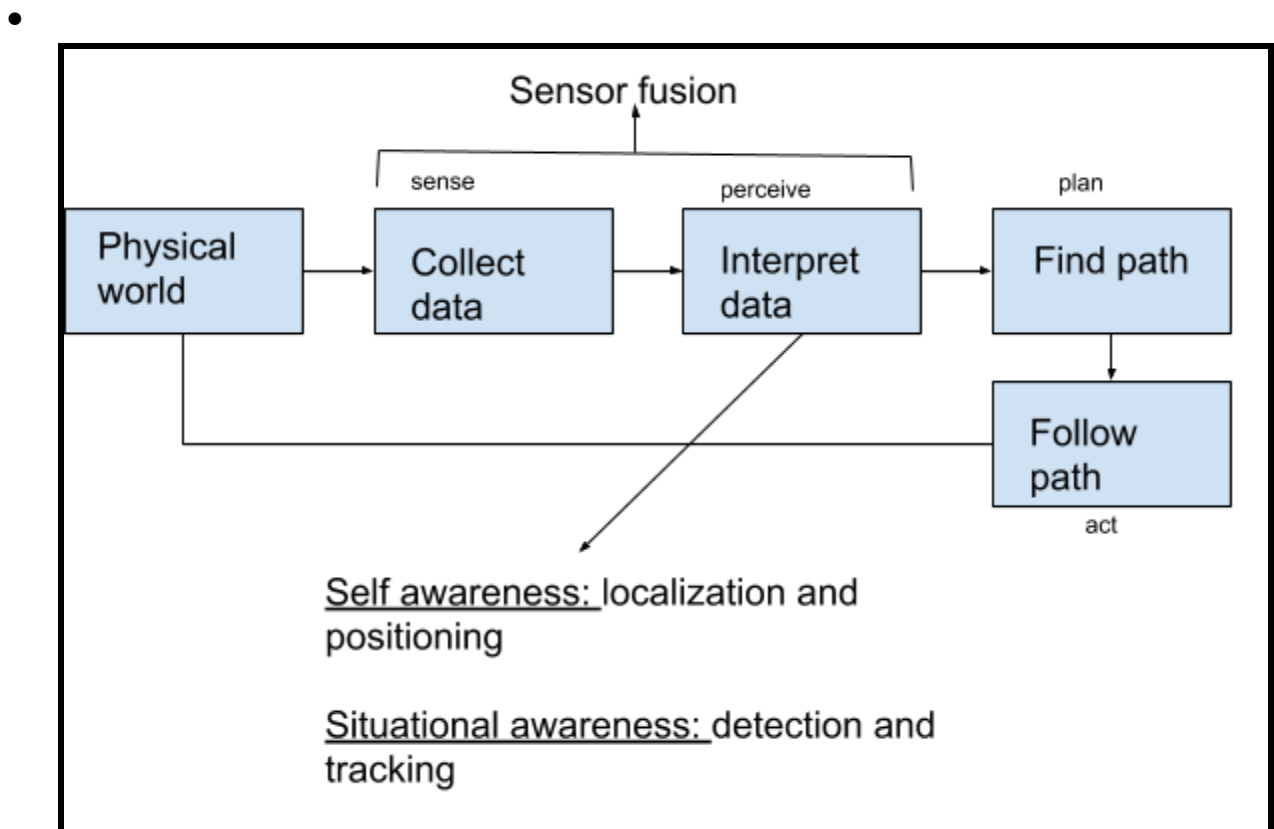
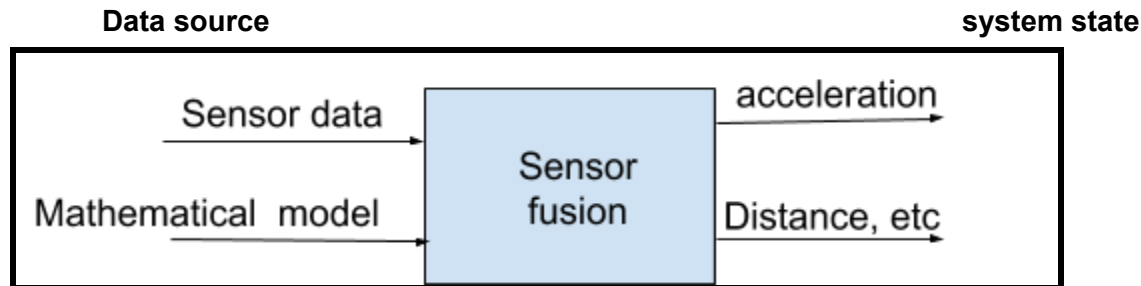


# Sensor fusion

- Broad definition: Combining two or more data sources that generate a better understanding of the system. (more consistent, more accurate, more dependable)



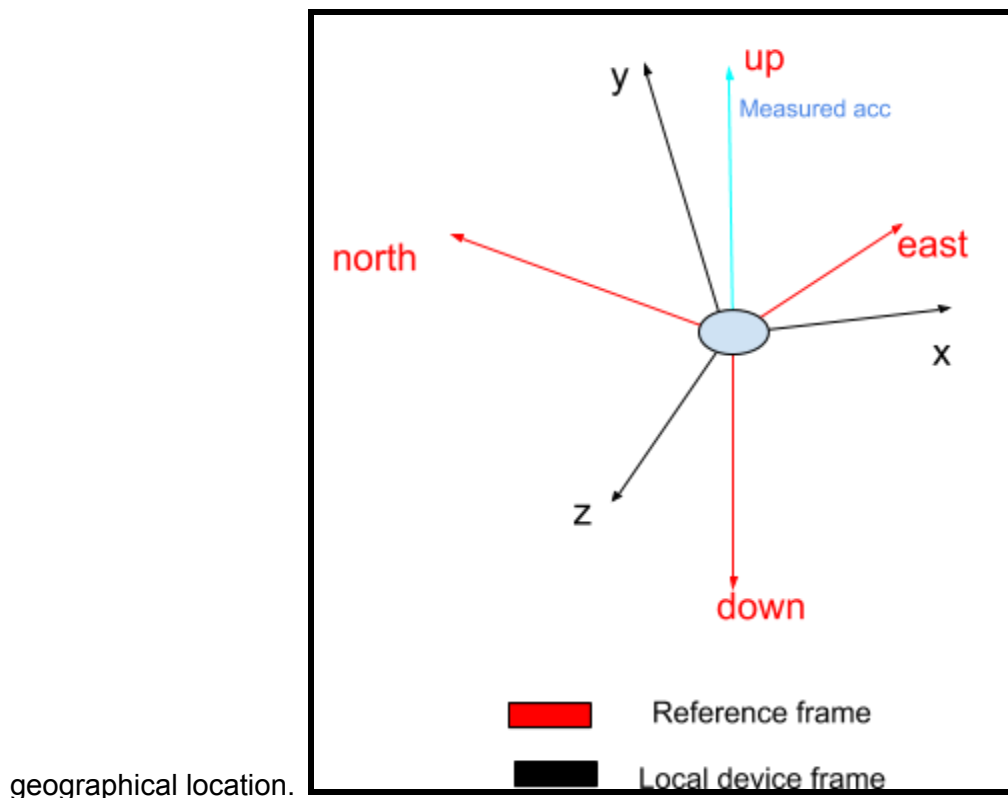
- Sensors are needed to increase the quality of data. For accurate data, a fusion of many sensors can average out the random noise generated, if the noise is uncorrelated.  

$$\text{Noise reduction} = \sqrt{\text{number of sensors}}$$
- Averaging functions don't work on correlated data. For such cases fusion with different sensors is preferred to reduce the noise. For example, a magnetometer reading can be fused with a gyroscope reading to filter out the noise by electronics. Kalman filters are common fusion algorithms that help to fuse different sensor readings

## Sensor fusion to estimate the orientation

- Orientation -----> attitude/heading. Thus fusion algorithm can be referred to as Attitude and Heading Reference System (AHRS)
- Orientation is how far the object has been rotated wrt some reference frame. To define orientation, we need
  1. Reference frame
  2. Specifying of rotation

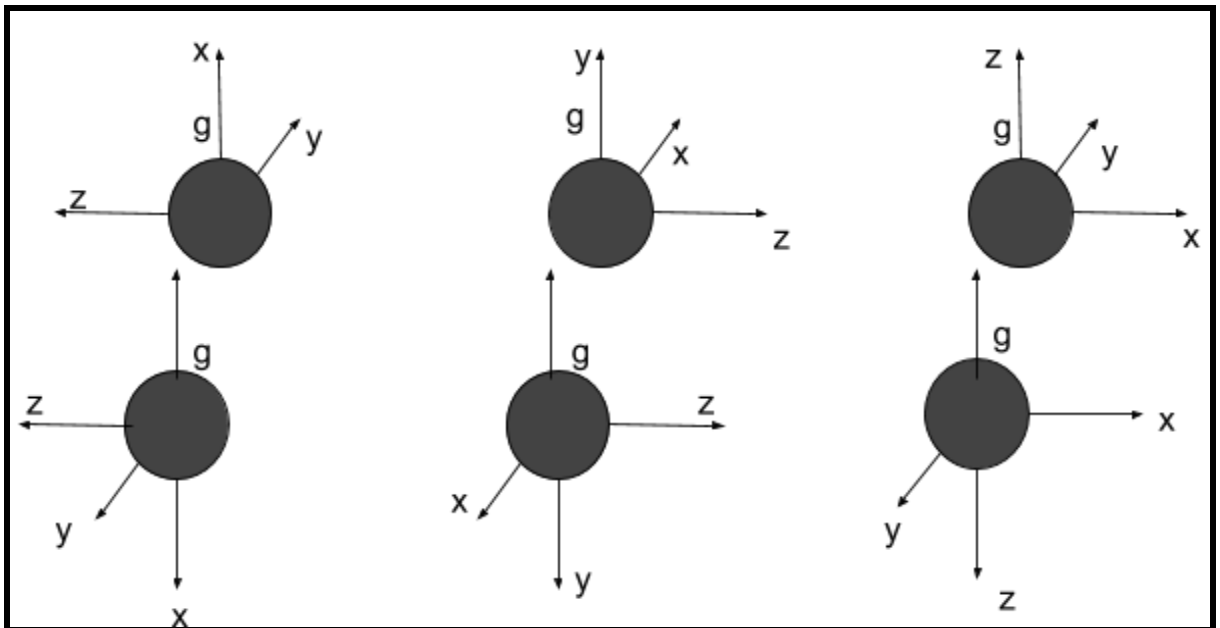
For example, roll, pitch, yaw(reference: local horizon, rotation: angular deviation)  
Rotation can also be defined by direction cosine matrices (DCM) or quaternion. These Two represent a 3-dimensional rotation between two coordinate frames.
- We can find an absolute orientation of a device at rest using an accelerometer, magnetometer, and a gyroscope using the device's body coordinate frame relative to the local north, east, and down coordinate.  
When at rest, the accelerometer measures the device's acceleration due to gravity, and the direction of this acceleration is 'up'. While magnetometer measures the direction of the magnetic north, which can move up or down of the north-east plane due to



- To correct for this deflection, we need to apply cross-products. Since the sensor returns directional vectors, the cross product of the down direction and the measured magnetic

field is east. Similarly, the north is the cross product of the east and down. Direction cosines can be built from the north, east, and down vectors.

- Accelerometers measure acc in all directions so that when a system is moving the down direction gets an offset with the motion. Also if the accelerometers are not located at the COG of the system any rotational motion will change the linear acceleration vectors; even if the system has none and is simply rotating along its axis. To solve this, we provide a threshold for the max and min values in terms of g that are not to be exceeded, else the device ignores it. Or add the gyroscope to the combination of system, incorporating the dead reckoning essentially making it an IMU.
- Calibration: let  $\max_x$ ,  $\max_y$ ,  $\max_z$ , and  $\min_x$ ,  $\min_y$ ,  $\min_z$  be the maximum and minimum magnitudes of vector recorded by the accelerometer. For a device at rest, it can have 6 possible attitudes



Once that we have all 6 values, we have to do some basic calculations to get the offsets and scales. Accelerometer calibration is done using these two values (per axis).

For offsets:

$$offset_x = \frac{\min_x + \max_x}{2}$$

$$offset_y = \frac{\min_y + \max_y}{2}$$

$$offset_z = \frac{\min_z + \max_z}{2}$$

For scales,

$$scale_x = \frac{1}{max_x - offset_x}$$

$$scale_y = \frac{1}{max_y - offset_y}$$

$$scale_z = \frac{1}{max_z - offset_z}$$

Once we have the offsets and scales, we have to just apply them to our accelerometer measurement. This just squeezes or expands the measurements and adds an offset.

$$calibrated_x = (acc_x - offset_x)scale_x$$

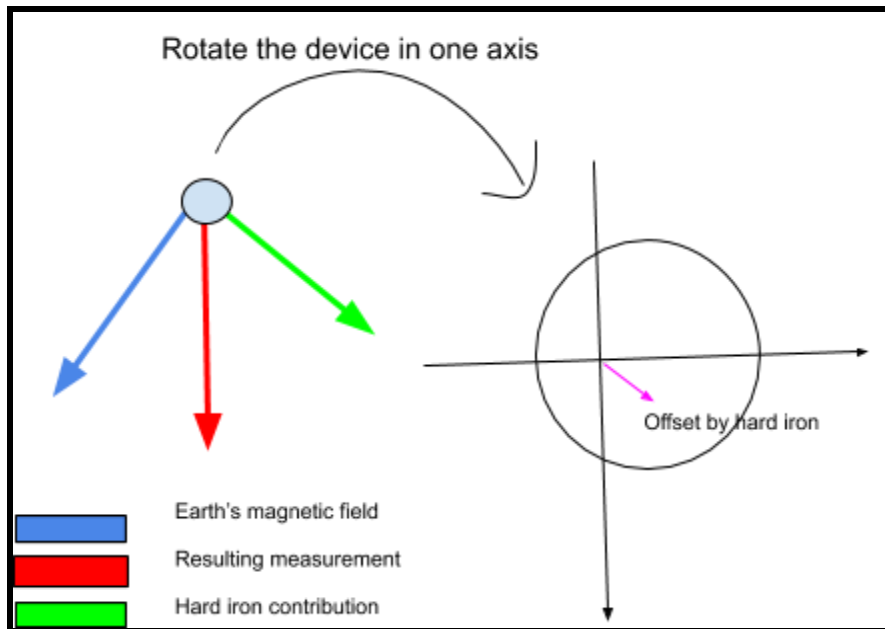
$$calibrated_y = (acc_y - offset_y)scale_y$$

$$calibrated_z = (acc_z - offset_z)scale_z$$

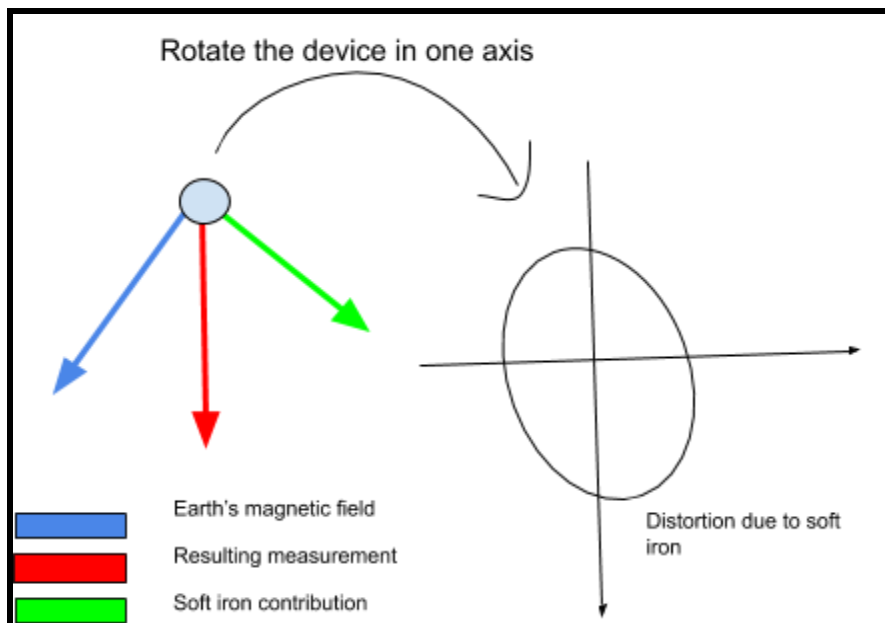
- Magnetometers are affected by disturbances in the magnetic field. If this magnetic field is part of the system, and rotates with the magnetometer then it can be calibrated out.

These disturbances are caused by two kinds of hardware; the hard iron source and the soft iron source.

Hard iron sources are stuff that generates their own magnetic field like permanent magnets and device circuitry. When measuring the earth's magnetic field, the hard iron magnetic field would contribute to the resulting measurement and when rotating the system in a single axis while still measuring the field, the result would be a circle that is offset from the origin.



Soft iron sources do not have their own magnetic fields but interact with external fields to create disturbances.



If the device is rotated in all of the 4 pi radian directions without any of these sources, then it would be creating a perfect sphere with the radius as the magnitude of the field. When distorted and shifted by soft and hard sources, the resulting shape is an irregular spheroid. To correct the spheroid back to sphere, so as to calibrate it, we need to transform the received data matrix with the hard and soft iron bias.

$$X_{\text{corrected}} = (X.b).A$$

Where  $b = (3 \times 1)$  vector, hard iron bias

$A = (3 \times 3)$  matrix, soft iron bias

For the soft iron core, the equation of an ellipsoid is

$$Ax^2 + By^2 + Cz^2 + 2Dxy + 2Exz + 2Fyz + 2Gx + 2Hy + 2Iz$$

Here A, B, C, D, E, F, G, H, and I are ellipsoid parameters

With the following expression we can do this transformation. Note that  $c$  is the center of the transformation and  $e_{xy}$  is the XY element of the transformation matrix.

$Mag_x$ ,  $mag_y$ , and  $mag_z$  are magnetic field vectors from sensors.

$$\begin{pmatrix} mag_x \\ max_y \\ max_z \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{pmatrix} \begin{pmatrix} mag_x - c_x \\ max_y - c_y \\ mag_z - c_z \end{pmatrix}$$

<https://www.youtube.com/watch?v=0rlvvYgmTvl>

<https://www.youtube.com/watch?v=T9jXoG0QYIA>

<https://github.com/alrevuelta/sensor-calibration/blob/master/README.md>

<https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.110.5134&rep=rep1&type=pdf>

<https://learn.adafruit.com/adafruit-analog-accelerometer-breakouts/circuitpython-code>

[https://x-io.co.uk/res/doc/madgwick\\_internal\\_report.pdf](https://x-io.co.uk/res/doc/madgwick_internal_report.pdf)