a) $\lim_{x\to 1} \dfrac{x^3-1}{x^2-1}$

1-a) $\lim_{x\to 1} \dfrac{x^3-1}{x^2-1} = \dfrac{1-1}{1-1} = \dfrac{0}{0}$

$\lim_{x\to 1} \dfrac{\cancel{(x-1)}(x^2+x+1)}{(x+1)\cancel{(x-1)}}$

$\lim_{x\to 1} \dfrac{x^2+x+1}{x+1} = \dfrac{1+1+1}{1+1} = \dfrac{3}{2}$

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b) $\lim_{x\to 2} \dfrac{x^2-5x+6}{x^2-4x+4}$

1-b) $\lim_{x\to 2} \dfrac{x^2-5x+6}{x^2-4x+4} = \dfrac{4-10+6}{4-8+4} = \dfrac{0}{0}$

$\lim_{x\to 2} \dfrac{x^2-2x-3x+6}{x^2-2x-2x+4}$

$\lim_{x\to 2} \dfrac{x(x-2)-3(x-2)}{x(x-2)-2(x-2)}$

$\lim_{x\to 2} \dfrac{\cancel{(x-2)}(x-3)}{(x-2)^{\cancel{2}}}$

$\lim_{x\to 2} \dfrac{x-3}{x-2} = \dfrac{-1}{0} = \nexists$

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c) $\lim_{x\to 0} \dfrac{\sqrt{2-x}-\sqrt{2}}{x}$

1-c) $\lim_{x\to 0} \dfrac{\sqrt{2-x}-\sqrt{2}}{x} = \dfrac{0}{0}$

$\lim_{x\to 0} \dfrac{\sqrt{2-x}-\sqrt{2}}{x} \cdot \dfrac{\sqrt{2-x}+\sqrt{2}}{\sqrt{2-x}+\sqrt{2}}$

$\lim_{x\to 0} \dfrac{-\cancel{x}}{\cancel{x}(\sqrt{2-x}+\sqrt{2})}$

$\lim_{x\to 0} \dfrac{-1}{\sqrt{2-x}+\sqrt{2}} = \dfrac{-1}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{-\sqrt{2}}{2}$

d) $\lim_{h\to 1} \dfrac{\sqrt[3]{h}-1}{\sqrt[4]{h}-1}$

$h = u^{12}$

1-d) $\lim_{h\to 1} \dfrac{\sqrt[3]{h}-1}{\sqrt[4]{h}-1} = \dfrac{1-1}{1-1} = \dfrac{0}{0}$

$\sqrt[4]{h} = u^3$

$\sqrt[3]{h} = u^4$

$\lim_{u\to 1} \dfrac{u^4-1}{u^3-1}$

$\lim_{u\to 1} \dfrac{(u^2-1)(u^2+1)}{(u-1)(u^2+u+1)}$

$\lim_{u\to 1} \dfrac{\cancel{(u-1)}(u+1)(u^2+1)}{\cancel{(u-1)}(u^2+u+1)}$

$\lim_{u\to 1} \dfrac{(u+1)(u^2+1)}{u^2+u+1} = \dfrac{2\cdot 2}{3} = \dfrac{4}{3}$

Questão 3) (1,5 ponto) Calcule os limites.

a) $\lim_{x\to\infty} \dfrac{3x^3-x+1}{6x^3+2x^2-7}$ b) $\lim_{x\to-\infty} \dfrac{(3x+4)(x-1)}{(2x+7)(x+2)}$ c) $\lim_{x\to 0} \sqrt{x}.e^{sen(\frac{\pi}{x})}$

$\dfrac{\pi}{0} = \infty$

3-a) $\lim_{x\to\infty} \dfrac{3x^3-x+1}{6x^3+2x^2-7} = \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{\cancel{x^3}\left(3-\frac{1}{x^2}+\frac{1}{x^3}\right)}{\cancel{x^3}\left(6+\frac{2}{x}-\frac{7}{x^3}\right)}$

$\lim_{x\to\infty} \dfrac{3-\cancel{\frac{1}{x^2}}^{0}+\cancel{\frac{1}{x^3}}^{0}}{6+\cancel{\frac{2}{x}}^{0}-\cancel{\frac{7}{x^3}}^{0}} = \dfrac{3}{6} = \dfrac{1}{2}$

3-b) $\lim_{x\to-\infty} \dfrac{3x^2-3x+4x-4}{2x^2+4x+7x+14}$

$\lim_{x\to-\infty} \dfrac{\cancel{x^2}\left(3-\frac{3}{x}+\frac{4}{x}-\frac{4}{x^2}\right)}{\cancel{x^2}\left(2+\frac{4}{x}+\frac{7}{x}+\frac{14}{x^2}\right)}$

$\lim_{x\to-\infty} \dfrac{3-\cancel{\frac{3}{x}}^{0}+\cancel{\frac{4}{x}}^{0}-\cancel{\frac{4}{x^2}}^{0}}{2+\cancel{\frac{4}{x}}^{0}+\cancel{\frac{7}{x}}^{0}+\cancel{\frac{14}{x^2}}^{0}} = \dfrac{3}{2}$

3-c) $\lim_{x\to 0} \sqrt{x}\cdot e^{sen\left(\frac{\pi}{x}\right)}$

$sen\left(\frac{\pi}{x}\right)$ oscila entre 0 e 1.

$\lim_{x\to 0} \sqrt{x}\cdot e^1$ $\lim_{x\to 0} \sqrt{x}\cdot e^0$

$\lim_{x\to 0} \sqrt{x}\cdot e = 0$ $\lim_{x\to 0} \sqrt{x} = 0$

Questão 4) (2,0 ponto) Seja,

$$F(x) = \begin{cases} \sqrt{x^2 - 8x} - \sqrt{x^2} & \text{se} \quad x < -1 \\ x^3 + 2 & \text{se} \quad -1 \le x < 1 \\ 4 & \text{se} \quad x = 1 \\ 3x & \text{se} \quad 1 < x \le 2 \\ \frac{x^2-4}{2-x} + 10 & \text{se} \quad x > 2 \end{cases}$$

a) Calcule $\lim_{x \to -\infty} F(x)$

b) Calcule $\lim_{x \to +\infty} F(x)$.

c) F é contínua em -1 ?

d) F é contínua em 2?

4a) $\lim_{x \to -\infty} \sqrt{x^2-8x} - \cancel{\sqrt{x^2}}$

$\lim_{x \to -\infty} \sqrt{x^2-8x} - x = +\infty$

4-b) $\lim_{x \to +\infty} \frac{x^2-4}{2-x} + 10$

$\lim_{x \to +\infty} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} + 10$

$\lim_{x \to +\infty} -x-2+10 = -\infty$

4-c) Não.

4-d) Sim.

Questão 5) (1,5 pontos) Encontre, caso existam, as assíntotas verticais e as assíntotas horizontais ao gráfico da função f.

a) $B(x) = \frac{5x}{4-x^2}$

b) $S(x) = \frac{\sqrt{x^2+2}}{2x+1}$

5-a) $B(x) = \frac{5x}{4-x^2}$

$\lim_{x \to 2} \frac{5x}{4-x^2} = \frac{10}{0} = -\infty$

$\lim_{x \to -2} \frac{5x}{4-x^2} = \frac{-10}{0} = +\infty$

assíntotas vertical $= 2$ e -2

$\lim_{x \to \infty} \frac{5x}{4-x^2}$ $\quad \lim_{x \to \infty} \frac{5}{-x} = 0$ ← assíntota horizontal

$\lim_{x \to \infty} \frac{5}{\frac{4}{\cancel{x}} - x}$

5-b) $\lim_{x \to -\frac{1}{2}} \frac{\sqrt{x^2+2}}{2x+1}$ $\quad \frac{\sqrt{\frac{1}{4}+2}}{0} = -\infty$

$\frac{1}{2}$ é assíntota vertical

$\lim_{x \to \infty} \frac{\sqrt{x^2+2} \div x}{2x+1 \div x}$

$\lim_{x \to \infty} \frac{\sqrt{1+\cancel{\frac{2}{x^2}}}}{2+\cancel{\frac{1}{x}}} = \frac{1}{2}$ ou $-\frac{1}{2}$

assíntota horizontal