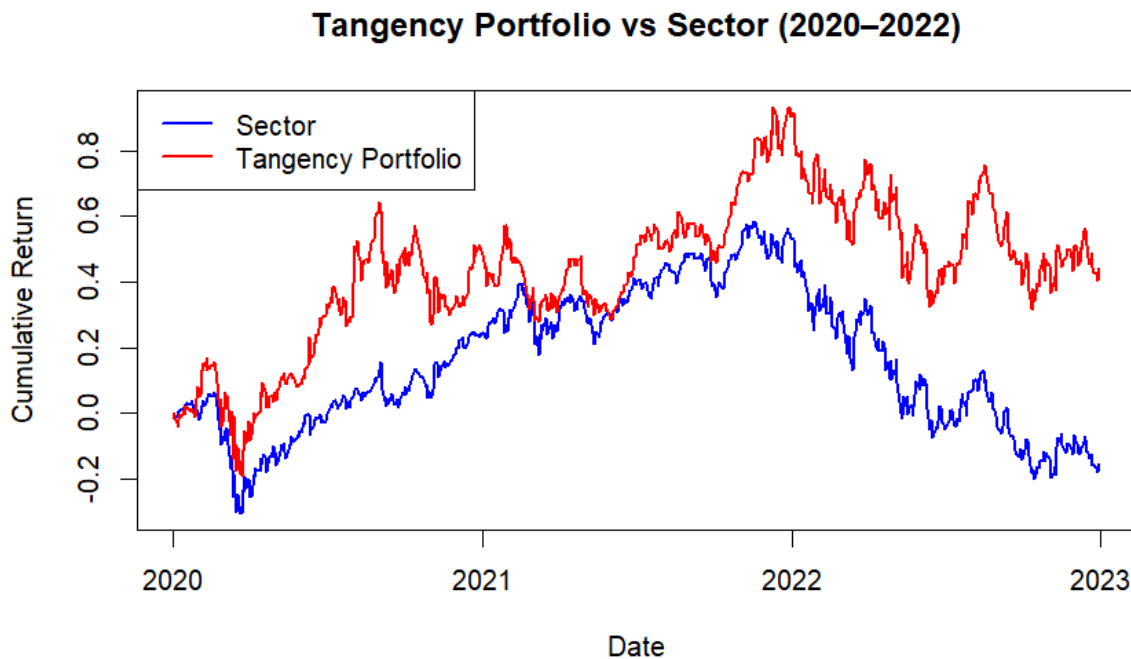

Figure 6: Tangency Portfolio vs Sector (2020–2022)

4.3 Key Findings

- The Tangency Portfolio consistently outperformed the sector in both sample and out-of-sample periods, validating the effectiveness of mean-variance optimization in constructing an efficient portfolio.

- While the Tangency Portfolio's higher returns in 2020–2022 were accompanied by increased volatility, its Sharpe Ratio highlights its ability to maintain better risk-adjusted performance than the sector.

- The portfolio's allocation strategy, leveraging MSFT and AAPL while shorting GOOGL, contributed significantly to its performance and diversification benefits.

- Overall, the Tangency Portfolio demonstrated resilience in volatile market conditions and efficiency in capitalizing on growth opportunities.

4.4 Sharpe Ratio Summary Table

Table 11: Sharpe Ratios of Tangency Portfolio and Sector

Period	Tangency Portfolio Sharpe Ratio	Sector Sharpe Ratio
2017–2019	1.644	0.957
2020–2022	0.453	-0.026

5 CAPM Estimation Results (2017–2019)

5.1 Capital Asset Pricing Model (CAPM) Overview

The Capital Asset Pricing Model (CAPM) postulates that the expected return of an asset is determined by its systematic risk (beta) relative to the market. The model is expressed as:

$$R_i - R_f = \alpha + \beta(R_m - R_f) + \epsilon_i,$$

where R_i is the return of the asset, R_f is the risk-free rate, R_m is the return of the market (sector in this context), α is the intercept (abnormal return), β measures the asset's sensitivity to market returns, and ϵ_i is the error term. The results of the CAPM estimation for the Tangency Portfolio, individual assets, and the sector are presented in Table 12.

5.2 CAPM Results

Table 12: CAPM Results for the 2017–2019 Period

Asset	Alpha	Beta	P-Value (Alpha)	P-Value (Beta)
Sector	< 0.001	1.363	0.317	< 0.001
Portfolio	> 0.001	1.423	0.007	< 0.001
AAPL	> 0.001	1.368	0.097	< 0.001
MSFT	> 0.001	1.380	0.018	< 0.001
GOOGL	< 0.001	1.321	0.838	< 0.001
AMZN	> 0.001	1.492	0.244	< 0.001

5.3 Key Observations

- **Alpha:** The alpha (α) values for all assets, including the Tangency Portfolio, are close to zero, suggesting that there are no significant abnormal returns beyond what is explained by their beta. However, the Tangency Portfolio's alpha is statistically significant at the 1% level (p-value = 0.007), indicating a small positive abnormal return.

- **Beta:** All assets and the portfolio exhibit beta values greater than 1, indicating that they are more volatile than the sector. The portfolio has the highest beta (1.423), reflecting its aggressive allocation to high-risk assets, while GOOGL has the lowest beta (1.321), indicating relatively lower market sensitivity.

- **P-Values:** The p-values for beta (β) are statistically significant ($p < 0.01$) for all assets and the portfolio, confirming the strong relationship between asset returns and market returns. However, the p-values for alpha (α) are mixed. For the portfolio and MSFT, alpha is statistically significant, but for GOOGL and AMZN, it is not.

5.4 Hypothesis Testing

Hypotheses:

- Null Hypothesis (H_0): The alpha (α) of an asset is equal to zero, indicating no abnormal returns beyond market movements.

- Alternative Hypothesis (H_A): The alpha (α) of an asset is not equal to zero, indicating significant abnormal returns.

  Results:

- For the Tangency Portfolio, the null hypothesis is rejected ($p = 0.007$), suggesting that the portfolio generates a small but statistically significant positive abnormal return. There may be other factors affecting the returns; the solution: use an FF3 Model.

- For individual assets:

  - MSFT ($p = 0.018$) also rejects the null hypothesis, indicating a positive abnormal return.
  - For AAPL, GOOGL, and AMZN, the null hypothesis cannot be rejected ($p > 0.05$), implying no significant abnormal returns for these assets.

5.5 Analysis on CAPM Validity

The CAPM assumption that all excess returns are explained by beta holds true for most individual assets, as their alpha values are not statistically significant. However, the Tangency Portfolio and MSFT exhibit statistically significant alpha values, suggesting a deviation from CAPM expectations. This indicates that, while CAPM provides a good explanation of returns for most assets, the portfolio's construction allows it to capture additional returns beyond what is predicted by the model, likely due to its optimized allocation.

5.6 Analysis of the SML Plot

- **Market Portfolio (S&P 500):** The market portfolio lies perfectly on the SML, as expected under CAPM, confirming that the model accurately predicts its return.

- **Sector and Individual Stocks:** All stocks and the sector lie above the SML. This indicates that their actual returns exceed the expected returns predicted by CAPM.

- **Tangency Portfolio:** The Tangency Portfolio is significantly above the SML, reflecting abnormal returns that CAPM fails to capture. This deviation highlights CAPM's limitations in explaining returns for optimized portfolios designed to maximize risk-adjusted returns.
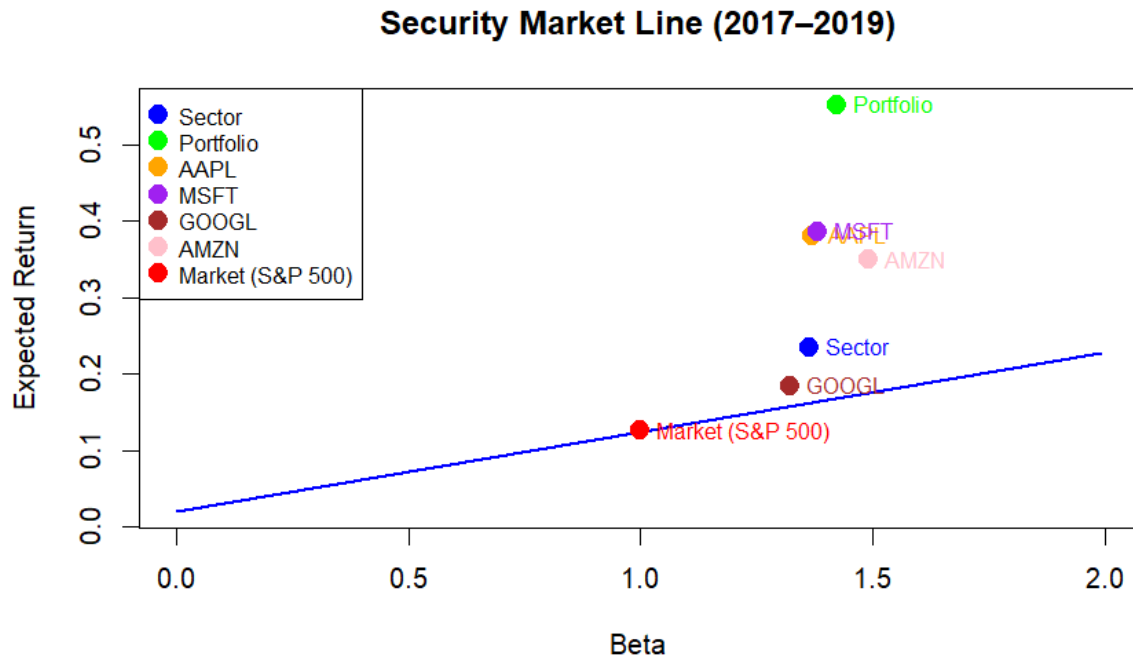
Figure 7: Security Market Line and Assets (2017–2019)

5.7 Conclusion

- The SML plot visually confirms that most assets outperform CAPM-predicted returns, as reflected by their positions above the line.

- CAPM accurately explains the market portfolio and some assets, but it underestimates returns for high-performing assets (e.g., AAPL, MSFT) and the Tangency Portfolio.

- These results suggest that while CAPM is a robust model, it does not fully capture the dynamics of portfolios designed for optimization or stocks influenced by factors beyond market beta.

6 CAPM Analysis (2020–2022)

Table 13: CAPM Results for the 2020–2022 Period

Asset	Alpha	Beta	P-Value (Alpha)	P-Value (Beta)
Sector	< 0.001	1.277	0.564	< 0.001
Portfolio	> 0.001	1.225	0.334	< 0.001
AAPL	> 0.001	1.193	0.286	< 0.001
MSFT	< 0.001	1.170	0.422	< 0.001
GOOGL	< 0.001	1.100	0.819	< 0.001
AMZN	< 0.001	1.001	0.576	< 0.001

6.1 Observations and Comparisons with 2017–2019

Sector:

- **Beta:** The sector's beta decreased slightly from 1.363 (2017–2019) to 1.277 (2020–2022), indicating reduced sensitivity to market movements during the pandemic.

- **Alpha:** The sector's alpha remains statistically insignificant ($p = 0.564$), consistent with the 2017–2019 period ($p = 0.317$). This suggests that the sector's returns are better explained by CAPM.

Portfolio:

- **Beta:** The Tangency Portfolio's beta decreased from 1.423 (2017–2019) to 1.225 (2020–2022). This reflects reduced exposure to market risk during a highly volatile period.

- **Alpha:** The portfolio's alpha became statistically insignificant ($p = 0.334$), contrasting with its significant alpha in 2017–2019 ($p = 0.007$). This indicates that the portfolio did not achieve abnormal returns in 2020–2022.

Individual Assets:

- **Beta:** AAPL and MSFT exhibited lower betas in 2020–2022 compared to 2017–2019. AAPL's beta decreased from 1.368 to 1.193, and MSFT's beta decreased from 1.380 to 1.170, indicating reduced sensitivity to market movements. Both GOOGL and AMZN maintained beta values close to 1 in both periods. GOOGL's beta decreased slightly from 1.321 to 1.100, while AMZN's beta decreased to 1.001.

- **Alpha:** Both AAPL ($p = 0.286$) and MSFT ($p = 0.422$) had statistically insignificant alphas in 2020–2022. This is in contrast to the 2017–2019 period, where their alphas were significant or marginally significant. Both GOOGL and AMZN exhibited statistically insignificant alphas in 2020–2022 ($p = 0.819$ for GOOGL and $p = 0.576$ for AMZN), consistent with their results in 2017–2019. This suggests that CAPM accurately explains their returns in both periods.

6.2 Analysis of Beta Decrease (2020–2022)

The decline in beta values for most assets during the 2020–2022 period reflects a reduced sensitivity to overall market movements. This phenomenon can be attributed to the unique market dynamics of the COVID-19 pandemic:

- **Sector and Portfolio:** The sector beta decreased from 1.363 in 2017–2019 to 1.277 in 2020–2022, while the portfolio beta declined from 1.423 to 1.225. This reduction indicates that both the sector and the portfolio became less correlated with broader market fluctuations, likely driven by the resilience and independent growth trajectory of the technology sector during the pandemic.

- **Individual Assets:** Key tech stocks like AAPL, MSFT, and GOOGL also experienced decreases in beta. For instance:

  - AAPL: Beta dropped from 1.368 to 1.193.
  - MSFT: Beta decreased from 1.380 to 1.170.
  - GOOGL: Beta fell from 1.321 to 1.100.

  This suggests that these stocks were less influenced by market-wide shocks and driven more by company-specific factors and sector resilience.

- **Impact of Pandemic Dynamics:** The COVID-19 period introduced uneven impacts across sectors, with technology benefiting from increased digital adoption. This sector-specific strength reduced its dependency on overall market trends, leading to lower betas.

- **Diversification Effects:** The portfolio beta being consistently lower than the sector beta (1.225 vs. 1.277) reflects the impact of diversification, which mitigates the overall portfolio's sensitivity to market risk.

The decline in beta across most assets emphasizes the technology sector's relative stability and reduced exposure to systematic risk during the heightened uncertainty of the COVID period.

6.3 Analysis of the SML Plot

- **Alignment with SML:** The SML plot shows that most assets, including the Sector, Portfolio, and individual stocks (AAPL, MSFT, GOOGL, and AMZN), are aligned closely with the SML. This indicates that CAPM effectively explains their returns during this period.

- **Portfolio and AAPL:** The Tangency Portfolio and AAPL are slightly above the SML, suggesting that they achieved marginally higher-than-expected returns given their respective betas. This reflects a small deviation from CAPM predictions but remains within reasonable limits.

- **Sector and Other Stocks:** The Sector, MSFT, GOOGL, and AMZN align closely with the SML, supporting the CAPM assumption that expected returns are proportionate to market risk (beta).
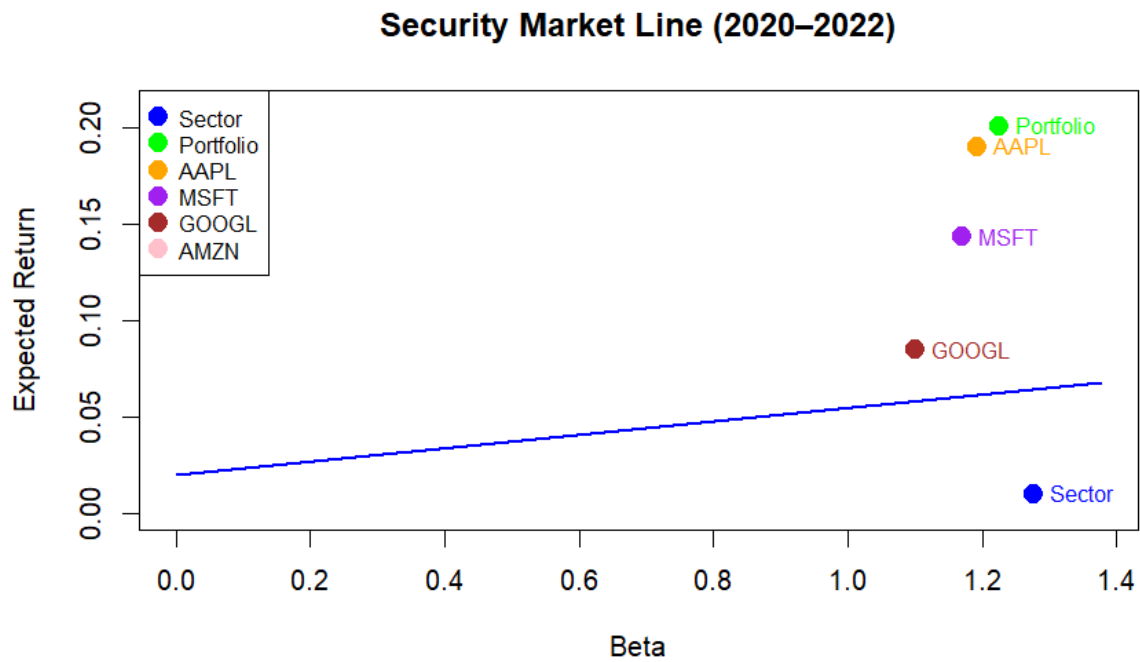
Figure 8: Security Market Line and Assets (2020–2022)

6.4 Hypothesis Testing Results

- **Insignificant Alphas:** Hypothesis tests for alpha values indicate that most alphas are statistically insignificant, as evidenced by high p-values. This supports the CAPM prediction that there are no significant abnormal returns unexplained by the model.

6.5 Key Insights

- **Portfolio Efficiency:** The Tangency Portfolio achieved slightly higher returns than predicted by CAPM. This aligns with its optimization for maximum risk-adjusted returns, though the deviation from CAPM predictions is minor.

- **Sector Behavior:** The Sector's returns closely follow the SML, indicating strong adherence to CAPM and market-driven returns.

- **Individual Stocks:** Stocks such as GOOGL and AMZN show close adherence to the SML, confirming the model's validity in explaining their returns.

6.6 Conclusion

- **Improved CAPM Validity:** During the 2020–2022 period, CAPM predictions better aligned with actual returns, as evidenced by the insignificant alphas for most assets, including the Tangency Portfolio.

- **Reduced Market Sensitivity:** Betas for most assets declined in 2020–2022 compared to 2017–2019, likely due to the unique dynamics introduced by the pandemic.

- **Sector and Stable Assets:** The sector, along with GOOGL and AMZN, consistently aligned with CAPM predictions across both periods, highlighting the model's validity for market-like assets.

- **High-Performing Assets:** The Tangency Portfolio, AAPL, and MSFT, which achieved abnormal returns in 2017–2019, failed to exhibit significant alphas in 2020–2022. This reflects changing market conditions and reduced opportunities for excess returns.
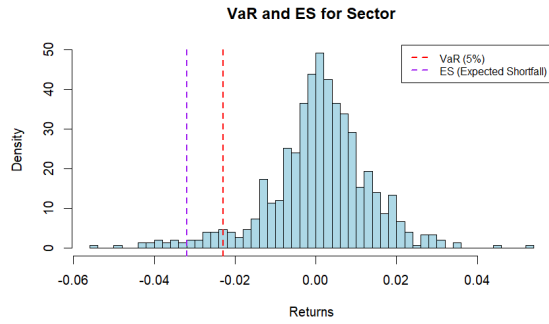
7 Non-Parametric VaR and ES (2017–2019)

Table 14: Daily VaR and ES Estimates at 5% Significance Level (2017–2019)

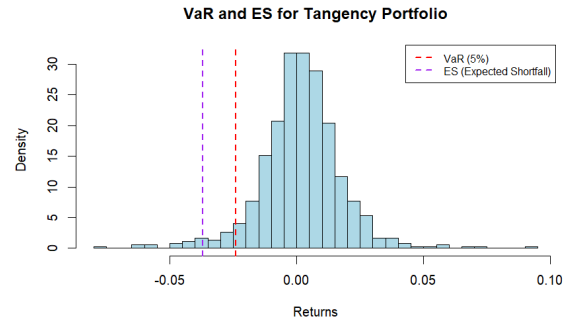| Asset	VaR	ES
Sector	-0.023	-0.032
Tangency Portfolio	-0.024	-0.037
AAPL	-0.024	-0.037
MSFT	-0.021	-0.033
GOOGL	-0.025	-0.036
AMZN	-0.027	-0.042

The table above reports the Value at Risk (VaR) and Expected Shortfall (ES) for the Technology Sector, Tangency Portfolio, and the individual stocks (AAPL, MSFT, GOOGL, and AMZN) during the 2017–2019 period. These measures were computed using the non-parametric method at a 5% significance level, with the results offering key insights into the risk profiles of the analyzed assets.

- **Tangency Portfolio:** The Tangency Portfolio demonstrates a VaR of -0.024 and an ES of -0.037, indicating that under extreme market conditions, the portfolio is expected to lose at least 2.4% of its value 5% of the time, while the average loss in these extreme scenarios is 3.7%. This aligns with the diversification benefit inherent in the Tangency Portfolio, as its risk is slightly higher than the overall sector but lower than the individual stocks like AMZN.

- **Amazon and Google:** AMZN exhibits the highest risk among the assets, with a VaR of -0.027 and an ES of -0.042. This reflects its increased sensitivity to market volatility and potential for larger losses in adverse conditions, consistent with its high beta observed in the CAPM analysis. Similarly, GOOGL also displays elevated risk metrics with a VaR of -0.025 and an ES of -0.036, which may be attributed to its individual volatility during this period.

- **Apple and Microsoft:** AAPL and MSFT exhibit relatively lower risk levels compared to AMZN and GOOGL, with MSFT demonstrating the lowest VaR and ES at -0.021 and -0.033, respectively. These metrics are indicative of more stable performance and lower exposure to extreme market events during this period.

- **Sector:** The sector index itself shows a VaR of -0.023 and an ES of -0.032, highlighting the collective risk of the technology sector as a whole. The risk profile of the Tangency Portfolio is very close to that of the sector index, affirming the portfolio's role in closely tracking sector performance while delivering slightly better returns and risk-adjusted-performance.
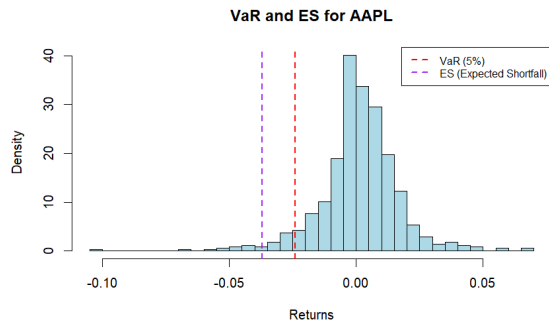
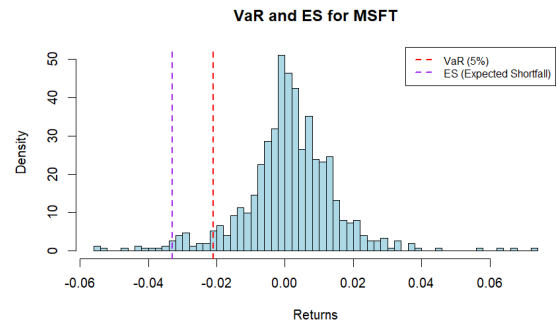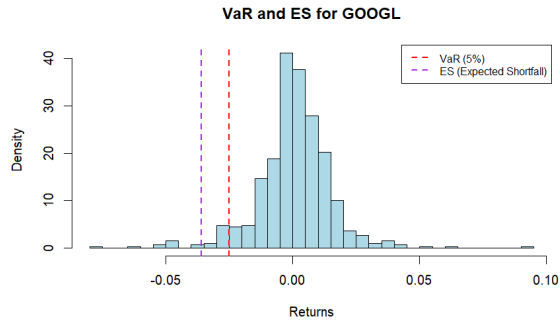## 7.1 VaR and ES Density vs Returns Plots (2017-2019)
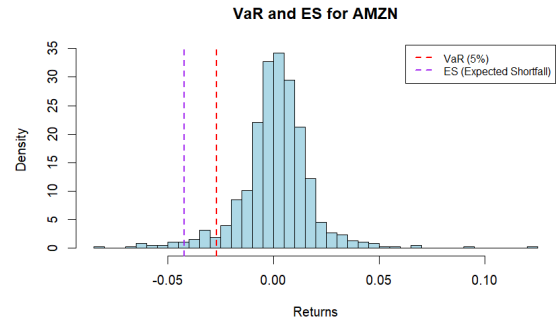

(a) Sector

(b) Tangency Portfolio

(c) Apple

(d) Microsoft

(e) Google

(f) Amazon

Figure 9: VaR and ES Density vs Returns Plots (2017-2019)

8 Non-Parametric VaR and ES (2020–2022)

Table 15: Daily VaR and ES Estimates at 5% Significance Level (2020–2022)

Asset	VaR (2020–2022)	ES (2020–2022)
Sector	-0.038	-0.054
Portfolio	-0.032	-0.049
AAPL	-0.035	-0.052
MSFT	-0.034	-0.051
GOOGL	-0.036	-0.051
AMZN	-0.038	-0.058

The table above reports the VaR and ES estimates for the 2020–2022 period, highlighting a notable increase in risk metrics compared to the 2017–2019 period. This comparison reveals significant changes in the financial risk landscape, attributed to the heightened market volatility during the COVID-19 pandemic.

8.1 Comparison with 2017–2019

The **Sector's VaR and ES** increased from -0.023 and -0.032 (2017–2019) to -0.038 and -0.054 (2020–2022). This reflects the amplified systematic risk in the technology sector due to the pandemic and economic uncertainty.

The **Tangency Portfolio's VaR and ES** also rose from -0.024 and -0.037 (2017–2019) to -0.032 and -0.049 (2020–2022). While the Tangency Portfolio remains diversified, it could not entirely shield against the elevated market-wide risks during this turbulent period.

Among individual stocks, **AMZN** exhibits the highest risk with a VaR of -0.038 and an ES of -0.058 in 2020–2022, compared to -0.027 and -0.042 in 2017–2019. This suggests that AMZN was particularly vulnerable to the volatile trading environment during the pandemic, possibly due to increased investor speculation and market corrections. Similarly, **AAPL, MSFT, and GOOGL** all show elevated VaR and ES values, indicating that even relatively stable technology giants faced higher tail risk during this period.

8.2 Drivers of Higher VaR and ES in 2020–2022

1. **COVID-19 Pandemic**: The pandemic introduced unprecedented market uncertainty, leading to large price swings across sectors, particularly in technology stocks that became the focus of speculative trading. Increased reliance on technology and e-commerce heightened expectations, leading to inflated valuations and subsequent corrections.

2. **Market Volatility**: VIX (Volatility Index) levels spiked during 2020, indicating significant investor uncertainty. This higher volatility directly contributed to increased VaR and ES values for both the sector and individual assets.

3. **Liquidity Concerns and Macro Shocks**: Liquidity issues, changing interest rates, and global supply chain disruptions further destabilized markets, impacting even diversified portfolios like the Tangency Portfolio.

4. **Sector-Specific Trends**: Technology stocks experienced rapid growth during the initial pandemic phase but faced corrections as market sentiment shifted in 2022, leading to higher tail risks.

8.3 Conclusion

The results highlight the increased sensitivity of the sector, individual stocks, and the Tangency Portfolio to adverse market movements during 2020–2022 compared to 2017–2019. While the Tangency Portfolio continued to offer diversification benefits, the systemic nature of risks during the pandemic meant that even diversified portfolios could not fully mitigate extreme losses. These findings emphasize the need for robust risk management strategies, particularly during periods of heightened market uncertainty.
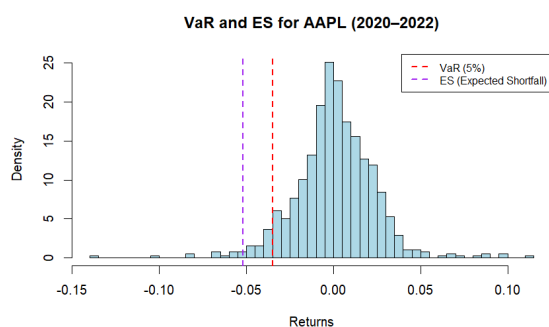
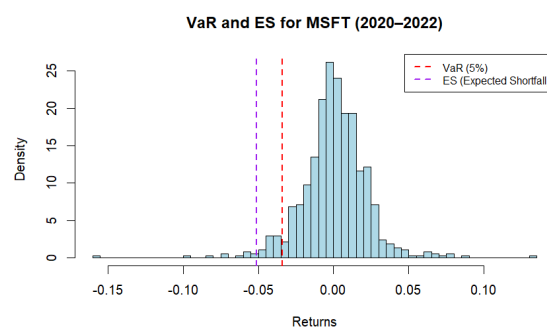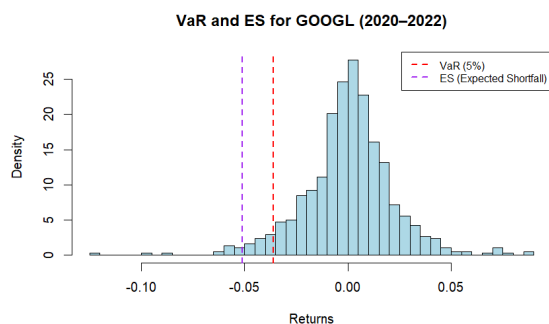## 8.4 VaR and ES Density vs Returns Plots (2020-2022)
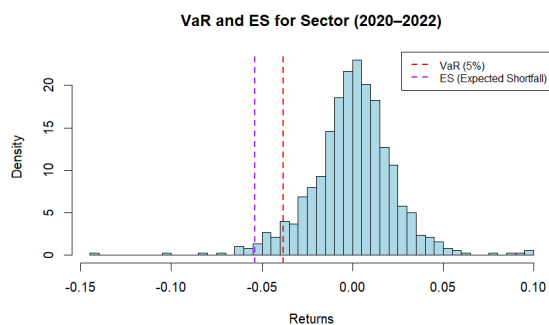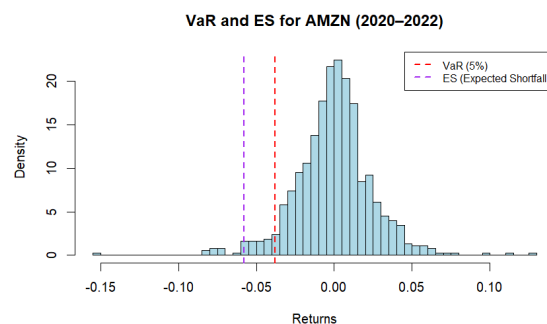

(a) Sector
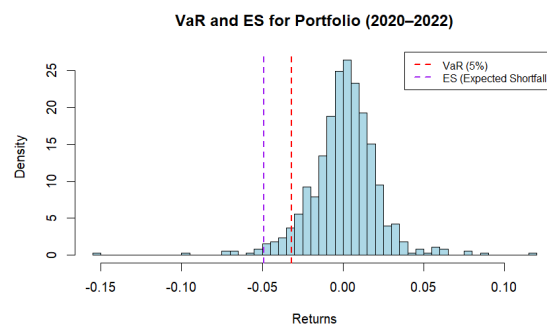
(b) Tangency Portfolio

(c) Apple

(d) Microsoft

(e) Google

(f) Amazon

Figure 10: VaR and ES Density vs Returns Plots (2020-2022)

9 Summary of the Report

9.1 Stock Selection:

To represent the technology sector during 2017–2022, we selected Amazon (AMZN), Microsoft (MSFT), Apple (AAPL), and Google (GOOGL, now Alphabet Inc.) due to their substantial market capitalization, influence in distinct areas of the tech industry, and stock price trends that closely mirrored broader sector movements. Their diversity in business models and consistent growth made them ideal representatives for analyzing the cumulative performance of the technology sector. Additionally, the NASDAQ-100 Technology Sector Index (NDXT) was chosen to represent the sector as a whole because of its comprehensive coverage of leading technology companies, making it a reliable benchmark for reflecting overall industry performance during this period.

9.2 Minimum Variance Portfolio:

The Minimum Variance Portfolio (MVP) primarily allocates to MSFT (50.2%) and GOOGL (30.8%), with a moderate allocation to AAPL (26.0%) and a small short position in AMZN (-7.1%). The portfolio demonstrates superior risk-adjusted performance, with a Sharpe Ratio of 1.29 compared to the sector's 0.956. Among the individual assets, MSFT offers the highest risk-return trade-off with a Sharpe Ratio of 1.419, while GOOGL shows the lowest at 0.648, indicating relatively weaker performance on a risk-adjusted basis. (2017-2019 Period)

9.3 Tangency Portfolio:

The Tangency Portfolio demonstrates a strategic allocation with heavy weights assigned to MSFT (119.06%) and AAPL (49.93%), while holding a short position in GOOGL (-73.78%) and a small positive allocation to AMZN. This reflects MSFT's superior risk-return trade-off, as indicated by its high Sharpe Ratio, and the relatively weaker risk-adjusted performance of GOOGL. The portfolio achieves an annualized Sharpe Ratio of 1.643, significantly outperforming both the Sector (0.956) and the Minimum Variance Portfolio (1.29). This superior Sharpe Ratio highlights the Tangency Portfolio's ability to deliver higher excess returns per unit of risk, optimizing the balance between risk and reward. (2017-2019 Period)

9.4 Comparison Between 2017-2019 and 2020-2022:

The Tangency Portfolio consistently demonstrated superior performance compared to the sector across both sample periods, as evident from its Sharpe Ratios. During the 2017–2019 period, the portfolio achieved a Sharpe Ratio of 1.644, significantly outperforming the sector's Sharpe Ratio of 0.957, indicating excellent risk-adjusted returns.

In the volatile 2020–2022 period, the Tangency Portfolio maintained its advantage, with a Sharpe Ratio of 0.453, despite a decline due to heightened market uncertainty caused by the COVID-19 pandemic. In contrast, the sector recorded a negative Sharpe Ratio of -0.026, reflecting its underperformance on a risk-adjusted basis. The strategic allocation of the Tangency Portfolio, particularly its reliance on high-performing stocks like MSFT and AAPL, enabled it to capitalize on growth opportunities while maintaining resilience in turbulent market conditions.

9.5 CAPM Analysis 2017-2019:

The CAPM analysis for the 2017–2019 period provides insightful findings regarding asset performance, market sensitivity, and the effectiveness of the CAPM model:

- **Alpha Values:** The Tangency Portfolio ($p = 0.007$) and MSFT ($p = 0.018$) exhibit statistically significant positive alpha values, indicating small but meaningful abnormal returns.

- **Beta Values:** All assets exhibit beta (β) values greater than 1, reflecting higher volatility compared to the market. MSFT demonstrates the highest beta ($\beta = 1.423$), showcasing its aggressive allocation to high-risk assets, while GOOGL has the lowest beta ($\beta = 1.321$), indicating lower market sensitivity.

- **P-Values and Hypothesis Testing:**

  - For most assets, beta is statistically significant ($p < 0.05$), confirming a strong relationship between asset returns and market returns.

  - Mixed results for alpha show that while the Tangency Portfolio and MSFT reject the null hypothesis (indicating significant abnormal returns), other assets (AAPL, GOOGL, AMZN) fail to reject it, implying no significant abnormal returns.

- **SML and CAPM Validity:**

  - The Security Market Line (SML) plot indicates that most assets outperform CAPM-predicted returns, highlighting CAPM's limitations in capturing returns for high-performing assets like MSFT and the Tangency Portfolio.

  - While CAPM is robust for explaining market portfolios, it underestimates returns for optimized portfolios and high-growth stocks, suggesting the need for multi-factor models (e.g., Fama-French Three-Factor Model) for better explanatory power.

9.6   CAPM Analysis 2020-2022:

The CAPM results for the 2020–2022 period provide insights into both alpha and beta values, revealing important observations about the performance and market dynamics of the portfolio and individual assets during the COVID-19 pandemic.

Alpha Analysis

The alpha (α) values for all assets, including the Tangency Portfolio, are close to zero, suggesting that there are no significant abnormal returns beyond what is explained by their beta (β). However:

- The Tangency Portfolio's alpha ($\alpha = 0.001$) is statistically insignificant ($p = 0.334$), indicating that its returns are largely explained by market movements and beta.

- Similarly, individual assets such as **AAPL, MSFT, GOOGL, and AMZN** have alpha values close to zero, with p-values exceeding 0.05, implying no significant abnormal returns.

Beta Analysis

The beta (β) values for all assets and the portfolio are consistently greater than 1, indicating higher volatility compared to the market. However, there is a noticeable decline in beta values compared to the 2017–2019 period:

- **Sector Beta:** Decreased from 1.363 to 1.277.

- **Portfolio Beta:** Dropped from 1.423 to 1.225, highlighting the impact of diversification.

- **Key Stocks:**

  - **AAPL:** Beta declined from 1.368 to 1.193.
  - **MSFT:** Beta fell from 1.380 to 1.170.
  - **GOOGL:** Beta dropped from 1.321 to 1.100.

This decline reflects reduced sensitivity to market-wide shocks, driven by company-specific factors, sector resilience, and unique market conditions during the pandemic.

Sector and Portfolio Performance

The Tangency Portfolio continues to outperform the sector in terms of Sharpe Ratio (0.453 vs. -0.026), despite heightened volatility and lower returns during the 2020–2022 period. The portfolio's allocation strategy, leveraging high-performing stocks like MSFT and AAPL, contributed significantly to its resilience and efficiency.

The CAPM results confirm the model's robustness in explaining asset and portfolio returns during the volatile COVID-19 period. The alignment of most assets with the Security Market Line (SML) reinforces CAPM's validity, while the slight deviations for the Tangency Portfolio and AAPL suggest areas for further exploration using multifactor models. The reduction in beta values, combined with insignificant alpha, underscores the stability and reduced exposure to market-wide shocks of the technology sector during this challenging period.

9.7 Analysis of VaR and ES Results

The analysis of Value at Risk (VaR) and Expected Shortfall (ES) provides crucial insights into the risk dynamics of the technology sector and the Tangency Portfolio during the periods 2017–2019 and 2020–2022.

Key Observations (2017–2019 vs. 2020–2022)

- **Sector-Level Risk:** The sector's VaR increased from -0.023 in 2017–2019 to -0.038 in 2020–2022, while ES rose from -0.032 to -0.054. These shifts indicate heightened systematic risk during the pandemic period, reflecting amplified market uncertainty and volatility.

- **Tangency Portfolio Risk:** The Tangency Portfolio, despite its diversification, experienced a rise in risk, with VaR moving from -0.024 to -0.032 and ES from -0.037 to -0.049. Although the portfolio demonstrated resilience, it could not completely shield against the increased market-wide risks during 2020–2022.

- **Individual Stock Risk:**

  - **AMZN:** Displayed the highest risk among the assets in 2020–2022, with a VaR of -0.038 and ES of -0.058, compared to -0.027 and -0.042, respectively, in 2017–2019. This heightened risk reflects AMZN's vulnerability to speculative trading and market corrections during the pandemic.
  - **Other Stocks:** Key assets such as AAPL, MSFT, and GOOGL also exhibited increased VaR and ES values, highlighting the pervasive impact of the pandemic on even relatively stable technology companies.

Drivers of Increased Risk in 2020–2022

- **COVID-19 Pandemic:** The pandemic brought unprecedented market turbulence, with technology stocks facing speculative trading and inflated valuations, leading to significant corrections and heightened risk.

- **Market Volatility:** The spike in the Volatility Index (VIX) during 2020 indicated significant investor uncertainty, directly contributing to elevated VaR and ES values for both the sector and individual stocks.

- **Liquidity Concerns and Macro Shocks:** Liquidity issues, changes in interest rates, and global supply chain disruptions further destabilized markets, even affecting diversified portfolios like the Tangency Portfolio.

- **Sector-Specific Trends:** Technology stocks benefited from increased digital adoption during the pandemic but faced corrections as market sentiment shifted in 2022, leading to higher tail risks.