1. Modeling of carbon cycle

In this problem, we will build a box model to understand the Earth's carbon cycle based on the framework in Tomizuka 2009.

1.1 [15 points] Following equation 1-2 (without the buffer effect), build a two-box model to compute the atmospheric CO_2 level in ppm (parts per million) from 1987 to 2004.

1.2 [20 points] Following equation 3-4 (with the buffer effect), build a two-box model to compute the atmospheric CO_2 level in ppm from 1987 to 2004.

1.3 [5 points] Based on your results from **1.1** and **1.2**, reproduce Figure 2 in Tomizuka (2009) as much as you can.

[Bonus] [15 points] Following equation 5-13, compute the atmospheric CO_2 level in ppm and reproduce Figure 4 in Tomizuka (2009).

Solution:
CODE:
1.1

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from sklearn.linear_model import LinearRegression

# Load the emissions data
emissions_data = pd.read_csv('global_1751_2016.csv')

# Filter relevant columns
emissions_data = emissions_data[['Year', 'Total carbon emissions from fossil fuel consumption and cement production (million metric tons of C)',
                                  'Carbon emissions from cement production']]

# Calculate the net carbon emissions by subtracting emissions from cement production
emissions_data['NetCarbonEmissions'] = emissions_data['Total carbon emissions from fossil fuel consumption and cement production (million metric tons of C)'] - emissions_data['Carbon emissions from cement production']

# Convert net carbon emissions to γ values
emissions_data['Gamma'] = emissions_data['NetCarbonEmissions'] / (1000 * 2.13)

# Define parameters
k12 = 105 / 740
k21 = 102 / 900
```

```
# Time parameters
start_year = 1986    # Adjusted start year
end_year = 2004        # Adjusted end year
time_step = 1

# Initial conditions
N1 = 740 / 2.13
N2 = 900 / 2.13

# Time array
time = np.arange(start_year, end_year + time_step, time_step)

# Lists to store results
atmosphere = [N1]
ocean = [N2]


# Euler's method
for t in range(start_year, end_year , time_step):
        # Get γ value for the current year
        gamma = emissions_data.loc[emissions_data['Year'] == t, 'Gamma'].values[0]

        dN1_dt = -k12 * atmosphere[-1] + k21 * ocean[-1] + gamma
        dN2_dt = k12 * atmosphere[-1] - k21 * ocean[-1]

        N1_new = atmosphere[-1] + dN1_dt * time_step
        N2_new = ocean[-1] + dN2_dt * time_step

        atmosphere.append(N1_new)
        ocean.append(N2_new)

# Perform linear regression
X = np.array(time[1:]).reshape(-1, 1)
y = np.array(atmosphere[1:]).reshape(-1, 1)

regressor = LinearRegression()
regressor.fit(X, y)
line = regressor.predict(X)

# Plotting the results
plt.scatter(time, atmosphere, label='Atmosphere (N1)')
plt.plot(time[1:], line, color='red', linestyle='--', label='Linear Regression')
plt.xlabel('Year')
plt.ylabel('Concentration (ppm)')
```

```
plt.title('Atmospheric CO2 Concentration with Linear Regression (1987-2004)')
plt.legend()

# Set x-axis ticks with 5-year intervals
plt.xticks(np.arange(start_year, end_year + 1, 5))

plt.show()
```

1.2

```
# Define parameters
k12 = 105 / 740
k21 = 102 / 900
N2_0 = 821 / 2.13

# Time parameters
start_year = 1986   # Adjusted start year
end_year = 2004     # Adjusted end year
time_step = 1

# Initial conditions
N1 = 740 / 2.13
N2 = 900 / 2.13


# Time array
time = np.arange(start_year, end_year + time_step, time_step)

# Lists to store results
atmosphere_with_buffer = [N1]
ocean_with_buffer = [N2]


# Euler's method
for t in range(start_year, end_year , time_step):
    # Get γ value for the current year
    gamma = emissions_data.loc[emissions_data['Year'] == t, 'Gamma'].values[0]

    # Calculate ξ value
    z = atmosphere_with_buffer[-1]   # Assuming z = N1 (atmosphere concentration)
    xi = 3.69 + 0.0189 * z - 0.0000018 * z**2

    dN1_dt = -k12 * atmosphere_with_buffer[-1] + k21 * (N2_0 + xi *
(ocean_with_buffer[-1] - N2_0)) + gamma
    dN2_dt = k12 * atmosphere_with_buffer[-1] - k21 * (N2_0 + xi *
```

```
(ocean_with_buffer[-1] - N2_0))

    N1_new = atmosphere_with_buffer[-1] + dN1_dt * time_step
    N2_new = ocean_with_buffer[-1] + dN2_dt * time_step

    atmosphere_with_buffer.append(N1_new)
    ocean_with_buffer.append(N2_new)

# Perform linear regression
X = np.array(time[1:]).reshape(-1, 1)
y = np.array(atmosphere_with_buffer[1:]).reshape(-1, 1)

regressor = LinearRegression()
regressor.fit(X, y)
line_with_buffer = regressor.predict(X)

# Plotting the results
plt.scatter(time, atmosphere_with_buffer, label='Atmosphere (N1)')
plt.plot(time[1:], line_with_buffer, color='red', linestyle='--', label='Linear Regression')
plt.xlabel('Year')
plt.ylabel('Concentration (ppm)')
plt.title('Atmospheric CO2 Concentration with Linear Regression (1987-2004)')
plt.legend()

# Set x-axis ticks with 5-year intervals
plt.xticks(np.arange(start_year, end_year + 1, 5))

plt.show()
```

1.3

```
# Load the emissions data
observations = pd.read_csv('co2_annmean_mlo.csv')

# Filter data for the years 1986-2004
filtered_data = observations[(observations['year'] >= 1986) & (observations['year'] <=
2004)]

# Plotting the scatter plot
plt.scatter(filtered_data['year'], filtered_data['mean'], label="observations")
plt.plot(time[1:], line, color='red', linestyle='-', label='Linear Regression')
plt.plot(time[1:], line_with_buffer, color='blue', linestyle='-', label='Linear Regression')
plt.xlabel('Year')
plt.ylabel('CO2 Concentration (ppm)')
plt.legend()
```

```
# Set x-axis ticks with 5-year intervals
plt.xticks(np.arange(start_year, end_year + 1, 5))
plt.ylim(340, 430)
plt.show()
```

Bouns

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

# Load the emissions data
observations = pd.read_csv('1750-2000CO2.csv')

# Load the emissions data
land_use = pd.read_excel('Global_land-use_flux-1750_2005.xls')
emissions_data = pd.read_csv('global_1751_2016.csv')

# Extract relevant columns
land_use_data = land_use[['Year', 'Global']]
land_use_data['Delta'] = land_use_data['Global']/(1000*2.13)

# Filter relevant columns
emissions_data = emissions_data[['Year', 'Total carbon emissions from fossil fuel consumption and cement production (million metric tons of C)',
                                 'Carbon emissions from cement production']]

# Calculate the net carbon emissions by subtracting emissions from cement production
emissions_data['NetCarbonEmissions'] = emissions_data['Total carbon emissions from fossil fuel consumption and cement production (million metric tons of C)'] - emissions_data['Carbon emissions from cement production']

# Convert net carbon emissions to γ values
emissions_data['Gamma'] = emissions_data['NetCarbonEmissions'] / (1000 * 2.13)

# Define parameters
k12 = 60 / 615
k21 = 60 / 842
k23 = 9 / 842
k24 = 43 / 842
k32 = 52 / 9744
k34 = 162 / 9744
k43 = 205 / 26280
k45 = 0.2 / 26280
```

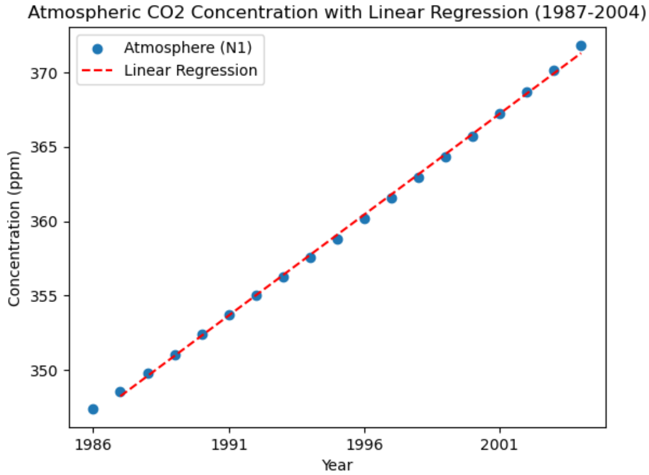
```
k51 = 0.2 / 90000000
k67 = 62 / 731
k71 = 62 / 1238
N2_0 = 842 / 2.13

# Time parameters
start_year = 1750    # Adjusted start year
end_year = 2000        # Adjusted end year
time_step = 1

# Initial conditions
N1 = 615 / 2.13
N2 = 842 / 2.13
N3 = 9744 / 2.13
N4 = 26280 / 2.13
N5 = 90000000 / 2.13
N6 = 731 / 2.13
N7 = 1238 / 2.13

f0 = 62 / 2.13
P = atmosphere    # Replace with the appropriate variable
P0 = 615 / 2.13

# Beta values to explore
beta_values = [0.38, 0.5]

# Lists to store results for each beta
atmosphere_results = []

# Loop over beta values
for beta in beta_values:
    atmosphere = [N1]
    surface_ocean = [N2]
    intermediate_ocean = [N3]
    deep_ocean = [N4]
    sediments = [N5]
    biosphere = [N6]
    soil = [N7]
    for t in range(start_year, end_year , time_step):
        f = [f0 * (1 + beta * np.log(atmosphere[-1] / P0))]
        gamma = emissions_data.loc[emissions_data['Year'] == t, 'Gamma'].values[0]
        delta = land_use_data.loc[land_use_data['Year'] == t, 'Delta'].values[0]

        # Calculate ξ value
```

```
        z = atmosphere[-1]    # Assuming z = N2 (ocean concentration)
        xi = 3.69 + 0.0186 * z - 0.0000018 * z**2

        # Update additional parameters for the formula f=f0(1+beta*ln（P/P0））
        P = atmosphere[-1]    # Replace with the appropriate variable

        dN1_dt = -k12 * atmosphere[-1] + k21 * (N2_0 + xi * (surface_ocean[-1] - N2_0)) + gamma - f[-1] + delta + k51 * sediments[-1] + k71 * soil[-1]
        dN2_dt = k12 * atmosphere[-1] - k21 * (N2_0 + xi * (surface_ocean[-1] - N2_0)) - k23 * surface_ocean[-1] + k32 * intermediate_ocean[-1] - k24 * surface_ocean[-1]
        dN3_dt = k23 * surface_ocean[-1] - k32 * intermediate_ocean[-1] - k34 * intermediate_ocean[-1] + k43 * deep_ocean[-1]
        dN4_dt = k34 * intermediate_ocean[-1] - k43 * deep_ocean[-1] + k24 * surface_ocean[-1] - k45 * deep_ocean[-1]
        dN5_dt = k45 * deep_ocean[-1] - k51 * sediments[-1]
        dN6_dt = f[-1] - k67 * biosphere[-1] - 2 * delta
        dN7_dt = k67 * biosphere[-1] - k71 * soil[-1] + delta

        N1_new = atmosphere[-1] + dN1_dt * time_step
        N2_new = surface_ocean[-1] + dN2_dt * time_step
        N3_new = intermediate_ocean[-1] + dN3_dt * time_step
        N4_new = deep_ocean[-1] + dN4_dt * time_step
        N5_new = sediments[-1] + dN5_dt * time_step
        N6_new = biosphere[-1] + dN6_dt * time_step
        N7_new = soil[-1] + dN7_dt * time_step

        atmosphere.append(N1_new)
        surface_ocean.append(N2_new)
        intermediate_ocean.append(N3_new)
        deep_ocean.append(N4_new)
        sediments.append(N5_new)
        biosphere.append(N6_new)
        soil.append(N7_new)

    # Append atmospheric concentrations for the current beta
    atmosphere_results.append(atmosphere)

# Plotting the results
plt.figure(figsize=(12, 8))

# Plotting the scatter plot
plt.scatter(observations['year'], observations['mean'])
```

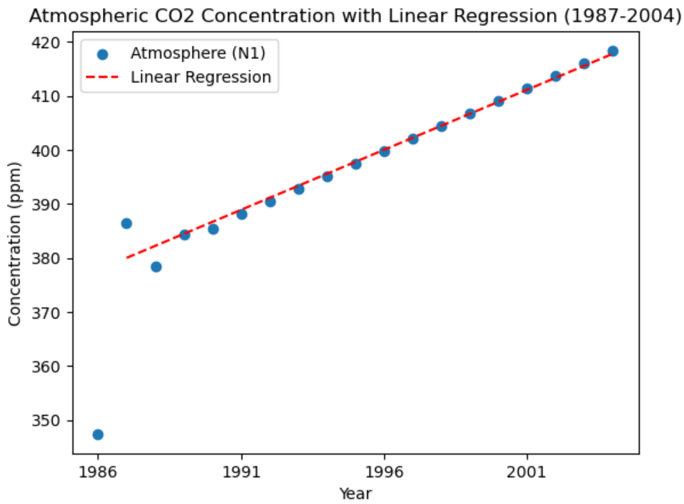
```
plt.text(1850, 300, 'calcluations', fontsize=12, fontweight='bold')
plt.text(1900, 290, 'observations', fontsize=12, fontweight='bold')
plt.text(1950, 345, 'β=0.38', fontsize=12, fontweight='bold',color='red')
plt.text(1980, 320, 'β=0.50', fontsize=12, fontweight='bold',color='blue')

plt.plot(range(start_year, end_year + time_step, time_step), atmosphere_results[0],color='red')
plt.plot(range(start_year, end_year + time_step, time_step), atmosphere_results[1],color='blue')

plt.xlabel('Year', fontsize=16)
plt.ylabel('CO2 Concentration (ppm)', fontsize=16)
plt.ylim(260,380)
plt.legend()
plt.show()
```

RESULT

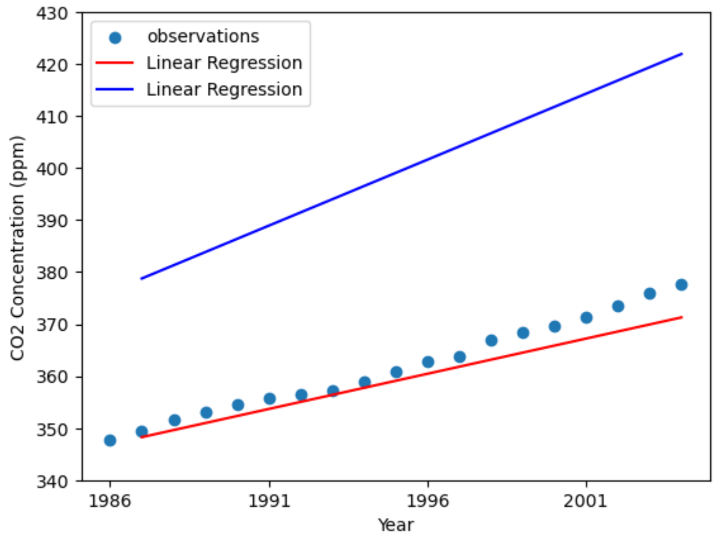
1.1



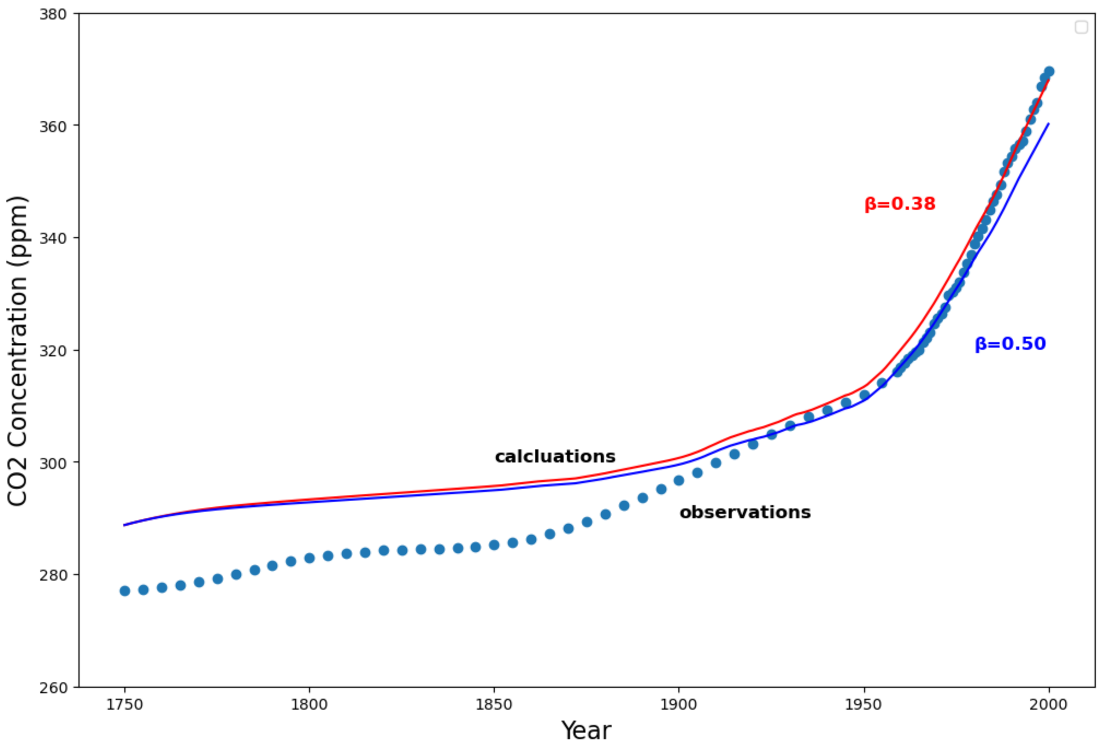
1.2



1.3



Bonus



Problem-solving ideas

1.Understanding the Code:

Familiarize yourself with the code and its components. Understand the purpose of each section, such as data loading, parameter definition, Euler's method implementation, linear regression, and data visualization.

2.Box model Method:

Ensure a clear understanding of the Euler's method implementation. Verify the correctness of the differential equations used to model the changes in atmospheric CO2 concentration.

3.Parameter Definitions:
Check the accuracy of the parameter values (e.g., k12, k21, N2_0) used in the model. Ensure they align with the physical interpretation of the problem.
4.Linear Regression:
Review the linear regression analysis. Understand how it is applied to the modeled data and observational data. Check if the linear regression assumptions are met.
5.Buffer Effect:
Understand the purpose of introducing a buffer effect in the model. Verify the correctness of the equations related to the buffer effect.
6.Observational Data:
Understand how observational data is loaded and filtered. Verify the consistency of the observational data with the modeled data.