Teoría de Autómatas y Lenguajes Formales

Práctica 3: Turing Machine, recursive functions and WHILE language

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1 Define the TM solution of exercise 3.4 of the problem list and test its correct behaviour

Ejemplo 1.1. Máquina de Turing en JFLAP

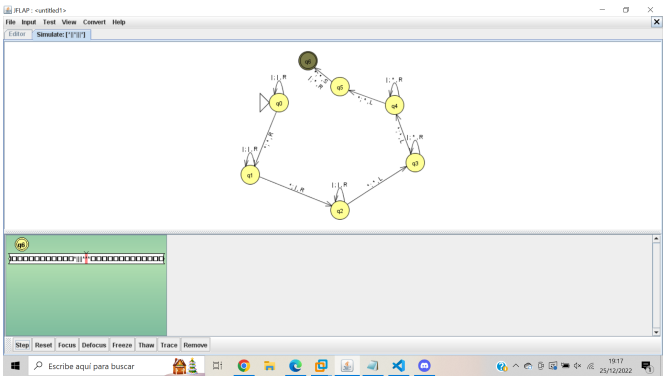

Figure 1: Máquina de Turing

2 Define a recursive function for the sum of three values

Definición 2.1. Composition composición de funciones
Let $m > 0, k \geq 0$ and the functions:
$g : N^m \to N$
$h_1, \ldots, h_m : N^k \to N$
If the function $f : N^k \to N$ is

$$f(\vec{n}) = g(h_1(\vec{n}), \ldots, h_m(\vec{n}))$$

then f is a composition of g and $h_1, \ldots, h_m$, expressed as $f(\vec{n}) = g(h_1, \ldots, h_m)(\vec{n})$, or simply $f = g(h_1, \ldots, h_m)$.

Definición 2.2. Primitive recursion recursión primitiva
Let $k \geq 0$ and the functions
$g : N^k \to N$
$h : N^{k+2} \to N$
If the function $f : N^{k+1} \to N$ is

$$f(\vec{n}, m) = \begin{cases} g(\vec{n}) & \text{if} \quad m = 0 \\ h(\vec{n}, m-1, f(\vec{n}, m-1)) & \text{if} \quad m > 0 \end{cases}$$

then f is obtained from g and h by primitive recursion.
We will express it as $f(\vec{n}) = gh(\vec{n})$, or simply $f = gh$.

Ejemplo 2.1. $add3 = \langle\langle \pi_1^1 | successor_3 \rangle | successor_4 \rangle$

```
addition3(1,2,3)
<<π¹₁|σ(π³₃)>|σ(π⁴₄)>(1,2,3)
<<π¹₁|σ(π³₃)>|σ(π⁴₄)>(1,2,1)
<<π¹₁|σ(π³₃)>|σ(π⁴₄)>(1,2,1)
<<π¹₁|σ(π³₃)>|σ(π⁴₄)>(1,2,0)
<π¹₁|σ(π³₃)>(1,2)
<π¹₁|σ(π³₃)>(1,1)
π¹₁(1) = 1
π¹₁(1) = 1
σ(π³₃)(1,0,1)
π³₃(1,0,1) = 1

σ(1) = 2
σ(π³₃)(1,1,2)
π³₃(1,1,2) = 2

σ(2) = 3
σ(π⁴₄)(1,2,0,3)
π⁴₄(1,2,0,3) = 3

σ(3) = 4
σ(π⁴₄)(1,2,1,4)
π⁴₄(1,2,1,4) = 4

σ(4) = 5
σ(π⁴₄)(1,2,2,5)
π⁴₄(1,2,2,5) = 5

σ(5) = 6
ans = 6
```

Figure 2: Suma de tres números

Ejemplo 2.2. Función recursiva en Octave

3 Implement a WHILE program that computes the sum of three values, you must use an auxiliary variable that accumulates the result of the sum

$X_4 := X_1;$
while $X_2 \neq 0$ **do**
    $X_4 := X_4 + 1;$
    $X_2 := X_2 - 1;$
od
while $X_3 \neq 0$ **do**
    $X_4 := X_4 + 1;$
    $X_3 := X_3 - 1;$
od
$X_1 := X_4;$