A Review of the Literature Concerning the Use of Metaphors, Metonymies, and Prototypes to Assess Students' Conceptual Understandings

Allison Theobold

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# Abstract

# Introduction

Research in students usage of prototypes, metaphors, and metonymies was largely influential during the years 1990 to 2000, with subsequent declines since. During the early parts of the 1990's foundational works in students' usage of prototypes, metaphors, and metonymies were published by Presmeg, which are still the building blocks of research studies today. The purpose of this literature review is to understand the foundational concepts behind individual's usage of prototypes, metaphors, and metonymies and how these understandings have been and can be applied to assess students' conceptual understandings.

When learning mathematics students employ their imaginations in reasoning through concepts and tasks. These imageries are far more than peripheral, instead playing a central role in how students reason through mathematics. With the central role these imageries play, implications can be found for the advancement of teaching and learning mathematics. The imageries can inform classroom assessment in a variety of ways. First, in assessing students' conceptual understanding educators can elicit students' metaphors or metonymies regarding the concept at hand. Through understanding how students construct these imageries we can better create classroom conversations and teaching methods that can prevent students constructing unhelpful imageries. Following these preliminary assessments and classroom conversations, secondary assessments of students' conceptual understandings could be used to verify student imagery of mathematical concepts.

In the following section we will discuss the foundational literatures that have informed this study. First, we review the foundational literature on individual's usage of prototypes, metaphors, and metonymies. We then discuss how these imageries can be utilized to assess students' conceptual understandings, using illustrations from practitioner research.

# Review of Literature

## Influence of Imagery on Mathematical Reasoning and Cognition

This widely cited article by Presmeg describes high school case studies used to understand students' uses of prototypes, metaphors, and metonymies in mathematical reasoning. Prototypes are described as mental representations of categories, which can be helpful or unhelpful, and can be used as the basis for metaphors and metonymies.

## Types of Imageries Employed by Students

* Metaphor
* Metonymy
* Prototype

## Using Imagery to Assess Student Understanding

Presmeg: The authors present an example, in which it is common for students (and individuals) to have a prototypical image of a triangle, which can be beneficial in certain settings and unhelpful in others. These prototypes can be used in metaphorical ways "to guide the reasoning process of visualizers," as well as metonymic ways where "some category or member or submodel is used to comprehend the category as a whole" (Presmeg, 1992). The author then explores the usage of imagery "to depict abstract [mathematical] situations," discussing how pattern imagery is experienced by these students.  
Hancock: This article investigates introductory statistics students usage of metonymy in conceptualizing distributions, their relation to sampling, and their usage in informal statistical inference. The authors specifically state that they considered metonymy and not metaphors because this sample of students did not use metaphors, instead "their speech remained within the same conceptual structure rather than mapping between different structures" (Noll & Hancock, 2015). In this sample of students two primary metonymies for distributions emerged: the paradigmatic metonymy, where "one part of the concept is a prototype for the whole concept" and the proper metonymy, where "one part of the concept stands in for the whole concept." When students were asked to reason about distributions they often employed methods used for the Normal distribution, even when the distribution in the task was bimodal and right skewed. Additionally, when students were asked to reason through questions regarding a sampling distribution the authors found that many students referred "to sampling distributions as a *collection of many samples* or *differences from sample to sample*," substituting the term sample for the sample statistic. Through understanding how introductory statistics students (students similar to mine) construct metonymies of statistical concepts, we can better construct classroom conversations that can prevent students constructing unhelpful metonymies.

Groth: This article "describes the nature of pre-service teachers' idiosyncratic metaphors for the concept of a statistical sample," (Groth, 2005) in an attempt to shed light on pre-service teachers knowledge of samples and how they may impact students' understandings. In this sample of 54 pre-service elementary teachers from a university-level mathematics teaching methods course, students participated in a discussion on the use of metaphors in mathematics. Following this discussion, students were given a writing prompt to construct a "metaphor for the concept of a statistical sample, and to identify the ground and tension inherent in the metaphor they had written." The definitions form seven categories, among which are descriptions of a "sample as a collection of objects," a "sample as a part of a whole," a "sample as a representative part of a whole," and "actions to be taken upon samples."

Turkey: This article, from the conference for New Horizons in Education, describes a study of 62 mathematics pre-service teachers at a public university in Turkey, and their metaphorical concepts of the field of statistics. The researchers asked these pre-service teachers to anonymously complete the phrase "statistics is like...because..." (Memnun, 2003). Of the metaphors given by the participants, 32 were deemed "valid," among which were metaphors using a puzzle, a maze, and an ocean.

* Results from research-findings and conclusions from research studies.
  + Provide essential facts about what and who was studied and how the study was conducted - then focus on the results.

# Conclusions

Describe what you (or other experts) think should happen next.

* What further research should be conducted on this topic?
* How should assessment practice be changed based on what you've learned?

# References

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